On the study of some Caccioppoli's equations. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory, Theoretical Cosmology and String Theory

Michele Nardelli[1], Antonio Nardelli

Abstract

In this paper, we analyze some Caccioppoli's equations. We obtain new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory, Theoretical Cosmology and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan

(1887 - 1920)

Introduction

In this paper, an octahedron could serve as a mathematical or conceptual model of the universe in the quantic phase, while the spherical surface could be used to describe the geometry of the bubble-universe.

The values $(2\sqrt{2})/\pi$, the golden ratio φ, $\zeta(2)$ and π, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$ the so called DN Constant:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and bubbles universe in eternal inflation.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and bubbles universe.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of bubbles universe and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

The above values $(2\sqrt{2})/\pi$, the golden ratio φ and π, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

The various mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe.

Regarding the fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the quantum universe and bubbles universe.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

Proposal:

The initial octahedron: Let's imagine a regular octahedron, with perfectly symmetrical faces. Each face represents an ideal symmetry.

The emerging sphere: Inside the octahedron, there is an inscribed sphere. This sphere represents the bubble of the universe that emerges from the perturbations of the quantum vacuum during eternal inflation.

Expansion and transitions: As time passes, the universe expands. The faces of the octahedron begin to break, symbolizing "symmetry breaks." The sphere continues to grow, representing the expanding universe.

Constants and numbers: We integrate the mathematical results you obtained. For example, the golden ratio (φ) could be represented by a proportion between the dimensions of the octahedron and the sphere.

Entropy and complexity: Entropy increases as the universe evolves. We can represent this with a disordered growth of structures within the emerging sphere. Imagine this scene as an abstract work of art, where geometric shapes and cosmological concepts merge

In Fig.1 and Fig.2 let's imagine a regular octahedron representing the universe in its phase of high symmetry and very low entropy. Inside the octahedron we have an inscribed sphere that emerges from perturbations of the quantum vacuum during eternal inflation. As time passes, the universe expands, the faces of the octahedron break (symmetry breaks), and entropy increases. Spheres emerge from the octahedra, symbolizing the transition phases from a regime of very low entropy to a universe in which, with the passage of time, entropy increases, increasing the complexity of the universe itself.

Fig. 1

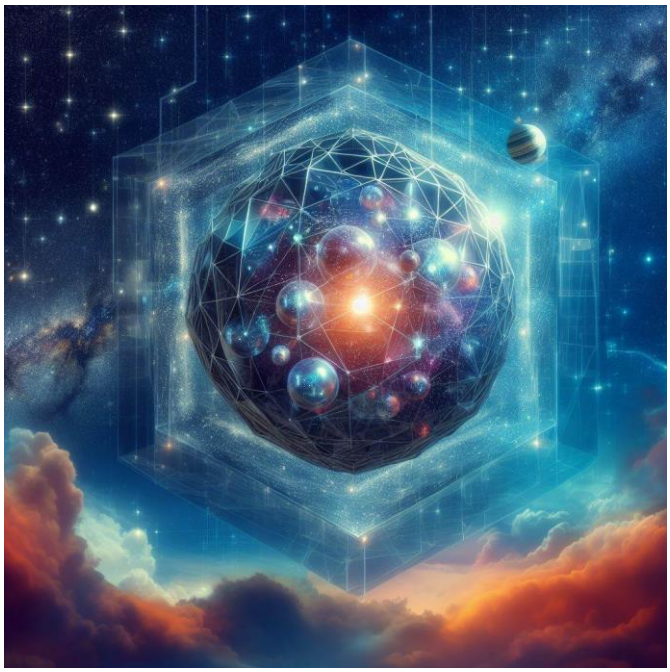
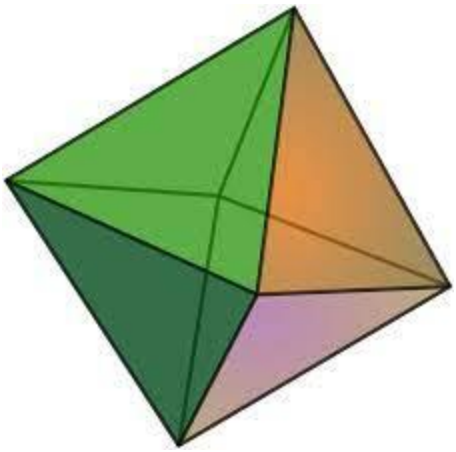
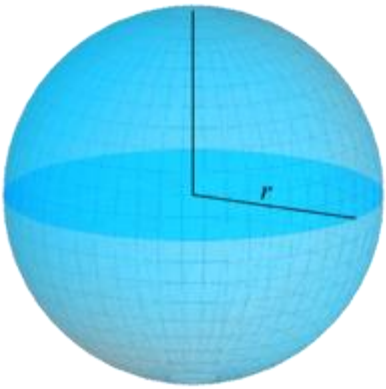


Fig. 2

Now, we have that:



Octahedron



Sphere

From the octahedron volume V = 1/3*√2 l^3 and, from the sphere volume,

V = (4/3*π*r^3) , we consider the following relationship, for r = x:

4/3*π*x^3 = 1/3*√2*l^3

Input

$$\frac{4}{3}\pi x^3 = \frac{1}{3}\sqrt{2}\, l^3$$

Exact result

$$\frac{4\pi x^3}{3} = \frac{\sqrt{2}\, l^3}{3}$$

Alternate forms

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$\frac{4\,\pi\,x^3}{3} - \frac{\sqrt{2}\,l^3}{3} = 0$$

Real solution

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Solutions

$$x = -\frac{\sqrt[3]{-\frac{1}{\pi}}\,l}{\sqrt{2}}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Integer solution

$$l = 0\,, \quad x = 0$$

Implicit derivatives

$$\frac{\partial x(l)}{\partial l} = \frac{l^2}{2\sqrt{2}\,\pi\,x^2}$$

$$\frac{\partial l(x)}{\partial x} = \frac{2\sqrt{2}\,\pi\,x^2}{l^2}$$

From the alternate form

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

for l = 8, we have that:

8/(sqrt(2) π^(1/3)) = 8/(2sqrt2 * Pi)^1/3

Input

$$\frac{8}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{8}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Result

True

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Thence:

l/(sqrt(2) π^(1/3)) = l/(2sqrt2 * Pi)^1/3

Input

$$\frac{l}{\sqrt{2}\ \sqrt[3]{\pi}} = \frac{l}{\sqrt[3]{2\sqrt{2}\ \pi}}$$

Logarithmic form

$$\log_{\sqrt{2}\ \sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\ \sqrt[3]{\pi}}\left(\sqrt{2}\ \sqrt[3]{\pi}\right) = \log_{\sqrt{2}\ \sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\ \sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\ \pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Now, we have that:

l/(2 sqrt(2) π)^(1/3) = (2sqrt2)/Pi

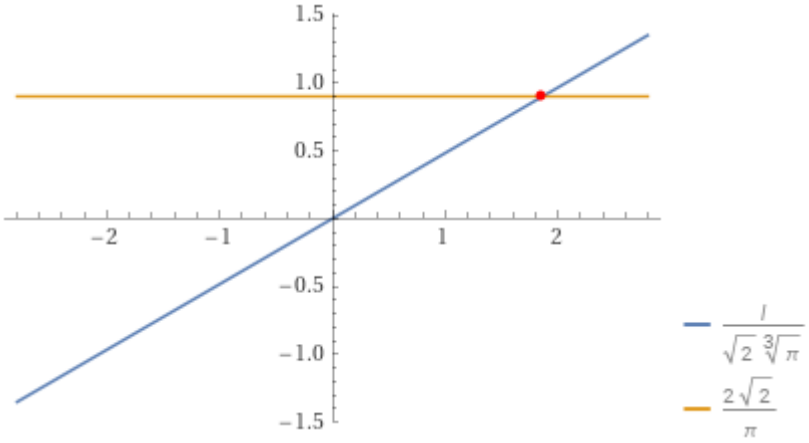
Input

$$\frac{l}{\sqrt[3]{2\sqrt{2}\ \pi}} = \frac{2\sqrt{2}}{\pi}$$

Exact result

$$\frac{l}{\sqrt{2}\ \sqrt[3]{\pi}} = \frac{2\sqrt{2}}{\pi}$$

Plot



Solution

$$l = \frac{4}{\pi^{2/3}}$$

$$l = \frac{4}{\sqrt[3]{\pi^2}}$$

From

ESISTENZA E LIMITAZIONE DELLO SPETTRO IN UN PROBLEMA AI LIMITI PER UN'EQUAZIONE DIFFERENZIALE ORDINARIA NON LINEARE – *Renato Caccioppoli* - 1942

We analyze the following equation:

$$-\left[ug'(u)-\frac{1}{2}g(u)\right]=\int_0^1\frac{1-s^2}{[2(1-s^2)-u(1-s^2)^2]^{3/2}}ds<\int_0^1\frac{ds}{\sqrt{1-s^2}}=\frac{\pi}{2},$$

We consider:

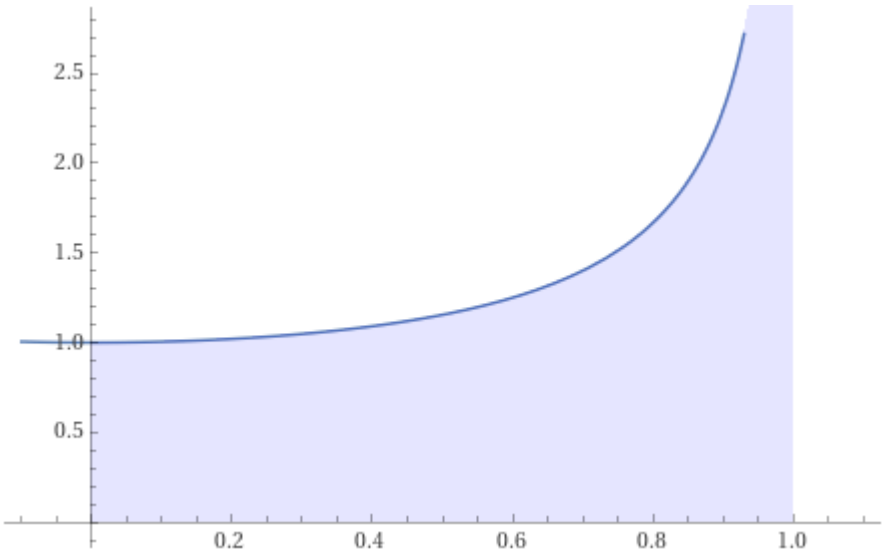
Integrate(1/(sqrt(1-s^2))ds, s=0..1

Definite integral

$$\int_0^1\frac{1}{\sqrt{1-s^2}}\,ds=\frac{\pi}{2}\approx 1.5708$$

1.5708

Visual representation of the integral



Thence, we obtain:

1/6*(2(Integrate(1/(sqrt(1-s^2))ds, s=0..1)))^2

Input

$$\frac{1}{6}\left(2\int_0^1 \frac{1}{\sqrt{1-s^2}}\,ds\right)^2$$

Computation result

$$\frac{1}{6}\left(2\int_0^1 \frac{1}{\sqrt{1-s^2}}\,ds\right)^2 = \frac{\pi^2}{6} \approx 1.64493$$

1.64493 ≈ $\zeta(2) = \pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

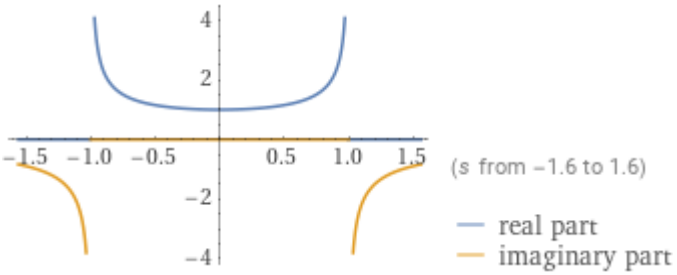
We have also:

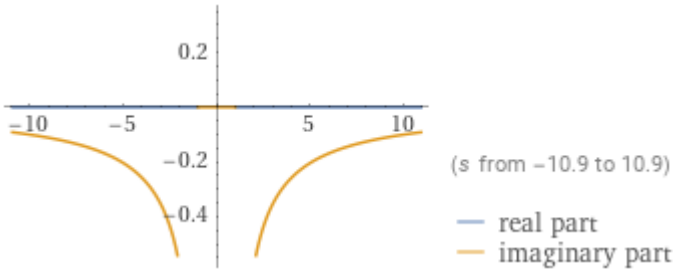
1/(sqrt(1-s^2))

Input

$$\frac{1}{\sqrt{1-s^2}}$$

Plots **(figures that can be related to the open strings)**





Alternate form

$$\frac{1}{\sqrt{1-s}\ \sqrt{s+1}}$$

Roots

(no roots exist)

Property as a function

Parity

even

Series expansion at s=-1

$$\frac{1}{\sqrt{2}\ \sqrt{s+1}} + \frac{\sqrt{s+1}}{4\sqrt{2}} + \frac{3\,(s+1)^{3/2}}{32\sqrt{2}} + \frac{5\,(s+1)^{5/2}}{128\sqrt{2}} + O\left((s+1)^{7/2}\right)$$

(Puiseux series)

Series expansion at s=0

$$1 + \frac{s^2}{2} + \frac{3\,s^4}{8} + O\left(s^5\right)$$

(Taylor series)

Series expansion at s=1

$$\frac{\sqrt{s-1}}{\sqrt{2-2s}\ \sqrt{s-1}} - \frac{\sqrt{s-1}\ \sqrt{s-1}}{4\sqrt{2-2s}} + \frac{3\sqrt{s-1}\ (s-1)^{3/2}}{32\sqrt{2-2s}} - \frac{5\sqrt{s-1}\ (s-1)^{5/2}}{128\sqrt{2-2s}} + O\left((s-1)^{7/2}\right)$$

(generalized Puiseux series)

Series expansion at s=∞

$$\frac{s}{\sqrt{-s^2}\ s} + \frac{s}{2\sqrt{-s^2}\ s^3} + \frac{3s}{8\sqrt{-s^2}\ s^5} + \frac{5s}{16\sqrt{-s^2}\ s^7} + O\left(\left(\frac{1}{s}\right)^9\right)$$

(generalized Puiseux series)

Derivative

$$\frac{d}{ds}\left(\frac{1}{\sqrt{1-s^2}}\right) = \frac{s}{(1-s^2)^{3/2}}$$

Indefinite integral

$$\int \frac{1}{\sqrt{1-s^2}}\,ds = \sin^{-1}(s) + \text{constant}$$

$\sin^{-1}(x)$ is the inverse sine function

Global minimum

$$\min\left\{\frac{1}{\sqrt{1-s^2}}\right\} = 1 \text{ at } s = 0$$

Limit

$$\lim_{s\to\pm\infty} \frac{1}{\sqrt{1-s^2}} = 0$$

Series representations

$$\frac{1}{\sqrt{1-s^2}} = \frac{1}{\sqrt{-s^2}\sum_{k=0}^{\infty}\left(-s^2\right)^{-k}\binom{\frac{1}{2}}{k}} \quad \text{for } |s| > 1$$

$$\frac{1}{\sqrt{1-s^2}} = \frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-s^2\right)^k\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } |s|^2 < 1$$

$$\frac{1}{\sqrt{1-s^2}} = \frac{1}{\sqrt{-s^2}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-s^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } |s| > 1$$

$\binom{n}{m}$ is the binomial coefficient

$|z|$ is the absolute value of z

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

Definite integral

$$\int_{-1}^{1} \frac{1}{\sqrt{1-s^2}}\,ds = \pi \approx 3.14159$$

From the above definite integral

$$\int_{-1}^{1} \frac{1}{\sqrt{1-s^2}}\, ds = \pi \approx 3.14159$$

we obtain:

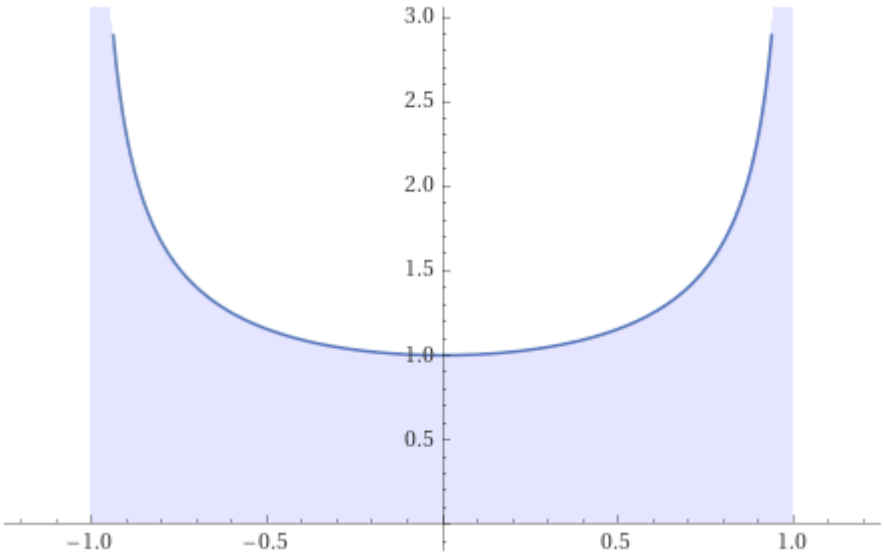
Integrate(1/sqrt(1 - s^2)) ds, s=-1..1

Definite integral

$$\int_{-1}^{1} \frac{1}{\sqrt{1-s^2}}\, ds = \pi \approx 3.1416$$

3.1416 ≈ π (Ramanujan Recurring Number)

Visual representation of the integral



From which:

1/6*((Integrate(1/sqrt(1 - s^2)) ds, s=-1..1))^2

Input

$$\frac{1}{6}\left(\int_{-1}^{1}\frac{1}{\sqrt{1-s^2}}\,ds\right)^2$$

Computation result

$$\frac{1}{6}\left(\int_{-1}^{1}\frac{1}{\sqrt{1-s^2}}\,ds\right)^2=\frac{\pi^2}{6}\approx 1.64493$$

1.64493 ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Now, we analyze the following equation:

$$\varphi(k) > \int_{\sqrt{\frac{k-1}{k}}}^{1}\frac{1-k(1-s^2)}{\sqrt{2k(1-s^2)}}\,ds=\frac{\sqrt{k-1}+(2-k)\,\text{arc sen}\,\frac{1}{\sqrt{k}}}{2\sqrt{2k}}.$$

We consider:

((sqrt(k-1))+(2-k) asin(1/√k))/((2sqrt(2k)))

Input

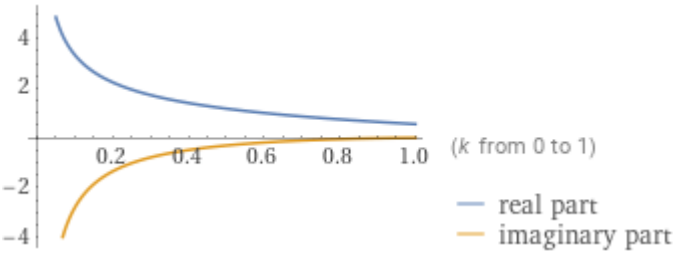
$$\frac{\sqrt{k-1}+(2-k)\sin^{-1}\left(\frac{1}{\sqrt{k}}\right)}{2\sqrt{2k}}$$

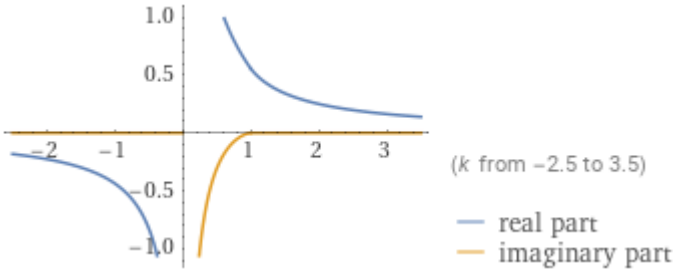
$\sin^{-1}(x)$ is the inverse sine function

Exact result

$$\frac{\sqrt{k-1}+(2-k)\sin^{-1}\left(\frac{1}{\sqrt{k}}\right)}{2\sqrt{2}\sqrt{k}}$$

Plots (figures that can be related to the open strings)





Alternate forms

$$\frac{\sqrt{k-1}-(k-2)\csc^{-1}\left(\sqrt{k}\right)}{2\sqrt{2}\sqrt{k}}$$

$$\frac{\sqrt{k-1}-(k-2)\sin^{-1}\left(\frac{1}{\sqrt{k}}\right)}{2\sqrt{2}\sqrt{k}}$$

$$\frac{\sqrt{k-1}+(2-k)\csc^{-1}\left(\sqrt{k}\right)}{2\sqrt{2}\sqrt{k}}$$

$\csc^{-1}(x)$ is the inverse cosecant function

Expanded form

$$\frac{\sqrt{k-1}}{2\sqrt{2}\ \sqrt{k}} - \frac{\sqrt{k}\ \sin^{-1}\left(\frac{1}{\sqrt{k}}\right)}{2\sqrt{2}} + \frac{\sin^{-1}\left(\frac{1}{\sqrt{k}}\right)}{\sqrt{2}\ \sqrt{k}}$$

Series expansion at k=0

$$\begin{cases} \left(\frac{1}{2\sqrt{2}\ \sqrt{k}} + O\left(k^{11/2}\right)\right) & \text{Im}(k) < 0 \\ \left((-1)^{\lfloor \arg(k)/(2\pi) \rfloor} \left\lfloor \frac{\arg(k)}{2\pi} \right\rfloor \left(2\pi - \pi k + O\left(k^6\right)\right) + (-1)^{\lfloor \arg(k)/(2\pi) \rfloor} \right. \\ \quad \left((i\log(k) - i\log(4) + \pi) + \frac{1}{2}(-i\log(k) + i\log(4) - \pi + i)k - \right. \\ \quad\quad \left. \frac{ik^2}{16} + \frac{ik^3}{96} + \frac{25ik^4}{1536} + \frac{77ik^5}{5120} + O\left(k^6\right) \right) + \\ \quad \left. \left(-i + \frac{ik}{2} + \frac{ik^2}{8} + \frac{ik^3}{16} + \frac{5ik^4}{128} + \frac{7ik^5}{256} + O\left(k^6\right)\right)\right) \\ \left(\frac{1}{2\sqrt{2}\ \sqrt{k}} + O\left(k^{11/2}\right)\right) & \text{(otherwise)} \\ \left((-1)^{\lfloor \arg(k)/(2\pi) \rfloor} \left\lfloor \frac{\arg(k)}{2\pi} \right\rfloor \left(2\pi - \pi k + O\left(k^6\right)\right) + (-1)^{\lfloor \arg(k)/(2\pi) \rfloor} \right. \\ \quad \left((i\log(k) - i\log(4) + \pi) + \frac{1}{2}(-i\log(k) + i\log(4) - \pi + i)k - \right. \\ \quad\quad \left. \frac{ik^2}{16} + \frac{ik^3}{96} + \frac{25ik^4}{1536} + \frac{77ik^5}{5120} + O\left(k^6\right) \right) + \\ \quad \left. \left(i - \frac{ik}{2} - \frac{ik^2}{8} - \frac{ik^3}{16} - \frac{5ik^4}{128} - \frac{7ik^5}{256} + O\left(k^6\right)\right)\right) \end{cases}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$\log(x)$ is the natural logarithm

$\text{Im}(z)$ is the imaginary part of z

Series expansion at k=1

$$(-1)^{\left\lfloor -\left(\arg(k-1)+\arg\left(\frac{1}{\left(\sqrt{k}+1\right)\sqrt{k}}\right)-\pi\right)/(2\pi)\right\rfloor}\left(-\frac{\sqrt{k-1}}{2\sqrt{2}}+\frac{11\,(k-1)^{3/2}}{12\sqrt{2}}-\frac{63\,(k-1)^{5/2}}{80\sqrt{2}}+\frac{2389\,(k-1)^{7/2}}{3360\sqrt{2}}-\frac{53\,041\,(k-1)^{9/2}}{80\,640\sqrt{2}}+O\left((k-1)^{11/2}\right)\right)+\left(\frac{\pi}{4\sqrt{2}}+\frac{\sqrt{k-1}}{2\sqrt{2}}-\frac{3\,\pi\,(k-1)}{8\sqrt{2}}-\frac{(k-1)^{3/2}}{4\sqrt{2}}+\frac{7\,\pi\,(k-1)^2}{32\sqrt{2}}+\frac{3\,(k-1)^{5/2}}{16\sqrt{2}}-\frac{11\,\pi\,(k-1)^3}{64\sqrt{2}}-\frac{5\,(k-1)^{7/2}}{32\sqrt{2}}+\frac{75\,\pi\,(k-1)^4}{512\sqrt{2}}+\frac{35\,(k-1)^{9/2}}{256\sqrt{2}}-\frac{133\,\pi\,(k-1)^5}{1024\sqrt{2}}+O\left((k-1)^{11/2}\right)\right)$$

Series expansion at k=∞

$$\frac{\sqrt{2}}{3\,k}+\frac{1}{15\sqrt{2}\,k^2}+\frac{3}{140\sqrt{2}\,k^3}+\frac{5}{504\sqrt{2}\,k^4}+\frac{35}{6336\sqrt{2}\,k^5}+O\left(\left(\frac{1}{k}\right)^6\right)$$

(Laurent series)

Derivative

$$\frac{d}{dk}\left(\frac{\sqrt{k-1}\,+(2-k)\sin^{-1}\left(\frac{1}{\sqrt{k}}\right)}{2\sqrt{2\,k}}\right)=\frac{\left(\frac{k-1}{k}\right)^{3/2}\left((k-1)^{3/2}-\left(\sqrt{k-1}\,\sqrt{\frac{k-1}{k}}\,k^{3/2}+2\,k-2\right)\sin^{-1}\left(\frac{1}{\sqrt{k}}\right)\right)}{4\sqrt{2}\,(k-1)^{5/2}}$$

Indefinite integral

$$\int \frac{\sqrt{-1+k}\ +(2-k)\sin^{-1}\left(\frac{1}{\sqrt{k}}\right)}{2\sqrt{2}\ \sqrt{k}}\,dk =$$
$$\frac{1}{6\sqrt{2}}\left(-\sqrt{\frac{k-1}{k}}\ k+3\sqrt{k-1}\ \sqrt{k}\ -3\log\left(\sqrt{k-1}\ +\sqrt{k}\right)+11\log\left(\left(\sqrt{\frac{k-1}{k}}\ +1\right)\right.\right.$$
$$\left.\left.\sqrt{k}\right)-2(k-6)\sqrt{k}\ \csc^{-1}\left(\sqrt{k}\right)\right)+\text{constant}$$

(assuming a complex-valued logarithm)

From the indefinite integral result

$$\int \frac{\sqrt{-1+k}\ +(2-k)\sin^{-1}\left(\frac{1}{\sqrt{k}}\right)}{2\sqrt{2}\ \sqrt{k}}\,dk =$$
$$\frac{1}{6\sqrt{2}}\left(-\sqrt{\frac{k-1}{k}}\ k+3\sqrt{k-1}\ \sqrt{k}\ -3\log\left(\sqrt{k-1}\ +\sqrt{k}\right)+11\log\left(\left(\sqrt{\frac{k-1}{k}}\ +1\right)\right.\right.$$
$$\left.\left.\sqrt{k}\right)-2(k-6)\sqrt{k}\ \csc^{-1}\left(\sqrt{k}\right)\right)+\text{constant}$$

(assuming a complex-valued logarithm)

we obtain:

(3 sqrt(-1 + k) sqrt(k) - sqrt((-1 + k)/k) k - 2 (-6 + k) sqrt(k) csc^(-1)(sqrt(k)) - 3 log(sqrt(-1 + k) + sqrt(k)) + 11 log((1 + sqrt((-1 + k)/k)) sqrt(k)))/(6 sqrt(2))

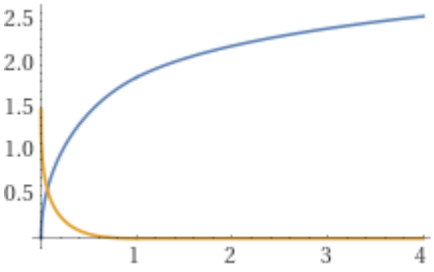
Input

$$\frac{1}{6\sqrt{2}}\left(3\sqrt{-1+k}\ \sqrt{k}\ -\sqrt{\frac{-1+k}{k}}\ k-2(-6+k)\sqrt{k}\ \csc^{-1}\left(\sqrt{k}\right)-\right.$$
$$\left.3\log\left(\sqrt{-1+k}\ +\sqrt{k}\right)+11\log\left(\left(1+\sqrt{\frac{-1+k}{k}}\right)\sqrt{k}\right)\right)$$

$\csc^{-1}(x)$ is the inverse cosecant function

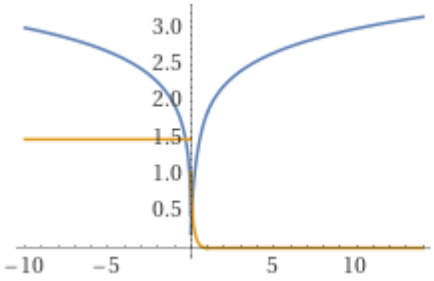
$\log(x)$ is the natural logarithm

Plots (figures that can be related to the open strings)



(k from 0 to 4)

real part
imaginary part



(k from −10 to 14)

real part
imaginary part

Alternate forms

$$\frac{\sqrt{k}\left(\sqrt{k-1}-(k-6)\csc^{-1}\left(\sqrt{k}\right)\right)+4\sinh^{-1}\left(\sqrt{k-1}\right)}{3\sqrt{2}}$$

$$-\frac{1}{6\sqrt{2}}\left(\sqrt{-\frac{1-k}{k}}\,k-3\sqrt{k-1}\,\sqrt{k}+3\log\left(\sqrt{k-1}+\sqrt{k}\right)-11\log\left(\left(\sqrt{\frac{k-1}{k}}+1\right)\sqrt{k}\right)+2(k-6)\sqrt{k}\,\sin^{-1}\left(\frac{1}{\sqrt{k}}\right)\right)$$

$$\frac{1}{6\sqrt{2}}\left(-\sqrt{\frac{k-1}{k}}\,k+3\sqrt{k-1}\,\sqrt{k}+2i(k-6)\sqrt{k}\,\log\left(\sqrt{1-\frac{1}{k}}+\frac{i}{\sqrt{k}}\right)-3\log\left(\sqrt{k-1}+\sqrt{k}\right)+11\log\left(\left(\sqrt{\frac{k-1}{k}}+1\right)\sqrt{k}\right)\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

$\sin^{-1}(x)$ is the inverse sine function

Alternate forms assuming k>0

$$\frac{1}{12\sqrt{2}}\left(4\sqrt{k-1}\ \sqrt{k}+22\log\left(\sqrt{\frac{k-1}{k}}+1\right)-6\log\left(\sqrt{k-1}+\sqrt{k}\right)+11\log(k)-4\right.$$
$$\left.(k-6)\sqrt{k}\ \csc^{-1}\left(\sqrt{k}\right)\right)$$

$$\frac{\sqrt{k-1}\ \sqrt{k}}{3\sqrt{2}}-\frac{\log\left(\sqrt{k-1}+\sqrt{k}\right)}{2\sqrt{2}}+\frac{11\left(\log\left(\frac{\sqrt{k-1}}{\sqrt{k}}+1\right)+\frac{\log(k)}{2}\right)}{6\sqrt{2}}-$$
$$\frac{(k-6)\sqrt{k}\ \csc^{-1}\left(\sqrt{k}\right)}{3\sqrt{2}}$$

Expanded logarithmic form

$$-\frac{1}{12}\sqrt{2-\frac{2}{k}}\ k+\frac{1}{4}\sqrt{k}\ \sqrt{-2+2k}+\sqrt{2k}\ \csc^{-1}\left(\sqrt{k}\right)-\frac{1}{6}\sqrt{2}\ k^{3/2}\csc^{-1}\left(\sqrt{k}\right)+$$
$$\frac{11}{12}\sqrt{2}\log\left(1+\sqrt{\frac{-1+k}{k}}\right)-\frac{1}{4}\sqrt{2}\log\left(\sqrt{-1+k}+\sqrt{k}\right)+\frac{11\log(k)\sqrt{2}}{12\times 2}$$

Alternate form assuming k≥1

$$\frac{1}{6\sqrt{2}}\left(3\sqrt{-1+k}\ \sqrt{k}-\sqrt{\frac{-1+k}{k}}\ k-2(-6+k)\sqrt{k}\ \csc^{-1}\left(\sqrt{k}\right)+\log\left(\right.\right.$$
$$\left.\left.\frac{1}{\left(\sqrt{-1+k}+\sqrt{k}\right)^3}\right)+\log\left(\left(\left(1+\sqrt{\frac{-1+k}{k}}\right)^{11}k^{11/2}\right)\right)\right.$$

Derivative

$$\frac{d}{dk}\left(\frac{1}{6\sqrt{2}}\left(3\sqrt{-1+k}\ \sqrt{k}-\sqrt{\frac{-1+k}{k}}\ k-2(-6+k)\sqrt{k}\ \csc^{-1}\left(\sqrt{k}\right)-\right.\right.$$
$$\left.\left.3\log\left(\sqrt{-1+k}+\sqrt{k}\right)+11\log\left(\left(1+\sqrt{\frac{-1+k}{k}}\right)\sqrt{k}\right)\right)\right)=$$
$$-\frac{\sqrt{-1+k}\left(\sqrt{\frac{k-1}{k}}\ k+(-1+k)\right)\left(-k+(k-2)\sqrt{-1+k}\ \csc^{-1}\left(\sqrt{k}\right)+1\right)}{2\sqrt{2}\ k^{5/2}\left(\frac{-1+k}{k}\right)^{3/2}\left(\sqrt{\frac{-1+k}{k}}+1\right)}$$

Indefinite integral

$$\int\frac{1}{6\sqrt{2}}\left(3\sqrt{-1+k}\ \sqrt{k}-\sqrt{\frac{-1+k}{k}}\ k-2(-6+k)\sqrt{k}\ \csc^{-1}\left(\sqrt{k}\right)-3\log\left(\right.\right.dk=$$
$$\left.\left.\sqrt{-1+k}+\sqrt{k}\right)+11\log\left(\left(1+\sqrt{\frac{-1+k}{k}}\right)\sqrt{k}\right)\right)$$
$$\frac{1}{60\sqrt{2}}\left(-8(k-10)k^{3/2}\csc^{-1}\left(\sqrt{k}\right)-\sqrt{\frac{k-1}{k}}\ (2k-37)k+\right.$$
$$5\sqrt{k-1}\ (2k-1)\sqrt{k}-5\log\left(\sqrt{k-1}+\sqrt{k}\right)+$$
$$37\log\left(\left(\sqrt{\frac{k-1}{k}}+1\right)\sqrt{k}\right)-\frac{40(k-1)\left(\sqrt{-(k-1)k}+\sin^{-1}\left(\sqrt{1-k}\right)\right)}{\sqrt{-(k-1)^2}}+$$
$$\left.80k\sinh^{-1}\left(\sqrt{k-1}\right)\right)+\text{constant}$$

(assuming a complex-valued logarithm)

Again, from the indefinite integral result

$$\int \frac{1}{6\sqrt{2}}\left(3\sqrt{-1+k}\,\sqrt{k}-\sqrt{\frac{-1+k}{k}}\,k-2(-6+k)\sqrt{k}\,\csc^{-1}\left(\sqrt{k}\right)-3\log\left(dk=\right.\right.$$
$$\left.\sqrt{-1+k}+\sqrt{k}\right)+11\log\left(\left(1+\sqrt{\frac{-1+k}{k}}\right)\sqrt{k}\right)\Bigg)$$
$$\frac{1}{60\sqrt{2}}\left(-8(k-10)k^{3/2}\csc^{-1}\left(\sqrt{k}\right)-\sqrt{\frac{k-1}{k}}\,(2k-37)k+\right.$$
$$5\sqrt{k-1}\,(2k-1)\sqrt{k}-5\log\left(\sqrt{k-1}+\sqrt{k}\right)+$$
$$37\log\left(\left(\sqrt{\frac{k-1}{k}}+1\right)\sqrt{k}\right)-\frac{40(k-1)\left(\sqrt{-(k-1)k}+\sin^{-1}\left(\sqrt{1-k}\right)\right)}{\sqrt{-(k-1)^2}}+$$
$$\left.80k\sinh^{-1}\left(\sqrt{k-1}\right)\right)+\text{constant}$$

(assuming a complex-valued logarithm)

(-8 (k - 10) k^(3/2) csc^(-1)(sqrt(k)) - sqrt((k - 1)/k) (2 k - 37) k + 5 sqrt(k - 1) (2 k - 1) sqrt(k) - 5 log(sqrt(k - 1) + sqrt(k)) + 37 log((sqrt((k - 1)/k) + 1) sqrt(k)) - (40 (k - 1) (sqrt(-(k - 1) k) + sin^(-1)(sqrt(1 - k))))/sqrt(-(k - 1)^2) + 80 k sinh^(-1)(sqrt(k - 1)))/(60 sqrt(2))

for k = 2:

(-8(2-10) 2^(3/2) csc^(-1)(√(2)) - √((2 - 1)/2)(2*2-37) 2+5 √(2-1) (2*2-1) √(2) - 5 ln(√(2-1)+√(2))+37 ln((√((2-1)/2)+1) √(2))-(40(2-1)(√(-(2-1)2)+sin^(-1)(√(1-2))))

Input

$$(-8(2-10))\times 2^{3/2}\csc^{-1}\left(\sqrt{2}\right)-\left(\sqrt{\frac{2-1}{2}}\,(2\times 2-37)\right)\times 2+$$
$$5\sqrt{2-1}\,(2\times 2-1)\sqrt{2}-5\log\left(\sqrt{2-1}+\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}+1\right)\sqrt{2}\right)-40(2-1)\left(\sqrt{-(2-1)\times 2}+\sin^{-1}\left(\sqrt{1-2}\right)\right)$$

$\csc^{-1}(x)$ is the inverse cosecant function

$\log(x)$ is the natural logarithm

$\sin^{-1}(x)$ is the inverse sine function

Exact Result

$$48\sqrt{2}+32\sqrt{2}\,\pi+37\log\left(\sqrt{2}\left(1+\frac{1}{\sqrt{2}}\right)\right)-5\log\left(1+\sqrt{2}\right)-40\left(i\sqrt{2}+i\sinh^{-1}(1)\right)$$

(result in radians)

Decimal approximation

$$238.258459799601659054432748843361060015913274582487927795318746 3\ldots$$
$$-$$
$$91.823485975705522961371921967579615503913288145543339357199013 86\ldots\,i$$

(result in radians)

Alternate complex forms

$$\left(48\sqrt{2}+32\sqrt{2}\,\pi+37\log\left(\sqrt{2}\left(1+\frac{1}{\sqrt{2}}\right)\right)-5\log\left(1+\sqrt{2}\right)\right)-\left(40\left(\sqrt{2}+\sinh^{-1}(1)\right)\right)i$$

$$8\sqrt{25\left(\sqrt{2}+\log\left(1+\sqrt{2}\right)\right)^2+4\left(3\sqrt{2}+2\sqrt{2}\,\pi+2\log\left(1+\sqrt{2}\right)\right)^2}\left(\cos\left(-\tan^{-1}\left(\frac{40\left(\sqrt{2}+\sinh^{-1}(1)\right)}{48\sqrt{2}+32\sqrt{2}\,\pi+37\log\left(\sqrt{2}\left(1+\frac{1}{\sqrt{2}}\right)\right)-5\log\left(1+\sqrt{2}\right)}\right)\right)+i\sin\left(-\tan^{-1}\left(\frac{40\left(\sqrt{2}+\sinh^{-1}(1)\right)}{48\sqrt{2}+32\sqrt{2}\,\pi+37\log\left(\sqrt{2}\left(1+\frac{1}{\sqrt{2}}\right)\right)-5\log\left(1+\sqrt{2}\right)}\right)\right)\right)$$

$$8\sqrt{25\left(\sqrt{2}+\log\left(1+\sqrt{2}\right)\right)^2+4\left(3\sqrt{2}+2\sqrt{2}\,\pi+2\log\left(1+\sqrt{2}\right)\right)^2}\exp\left(-i\tan^{-1}\left(\frac{40\left(\sqrt{2}+\sinh^{-1}(1)\right)}{48\sqrt{2}+32\sqrt{2}\,\pi+37\log\left(\sqrt{2}\left(1+\frac{1}{\sqrt{2}}\right)\right)-5\log\left(1+\sqrt{2}\right)}\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r = 8\sqrt{25\left(\sqrt{2}+\log\left(1+\sqrt{2}\right)\right)^2+4\left(3\sqrt{2}+2\sqrt{2}\,\pi+2\log\left(1+\sqrt{2}\right)\right)^2}$$

(radius),

$$\theta = -\tan^{-1}\left(\frac{40\left(\sqrt{2}+\sinh^{-1}(1)\right)}{48\sqrt{2}+32\sqrt{2}\,\pi+37\log\left(\sqrt{2}\left(1+\frac{1}{\sqrt{2}}\right)\right)-5\log\left(1+\sqrt{2}\right)}\right)$$ (angle)

Alternate forms

$$8\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+(4-5\,i)\sinh^{-1}(1)\right)$$

$$(48-40\,i)\sqrt{2}+32\sqrt{2}\,\pi+32\log\left(1+\sqrt{2}\right)-40\,i\sinh^{-1}(1)$$

$$8\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+4\log\left(1+\sqrt{2}\right)-5\,i\sinh^{-1}(1)\right)$$

Alternative representations

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\,(2\times 2-37)+$$
$$5\sqrt{2-1}\,(2\times 2-1)\sqrt{2}-5\log\left(\sqrt{2-1}+\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}+1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}+\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$37\log(a)\log_a\left(\left(1+\sqrt{\frac{1}{2}}\right)\sqrt{2}\right)-5\log(a)\log_a\left(\sqrt{1}+\sqrt{2}\right)+$$
$$64\csc^{-1}\left(\sqrt{2}\right)2^{3/2}-40\left(\sin^{-1}\left(\sqrt{-1}\right)+\sqrt{-2}\right)+66\sqrt{\frac{1}{2}}+15\sqrt{1}\,\sqrt{2}$$

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\ (2\times 2-37)+$$
$$5\sqrt{2-1}\ (2\times 2-1)\sqrt{2}\ -5\log\left(\sqrt{2-1}\ +\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}\ +1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}\ +\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$37\log_e\left(\left(1+\sqrt{\frac{1}{2}}\right)\sqrt{2}\right)-5\log_e\left(\sqrt{1}\ +\sqrt{2}\right)+64\csc^{-1}\left(\sqrt{2}\right)2^{3/2}-$$
$$40\left(\sin^{-1}\left(\sqrt{-1}\right)+\sqrt{-2}\right)+66\sqrt{\frac{1}{2}}\ +15\sqrt{1}\ \sqrt{2}$$

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\ (2\times 2-37)+$$
$$5\sqrt{2-1}\ (2\times 2-1)\sqrt{2}\ -5\log\left(\sqrt{2-1}\ +\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}\ +1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}\ +\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$37\log(a)\log_a\left(\left(1+\sqrt{\frac{1}{2}}\right)\sqrt{2}\right)-5\log(a)\log_a\left(\sqrt{1}\ +\sqrt{2}\right)+$$
$$64\,i\,\mathrm{csch}^{-1}\left(i\sqrt{2}\right)2^{3/2}-40\left(\sin^{-1}\left(\sqrt{-1}\right)+\sqrt{-2}\right)+66\sqrt{\frac{1}{2}}\ +15\sqrt{1}\ \sqrt{2}$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

Series representations

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\ (2\times 2-37)+$$
$$5\sqrt{2-1}\ (2\times 2-1)\sqrt{2}\ -5\log\left(\sqrt{2-1}\ +\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}\ +1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}\ +\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$(48-40\,i)\sqrt{2}\ +32\sqrt{2}\ \pi-40\,i\sinh^{-1}(1)+8\log(4)-32\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}$$

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\,(2\times 2-37)+$$
$$5\sqrt{2-1}\,(2\times 2-1)\sqrt{2}-5\log\left(\sqrt{2-1}+\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}+1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}+\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$4\left((12-10\,i)\sqrt{2}+8\sqrt{2}\,\pi+\log(16)-8\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}-\right.$$
$$\left.5\,i\sqrt{\pi}\sum_{k=0}^{\infty}\frac{{}_3\tilde{F}_2\left(\frac{1}{2},\frac{1}{2},1;1-\frac{k}{2},\frac{3-k}{2};-z_0^2\right)(2-2\,z_0)^k\,z_0^{1-k}}{k!}\right)$$
for $(i\,z_0\notin\mathbb{R}$ or $((\text{not } 1\le i\,z_0<\infty)$ and $(\text{not } -\infty<i\,z_0\le -1)))$

$n!$ is the factorial function

${}_p\tilde{F}_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)$ is the regularized generalized hypergeometric function

$\mathbb{R}$ is the set of real numbers

Integral representations

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\,(2\times 2-37)+$$
$$5\sqrt{2-1}\,(2\times 2-1)\sqrt{2}-5\log\left(\sqrt{2-1}+\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}+1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}+\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$(48-40\,i)\sqrt{2}+32\sqrt{2}\,\pi+\int_1^{1+\sqrt{2}}\left(\frac{32}{t}-\frac{40\,i}{\sqrt{3-2\,t+t^2}}\right)dt$$

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\,(2\times 2-37)+$$
$$5\sqrt{2-1}\,(2\times 2-1)\sqrt{2}-5\log\left(\sqrt{2-1}+\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}+1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}+\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$\frac{1}{\pi^{3/2}}2\left((24-20\,i)\sqrt{2}\,\pi^{3/2}+16\sqrt{2}\,\pi^{5/2}+16\pi^{3/2}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt-\right.$$
$$\left.5\times\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}2^{-s}\,\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)\,\Gamma\left(\frac{1}{2}+s\right)ds\right)\quad\text{for } 0<\gamma<\frac{1}{2}$$

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\,(2\times 2-37)+$$
$$5\sqrt{2-1}\,(2\times 2-1)\sqrt{2}\,-5\log\left(\sqrt{2-1}\,+\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}\,+1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}\,+\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$\frac{1}{\pi^{3/2}}\,2\left((24-20\,i)\sqrt{2}\,\pi^{3/2}+16\sqrt{2}\,\pi^{5/2}+16\pi^{3/2}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt-\right.$$
$$\left.5\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)\quad\text{for } 0<\gamma<\frac{1}{2}$$

$\Gamma(x)$ is the gamma function

Continued fraction representations

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\,(2\times 2-37)+$$
$$5\sqrt{2-1}\,(2\times 2-1)\sqrt{2}\,-5\log\left(\sqrt{2-1}\,+\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}\,+1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}\,+\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$8\left(4\sqrt{2}\,\pi+4\log\left(1+\sqrt{2}\right)+\sqrt{2}\left((6-5\,i)-\frac{5\,i}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{1+2k}}\right)\right)=$$
$$8\left(4\sqrt{2}\,\pi+4\log\left(1+\sqrt{2}\right)+\sqrt{2}\left((6-5\,i)-\cfrac{5\,i}{1+\cfrac{2}{3+\cfrac{2}{5+\cfrac{12}{7+\cfrac{12}{9+\ddots}}}}}\right)\right)$$

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\,(2\times 2-37)+$$
$$5\sqrt{2-1}\,(2\times 2-1)\sqrt{2}\,-5\log\left(\sqrt{2-1}\,+\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}\,+1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}\,+\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$8\sqrt{2}\left((6-5\,i)+4\,\pi+\frac{4}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\sqrt{2}\left|\frac{1+k}{2}\right|^2}{1+k}}-\frac{5\,i}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{1+2k}}\right)=$$
$$8\sqrt{2}\left((6-5\,i)+4\,\pi+\cfrac{4}{1+\cfrac{\sqrt{2}}{2+\cfrac{\sqrt{2}}{3+\cfrac{4\sqrt{2}}{4+\cfrac{4\sqrt{2}}{5+\ldots}}}}}-\cfrac{5\,i}{1+\cfrac{2}{3+\cfrac{2}{5+\cfrac{12}{7+\cfrac{12}{9+\ldots}}}}}\right)$$

$$\left(2^{3/2}\csc^{-1}\left(\sqrt{2}\right)\right)(-8)(2-10)-2\sqrt{\frac{2-1}{2}}\,(2\times 2-37)+$$
$$5\sqrt{2-1}\,(2\times 2-1)\sqrt{2}\,-5\log\left(\sqrt{2-1}\,+\sqrt{2}\right)+$$
$$37\log\left(\left(\sqrt{\frac{2-1}{2}}\,+1\right)\sqrt{2}\right)-40\left((2-1)\left(\sqrt{-(2-1)\,2}\,+\sin^{-1}\left(\sqrt{1-2}\right)\right)\right)=$$
$$8\sqrt{2}\left((6-5\,i)+4\,\pi+\frac{4}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{\sqrt{2}\left\lfloor\frac{1+k}{2}\right\rfloor}{\frac{1}{2}\left(3+(-1)^k(-1+k)+k\right)}}-\frac{5\,i}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{2\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{1+2k}}\right)=$$
$$8\sqrt{2}\left((6-5\,i)+4\,\pi+\cfrac{4}{1+\cfrac{\sqrt{2}}{2+\cfrac{\sqrt{2}}{3+\cfrac{2\sqrt{2}}{2+\cfrac{2\sqrt{2}}{5+\ldots}}}}}-\cfrac{5\,i}{1+\cfrac{2}{3+\cfrac{2}{5+\cfrac{12}{7+\cfrac{12}{9+\ldots}}}}}\right)$$

$\lfloor x \rfloor$ is the floor function

$\operatorname{K}_{k=k_1}^{k_2} a_k / b_k$ is a continued fraction

((sqrt(-(2 - 1)^2)) + 80*2 sinh^(-1)(sqrt(2 - 1)))/(60 sqrt(2))

Input

$$\frac{\sqrt{-(2-1)^2} + 80 \times 2 \sinh^{-1}\left(\sqrt{2-1}\right)}{60\sqrt{2}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Exact result

$$\frac{160 \sinh^{-1}(1) + i}{60\sqrt{2}}$$

Decimal approximation

1.661933973707281369050720214001514673735187499590846007729906 0… +
0.011785113019775792073347406035080817321413932294807900609805664…
i

Property

$\frac{i + 160 \sinh^{-1}(1)}{60\sqrt{2}}$ is a transcendental number

Alternate complex forms

$$\frac{4}{3}\sqrt{2}\,\sinh^{-1}(1) + \frac{i}{60\sqrt{2}}$$

$$\frac{1}{60}\sqrt{\frac{1}{2}+12\,800\log^2\left(1+\sqrt{2}\right)}\left(\cos\left(\tan^{-1}\left(\frac{1}{160\sinh^{-1}(1)}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{1}{160\sinh^{-1}(1)}\right)\right)\right)$$

$$\frac{1}{60}\sqrt{\frac{1}{2}+12\,800\log^2\left(1+\sqrt{2}\right)}\;e^{i\tan^{-1}(1/(160\sinh^{-1}(1)))}$$

i is the imaginary unit

$\log(x)$ is the natural logarithm

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=\frac{1}{60}\sqrt{\frac{1}{2}+12\,800\log^2\left(1+\sqrt{2}\right)}\ \text{(radius)},\quad \theta=\tan^{-1}\left(\frac{1}{160\sinh^{-1}(1)}\right)\ \text{(angle)}$$

Expanded form

$$\frac{4}{3}\sqrt{2}\,\sinh^{-1}(1)+\frac{i}{60\sqrt{2}}$$

Alternate form

$$\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}$$

Alternative representations

$$\frac{\sqrt{-(2-1)^2}+80\times 2\sinh^{-1}\left(\sqrt{2-1}\right)}{60\sqrt{2}}=\frac{160\log\left(\sqrt{1}+\sqrt{1+\sqrt{1}^{\,2}}\right)+\sqrt{-1}}{60\sqrt{2}}$$

$$\frac{\sqrt{-(2-1)^2}+80\times 2\sinh^{-1}\left(\sqrt{2-1}\right)}{60\sqrt{2}}=\frac{160\sinh^{-1}\left(\sqrt{1}\right)+\sqrt{-1}\ \sqrt{1}}{60\sqrt{2}}$$

$$\frac{\sqrt{-(2-1)^2}+80\times 2\sinh^{-1}\left(\sqrt{2-1}\right)}{60\sqrt{2}}=\frac{160\log\left(\sqrt{1}+\sqrt{1+\sqrt{1}^2}\right)+\sqrt{-1}\ \sqrt{1}}{60\sqrt{2}}$$

Series representations

$$\frac{\sqrt{-(2-1)^2}+80\times 2\sinh^{-1}\left(\sqrt{2-1}\right)}{60\sqrt{2}}=$$
$$\frac{i}{60\sqrt{2}}+\frac{2}{3}i\sqrt{2}\,\pi-\frac{8}{3}i\sqrt{1+i}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}+\frac{i}{2}\right)^k\left(\frac{1}{2}\right)_k}{k!+2k\,k!}$$

$$\frac{\sqrt{-(2-1)^2}+80\times 2\sinh^{-1}\left(\sqrt{2-1}\right)}{60\sqrt{2}}=$$
$$\frac{i}{60\sqrt{2}}-\frac{2}{3}i\sqrt{2}\,\pi+\frac{8}{3}i\sqrt{1-i}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}-\frac{i}{2}\right)^k\left(\frac{1}{2}\right)_k}{k!+2k\,k!}$$

$$\frac{\sqrt{-(2-1)^2}+80\times 2\sinh^{-1}\left(\sqrt{2-1}\right)}{60\sqrt{2}}=$$
$$\frac{i}{60\sqrt{2}}+\frac{2}{3}\sqrt{2\pi}\sum_{k=0}^{\infty}\frac{{}_3\tilde{F}_2\left(\frac{1}{2},\frac{1}{2},1;1-\frac{k}{2},\frac{3-k}{2};-z_0^2\right)(2-2z_0)^k\,z_0^{1-k}}{k!}$$
for $(i\,z_0\notin\mathbb{R}$ or $((\text{not } 1\le i\,z_0<\infty)$ and $(\text{not } -\infty<i\,z_0\le -1)))$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

${}_p\tilde{F}_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)$ is the regularized generalized hypergeometric function

$\mathbb{R}$ is the set of real numbers

Integral representations

$$\frac{\sqrt{-(2-1)^2}+80\times 2\sinh^{-1}\left(\sqrt{2-1}\right)}{60\sqrt{2}}=\frac{i}{60\sqrt{2}}+\frac{4\sqrt{2}}{3}\int_0^1\frac{1}{\sqrt{1+t^2}}\,dt$$

$$\frac{\sqrt{-(2-1)^2}+80\times 2\sinh^{-1}\left(\sqrt{2-1}\right)}{60\sqrt{2}}=$$
$$\frac{i}{60\sqrt{2}}-\frac{i\sqrt{2}}{3\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}2^{-s}\,\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)\,\Gamma\left(\frac{1}{2}+s\right)ds\quad\text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{\sqrt{-(2-1)^2}+80\times 2\sinh^{-1}\left(\sqrt{2-1}\right)}{60\sqrt{2}}=\frac{i}{60\sqrt{2}}-\frac{i\sqrt{2}}{3\pi^{3/2}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds$$
$$\text{for } 0<\gamma<\frac{1}{2}$$

$\Gamma(x)$ is the gamma function

From the following alternate form:

$$8\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+(4-5\,i)\sinh^{-1}(1)\right)$$

and

$$\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}$$

we obtain:

((8 ((6 - 5 i) sqrt(2) + 4 sqrt(2) π + (4 - 5 i) sinh^(-1)(1)))) / ((4/3 sqrt(2) log(1 + sqrt(2)) + i/(60 sqrt(2))))

Input

$$\frac{8\left((6-5\,i)\,\sqrt{2}+4\,\sqrt{2}\,\pi+(4-5\,i)\,\sinh^{-1}(1)\right)}{\frac{4}{3}\,\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\,\sqrt{2}}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Decimal approximation

142.963184430035725996018947741488184350813519605878708225953565 0…

− 56.264764269359770343869866012031876842038489579407924001011094 82… i

Alternate complex forms

$$\left(\frac{\sqrt{2}\,\left(-5\,\sqrt{2}-5\,\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\,\log^2\left(1+\sqrt{2}\right)\right)}+\frac{32\,\sqrt{2}\,\left(6\,\sqrt{2}+4\,\sqrt{2}\,\pi+4\,\sinh^{-1}(1)\right)\log\left(1+\sqrt{2}\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\,\log^2\left(1+\sqrt{2}\right)\right)}\right)+\left(-\frac{\sqrt{2}\,\left(6\,\sqrt{2}+4\,\sqrt{2}\,\pi+4\,\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\,\log^2\left(1+\sqrt{2}\right)\right)}+\frac{32\,\sqrt{2}\,\left(-5\,\sqrt{2}-5\,\sinh^{-1}(1)\right)\log\left(1+\sqrt{2}\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\,\log^2\left(1+\sqrt{2}\right)\right)}\right)i$$

$$\Bigg(480\left(122\sqrt{2}+96\sqrt{2}\,\pi+32\sqrt{2}\,\pi^2+41\sqrt{2}\,\log^2\left(1+\sqrt{2}\right)+196\log\left(1+\sqrt{2}\right)+64\,\pi\log\left(1+\sqrt{2}\right)\right)\left(\cos\left(\tan^{-1}\left(\frac{\frac{4\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}-\frac{6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)}{60\sqrt{2}\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}}{\frac{4\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}-\frac{5\sqrt{2}+5\sinh^{-1}(1)}{60\sqrt{2}\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\frac{4\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}-\frac{6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)}{60\sqrt{2}\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}}{\frac{4\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}-\frac{5\sqrt{2}+5\sinh^{-1}(1)}{60\sqrt{2}\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}}\right)\right)\Bigg)\Bigg/\left(\sqrt{\left(\left(1+25\,600\log^2\left(1+\sqrt{2}\right)\right)\left(122+32\pi^2+41\log^2\left(1+\sqrt{2}\right)+98\sqrt{2}\,\log\left(1+\sqrt{2}\right)+32\pi\left(3+\sqrt{2}\,\log\left(1+\sqrt{2}\right)\right)\right)\right)}\right)$$

$$\Bigg(480\left(122\sqrt{2}+96\sqrt{2}\,\pi+32\sqrt{2}\,\pi^2+41\sqrt{2}\,\log^2\left(1+\sqrt{2}\right)+196\log\left(1+\sqrt{2}\right)+64\,\pi\log\left(1+\sqrt{2}\right)\right)\exp\left(i\tan^{-1}\left(\frac{\frac{4\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}-\frac{6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)}{60\sqrt{2}\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}}{\frac{4\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}-\frac{5\sqrt{2}+5\sinh^{-1}(1)}{60\sqrt{2}\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}}\right)\right)\Bigg)\Bigg/\left(\sqrt{\left(\left(1+25\,600\log^2\left(1+\sqrt{2}\right)\right)\left(122+32\pi^2+41\log^2\left(1+\sqrt{2}\right)+98\sqrt{2}\,\log\left(1+\sqrt{2}\right)+32\pi\left(3+\sqrt{2}\,\log\left(1+\sqrt{2}\right)\right)\right)\right)}\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r = \Big(480\Big(122\sqrt{2} + 96\sqrt{2}\,\pi + 32\sqrt{2}\,\pi^2 + 41\sqrt{2}\,\log^2\big(1+\sqrt{2}\big) + 196\log\big(1+\sqrt{2}\big) + 64\,\pi\log\big(1+\sqrt{2}\big)\Big)\Big)\Big/\Big(\sqrt{\big(\big(1+25\,600\log^2\big(1+\sqrt{2}\big)\big)\big(122+32\,\pi^2+41\log^2\big(1+\sqrt{2}\big)+98\sqrt{2}\,\log\big(1+\sqrt{2}\big)+32\,\pi\big(3+\sqrt{2}\,\log\big(1+\sqrt{2}\big)\big)\big)\big)}\Big)$$ (radius),

$$\theta = \tan^{-1}\left(\frac{\frac{4\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)} - \frac{6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)}{60\sqrt{2}\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}}{\frac{4\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)} - \frac{5\sqrt{2}+5\sinh^{-1}(1)}{60\sqrt{2}\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}}\right)$$

(angle)

$r \approx 153.64$ (radius), $\theta \approx -0.37494$ (angle)

153.64

Alternate forms

$$\frac{480\left((12-10\,i)+8\,\pi+(4-5\,i)\sqrt{2}\,\sinh^{-1}(1)\right)}{160\sinh^{-1}(1)+i}$$

$$\frac{480\sqrt{2}\,\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+(4-5\,i)\sinh^{-1}(1)\right)}{160\log\left(1+\sqrt{2}\right)+i}$$

$$\frac{480\sqrt{2}\,\left(4\sqrt{2}\,\pi-i\left((5+6\,i)\sqrt{2}+(5+4\,i)\sinh^{-1}(1)\right)\right)}{160\log\left(1+\sqrt{2}\right)+i}$$

Expanded form

$$\frac{(48-40\,i)\,\sqrt{2}}{\frac{4}{3}\,\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\,\sqrt{2}}}+\frac{32\,\sqrt{2}\,\pi}{\frac{4}{3}\,\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\,\sqrt{2}}}+\frac{(32-40\,i)\,\sinh^{-1}(1)}{\frac{4}{3}\,\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\,\sqrt{2}}}$$

Alternative representations

$$\frac{8\left((6-i\,5)\,\sqrt{2}+4\,\sqrt{2}\,\pi+(4-i\,5)\,\sinh^{-1}(1)\right)}{\frac{4}{3}\,\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\,\sqrt{2}}}=\frac{8\left((4-5\,i)\,\log\left(1+\sqrt{2}\right)+(6-5\,i)\,\sqrt{2}+4\,\pi\,\sqrt{2}\right)}{\frac{i}{60\,\sqrt{2}}+\frac{4}{3}\,\log\left(1+\sqrt{2}\right)\sqrt{2}}$$

$$\frac{8\left((6-i\,5)\,\sqrt{2}+4\,\sqrt{2}\,\pi+(4-i\,5)\,\sinh^{-1}(1)\right)}{\frac{4}{3}\,\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\,\sqrt{2}}}=\frac{8\left((4-5\,i)\,\log\left(1+\sqrt{2}\right)+(6-5\,i)\,\sqrt{2}+4\,\pi\,\sqrt{2}\right)}{\frac{i}{60\,\sqrt{2}}+\frac{4}{3}\,\log_e\left(1+\sqrt{2}\right)\sqrt{2}}$$

$$\frac{8\left((6-i\,5)\,\sqrt{2}+4\,\sqrt{2}\,\pi+(4-i\,5)\,\sinh^{-1}(1)\right)}{\frac{4}{3}\,\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\,\sqrt{2}}}=\frac{8\left((4-5\,i)\,\log\left(1+\sqrt{2}\right)+(6-5\,i)\,\sqrt{2}+4\,\pi\,\sqrt{2}\right)}{\frac{i}{60\,\sqrt{2}}+\frac{4}{3}\,\log(a)\,\log_a\left(1+\sqrt{2}\right)\sqrt{2}}$$

$\log_b(x)$ is the base- b logarithm

Series representations

$$\frac{8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}=\frac{480\sqrt{2}\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+(4-5\,i)\sinh^{-1}(1)\right)}{i+80\log(2)-160\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}$$

$$\frac{8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}=\frac{240\left((24-20\,i)+16\,\pi+(4-5\,i)\sqrt{2\,\pi}\,\sum_{k=0}^{\infty}\frac{3\tilde{F}_2\left(\frac{1}{2},\frac{1}{2},1;1-\frac{k}{2},\frac{3-k}{2};-z_0^2\right)(2-2\,z_0)^k\,z_0^{1-k}}{k!}\right)}{i+80\log(2)-160\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}$$

for $(i\,z_0\notin\mathbb{R}$ or $((\text{not }1\le i\,z_0<\infty)$ and $(\text{not }-\infty<i\,z_0\le-1)))$

$n!$ is the factorial function

${}_p\tilde{F}_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)$ is the regularized generalized hypergeometric function

$\mathbb{R}$ is the set of real numbers

Integral representations

$$\frac{8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}=\left(120\left((48-40\,i)\,\pi^{3/2}+32\,\pi^{5/2}-(5+4\,i)\sqrt{2}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}2^{-s}\,\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)\,\Gamma\left(\frac{1}{2}+s\right)ds\right)\right)\Big/\left(\pi^{3/2}\left(i+160\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt\right)\right)\quad\text{for }0<\gamma<\frac{1}{2}$$

$$\frac{8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}=$$
$$\frac{120\left((48-40\,i)\,\pi^{3/2}+32\,\pi^{5/2}-(5+4\,i)\sqrt{2}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)}{\pi^{3/2}\left(i+160\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt\right)}\quad\text{for}$$
$$0<\gamma<\frac{1}{2}$$

$$\frac{8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}=$$
$$-\frac{480\,i\,\pi\left((12-10\,i)+8\,\pi+(4-5\,i)\sqrt{2}\int_0^1\frac{1}{\sqrt{1+t^2}}\,dt\right)}{\pi-80\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\quad\text{for}\quad -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

From which:

11((((8 ((6 - 5 i) sqrt(2) + 4 sqrt(2) π + (4 - 5 i) sinh^(-1)(1)))) / ((4/3 sqrt(2) log(1 + sqrt(2)) + i/(60 sqrt(2))))))+34+5+e

Input

$$11\times\frac{8\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+(4-5\,i)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result

$$39 + e + \frac{88\left((6 - 5\,i)\sqrt{2} + 4\sqrt{2}\,\pi + (4 - 5\,i)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1 + \sqrt{2}\right) + \frac{i}{60\sqrt{2}}}$$

Decimal approximation

1614.313310558852031191568712627722690356705962758365750060456182…

$-$

618.912406962957473782568526132350645262423385373487164011122043 0…

i

Alternate complex forms

$$\left(39 + e + \frac{11\sqrt{2}\left(-5\sqrt{2} - 5\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200} + \frac{32}{9}\log^2\left(1 + \sqrt{2}\right)\right)} + \frac{352\sqrt{2}\left(6\sqrt{2} + 4\sqrt{2}\,\pi + 4\sinh^{-1}(1)\right)\log\left(1 + \sqrt{2}\right)}{3\left(\frac{1}{7200} + \frac{32}{9}\log^2\left(1 + \sqrt{2}\right)\right)}\right) + \left(-\frac{11\sqrt{2}\left(6\sqrt{2} + 4\sqrt{2}\,\pi + 4\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200} + \frac{32}{9}\log^2\left(1 + \sqrt{2}\right)\right)} + \frac{352\sqrt{2}\left(-5\sqrt{2} - 5\sinh^{-1}(1)\right)\log\left(1 + \sqrt{2}\right)}{3\left(\frac{1}{7200} + \frac{32}{9}\log^2\left(1 + \sqrt{2}\right)\right)}\right) i$$

$$\left(\cos\left(\tan^{-1}\left(\frac{\frac{352\sqrt{2}\,\log(1+\sqrt{2})(-5\sqrt{2}-5\sinh^{-1}(1))}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}-\frac{11\sqrt{2}\,(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1))}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}}{39+e+\frac{11\sqrt{2}\,(-5\sqrt{2}-5\sinh^{-1}(1))}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}+\frac{352\sqrt{2}\,\log(1+\sqrt{2})(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1))}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\frac{352\sqrt{2}\,\log(1+\sqrt{2})(-5\sqrt{2}-5\sinh^{-1}(1))}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}-\frac{11\sqrt{2}\,(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1))}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}}{39+e+\frac{11\sqrt{2}\,(-5\sqrt{2}-5\sinh^{-1}(1))}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}+\frac{352\sqrt{2}\,\log(1+\sqrt{2})(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1))}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}}\right)\right)\Bigg/$$

$$\left(\sqrt{\left(\left(1+25\,600\log^2(1+\sqrt{2})\right)\Big/\left(e^2\left(1+25\,600\log^2(1+\sqrt{2})\right)+6e\left(-17\,587+25\,600\left(13+44\sqrt{2}\right)\log^2(1+\sqrt{2})-8800\left(\sqrt{2}-384\right)\log(1+\sqrt{2})+2\,252\,800\pi\log(1+\sqrt{2})\right)+9\left(755\,356\,969+198\,246\,400\pi^2+25\,600\left(10\,091+1144\sqrt{2}\right)\log^2(1+\sqrt{2})+17\,600\left(4992+34\,483\sqrt{2}\right)\log(1+\sqrt{2})+4\,505\,600\pi\left(132+\left(13+44\sqrt{2}\right)\log(1+\sqrt{2})\right)\right)\right)\right)}\right)$$

$$\exp\left(i\tan^{-1}\left(\frac{\frac{352\sqrt{2}\,\log(1+\sqrt{2})(-5\sqrt{2}-5\sinh^{-1}(1))}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}-\frac{11\sqrt{2}\,(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1))}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}}{39+e+\frac{11\sqrt{2}\,(-5\sqrt{2}-5\sinh^{-1}(1))}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}+\frac{352\sqrt{2}\,\log(1+\sqrt{2})(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1))}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}}\right)\Bigg/$$
$$\left(\sqrt{\left((1+25\,600\log^2(1+\sqrt{2})\right)/\left(e^2\left(1+25\,600\log^2(1+\sqrt{2})\right)+\right.\right.$$
$$6e\left(-17\,587+25\,600\left(13+44\sqrt{2}\right)\log^2(1+\sqrt{2})-\right.$$
$$8800\left(\sqrt{2}-384\right)\log(1+\sqrt{2})+2\,252\,800\pi\log(1+\sqrt{2})\Big)+$$
$$9\left(755\,356\,969+198\,246\,400\pi^2+25\,600\left(10\,091+1144\sqrt{2}\right)\right.$$
$$\log^2(1+\sqrt{2})+17\,600\left(4992+34\,483\sqrt{2}\right)\log(1+\sqrt{2})+$$
$$\left.\left.4\,505\,600\pi\left(132+\left(13+44\sqrt{2}\right)\log(1+\sqrt{2})\right)\right)\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=1\Big/\left(\sqrt{\left((1+25\,600\log^2(1+\sqrt{2})\right)/\left(e^2\left(1+25\,600\log^2(1+\sqrt{2})\right)+\right.\right.$$
$$6e\left(-17\,587+25\,600\left(13+44\sqrt{2}\right)\log^2(1+\sqrt{2})-\right.$$
$$8800\left(\sqrt{2}-384\right)\log(1+\sqrt{2})+2\,252\,800\pi\log(1+\sqrt{2})\Big)+$$
$$9\left(755\,356\,969+198\,246\,400\pi^2+\right.$$
$$25\,600\left(10\,091+1144\sqrt{2}\right)\log^2(1+\sqrt{2})+$$
$$17\,600\left(4992+34\,483\sqrt{2}\right)\log(1+\sqrt{2})+4\,505\,600\pi$$
$$\left.\left.\left(132+\left(13+44\sqrt{2}\right)\log(1+\sqrt{2})\right)\right)\right)\right)$$ (radius), $\theta=$
$$\tan^{-1}\left(\frac{\frac{352\sqrt{2}\,\log(1+\sqrt{2})(-5\sqrt{2}-5\sinh^{-1}(1))}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}-\frac{11\sqrt{2}\,(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1))}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}}{39+e+\frac{11\sqrt{2}\,(-5\sqrt{2}-5\sinh^{-1}(1))}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}+\frac{352\sqrt{2}\,\log(1+\sqrt{2})(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1))}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2(1+\sqrt{2})\right)}}\right)$$

(angle)

$r\approx 1728.9$ (radius), $\theta\approx -0.36611$ (angle)

1728.9

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728=8^2\times 3^3$.

The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$39 + e + \frac{5280\left((12 - 10\,i) + 8\,\pi + (4 - 5\,i)\,\sqrt{2}\,\sinh^{-1}(1)\right)}{160\sinh^{-1}(1) + i}$$

$$39 + e + \frac{5280\,\sqrt{2}\,\left((6 - 5\,i)\,\sqrt{2} + 4\,\sqrt{2}\,\pi + (4 - 5\,i)\sinh^{-1}(1)\right)}{160\log\left(1 + \sqrt{2}\right) + i}$$

$$e + \frac{1}{160\log\left(1 + \sqrt{2}\right) + i}3\left(14\,080\,\pi + 2080\log\left(1 + \sqrt{2}\right) - i\left((17\,587 + 21\,120\,i) + (8800 + 7040\,i)\,\sqrt{2}\,\sinh^{-1}(1)\right)\right)$$

Expanded form

$$39 + e + \frac{(528 - 440\,i)\,\sqrt{2}}{\frac{4}{3}\,\sqrt{2}\,\log\left(1 + \sqrt{2}\right) + \frac{i}{60\,\sqrt{2}}} + \frac{352\,\sqrt{2}\,\pi}{\frac{4}{3}\,\sqrt{2}\,\log\left(1 + \sqrt{2}\right) + \frac{i}{60\,\sqrt{2}}} + \frac{(352 - 440\,i)\sinh^{-1}(1)}{\frac{4}{3}\,\sqrt{2}\,\log\left(1 + \sqrt{2}\right) + \frac{i}{60\,\sqrt{2}}}$$

Alternative representations

$$\frac{11\left(8\left((6 - i\,5)\,\sqrt{2} + 4\,\sqrt{2}\,\pi + (4 - i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\,\sqrt{2}\,\log\left(1 + \sqrt{2}\right) + \frac{i}{60\,\sqrt{2}}} + 34 + 5 + e = 39 + e + \frac{88\left((4 - 5\,i)\log\left(1 + \sqrt{2}\right) + (6 - 5\,i)\,\sqrt{2} + 4\,\pi\,\sqrt{2}\right)}{\frac{i}{60\,\sqrt{2}} + \frac{4}{3}\log\left(1 + \sqrt{2}\right)\sqrt{2}}$$

$$\frac{11\left(8\left((6 - i\,5)\,\sqrt{2} + 4\,\sqrt{2}\,\pi + (4 - i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\,\sqrt{2}\,\log\left(1 + \sqrt{2}\right) + \frac{i}{60\,\sqrt{2}}} + 34 + 5 + e = 39 + e + \frac{88\left((4 - 5\,i)\log\left(1 + \sqrt{2}\right) + (6 - 5\,i)\,\sqrt{2} + 4\,\pi\,\sqrt{2}\right)}{\frac{i}{60\,\sqrt{2}} + \frac{4}{3}\log_e\left(1 + \sqrt{2}\right)\sqrt{2}}$$

$$\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e=$$
$$39+e+\frac{88\left((4-5\,i)\log\left(1+\sqrt{2}\right)+(6-5\,i)\sqrt{2}+4\,\pi\sqrt{2}\right)}{\frac{i}{60\sqrt{2}}+\frac{4}{3}\log(a)\log_a\left(1+\sqrt{2}\right)\sqrt{2}}$$

$\log_b(x)$ is the base-b logarithm

Series representations

$$\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e=$$
$$39+e+\frac{5280\sqrt{2}\,\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+(4-5\,i)\sinh^{-1}(1)\right)}{i+80\log(2)-160\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}}$$

$$\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e=$$
$$\left((63\,360-52\,761\,i)+i\,e+42\,240\,\pi+3120\log(2)+80\,e\log(2)-\right.$$
$$6240\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}-160\,e\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}+(10\,560-13\,200\,i)$$
$$\left.\sqrt{2\,\pi}\sum_{k=0}^{\infty}\frac{{}_3\tilde{F}_2\left(\frac{1}{2},\frac{1}{2},1;1-\frac{k}{2},\frac{3-k}{2};-z_0^2\right)(2-2\,z_0)^k\,z_0^{1-k}}{k!}\right)\Big/$$
$$\left(i+80\log(2)-160\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}\right)$$

for $(i\,z_0\notin\mathbb{R}$ or $((\text{not }1\le i\,z_0<\infty)$ and $(\text{not }-\infty<i\,z_0\le-1)))$

$$\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e=$$
$$\left((63\,360-52\,761\,i)\sqrt{2}+i\sqrt{2}\,e+42\,240\sqrt{2}\,\pi+\right.$$
$$3120\sqrt{2}\,\log(2)+80\sqrt{2}\,e\log(2)-6240\sqrt{2}\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}-$$
$$160\sqrt{2}\,e\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}+(21\,120-26\,400\,i)\sqrt{\pi}$$
$$\left.\sum_{k=0}^{\infty}\frac{{}_3\tilde{F}_2\left(\frac{1}{2},\frac{1}{2},1;1-\frac{k}{2},\frac{3-k}{2};-z_0^2\right)(2-2\,z_0)^k\,z_0^{1-k}}{k!}\right)\Big/$$
$$\left(\sqrt{2}\left(i+80\log(2)-160\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}\right)\right)$$
for $(i\,z_0\notin\mathbb{R}$ or $((\text{not } 1\le i\,z_0<\infty)$ and $(\text{not } -\infty<i\,z_0\le -1)))$

$n!$ is the factorial function

${}_p\tilde{F}_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)$ is the regularized generalized hypergeometric function

$\mathbb{R}$ is the set of real numbers

Integral representations

$$\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e=$$
$$\left((63\,360-52\,761\,i)\,\pi^{3/2}+i\,e\,\pi^{3/2}+42\,240\,\pi^{5/2}+\right.$$
$$6240\,\pi^{3/2}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt+160\,e\,\pi^{3/2}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt-$$
$$\left.(6600+5280\,i)\sqrt{2}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}2^{-s}\,\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)\,\Gamma\left(\frac{1}{2}+s\right)ds\right)\Big/$$
$$\left(\pi^{3/2}\left(i+160\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt\right)\right)\quad\text{for } 0<\gamma<\frac{1}{2}$$

$$\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e=$$
$$\left((63360-52761\,i)\sqrt{2}\,\pi^{3/2}+i\sqrt{2}\,e\,\pi^{3/2}+42240\sqrt{2}\,\pi^{5/2}+\right.$$
$$6240\sqrt{2}\,\pi^{3/2}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt+160\sqrt{2}\,e\,\pi^{3/2}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt-$$
$$\left.13200+10560\,i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)\Big/$$
$$\left(\sqrt{2}\,\pi^{3/2}\left(i+160\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt\right)\right)\text{ for }0<\gamma<\frac{1}{2}$$

$$\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e=$$
$$\left((-52761-63360\,i)\,\pi+e\,\pi-42240\,i\,\pi^2-(26400+21120\,i)\sqrt{2}\,\pi\right.$$
$$\int_0^1\frac{1}{\sqrt{1+t^2}}\,dt-3120\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds-$$
$$\left.80\,e\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\Big/$$
$$\left(\pi-80\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\text{ for }-1<\gamma<0$$

$\Gamma(x)$ is the gamma function

(11((((8 ((6 - 5 i) sqrt(2) + 4 sqrt(2) π + (4 - 5 i) sinh^(-1)(1)))) / ((4/3 sqrt(2) log(1 + sqrt(2)) + i/(60 sqrt(2))))))+34+5+e)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{11\times\frac{8\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+(4-5\,i)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

$\log(x)$ is the natural logarithm

i is the imaginary unit

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \sqrt[15]{39 + e + \frac{88\left((6-5\,i)\sqrt{2} + 4\sqrt{2}\,\pi + (4-5\,i)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right) + \frac{i}{60\sqrt{2}}}}$$

Decimal approximation

1.64444145189668333773407246539356089853010502695038442471828 20… −
0.04011659454714472227367978808648872497032227298052034752560 6102…
i

Alternate complex forms

$1.6444 - 0.0401\,i$

$1.6449\,(\cos(-0.024390) + i\sin(-0.024390))$

$1.6449\,e^{-0.024390\,i}$

Polar coordinates

$r = 1.64489$ (radius), $\theta = -0.024381$ (angle)

1.64489 ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \sqrt[15]{39 + e + \frac{5280\sqrt{2}\left((6-5i)\sqrt{2} + 4\sqrt{2}\,\pi + (4-5i)\sinh^{-1}(1)\right)}{160\log\left(1+\sqrt{2}\right)+i}}$$

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \sqrt[15]{39 + e + \frac{88\left((6-5i)\sqrt{2} + 4\sqrt{2}\,\pi + (4-5i)\log\left(1+\sqrt{2}\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}}$$

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \left(\frac{1}{160\log\left(1+\sqrt{2}\right)+i}\left(e\left(160\log\left(1+\sqrt{2}\right)+i\right) + 3\left(14080\,\pi + 2080\right.\right.\right.$$
$$\left.\left.\left.\log\left(1+\sqrt{2}\right) - i\left((17587 + 21120\,i) + (8800 + 7040\,i)\sqrt{2}\,\sinh^{-1}(1)\right)\right)\right)\right)\text{^ }(1/15)$$

(1/27((11((((8 ((6 - 5 i) sqrt(2) + 4 sqrt(2) π + (4 - 5 i) sinh^(-1)(1)))) / ((4/3 sqrt(2) log(1 + sqrt(2)) + i/(60 sqrt(2))))))+34+5+e)-1))^2+1/5

Input

$$\left(\frac{1}{27}\left(\left(11\times\frac{8\left((6-5i)\sqrt{2} + 4\sqrt{2}\,\pi + (4-5i)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}} + 34 + 5 + e\right) - 1\right)\right)^2 + \frac{1}{5}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result

$$\frac{1}{5} + \frac{1}{729}\left(38 + e + \frac{88\left((6-5i)\sqrt{2} + 4\sqrt{2}\,\pi + (4-5i)\sinh^{-1}(1)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}\right)^2$$

Decimal approximation

3045.093375217670027324981631605050887552596123246951818790937674…

$-$

2739.367967690964129432936702929055547248091593353915940928607411…

i

Alternate complex forms

$$\left(\frac{1}{5}+\frac{1}{729}\left(-\left(-\frac{11\sqrt{2}\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}+\frac{352\sqrt{2}\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)\log\left(1+\sqrt{2}\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)^2+\left(38+e+\frac{11\sqrt{2}\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}+\frac{352\sqrt{2}\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)\log\left(1+\sqrt{2}\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)^2\right)\right)+\frac{1}{729}\left(2\left(-\frac{11\sqrt{2}\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}+\frac{352\sqrt{2}\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)\log\left(1+\sqrt{2}\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)\left(38+e+\frac{11\sqrt{2}\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}+\frac{352\sqrt{2}\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)\log\left(1+\sqrt{2}\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)\right)i$$

$$\left(\sqrt{\left(\frac{1}{7290000}121\left(3+2\pi+200\sqrt{2}\,\log^2\left(1+\sqrt{2}\right)+\left(400+\sqrt{2}\right)\log\left(1+\sqrt{2}\right)\right)^2\right.\right.$$
$$\left(-52762+e+972800\log^2\left(1+\sqrt{2}\right)+3379200\sqrt{2}\,\log^2\left(1+\sqrt{2}\right)+\right.$$
$$25600\,e\log^2\left(1+\sqrt{2}\right)+10137600\log\left(1+\sqrt{2}\right)-$$
$$\left.26400\sqrt{2}\,\log\left(1+\sqrt{2}\right)+6758400\,\pi\log\left(1+\sqrt{2}\right)\right)^2+$$
$$\left(729\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)^2-\frac{1936}{45}\left(3+2\pi+200\sqrt{2}\right.\right.$$
$$\left.\log^2\left(1+\sqrt{2}\right)+\left(400+\sqrt{2}\right)\log\left(1+\sqrt{2}\right)\right)^2+$$
$$\frac{5}{9}\left(-\frac{26381}{1200}+\frac{1216}{3}\log^2\left(1+\sqrt{2}\right)+3e\left(\frac{1}{7200}+\right.\right.$$
$$\left.\left.\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)-11\sqrt{2}\,\log\left(1+\sqrt{2}\right)+$$
$$\left.\left.1408\log\left(1+\sqrt{2}\right)\left(3+2\pi+\sqrt{2}\,\log\left(1+\sqrt{2}\right)\right)\right)^2\right)^2\Bigg)$$
$$\exp\left(i\tan^{-1}\left(\left(2\left(\frac{352\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}-\right.\right.\right.$$
$$\left.\frac{11\sqrt{2}\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)$$
$$\left(38+e+\frac{11\sqrt{2}\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}+\right.$$
$$\left.\left.\left.\frac{352\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)\right)\Big/$$
$$\left(729\left(\frac{1}{5}+\frac{1}{729}\left(\left(38+e+\frac{11\sqrt{2}\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}+\left(352\sqrt{2}\right.\right.\right.\right.$$
$$\log\left(1+\sqrt{2}\right)\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(\right.$$
$$\left.\left.\left.\left.1)\right)\right)\Big/\left(3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)\right)\right)^2-$$
$$\left.\left.\left.\left.\left(\frac{352\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{11\sqrt{2}\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)^2\right)\right)\right)\right)\right)\Bigg)$$
$$\Big/\left(3645\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)^2\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r = \left(\sqrt{\left(\frac{1}{7290000}121\left(3+2\pi+200\sqrt{2}\,\log^2\left(1+\sqrt{2}\right)+\left(400+\sqrt{2}\right)\log\left(1+\sqrt{2}\right)\right)^2\right.\right.$$
$$\left(-52762+e+972800\log^2\left(1+\sqrt{2}\right)+\right.$$
$$3379200\sqrt{2}\,\log^2\left(1+\sqrt{2}\right)+$$
$$25600\,e\log^2\left(1+\sqrt{2}\right)+10137600\log\left(1+\sqrt{2}\right)-$$
$$26400\sqrt{2}\,\log\left(1+\sqrt{2}\right)+6758400\,\pi\log\left(1+\sqrt{2}\right)\Big)^2+$$
$$\left(729\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)^2-\frac{1936}{45}\left(3+2\pi+200\sqrt{2}\right.\right.$$
$$\log^2\left(1+\sqrt{2}\right)+\left(400+\sqrt{2}\right)\log\left(1+\sqrt{2}\right)\Big)^2+$$
$$\frac{5}{9}\left(-\frac{26381}{1200}+\frac{1216}{3}\log^2\left(1+\sqrt{2}\right)+3\,e\left(\frac{1}{7200}+\frac{32}{9}\log^2\right.\right.$$
$$\left.\left.1+\sqrt{2}\right)\right)-11\sqrt{2}\,\log\left(1+\sqrt{2}\right)+1408$$
$$\left.\left.\log\left(1+\sqrt{2}\right)\left(3+2\pi+\sqrt{2}\,\log\left(1+\sqrt{2}\right)\right)\right)^2\right)^2\Bigg)\Bigg)\Big/$$
$$\left.\left(3645\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)^2\right)\right.$$

(radius)

,

$$\theta = \tan^{-1}\left(\left(2\left(\frac{352\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}-\right.\right.\right.$$
$$\left.\left.\frac{11\sqrt{2}\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)$$
$$\left(38+e+\frac{11\sqrt{2}\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}+\right.$$
$$\left.\left.\frac{352\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)\right)\Big/$$
$$\left(729\left(\frac{1}{5}+\frac{1}{729}\left(\left(\left(38+e+\frac{11\sqrt{2}\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}+\right.\right.\right.\right.$$
$$\left.\left.\frac{352\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)^2-$$
$$\left(\frac{352\sqrt{2}\,\log\left(1+\sqrt{2}\right)\left(-5\sqrt{2}-5\sinh^{-1}(1)\right)}{3\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}-\right.$$
$$\left.\left.\left.\left.\left.\frac{11\sqrt{2}\left(6\sqrt{2}+4\sqrt{2}\,\pi+4\sinh^{-1}(1)\right)}{15\left(\frac{1}{7200}+\frac{32}{9}\log^2\left(1+\sqrt{2}\right)\right)}\right)^2\right)\right)\right)\right)\right)$$ (angle)

$r \approx 4095.9$ (radius), $\theta \approx -0.73259$ (angle)

$4095.9 \approx 4096$

The number $4096 = 64^2$, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{1}{5}+\frac{1}{729}\left(38+e+\frac{5280\sqrt{2}\left((6-5i)\sqrt{2}+4\sqrt{2}\,\pi+(4-5i)\sinh^{-1}(1)\right)}{160\log\left(1+\sqrt{2}\right)+i}\right)^2$$

$$\frac{1}{5}+\frac{1}{729}\left(38+e+\frac{88\left((6-5i)\sqrt{2}+4\sqrt{2}\,\pi+(4-5i)\log\left(1+\sqrt{2}\right)\right)}{\frac{4}{3}\sqrt{2}\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}\right)^2$$

$$\Big(8\,921\,088\,000\,\pi^2 + 203\,494\,400\log^2\big(1+\sqrt{2}\big) + 5\,e^2\big(160\log\big(1+\sqrt{2}\big)+i\big)^2 - i\Big((33\,430\,003\,200 + 6\,153\,304\,051\,i) + (27\,870\,374\,400 - 547\,536\,000\,i)\sqrt{2}\,\sinh^{-1}(1) + (11\,151\,360\,000 - 2\,509\,056\,000\,i)\sinh^{-1}(1)^2\Big) - 320\,i\log\big(1+\sqrt{2}\big)\Big((10\,024\,051 + 12\,038\,400\,i) + (5\,016\,000 + 4\,012\,800\,i)\sqrt{2}\,\sinh^{-1}(1)\Big) + 844\,800\,\pi\Big(3040\log\big(1+\sqrt{2}\big) - i\Big((26\,381 + 31\,680\,i) + (13\,200 + 10\,560\,i)\sqrt{2}\,\sinh^{-1}(1)\Big)\Big) + 20\,e\big(160\log\big(1+\sqrt{2}\big)+i\big)\Big(21\,120\,\pi + 3040\log\big(1+\sqrt{2}\big) - i\Big((26\,381 + 31\,680\,i) + (13\,200 + 10\,560\,i)\sqrt{2}\,\sinh^{-1}(1)\Big)\Big)\Big)\Big/\Big(3645\big(160\log\big(1+\sqrt{2}\big)+i\big)^2\Big)$$

Expanded form

$$\frac{7949}{3645} + \frac{76\,e}{729} + \frac{e^2}{729} + \frac{\frac{170\,368}{729} - \frac{309\,760\,i}{243}}{\left(\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}\right)^2} + \frac{\left(\frac{247\,808}{243} - \frac{619\,520\,i}{729}\right)\pi}{\left(\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}\right)^2} + \frac{247\,808\,\pi^2}{729\left(\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}\right)^2} + \frac{\left(\frac{13\,376}{243} - \frac{33\,440\,i}{729}\right)\sqrt{2}}{\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}} + \frac{\left(\frac{352}{243} - \frac{880\,i}{729}\right)\sqrt{2}\,e}{\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}} + \frac{26\,752\sqrt{2}\,\pi}{729\left(\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}\right)} + \frac{704\sqrt{2}\,e\,\pi}{729\left(\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}\right)} - \frac{\left(\frac{15\,488}{729} + \frac{774\,400\,i}{729}\right)\sqrt{2}\,\sinh^{-1}(1)}{\left(\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}\right)^2} + \frac{\left(\frac{247\,808}{729} - \frac{309\,760\,i}{729}\right)\sqrt{2}\,\pi\sinh^{-1}(1)}{\left(\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}\right)^2} - \frac{\left(\frac{7744}{81} + \frac{309\,760\,i}{729}\right)\sinh^{-1}(1)^2}{\left(\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}\right)^2} + \frac{\left(\frac{26\,752}{729} - \frac{33\,440\,i}{729}\right)\sinh^{-1}(1)}{\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}} + \frac{\left(\frac{704}{729} - \frac{880\,i}{729}\right)e\,\sinh^{-1}(1)}{\frac{4}{3}\sqrt{2}\,\log\big(1+\sqrt{2}\big) + \frac{i}{60\sqrt{2}}}$$

Alternative representations

$$\left(\frac{1}{27}\left(\left(\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e\right)-1\right)\right)^2+\frac{1}{5}=$$
$$\frac{1}{5}+\left(\frac{1}{27}\left(38+e+\frac{88\left((4-5\,i)\log\left(1+\sqrt{2}\right)+(6-5\,i)\sqrt{2}+4\pi\sqrt{2}\right)}{\frac{i}{60\sqrt{2}}+\frac{4}{3}\log\left(1+\sqrt{2}\right)\sqrt{2}}\right)\right)^2$$

$$\left(\frac{1}{27}\left(\left(\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e\right)-1\right)\right)^2+\frac{1}{5}=$$
$$\frac{1}{5}+\left(\frac{1}{27}\left(38+e+\frac{88\left((4-5\,i)\log\left(1+\sqrt{2}\right)+(6-5\,i)\sqrt{2}+4\pi\sqrt{2}\right)}{\frac{i}{60\sqrt{2}}+\frac{4}{3}\log_e\left(1+\sqrt{2}\right)\sqrt{2}}\right)\right)^2$$

$$\left(\frac{1}{27}\left(\left(\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e\right)-1\right)\right)^2+\frac{1}{5}=$$
$$\frac{1}{5}+\left(\frac{1}{27}\left(38+e+\frac{88\left((4-5\,i)\log\left(1+\sqrt{2}\right)+(6-5\,i)\sqrt{2}+4\pi\sqrt{2}\right)}{\frac{i}{60\sqrt{2}}+\frac{4}{3}\log(a)\log_a\left(1+\sqrt{2}\right)\sqrt{2}}\right)\right)^2$$

$\log_b(x)$ is the base-b logarithm

Series representations

$$\left(\frac{1}{27}\left(\left(\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e\right)-1\right)\right)^2+\frac{1}{5}=$$
$$\frac{1}{5}+\frac{1}{729}\left(38+e+\frac{88\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+(4-5\,i)\sinh^{-1}(1)\right)}{\frac{i}{60\sqrt{2}}+\frac{4}{3}\sqrt{2}\left(\frac{\log(2)}{2}-\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}\right)}\right)^2$$

$$\left(\frac{1}{27}\left(\left(\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e\right)-1\right)\right)^2+\frac{1}{5}=$$
$$\frac{1}{5}+\frac{1}{729}\left(38+e+\left(88\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+\left(2-\frac{5\,i}{2}\right)\sqrt{\pi}\right.\right.\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\frac{{}_3\tilde{F}_2\left(\frac{1}{2},\frac{1}{2},1;1-\frac{k}{2},\frac{3-k}{2};-z_0^2\right)(2-2\,z_0)^k\,z_0^{1-k}}{k!}\right)\right)\Big/$$
$$\left.\left(\frac{i}{60\sqrt{2}}+\frac{4}{3}\sqrt{2}\left(\frac{\log(2)}{2}-\sum_{k=1}^{\infty}\frac{(-1)^k\,2^{-k/2}}{k}\right)\right)\right)^2$$
for $(i\,z_0\notin\mathbb{R}$ or $((\text{not } 1\le i\,z_0<\infty)$ and $(\text{not } -\infty<i\,z_0\le -1)))$

$n!$ is the factorial function

${}_p\tilde{F}_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)$ is the regularized generalized hypergeometric function

$\mathbb{R}$ is the set of real numbers

Integral representations

$$\left(\frac{1}{27}\left(\left(\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e\right)-1\right)\right)^2+\frac{1}{5}=$$
$$\frac{1}{5}+\frac{1}{729}\left(38+e+\frac{88\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+4-5\,i\int_0^1\frac{1}{\sqrt{1+t^2}}\,dt\right)}{\frac{i}{60\sqrt{2}}+\frac{4\sqrt{2}}{3}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt}\right)^2$$

$$\left(\frac{1}{27}\left(\left(\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e\right)-1\right)\right)^2+\frac{1}{5}=$$
$$\frac{1}{5}+\frac{1}{729}\left(38+e+\left(88\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi-\right.\right.\right.$$
$$\left.\left.\left.\frac{\frac{5}{4}+i}{\pi^{3/2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}2^{-s}\,\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)\,\Gamma\left(\frac{1}{2}+s\right)ds\right)\right)\Big/$$
$$\left.\left(\frac{i}{60\sqrt{2}}+\frac{4\sqrt{2}}{3}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt\right)\right)^2 \quad \text{for } 0<\gamma<\frac{1}{2}$$

$$\left(\frac{1}{27}\left(\left(\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e\right)-1\right)\right)^2+\frac{1}{5}=$$

$$\frac{1}{5}+\frac{1}{729}\left(38+e+\frac{88\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi-\frac{\frac{5}{4}+i}{\pi^{3/2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds\right)}{\frac{i}{60\sqrt{2}}+\frac{4\sqrt{2}}{3}\int_1^{1+\sqrt{2}}\frac{1}{t}\,dt}\right)^2$$

for $0<\gamma<\frac{1}{2}$

$$\left(\frac{1}{27}\left(\left(\frac{11\left(8\left((6-i\,5)\sqrt{2}+4\sqrt{2}\,\pi+(4-i\,5)\sinh^{-1}(1)\right)\right)}{\frac{4}{3}\sqrt{2}\,\log\left(1+\sqrt{2}\right)+\frac{i}{60\sqrt{2}}}+34+5+e\right)-1\right)\right)^2+\frac{1}{5}=$$

$$\frac{1}{5}+\frac{1}{729}\left(38+e+\frac{88\left((6-5\,i)\sqrt{2}+4\sqrt{2}\,\pi+4-5\,i\int_0^1\frac{1}{\sqrt{1+t^2}}\,dt\right)}{\frac{i}{60\sqrt{2}}-\frac{2\,i\sqrt{2}}{3\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{2^{-s/2}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}\right)^2 \quad \text{for}$$

$-1<\gamma<0$

$\Gamma(x)$ is the gamma function

Now, we analyze the following integral:

$$x=\int_y^\eta \frac{1-\mu\,(\eta^2-t^2)}{\sqrt{2\mu\,(\eta^2-t^2)-\mu^2\,(\eta^2-t^2)^2}}\,dt\,.$$

We consider:

Input

$$\frac{1-u\left(v^2-t^2\right)}{\sqrt{2\,u\left(v^2-t^2\right)-u^2\left(v^2-t^2\right)^2}}$$

Alternate forms

$$\frac{u\,(t-v)\,(t+v)+1}{\sqrt{-u\,(t-v)\,(t+v)\,(u\,(t-v)\,(t+v)+2)}}$$

$$\frac{t^2\,u-u\,v^2+1}{\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}$$

Expanded forms

$$\frac{t^2\,u}{\sqrt{t^4\left(-u^2\right)+2\,t^2\,u^2\,v^2-2\,t^2\,u-u^2\,v^4+2\,u\,v^2}}-\frac{u\,v^2}{\sqrt{t^4\left(-u^2\right)+2\,t^2\,u^2\,v^2-2\,t^2\,u-u^2\,v^4+2\,u\,v^2}}+\frac{1}{\sqrt{t^4\left(-u^2\right)+2\,t^2\,u^2\,v^2-2\,t^2\,u-u^2\,v^4+2\,u\,v^2}}$$

$$\frac{t^2\,u}{\sqrt{2\,u\left(v^2-t^2\right)-u^2\left(v^2-t^2\right)^2}}-\frac{u\,v^2}{\sqrt{2\,u\left(v^2-t^2\right)-u^2\left(v^2-t^2\right)^2}}+\frac{1}{\sqrt{2\,u\left(v^2-t^2\right)-u^2\left(v^2-t^2\right)^2}}$$

Alternate form assuming t, u, and v are positive

$$\frac{1}{\sqrt{2\,u\left(v^2-t^2\right)-u^2\left(v^2-t^2\right)^2}}-\frac{u\left(v^2-t^2\right)}{\sqrt{2\,u\left(v^2-t^2\right)-u^2\left(v^2-t^2\right)^2}}$$

Roots

$$u\neq 0\,,\quad v=-\frac{\sqrt{t^2\,u+1}}{\sqrt{u}}$$

$$u\neq 0\,,\quad v=\frac{\sqrt{t^2\,u+1}}{\sqrt{u}}$$

Integer roots

$t = \pm 1, \quad u = -1, \quad v = 0$

$t = 0, \quad u = 1, \quad v = \pm 1$

Root for the variable u

$$u = \frac{1}{v^2 - t^2}$$

Series expansion at u=0

$$\frac{1}{\sqrt{2}\ \sqrt{u}\ \sqrt{v^2 - t^2}} - \frac{3\sqrt{u}\ \sqrt{v^2 - t^2}}{4\sqrt{2}} - \frac{5\,u^{3/2}\left(v^2 - t^2\right)^{3/2}}{32\sqrt{2}} - \frac{7\,u^{5/2}\left(v^2 - t^2\right)^{5/2}}{128\sqrt{2}} + O\left(u^3\right)$$

(Puiseux series)

Series expansion at u=∞

$$\frac{u\left(t^2 - v^2\right)}{\sqrt{-u^2\left(t^2 - v^2\right)^2}} + \frac{u}{2\,u^2\,(t - v)\,(t + v)\sqrt{-u^2\left(t^2 - v^2\right)^2}} - \frac{u}{u^3\,(t - v)^2\,(t + v)^2\sqrt{-u^2\left(t^2 - v^2\right)^2}} + O\left(\left(\frac{1}{u}\right)^4\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial u}\left(\frac{1 - u\left(v^2 - t^2\right)}{\sqrt{2\,u\left(v^2 - t^2\right) - u^2\left(v^2 - t^2\right)^2}}\right) = -\frac{v^2 - t^2}{\left(-u\left(t^2 - v^2\right)\left(t^2\,u - u\,v^2 + 2\right)\right)^{3/2}}$$

Indefinite integral

$$\int \frac{1-u\left(v^2-t^2\right)}{\sqrt{2\,u\left(v^2-t^2\right)-u^2\left(v^2-t^2\right)^2}}\,du=-\frac{\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{t^2-v^2}+\text{constant}$$

Limit

$$\lim_{t\to\pm\infty}\frac{1-u\left(-t^2+v^2\right)}{\sqrt{2\,u\left(-t^2+v^2\right)-u^2\left(-t^2+v^2\right)^2}}=\frac{u}{\sqrt{-u^2}}$$

$$\lim_{u\to-\infty}\frac{1-u\left(-t^2+v^2\right)}{\sqrt{2\,u\left(-t^2+v^2\right)-u^2\left(-t^2+v^2\right)^2}}=\frac{v^2-t^2}{\sqrt{-\left(t^2-v^2\right)^2}}$$

$$\lim_{v\to\pm\infty}\frac{1-u\left(-t^2+v^2\right)}{\sqrt{2\,u\left(-t^2+v^2\right)-u^2\left(-t^2+v^2\right)^2}}=-\frac{u}{\sqrt{-u^2}}$$

$$\lim_{u\to\infty}\frac{1-u\left(-t^2+v^2\right)}{\sqrt{2\,u\left(-t^2+v^2\right)-u^2\left(-t^2+v^2\right)^2}}=\frac{t^2-v^2}{\sqrt{-\left(t^2-v^2\right)^2}}$$

From the indefinite integral result

$$\int \frac{1-u\left(v^2-t^2\right)}{\sqrt{2\,u\left(v^2-t^2\right)-u^2\left(v^2-t^2\right)^2}}\,du = -\frac{\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{t^2-v^2} + \text{constant}$$

we obtain:

-sqrt(-u (t^2 - v^2) (2 + t^2 u - u v^2))/(t^2 - v^2)

Input

$$-\frac{\sqrt{-u\left(t^2-v^2\right)\left(2+t^2\,u-u\,v^2\right)}}{t^2-v^2}$$

Alternate forms

$$\frac{u\,(u\,(t-v)\,(t+v)+2)}{\sqrt{-u\,(t-v)\,(t+v)\,(u\,(t-v)\,(t+v)+2)}}$$

$$-\frac{\sqrt{-u\left(t^2-v^2\right)\left(u\left(t^2-v^2\right)+2\right)}}{t^2-v^2}$$

Alternate forms assuming t, u, and v are positive

$$-\frac{i\sqrt{u}\ \sqrt{t^2\,u-u\,v^2+2}\ (-1)^{\left\lceil\left(\arg\left(t^2-v^2\right)+\arg\left(u\,t^2-u\,v^2+2\right)\right)/(2\pi)\right\rceil}}{\sqrt{t^2-v^2}}$$

$$-\frac{i\sqrt{u}\ \sqrt{t^2\,u-u\,v^2+2}\ \exp\left(i\,\pi\left\lfloor-\frac{\arg\left(t^2-v^2\right)}{2\pi}-\frac{\arg\left(u\,t^2-u\,v^2+2\right)}{2\pi}\right\rfloor\right)}{\sqrt{t^2-v^2}}$$

$\arg(z)$ is the complex argument

$\lceil x \rceil$ is the ceiling function

$\lfloor x \rfloor$ is the floor function

Roots

$$u \neq 0, \quad v = -\frac{\sqrt{t^2 u + 2}}{\sqrt{u}}$$

$$u \neq 0, \quad v = \frac{\sqrt{t^2 u + 2}}{\sqrt{u}}$$

$$t^2 - v^2 \neq 0, \quad u = 0$$

Root for the variable u

$$u = -\frac{2}{t^2 - v^2}$$

Series expansion at u=0

$$\frac{\sqrt{2}\,\sqrt{u}}{\sqrt{v^2 - t^2}} - \frac{u^{3/2}\sqrt{v^2 - t^2}}{2\sqrt{2}} - \frac{u^{5/2}\left(v^2 - t^2\right)^{3/2}}{16\sqrt{2}} - \frac{u^{7/2}\left(v^2 - t^2\right)^{5/2}}{64\sqrt{2}} - \frac{5\,u^{9/2}\left(v^2 - t^2\right)^{7/2}}{1024\sqrt{2}} + O\left(u^{11/2}\right)$$

(Puiseux series)

Series expansion at u=∞

$$\frac{u\,u\left(t^2 - v^2\right)}{\sqrt{-u^2\left(t^2 - v^2\right)^2}} - \frac{\sqrt{-u^2\left(t^2 - v^2\right)^2}}{u\,(t - v)^2\,(t + v)^2} + \frac{\sqrt{-u^2\left(t^2 - v^2\right)^2}}{2\,u\,u\,(t - v)^3\,(t + v)^3} - \frac{\sqrt{-u^2\left(t^2 - v^2\right)^2}}{2\,u^2\left(u\,(t - v)^4\,(t + v)^4\right)} + O\left(\left(\frac{1}{u}\right)^3\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial u}\left(-\frac{\sqrt{-u\left(t^2-v^2\right)\left(2+t^2\,u-u\,v^2\right)}}{t^2-v^2}\right)=\frac{t^2\,u-u\,v^2+1}{\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}$$

Indefinite integral

$$\int-\frac{\sqrt{-u\left(t^2-v^2\right)\left(2+t^2\,u-u\,v^2\right)}}{t^2-v^2}\,du=$$
$$\left(u\left(t^2-v^2\right)\left(t^4\,u^2+t^2\,u\left(3-2\,u\,v^2\right)+u^2\,v^4-3\,u\,v^2+2\right)-\right.$$
$$\left.2\sqrt{u}\,\sqrt{t^2-v^2}\,\sqrt{t^2\,u-u\,v^2+2}\,\sinh^{-1}\left(\frac{\sqrt{u}\,\sqrt{t^2-v^2}}{\sqrt{2}}\right)\right)\Big/$$
$$\left(2\left(t^2-v^2\right)^2\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}\right)+\text{constant}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

From the indefinite integral result

$$\int-\frac{\sqrt{-u\left(t^2-v^2\right)\left(2+t^2\,u-u\,v^2\right)}}{t^2-v^2}\,du=$$
$$\left(u\left(t^2-v^2\right)\left(t^4\,u^2+t^2\,u\left(3-2\,u\,v^2\right)+u^2\,v^4-3\,u\,v^2+2\right)-\right.$$
$$\left.2\sqrt{u}\,\sqrt{t^2-v^2}\,\sqrt{t^2\,u-u\,v^2+2}\,\sinh^{-1}\left(\frac{\sqrt{u}\,\sqrt{t^2-v^2}}{\sqrt{2}}\right)\right)\Big/$$
$$\left(2\left(t^2-v^2\right)^2\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}\right)+\text{constant}$$

we obtain:

(u (t^2-v^2) (t^4 u^2+t^2 u (3-2 u v^2)+u^2 v^4-3 u v^2+2)-2√(u) √(t^2 - v^2) √(t^2 u-u v^2+2) sinh^(-1)((√(u) √(t^2-v^2))/√(2)))/(2 (t^2-v^2)^2 √(-u (t^2-v^2) (t^2 u-u v^2+2)))

Input

$$\left(u\left(t^2-v^2\right)\left(t^4\,u^2+t^2\,u\left(3-2\,u\,v^2\right)+u^2\,v^4-3\,u\,v^2+2\right)-\left(2\sqrt{u}\right)\sqrt{t^2-v^2}\ \sqrt{t^2\,u-u\,v^2+2}\ \sinh^{-1}\left(\frac{\sqrt{u}\ \sqrt{t^2-v^2}}{\sqrt{2}}\right)\right)\Big/\left(2\left(t^2-v^2\right)^2\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Exact result

$$\left(u\left(t^2-v^2\right)\left(t^4\,u^2+t^2\,u\left(3-2\,u\,v^2\right)+u^2\,v^4-3\,u\,v^2+2\right)-2\sqrt{u}\ \sqrt{t^2-v^2}\ \sqrt{t^2\,u-u\,v^2+2}\ \sinh^{-1}\left(\frac{\sqrt{u}\ \sqrt{t^2-v^2}}{\sqrt{2}}\right)\right)\Big/\left(2\left(t^2-v^2\right)^2\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}\right)$$

Alternate forms

$$\left(u\,(t-v)\,(t+v)\,(u\,(t-v)\,(t+v)+1)\,(u\,(t-v)\,(t+v)+2)-2\sqrt{u}\ \sqrt{(t-v)\,(t+v)}\ \sqrt{u\,(t-v)\,(t+v)+2}\ \sinh^{-1}\left(\frac{\sqrt{u}\ \sqrt{t^2-v^2}}{\sqrt{2}}\right)\right)\Big/\left(2\,(t-v)^2\,(t+v)^2\sqrt{-u\,(t-v)\,(t+v)\,(u\,(t-v)\,(t+v)+2)}\right)$$

$$-\left(\left(\sqrt{u}\left(2\sqrt{u\left(t^2-v^2\right)+2}\,\sinh^{-1}\left(\frac{\sqrt{u}\,\sqrt{t^2-v^2}}{\sqrt{2}}\right)-\right.\right.\right.$$
$$\left.\left.\left.\sqrt{u}\,\sqrt{t^2-v^2}\,\left(u^2\left(t^4+v^4\right)+u\left(t^2\left(-\left(2\,u\,v^2-3\right)\right)-3\,v^2\right)+2\right)\right)\right)\Big/$$
$$\left(2\left(t^2-v^2\right)^{3/2}\sqrt{-u\left(t^2-v^2\right)\left(u\left(t^2-v^2\right)+2\right)}\right)\Bigg)$$

$$\left(u\left(t^2-v^2\right)\left(t^4\,u^2+t^2\,u\left(3-2\,u\,v^2\right)+u^2\,v^4-3\,u\,v^2+2\right)-2\sqrt{u}\,\sqrt{t^2-v^2}\right.$$
$$\left.\sqrt{t^2\,u-u\,v^2+2}\,\log\left(\frac{\sqrt{u}\,\sqrt{t^2-v^2}}{\sqrt{2}}+\sqrt{\frac{1}{2}\,u\left(t^2-v^2\right)+1}\right)\right)\Big/$$
$$\left(2\left(t^2-v^2\right)^2\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}\right)$$

log(x) is the natural logarithm

Expanded form

$$\frac{u^2\,v^6\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{2\left(t^2-v^2\right)^3\left(t^2\,u-u\,v^2+2\right)}-\frac{3\,t^2\,u^2\,v^4\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{2\left(t^2-v^2\right)^3\left(t^2\,u-u\,v^2+2\right)}+$$
$$\frac{3\,t^2\,u\,v^2\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{\left(t^2-v^2\right)^3\left(t^2\,u-u\,v^2+2\right)}-$$
$$\frac{t^2\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{\left(t^2-v^2\right)^3\left(t^2\,u-u\,v^2+2\right)}+\frac{v^2\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{\left(t^2-v^2\right)^3\left(t^2\,u-u\,v^2+2\right)}+$$
$$\frac{\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}\,\sinh^{-1}\left(\frac{\sqrt{u}\,\sqrt{t^2-v^2}}{\sqrt{2}}\right)}{\sqrt{u}\,\left(t^2-v^2\right)^{5/2}\sqrt{t^2\,u-u\,v^2+2}}-$$
$$\frac{3\,u\,v^4\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{2\left(t^2-v^2\right)^3\left(t^2\,u-u\,v^2+2\right)}-\frac{t^6\,u^2\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{2\left(t^2-v^2\right)^3\left(t^2\,u-u\,v^2+2\right)}+$$
$$\frac{3\,t^4\,u^2\,v^2\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{2\left(t^2-v^2\right)^3\left(t^2\,u-u\,v^2+2\right)}-\frac{3\,t^4\,u\sqrt{-u\left(t^2-v^2\right)\left(t^2\,u-u\,v^2+2\right)}}{2\left(t^2-v^2\right)^3\left(t^2\,u-u\,v^2+2\right)}$$

Alternate forms assuming t, u, and v are positive

$$\left(i\left(2\sqrt{t^2-v^2}\ \sqrt{t^2u-uv^2+2}\ \sinh^{-1}\left(\frac{\sqrt{u}\ \sqrt{t^2-v^2}}{\sqrt{2}}\right)-\sqrt{u}\ \left(t^2-v^2\right)\left(t^4u^2+t^2u\left(3-2uv^2\right)+u^2v^4-3uv^2+2\right)\right)(-1)^{\left\lceil\left(\arg\left(t^2-v^2\right)+\arg\left(ut^2-uv^2+2\right)\right)/(2\pi)\right\rceil}\right)\Big/\left(2\left(t^2-v^2\right)^{5/2}\sqrt{t^2u-uv^2+2}\right)$$

$$\frac{i\sinh^{-1}\left(\frac{\sqrt{u}\ \sqrt{t^2-v^2}}{\sqrt{2}}\right)\exp\left(i\pi\left\lfloor-\frac{\arg\left(t^2-v^2\right)}{2\pi}-\frac{\arg\left(ut^2-uv^2+2\right)}{2\pi}\right\rfloor\right)}{\left(t^2-v^2\right)^2}-\left(i\sqrt{u}\ \left(t^4u^2+t^2u\left(3-2uv^2\right)+u^2v^4-3uv^2+2\right)\exp\left(i\pi\left\lfloor-\frac{\arg\left(t^2-v^2\right)}{2\pi}-\frac{\arg\left(ut^2-uv^2+2\right)}{2\pi}\right\rfloor\right)\right)\Big/\left(2\left(t^2-v^2\right)^{3/2}\sqrt{t^2u-uv^2+2}\right)$$

$\arg(z)$ is the complex argument

$\lceil x \rceil$ is the ceiling function

$\lfloor x \rfloor$ is the floor function

Derivative

$$\frac{\partial}{\partial u}\left(\left(u\left(t^2-v^2\right)\left(t^4u^2+t^2u\left(3-2uv^2\right)+u^2v^4-3uv^2+2\right)-\left(2\sqrt{u}\right)\sqrt{t^2-v^2}\ \sqrt{t^2u-uv^2+2}\ \sinh^{-1}\left(\frac{\sqrt{u}\ \sqrt{t^2-v^2}}{\sqrt{2}}\right)\right)\Big/\left(2\left(t^2-v^2\right)^2\sqrt{-u\left(t^2-v^2\right)\left(t^2u-uv^2+2\right)}\right)\right)=-\frac{\sqrt{-u\left(t^2-v^2\right)\left(t^2u-uv^2+2\right)}}{t^2-v^2}$$

Indefinite integral

$$\int\left(u\left(t^2-v^2\right)\left(2+t^4u^2-3uv^2+u^2v^4+t^2u\left(3-2uv^2\right)\right)-2\sqrt{u}\sqrt{t^2-v^2}\sqrt{2+t^2u-uv^2}\sinh^{-1}\left(\frac{\sqrt{u}\sqrt{t^2-v^2}}{\sqrt{2}}\right)\right)\Big/\left(2\left(t^2-v^2\right)^2\sqrt{-u\left(t^2-v^2\right)\left(2+t^2u-uv^2\right)}\right)du=\left(u\sqrt{t^2-v^2}\left(t^6u^3+t^4u^2\left(4-3uv^2\right)+t^2u\left(3u^2v^4-8uv^2+7\right)-u^3v^6+4u^2v^4-7uv^2+6\right)-6\sqrt{u}\left(t^2u-uv^2+1\right)\sqrt{t^2u-uv^2+2}\sinh^{-1}\left(\frac{\sqrt{u}\sqrt{t^2-v^2}}{\sqrt{2}}\right)\right)\Big/\left(6(t-v)^2(t+v)^2\sqrt{t^2-v^2}\sqrt{-u\left(t^2-v^2\right)\left(t^2u-uv^2+2\right)}\right)+\text{constant}$$

From the indefinite integral result

$$\int\left(u\left(t^2-v^2\right)\left(2+t^4u^2-3uv^2+u^2v^4+t^2u\left(3-2uv^2\right)\right)-2\sqrt{u}\sqrt{t^2-v^2}\sqrt{2+t^2u-uv^2}\sinh^{-1}\left(\frac{\sqrt{u}\sqrt{t^2-v^2}}{\sqrt{2}}\right)\right)\Big/\left(2\left(t^2-v^2\right)^2\sqrt{-u\left(t^2-v^2\right)\left(2+t^2u-uv^2\right)}\right)du=\left(u\sqrt{t^2-v^2}\left(t^6u^3+t^4u^2\left(4-3uv^2\right)+t^2u\left(3u^2v^4-8uv^2+7\right)-u^3v^6+4u^2v^4-7uv^2+6\right)-6\sqrt{u}\left(t^2u-uv^2+1\right)\sqrt{t^2u-uv^2+2}\sinh^{-1}\left(\frac{\sqrt{u}\sqrt{t^2-v^2}}{\sqrt{2}}\right)\right)\Big/\left(6(t-v)^2(t+v)^2\sqrt{t^2-v^2}\sqrt{-u\left(t^2-v^2\right)\left(t^2u-uv^2+2\right)}\right)+\text{constant}$$

(u √(t^2 - v^2) (t^6 u^3 + t^4 u^2 (4 - 3 u v^2) + t^2 u (3 u^2 v^4 - 8 u v^2 + 7) - u^3 v^6 + 4 u^2 v^4 - 7 u v^2 + 6) - 6 √(u) (t^2 u - u v^2 + 1) √(t^2 u - u v^2 + 2) sinh^(-1)((√(u) √(t^2 - v^2))/√(2)))/(6 (t - v)^2 (t + v)^2 √(t^2 - v^2) √(-u (t^2 - v^2) (t^2 u - u v^2 + 2)))

for t = 2, u = 4 and v = 8:

(4√(2^2 - 8^2) (2^6 4^3 + 2^4 4^2 (4 – 3*4*8^2) + 2^2*4 (3*4^2 8^4 – 8*4*8^2+7) - 4^3 8^6 + 4*4^2 8^4 – 7*4*8^2+6) - 6 √(4) (2^2*4 – 4*8^2 + 1) √(2^2*4 – 4*8^2 + 2) sinh^(-1)((√(4) √(2^2 - 8^2))/√(2)))/(6 (2 - 8)^2 (2 + 8)^2 √(2^2 - 8^2) √(-4 (2^2 - 8^2) (2^2*4 – 4*8^2+2)))

(4√(4-64)(64*64+16*16(4–12*64)+16(3*16*8^4–8*4*64+7)–64*8^6+4*16*8^4–7*4*64+6)-6√(4)(16–4*64+1) √(16–4*64+2) sinh^(-1)((√(4)√(4-64))/√(2)))/(6(2-8)^2 (2+8)^2 √(4-64) √(-4(4-64)(16–4*64+2)))

Input

$$\left(4\sqrt{4-64}\,\left(64\times 64+16\times 16\,(4-12\times 64)+16\left(3\times 16\times 8^4-8\times 4\times 64+7\right)-64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-\left(6\sqrt{4}\,(16-4\times 64+1)\right)\sqrt{16-4\times 64+2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\Big/\left(\left(6\,(2-8)^2\right)(2+8)^2\,\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-4\times 64+2)}\right)$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Exact result

$$-\frac{-2868\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)-108\,762\,192\,i\sqrt{15}}{2\,592\,000\sqrt{238}}$$

$\sin^{-1}(x)$ is the inverse sine function

Decimal approximation

0.0017380570467776865370198420569532434938035265890061349333634 53…

+

10.530751456431916419826906644471207768647190759365908396221 02… i

Property

$$-\frac{-108\,762\,192\,i\sqrt{15}\ -2868\sqrt{238}\ \sin^{-1}\left(2\sqrt{30}\right)}{2\,592\,000\sqrt{238}}$$ is a transcendental number

$\sin^{-1}(x)$ is the inverse sine function

Alternate complex forms

$0.0017 + 10.5308\,i$

$10.531\,(\cos(1.5706) + i\sin(1.5706))$

$10.531\,e^{1.5706\,i}$

Polar coordinates

$r = 10.5308$ (radius), $\theta = 1.57063$ (angle)

10.5308

Alternate forms

$$\frac{239\sin^{-1}\left(2\sqrt{30}\right)+38\,082\,i\sqrt{3570}}{216\,000}$$

$$\frac{239\sqrt{238}\ \sin^{-1}\left(2\sqrt{30}\right)+9\,063\,516\,i\sqrt{15}}{216\,000\sqrt{238}}$$

$$\frac{239\,\pi}{432\,000} + \frac{i\left(38\,082\,\sqrt{3570} - 239\log\left(2\,\sqrt{30} + \sqrt{119}\right)\right)}{216\,000}$$

log(x) is the natural logarithm

Expanded form

$$\frac{239\sin^{-1}\left(2\,\sqrt{30}\right)}{216\,000} + \frac{6347\,i\,\sqrt{\frac{119}{30}}}{1200}$$

Alternative representations

$$\left(4\sqrt{4-64}\,\left(64\times 64 + 16\times 16\,(4 - 12\times 64) + 16\left(3\times 16\times 8^4 - 8\times 4\times 64 + 7\right) - 64\times 8^6 + 4\times 16\times 8^4 - 7\times 4\times 64 + 6\right) - \left(6\sqrt{16 - 4\times 64 + 2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\,(16 - 4\times 64 + 1)\right)\Big/ \left((2+8)^2\,6\,(2-8)^2\,\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-4\times 64+2)}\right) = \left(4\left(-193\,274 + 64\times 8^4 + 16\left(-2041 + 48\times 8^4\right) - 64\times 8^6\right)\sqrt{-60} + 1434\,\mathrm{sc}^{-1}\left(\frac{\sqrt{-60}\,\sqrt{4}}{\sqrt{2}}\,\middle|\,1\right)\sqrt{-238}\,\sqrt{4}\right)\Big/\left(6\,(-6)^2\,10^2\,\sqrt{-57\,120}\,\sqrt{-60}\right)$$

$$\left(4\sqrt{4-64}\,\left(64\times 64 + 16\times 16\,(4 - 12\times 64) + 16\left(3\times 16\times 8^4 - 8\times 4\times 64 + 7\right) - 64\times 8^6 + 4\times 16\times 8^4 - 7\times 4\times 64 + 6\right) - \left(6\sqrt{16 - 4\times 64 + 2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\,(16 - 4\times 64 + 1)\right)\Big/ \left((2+8)^2\,6\,(2-8)^2\,\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-4\times 64+2)}\right) = \left(4\left(-193\,274 + 64\times 8^4 + 16\left(-2041 + 48\times 8^4\right) - 64\times 8^6\right)\sqrt{-60} + 1434\,\mathrm{sd}^{-1}\left(\frac{\sqrt{-60}\,\sqrt{4}}{\sqrt{2}}\,\middle|\,1\right)\sqrt{-238}\,\sqrt{4}\right)\Big/\left(6\,(-6)^2\,10^2\,\sqrt{-57\,120}\,\sqrt{-60}\right)$$

$$\left(4\sqrt{4-64}\,\left(64\times 64+16\times 16\,(4-12\times 64)+16\left(3\times 16\times 8^4-8\times 4\times 64+7\right)-\right.\right.$$
$$\left.64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-$$
$$\left.\left(6\sqrt{16-4\times 64+2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\,(16-4\times 64+1)\right)\Big/$$
$$\left((2+8)^2\,6\,(2-8)^2\,\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-4\times 64+2)}\right)=$$
$$\left(4\left(-193\,274+64\times 8^4+16\left(-2041+48\times 8^4\right)-64\times 8^6\right)\sqrt{-60}\,-\right.$$
$$\left.1434\,i\,\sin^{-1}\left(\frac{i\,\sqrt{-60}\,\sqrt{4}}{\sqrt{2}}\right)\sqrt{-238}\,\sqrt{4}\right)\Big/\left(6\,(-6)^2\,10^2\,\sqrt{-57\,120}\,\sqrt{-60}\right)$$

$\text{sc}^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function sc

$\text{sd}^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function sd

i is the imaginary unit

Series representations

$$\left(4\sqrt{4-64}\,\left(64\times 64+16\times 16\,(4-12\times 64)+16\left(3\times 16\times 8^4-8\times 4\times 64+7\right)-\right.\right.$$
$$\left.64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-$$
$$\left.\left(6\sqrt{16-4\times 64+2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\,(16-4\times 64+1)\right)\Big/$$
$$\left((2+8)^2\,6\,(2-8)^2\,\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-4\times 64+2)}\right)=$$
$$\frac{6347\,i\,\sqrt{\frac{119}{30}}}{1200}+\frac{239\sum_{k=0}^{\infty}\frac{T_{1+2k}\left(\sqrt{30}\right)}{(1+2k)^2}}{54\,000\,\pi}$$

$$\left(4\sqrt{4-64}\,\left(64\times 64+16\times 16\,(4-12\times 64)+16\left(3\times 16\times 8^4-8\times 4\times 64+7\right)-\right.\right.$$
$$\left.64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-$$
$$\left.\left(6\sqrt{16-4\times 64+2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\,(16-4\times 64+1)\right)\Big/$$
$$\left((2+8)^2\,6\,(2-8)^2\,\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-4\times 64+2)}\right)=$$
$$\frac{6347\,i\,\sqrt{\frac{119}{30}}}{1200}+\frac{239\,\sqrt{\pi}\,\sum_{k=0}^{\infty}\frac{{}_3\tilde{F}_2\left(\frac{1}{2},\frac{1}{2},1;1-\frac{k}{2},\frac{3-k}{2};z_0^2\right)\left(4\sqrt{30}-2\,z_0\right)^k\,z_0^{1-k}}{k!}}{432\,000}$$
for $(z_0 \notin \mathbb{R}$ or $((\text{not } 1 \le z_0 < \infty)$ and $(\text{not } -\infty < z_0 \le -1)))$

$$\left(4\sqrt{4-64}\,\left(64\times 64+16\times 16\,(4-12\times 64)+16\left(3\times 16\times 8^4-8\times 4\times 64+7\right)-\right.\right.$$
$$\left.64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-$$
$$\left.\left(6\sqrt{16-4\times 64+2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\,(16-4\times 64+1)\right)\Big/$$
$$\left((2+8)^2\,6\,(2-8)^2\,\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-4\times 64+2)}\right)=$$
$$\frac{6347\,i\,\sqrt{\frac{119}{30}}}{1200}-\frac{239\,\sum_{j=0}^{\infty}\operatorname{Res}_{s=\frac{1}{2}+j}\frac{(-120)^{-s}\,\Gamma\left(\frac{1}{2}-s\right)^2\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}{7200\,\sqrt{30\,\pi}}$$

$T_n(x)$ is the Chebyshev polynomial of the first kind

$n!$ is the factorial function

${}_p\tilde{F}_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)$ is the regularized generalized hypergeometric function

$\mathbb{R}$ is the set of real numbers

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Integral representation

$$\left(4\sqrt{4-64}\,\left(64\times 64+16\times 16\,(4-12\times 64)+16\left(3\times 16\times 8^4-8\times 4\times 64+7\right)-\right.\right.$$
$$\left.64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-$$
$$\left.\left(6\sqrt{16-4\times 64+2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\,(16-4\times 64+1)\right)\Big/$$
$$\left((2+8)^2\,6\,(2-8)^2\,\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-4\times 64+2)}\right)=$$
$$\frac{6347\,i\,\sqrt{\frac{119}{30}}}{1200}+\frac{239}{3600\sqrt{30}}\int_0^1\frac{1}{\sqrt{1-120\,t^2}}\,dt$$

From which, we obtain:

(((4√(4-64)(4096+256(4–12*64)+16(48*8^4–8*4*64+7)–64*8^6+4*16*8^4–7*4*64+6)-6√(4)(16–4*64+1) √(16–256+2) sinh^(-1)((√(4)√(4-64))/√(2)))/(6(2-8)^2 (2+8)^2 √(4-64) √(-4(4-64)(16–256+2))))^2+47)^2+13+1/5

Input

$$\left(\left(\left(4\sqrt{4-64}\,\left(4096+256\,(4-12\times 64)+16\left(48\times 8^4-8\times 4\times 64+7\right)-64\times 8^6+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.4\times 16\times 8^4-7\times 4\times 64+6\right)-\left(6\sqrt{4}\,(16-4\times 64+1)\right)\right.\right.\right.$$
$$\left.\left.\sqrt{16-256+2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\Big/$$
$$\left(\left(6\,(2-8)^2\right)(2+8)^2\,\sqrt{4-64}\right.$$
$$\left.\left.\sqrt{-4\,(4-64)\,(16-256+2)}\right)\right)^2+47\Bigg)^2+13+\frac{1}{5}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Exact result

$$\frac{66}{5}+\left(47+\frac{\left(-2868\,\sqrt{238}\,\sin^{-1}\left(2\,\sqrt{30}\right)-108\,762\,192\,i\,\sqrt{15}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2$$

$\sin^{-1}(x)$ is the inverse sine function

Decimal approximation

4095.989897774446813765914366067328044612341706657635069936221643…
$-$
4.678018855627271490586671845635936374651684816811434476331806315… i

Property

$$\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}-2868\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2$$
is a transcendental number

Alternate complex forms

$4096.0 - 4.7\,i$

$4096.0\,(\cos(-0.0011421) + i\sin(-0.0011421))$

$4096.0\,e^{-0.0011421\,i}$

Polar coordinates

$r = 4096.$ (radius), $\theta = -0.00114746$ (angle)

4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group.

The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{66}{5}+\left(47-\frac{\left(9\,063\,516\sqrt{15}-239\,i\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{11\,104\,128\,000\,000}\right)^2$$

$$\frac{66}{5}+\left(47+\frac{\left(2868\,i\sqrt{238}\,\log\left(2\,i\sqrt{30}+i\sqrt{119}\right)-108\,762\,192\,i\sqrt{15}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2$$

$$\Big(8\,936\,094\,617\,739\,967\,068\,302\,400-108\,655\,613\,959\,755\,994\,560\,i\sqrt{3570}\,\sin^{-1}\left(2\sqrt{30}\right)-1\,523\,899\,712\,066\,197\,680\,\sin^{-1}\left(2\sqrt{30}\right)^2+2\,079\,569\,517\,432\,i\sqrt{3570}\,\sin^{-1}\left(2\sqrt{30}\right)^3+3\,262\,808\,641\,\sin^{-1}\left(2\sqrt{30}\right)^4\Big)\Big/2\,176\,782\,336\,000\,000\,000\,000$$

$\log(x)$ is the natural logarithm

Expanded form

$$\frac{7\,661\,260\,817\,678\,298\,241}{1\,866\,240\,000\,000\,000} - \frac{4\,191\,960\,415\,114\,043\,i\sqrt{\frac{119}{30}}\,\sin^{-1}\left(2\sqrt{30}\right)}{2\,799\,360\,000\,000\,000} - \frac{235\,169\,708\,652\,191\,\sin^{-1}\left(2\sqrt{30}\right)^2}{335\,923\,200\,000\,000\,000} + \frac{86\,648\,729\,893\,i\sqrt{\frac{119}{30}}\,\sin^{-1}\left(2\sqrt{30}\right)^3}{3\,023\,308\,800\,000\,000\,000} + \frac{3\,262\,808\,641\,\sin^{-1}\left(2\sqrt{30}\right)^4}{2\,176\,782\,336\,000\,000\,000\,000}$$

Alternative representations

$$\left(\left(\left(4\sqrt{4-64}\,\left(4096+256\,(4-12\times 64)+16\left(48\times 8^4-8\times 4\times 64+7\right)-64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-\left(6\sqrt{16-256+2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\,(16-4\times 64+1)\right)\Big/\left((2+8)^2\,6\,(2-8)^2\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-256+2)}\right)\right)^2+47\right)^2+13+\frac{1}{5}=13+\frac{1}{5}+\left(47+\left(\left(4\left(-193\,274+64\times 8^4+16\left(-2041+48\times 8^4\right)-64\times 8^6\right)\sqrt{-60}+1434\,\mathrm{sc}^{-1}\left(\frac{\sqrt{-60}\,\sqrt{4}}{\sqrt{2}}\,\middle|\,1\right)\sqrt{-238}\,\sqrt{4}\right)\Big/\left(6\,(-6)^2\,10^2\sqrt{-57\,120}\,\sqrt{-60}\right)\right)^2\right)^2$$

$$\Bigg(\Bigg(\Bigg(\Bigg(4\sqrt{4-64}\ \left(4096+256\left(4-12\times 64\right)+16\left(48\times 8^4-8\times 4\times 64+7\right)-64\times 8^6+\right.$$
$$\left.4\times 16\times 8^4-7\times 4\times 64+6\right)-\left(6\sqrt{16-256+2}\right.$$
$$\left.\sinh^{-1}\left(\frac{\sqrt{4}\ \sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\ \left(16-4\times 64+1\right)\Bigg)\Big/$$
$$\left(\left(2+8\right)^2 6\left(2-8\right)^2\sqrt{4-64}\ \sqrt{-4\left(4-64\right)\left(16-256+2\right)}\right)\Bigg)^2+$$
$$47\Bigg)^2+13+\frac{1}{5}=13+\frac{1}{5}+$$
$$\Bigg(47+\Bigg(\Bigg(4\left(-193\,274+64\times 8^4+16\left(-2041+48\times 8^4\right)-64\times 8^6\right)\sqrt{-60}\ +$$
$$1434\ \mathrm{sd}^{-1}\left(\frac{\sqrt{-60}\ \sqrt{4}}{\sqrt{2}}\ \middle|\ 1\right)\sqrt{-238}\ \sqrt{4}\ \Bigg)\Big/$$
$$\left(6\left(-6\right)^2 10^2\sqrt{-57\,120}\ \sqrt{-60}\right)\Bigg)^2\Bigg)^2$$

$$\Bigg(\Bigg(\Bigg(\Bigg(4\sqrt{4-64}\ \left(4096+256\left(4-12\times 64\right)+16\left(48\times 8^4-8\times 4\times 64+7\right)-64\times 8^6+\right.$$
$$\left.4\times 16\times 8^4-7\times 4\times 64+6\right)-\left(6\sqrt{16-256+2}\right.$$
$$\left.\sinh^{-1}\left(\frac{\sqrt{4}\ \sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\ \left(16-4\times 64+1\right)\Bigg)\Big/$$
$$\left(\left(2+8\right)^2 6\left(2-8\right)^2\sqrt{4-64}\ \sqrt{-4\left(4-64\right)\left(16-256+2\right)}\right)\Bigg)^2+$$
$$47\Bigg)^2+13+\frac{1}{5}=13+\frac{1}{5}+$$
$$\Bigg(47+\Bigg(\Bigg(4\left(-193\,274+64\times 8^4+16\left(-2041+48\times 8^4\right)-64\times 8^6\right)\sqrt{-60}\ -$$
$$1434\ i\ \sin^{-1}\left(\frac{i\sqrt{-60}\ \sqrt{4}}{\sqrt{2}}\right)\sqrt{-238}\ \sqrt{4}\ \Bigg)\Big/$$
$$\left(6\left(-6\right)^2 10^2\sqrt{-57\,120}\ \sqrt{-60}\right)\Bigg)^2\Bigg)^2$$

$\mathrm{sc}^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function sc

$\mathrm{sd}^{-1}(x \mid m)$ is the inverse of the Jacobi elliptic function sd

i is the imaginary unit

Series representations

$$\left(\left(\left(4\sqrt{4-64}\,\left(4096+256\,(4-12\times 64)+16\left(48\times 8^4-8\times 4\times 64+7\right)-64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-\left(6\sqrt{16-256+2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\,(16-4\times 64+1)\right)\Big/\left((2+8)^2\,6\,(2-8)^2\,\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-256+2)}\right)\right)^2+47\right)^2+13+\frac{1}{5}=\frac{66}{5}+\left(47-\frac{\left(2\,265\,879\sqrt{15}\,\pi-239\,i\,\sqrt{238}\,\sum_{k=0}^{\infty}\frac{T_{1+2k}\left(\sqrt{30}\right)}{(1+2k)^2}\right)^2}{694\,008\,000\,000\,\pi^2}\right)^2$$

$$\left(\left(\left(4\sqrt{4-64}\,\left(4096+256\,(4-12\times 64)+16\left(48\times 8^4-8\times 4\times 64+7\right)-64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-\left(6\sqrt{16-256+2}\,\sinh^{-1}\left(\frac{\sqrt{4}\,\sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\,(16-4\times 64+1)\right)\Big/\left((2+8)^2\,6\,(2-8)^2\,\sqrt{4-64}\,\sqrt{-4\,(4-64)\,(16-256+2)}\right)\right)^2+47\right)^2+13+\frac{1}{5}=\frac{66}{5}+\left(47+\left(-108\,762\,192\,i\,\sqrt{15}+1434\,i\,\sqrt{238}\,\left(i\,\pi+\log(480)-\sum_{k=1}^{\infty}\frac{2^{-3k}\times 15^{-k}\left(\frac{1}{2}\right)_k}{k\,k!}\right)\right)^2\Big/\,1\,598\,994\,432\,000\,000\right)^2$$

$$\left(\left(\left(4\sqrt{4-64}\ \left(4096+256\left(4-12\times 64\right)+16\left(48\times 8^4-8\times 4\times 64+7\right)-64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-\left(6\sqrt{16-256+2}\ \sinh^{-1}\left(\frac{\sqrt{4}\ \sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\left(16-4\times 64+1\right)\right)\Big/\left(\left(2+8\right)^2 6\left(2-8\right)^2\sqrt{4-64}\ \sqrt{-4\left(4-64\right)\left(16-256+2\right)}\right)\right)^2+47\right)^2+13+\frac{1}{5}=\frac{66}{5}+\left(47+\frac{\left(4\,531\,758\,i-239\sqrt{\frac{119}{\pi}}\ \sum_{j=0}^{\infty}\operatorname{Res}_{s=\frac{1}{2}+j}\frac{(-120)^{-s}\,\Gamma\left(\frac{1}{2}-s\right)^2\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)^2}{185\,068\,800\,000}\right)^2$$

$T_n(x)$ is the Chebyshev polynomial of the first kind

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Integral representation

$$\left(\left(\left(4\sqrt{4-64}\ \left(4096+256\left(4-12\times 64\right)+16\left(48\times 8^4-8\times 4\times 64+7\right)-64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-\left(6\sqrt{16-256+2}\ \sinh^{-1}\left(\frac{\sqrt{4}\ \sqrt{4-64}}{\sqrt{2}}\right)\right)\sqrt{4}\left(16-4\times 64+1\right)\right)\Big/\left(\left(2+8\right)^2 6\left(2-8\right)^2\sqrt{4-64}\ \sqrt{-4\left(4-64\right)\left(16-256+2\right)}\right)\right)^2+47\right)^2+13+\frac{1}{5}=\frac{66}{5}+\left(47+\frac{\left(2\,265\,879\,i+239\sqrt{119}\ \int_0^1\frac{1}{\sqrt{1-120\,t^2}}\,dt\right)^2}{46\,267\,200\,000}\right)^2$$

√((((4√(4-64)(4096+256(4–12*64)+16(48*8^4–8*4*64+7)–64*8^6+4*16*8^4–7*4*64+6)-6√(4)(16–4*64+1)√(16–256+2) sinh^(-1)((√(4)√(4-64))/√(2)))/(6(2-8)^2(2+8)^2√(4-64)√(-4(4-64)(16–256+2))))^2+47)^2+13+1/5)

Input

$$\sqrt{\left(\left(\left(\left(\left(4\sqrt{4-64}\ \left(4096+256\,(4-12\times 64)+16\left(48\times 8^4-8\times 4\times 64+7\right)-64\times 8^6+4\times 16\times 8^4-7\times 4\times 64+6\right)-\left(6\sqrt{4}\ (16-4\times 64+1)\sqrt{16-256+2}\right)\sinh^{-1}\left(\frac{\sqrt{4}\ \sqrt{4-64}}{\sqrt{2}}\right)\right)\Big/\left(6\,(2-8)^2\left((2+8)^2\sqrt{4-64}\ \sqrt{-4\,(4-64)\,(16-256+2)}\right)\right)\right)^2+47\right)^2+13+\frac{1}{5}\right)}$$

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Exact result

$$\sqrt{\frac{66}{5}+\left(47+\frac{\left(-2868\sqrt{238}\ \sin^{-1}\left(2\sqrt{30}\right)-108\,762\,192\,i\sqrt{15}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}$$

$\sin^{-1}(x)$ is the inverse sine function

Decimal approximation

63.999931511386363677934602562007854493610383414598984642493992… −
0.036547061419862576143647398470222311645392685670740794837066754…
i

Property

$$\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}\ -2868\sqrt{238}\ \sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}$$

is a transcendental number

Alternate complex forms

$$64.000 - 0.037\,i$$

$$64.000\,(\cos(-0.00057105) + i\sin(-0.00057105))$$

$$64.000\,e^{-0.00057105\,i}$$

Alternate forms

$$\sqrt{\frac{66}{5} + \left(47 - \frac{\left(9\,063\,516\sqrt{15} - 239\,i\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{11\,104\,128\,000\,000}\right)^2}$$

$$\sqrt{\frac{66}{5} + \left(47 + \frac{\left(2868\,i\sqrt{238}\,\log\left(2\,i\sqrt{30} + i\sqrt{119}\right) - 108\,762\,192\,i\sqrt{15}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}$$

$$\frac{1}{46\,656\,000\,000}\left(\sqrt{\left(8\,936\,094\,617\,739\,967\,068\,302\,400 - 108\,655\,613\,959\,755\,994\,560\,i\sqrt{3570}\right.\right.$$
$$\sin^{-1}\left(2\sqrt{30}\right) - 1\,523\,899\,712\,066\,197\,680\,\sin^{-1}\left(2\sqrt{30}\right)^2 +$$
$$\left.\left.2\,079\,569\,517\,432\,i\sqrt{3570}\,\sin^{-1}\left(2\sqrt{30}\right)^3 + 3\,262\,808\,641\,\sin^{-1}\left(2\sqrt{30}\right)^4\right)\right)$$

27*(sqrt(66/5 + (47 + (-108762192 i sqrt(15) - 2868 sqrt(238) sin^(-1)(2 sqrt(30)))^2/1598994432000000)^2))+1

Input

$$27\sqrt{\frac{66}{5} + \left(47 + \frac{\left(-108\,762\,192\,i\sqrt{15} - 2868\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2} + 1$$

$\sin^{-1}(x)$ is the inverse sine function

i is the imaginary unit

Exact result

$$1 + 27\sqrt{\frac{66}{5} + \left(47 + \frac{\left(-2868\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right) - 108\,762\,192\,i\sqrt{15}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}$$

Decimal approximation

$$1728.998150807431819304234269174212071327480352194172585347337 80\ldots - 0.986770658336289555878479758696002414425602513110001460600802366\ldots\,i$$

(result in radians)

Property

$$1 + 27\sqrt{\frac{66}{5} + \left(47 + \frac{\left(-108\,762\,192\,i\sqrt{15} - 2868\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}$$

is a transcendental number

Alternate complex forms

$$1729.0 - 1.\,i$$

$$1729.0\,(\cos(-0.00057072) + i\sin(-0.00057072))$$

$$1729.0\,e^{-0.00057072\,i}$$

Polar coordinates

$r = 1729.$ (radius), $\theta = -0.000578369$ (angle)

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2.

Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$1+27\sqrt{\frac{66}{5}+\left(47-\frac{\left(9\,063\,516\sqrt{15}-239\,i\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{11\,104\,128\,000\,000}\right)^2}$$

$$1+27\sqrt{\frac{66}{5}+\left(47+\frac{\left(2868\,i\sqrt{238}\,\log\left(2\,i\sqrt{30}+i\sqrt{119}\right)-108\,762\,192\,i\sqrt{15}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}$$

$$\frac{1}{1\,728\,000\,000}\left(1\,728\,000\,000+\sqrt{\left(8\,936\,094\,617\,739\,967\,068\,302\,400-108\,655\,613\,959\,755\,994\,560\,i\sqrt{3570}\,\sin^{-1}\left(2\sqrt{30}\right)-1\,523\,899\,712\,066\,197\,680\sin^{-1}\left(2\sqrt{30}\right)^2+2\,079\,569\,517\,432\,i\sqrt{3570}\,\sin^{-1}\left(2\sqrt{30}\right)^3+3\,262\,808\,641\sin^{-1}\left(2\sqrt{30}\right)^4\right)}\right)$$

$\log(x)$ is the natural logarithm

Alternative representations

$$27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}-2868\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}+1=1+27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}-2868\,\mathrm{sd}^{-1}\left(2\sqrt{30}\,\middle|\,0\right)\sqrt{238}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}$$

$$27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}\ -2868\sqrt{238}\ \sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}\ +1=$$
$$1+27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}\ -2868\,\mathrm{sn}^{-1}\left(2\sqrt{30}\ \middle|\ 0\right)\sqrt{238}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}$$

$$27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}\ -2868\sqrt{238}\ \sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}\ +1=$$
$$1+27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}\ +2868\,i\,\sinh^{-1}\left(2\,i\sqrt{30}\right)\sqrt{238}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}$$

$\mathrm{sd}^{-1}(x\ |\ m)$ is the inverse of the Jacobi elliptic function sd

$\mathrm{sn}^{-1}(x\ |\ m)$ is the inverse of the Jacobi elliptic function sn

$\sinh^{-1}(x)$ is the inverse hyperbolic sine function

Series representations

$$27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}\ -2868\sqrt{238}\ \sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}\ +1=$$
$$1+27\sqrt{\frac{66}{5}+\left(47-\frac{\left(2\,265\,879\sqrt{15}\ \pi-239\,i\sqrt{238}\ \sum_{k=0}^{\infty}\frac{T_{1+2k}\left(\sqrt{30}\right)}{(1+2k)^2}\right)^2}{694\,008\,000\,000\,\pi^2}\right)^2}$$

$$27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}-2868\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}+1=$$
$$1+27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}-\frac{11472\sqrt{238}\,\sum_{k=0}^{\infty}\frac{T_{1+2k}\left(\sqrt{30}\right)}{(1+2k)^2}}{\pi}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}$$

$$27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}-2868\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}+1=1+$$
$$27\sqrt{\frac{66}{5}+\left(47+\frac{\left(4\,531\,758\,i-239\sqrt{\frac{119}{\pi}}\,\sum_{j=0}^{\infty}\operatorname{Res}_{s=\frac{1}{2}+j}\frac{(-120)^{-s}\,\Gamma\left(\frac{1}{2}-s\right)^2\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\right)^2}{185\,068\,800\,000}\right)^2}$$

$T_n(x)$ is the Chebyshev polynomial of the first kind

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Integral representation

$$27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}-2868\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}+1=$$
$$1+27\sqrt{\frac{66}{5}+\left(47+\frac{\left(2\,265\,879\,i+239\sqrt{119}\,\int_0^1\frac{1}{\sqrt{1-120\,t^2}}\,dt\right)^2}{46\,267\,200\,000}\right)^2}$$

Continued fraction representation

$$27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}\ -2868\sqrt{238}\ \sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}+1=$$

$$1+27\sqrt{\frac{66}{5}+\left(47-\frac{119\left(19\,041+\frac{239}{1+\underset{k=1}{\overset{\infty}{\mathrm{K}}}\frac{-240\left\lfloor\frac{1+k}{2}\right\rfloor\left(-1+2\left\lfloor\frac{1+k}{2}\right\rfloor\right)}{1+2k}}\right)^2}{388\,800\,000}\right)^2}=$$

$$1+27\sqrt{\frac{66}{5}+\left(47-\frac{119\left(19\,041+\frac{239}{1+-\frac{240}{3-\frac{240}{5-\frac{1440}{7-\frac{1440}{9+\ldots}}}}}\right)^2}{388\,800\,000}\right)^2}\quad\text{for}\ \infty\le 2\sqrt{30}$$

$\lfloor x \rfloor$ is the floor function

$\underset{k=k_1}{\overset{k_2}{\mathrm{K}}}\, a_k / b_k$ is a continued fraction

(27*(sqrt(66/5 + (47 + (-108762192 i sqrt(15) - 2868 sqrt(238) sin^(-1)(2 sqrt(30)))^2/1598994432000000)^2))+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-108\,762\,192\,i\sqrt{15}\ -2868\sqrt{238}\ \sin^{-1}\left(2\sqrt{30}\right)\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}+1}+C_{\mathrm{MRB}}^{\,1-1/(4\pi)+\pi}$$

$\sin^{-1}(x)$ is the inverse sine function

i is the imaginary unit

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \sqrt[15]{1+27\sqrt{\frac{66}{5}+\left(47+\frac{\left(-2868\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)-108\,762\,192\,i\sqrt{15}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}}$$

Decimal approximation

1.64493792358203696880413832724054904998825072066315850121 28… −
0.00006254367055086616079566786821016807812756224322896566451 2917…
i

(result in radians)

Alternate complex forms

$1.6449 - 0.\times 10^{-5}\, i$

$1.6449\,(\cos(-0.000038022) + i\sin(-0.000038022))$

$1.6449\, e^{-0.000038022\, i}$

Result

1.6449

1.6449 ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \sqrt[15]{1+27\sqrt{\frac{66}{5}+\left(47-\frac{\left(9\,063\,516\sqrt{15}-239\,i\sqrt{238}\,\sin^{-1}\left(2\sqrt{30}\right)\right)^2}{11\,104\,128\,000\,000}\right)^2}}$$

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \left(1+27\sqrt{\frac{66}{5}+\left(47+\frac{\left(2868\,i\sqrt{238}\,\log\left(2\,i\sqrt{30}+i\sqrt{119}\right)-108\,762\,192\,i\sqrt{15}\right)^2}{1\,598\,994\,432\,000\,000}\right)^2}\right)\text{^}(1/15)$$

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \frac{1}{2^{4/5}\,\sqrt[5]{3}\;5^{2/5}}\left(\left(1\,728\,000\,000+\sqrt{\left(8\,936\,094\,617\,739\,967\,068\,302\,400-108\,655\,613\,959\,755\,994\,560\,i\sqrt{3570}\,\sin^{-1}\left(2\sqrt{30}\right)-1\,523\,899\,712\,066\,197\,680\,\sin^{-1}\left(2\sqrt{30}\right)^2+2\,079\,569\,517\,432\,i\sqrt{3570}\,\sin^{-1}\left(2\sqrt{30}\right)^3+3\,262\,808\,641\,\sin^{-1}\left(2\sqrt{30}\right)^4\right)}\right)\text{^}(1/15)\right)$$

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

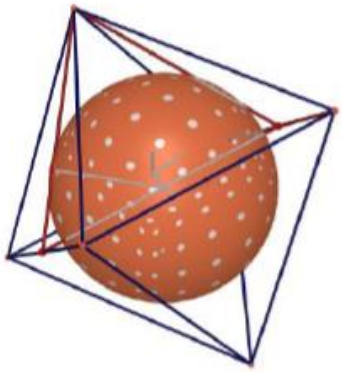


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$
$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

4096 = 64^2 , (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$).

The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^8\right)^{-k}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^8\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\,\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\,\sqrt{2}\,a^3}{\frac{4}{3}\,\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)}{3\times 3}}}} = \sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))) = 3.1415926535

Indeed:

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.1415926535884871624221932509858715278931025189245576170078100613
…

3.1415926535…. = π (Ramanujan Recurring Number)

From the following extended DN Constant (“Unitary Formula”)

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}}$$

with regard

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

for q = 1729 and p = 4096, we obtain by changing the sign in the algebraic sum of the aforementioned Cardano’s Formula and after some calculations:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

multiplied by

$$\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}$$

and performing the ninth root of the entire expression:

√((2^(-1/π)(5(3+√(5))π)^(1/(2π)))((∛(-1729/2+√(1729^2/4+4096^3/27)) -∛(-1729/2-√(1729^2/4+4096^3/27)))^1/9))

we obtain:

$$\sqrt{\left(\left(2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right)\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}\right)}$$

$$2^{-1/(2\pi)}\sqrt[18]{\sqrt[3]{\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}-\frac{1729}{2}}+\sqrt[3]{\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}}\Big/\sqrt[4\pi]{5\left(3+\sqrt{5}\right)\pi}$$

i.e.

2^(-1/(2π)) ((-1729/2+√(274958621851/3)/6)^(1/3)+(1729/2 + √(274958621851/3)/6)^(1/3))^(1/18) (5(3+√(5)) π)^(1/(4π))

= 1.61549140391….

The general "unitary" formula, which derives from DN Constant, is the following:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

Where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

C is any constant or solution to an equation, R is the radius of the Universe and $2.33*10^{-13}$ is the temperature of the universe expressed in GeV.

For example, $C = 9.9128$, inserting a radius of the Universe, which we have calculated, equal to $R = 2.06274*10^{12}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}$$

1.6180359123482642354401744088347098542545273508401733563064818107 …

1.618035912348…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We obtain also:

(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))dxdydz

Indefinite integral assuming all variables are real

$$0.809018\, x^2\, y\, z + \text{constant}$$

Definite integral over a cube of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} 1.61804\, dx\, dy\, dz = 12.9443\, L^3$$

Definite integral over a sphere of radius R

$$\iiint_{x^2+y^2+z^2<R^2} 1.61804\, dz\, dy\, dx = 6.77761\, R^3$$

From which, for L = R = 1 , dividing the two definite integral results by the original expression, we obtain:

12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))

Input interpretation

$$\frac{12.9443}{\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

8.00001…

8.00001…. ≈ 8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

And

3*(6.77761/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))

Input interpretation

$$3\times\frac{6.77761}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

12.5664…

12.5664…. ≈ 4π = Bekenstein-Hawking (S_{BH}) black hole entropy

New fundamental formula deriving from DN Constant

The DN Constant (Del Gaudio-Nardelli Constant) equals $(2\sqrt{2})/\pi$) is defined as the ratio of the volume of an octahedron to the volume of a sphere and is an intriguing mathematical concept. Michele Nardelli hypothesized that the regular octahedron represents a phase in which the universe is highly symmetrical and with very low entropy. On the other hand, the sphere (which is inscribed in the octahedron, i.e. is "inside" it) represents the universe emerging from the quantum vacuum, which over time increases entropy and undergoes various symmetry breakings. This occurs in a regime of eternal inflation.

From the following expression

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×C×R)))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\,C\,R\right)}}$$

which comes from the DN Constant, with $1.616255*10^{-35}$ which is equal to the Planck length, $1.1056*10^{-52}$ which is equal to the Cosmological Constant, C = 1729 which corresponds to the Hardy-Ramanujan number and R = $4.6018401361 \times 10^{-24}$, which represents the radius of the Universe, we obtain:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×4.6018401361 × 10^-24))) = 1.6180329973...

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 4.6018401361\times 10^{-24}\right)}}$$

1.618032997307532491506757016629746746422577260892967140791939590 3…

1.6180329973075… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We have also the following formula:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(4.4525642121 × 10^-24))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times10^{-35}}{1.1056\times10^{-52}}\times 1729\times 4.4525642121\times 10^{-24}\right)}}$$

Result

1.644932352102092130483898983704151168876621841655166877914166033 8…

1.64493235210209213…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

And again:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(1.2206935225 × 10^-24))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times10^{-35}}{1.1056\times10^{-52}}\times 1729\times 1.2206935225\times 10^{-24}\right)}}$$

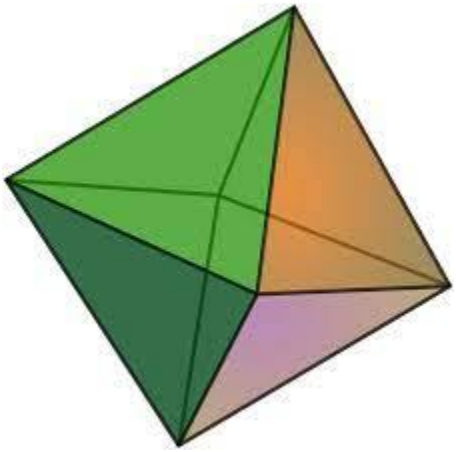
Result

3.14159225731469939939950039231605796469731171781644423955790024797

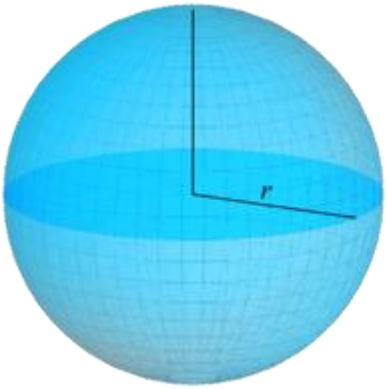
…

3.14159225731469…≈ π (Ramanujan Recurring Number)

Now, we have that:



Octahedron



Sphere

Given the value of a volume, independently of the solid, following the Poincaré Conjecture, we compare any solid "without holes" and a sphere. If we compare an octahedron with a sphere, we have:

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

If we consider the radius of the sphere as an unknown, we must find the value of the side of the octahedron which allows us to equalize the two volumes and which will give us the DN Constant as a result (which will therefore be equal to the radius of the sphere).

From

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

To find r we perform the following calculation:

$$r^3 = \frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi} = \frac{1}{3}\sqrt{2}a^3 \cdot \frac{3}{4\pi} = \frac{\sqrt{2}a^3}{4\pi} = \frac{\sqrt{2}\cdot\sqrt{2}a^3}{\sqrt{2}\cdot 4\pi} = \frac{2a^3}{\sqrt{2}\cdot 4\pi} = \frac{a^3}{2\sqrt{2}\pi}$$

$$r^3 = \frac{a^3}{2\sqrt{2}\pi}; \qquad r = \sqrt[3]{\frac{a^3}{2\sqrt{2}\pi}} = \frac{a}{\sqrt[3]{2\sqrt{2}\pi}}$$

To find *a*, we have, for

$$r = \frac{2\sqrt{2}}{\pi}; \qquad \frac{a}{\sqrt[3]{2\sqrt{2}\pi}} = \frac{2\sqrt{2}}{\pi};$$

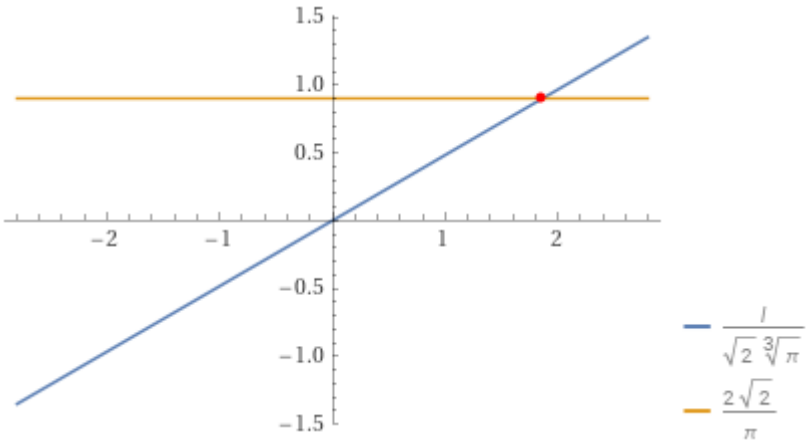
Thus, multiplying both the sides by $\sqrt[3]{2\sqrt{2}\pi}$, we obtain:

$$a = \frac{2\sqrt{2}}{\pi} \cdot \sqrt[3]{2\sqrt{2}\pi} = \sqrt[3]{\frac{(2\sqrt{2})^3 2\sqrt{2}\pi}{\pi^3}} =$$

$$= \sqrt[3]{\frac{8\cdot\sqrt{2^3}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{8\cdot\sqrt{2\cdot 2^2}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{16\cdot\sqrt{2}\cdot 2\sqrt{2}}{\pi^2}}$$

$$= \sqrt[3]{\frac{32\sqrt{2}\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{32 \cdot 2}{\pi^2}} = \frac{\sqrt[3]{64}}{\sqrt[3]{\pi^2}} = \frac{4}{\sqrt[3]{\pi^2}};\ a = \frac{4}{\sqrt[3]{\pi^2}}$$

Plot



Solution

$a = \frac{4}{\sqrt[3]{\pi^2}}$

for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), we obtain:

from the octahedron volume, we have: V = 1/3*√2*a^3 = (1/3*√2*($\frac{4}{\sqrt[3]{\pi^2}}$)^3)

(1/3*√2*(4/∛(π^2))^3)

Input

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3$$

$\sqrt[3]{x}$ is the real-valued 3rd root of x

Exact result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{3\pi^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And, from the sphere volume V = (4/3*π*r^3) = (4/3*π*((2√2)/π)^3)

(4/3*π*((2√2)/π)^3)

Input

$$\frac{4}{3}\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3$$

Result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\sqrt{z_0}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\exp^3\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3}{3\pi^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\left(\frac{1}{z_0}\right)^{3/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{3/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

$1.64381856858\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\right/\right.$$
$$\left.\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\text{^}(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right/\right.$$
$$\left.\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$\exp\left(-\pi\sqrt{18}\right)$

Exact result:

$e^{-3\sqrt{2}\,\pi}$

Decimal approximation:

$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10^{-6}$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141}=4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141}=4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141}=4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

–5.010882647757...

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\,\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\,\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - z_0)^k\,z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\,dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}-\varphi+1}}=1-\cfrac{e^{-\pi}}{1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-3\pi}}{1+\cfrac{e^{-4\pi}}{1+\ldots}}}}\approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.99040073270864402755097375571330141546073279617855555168 4...

0.99040073.... result very near to the dilaton value 0.989117352243 = ϕ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\;. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\;\phi\;e^{-\gamma\,\phi}\right)^2\;. \tag{4.36}$$

We analyzing the following equation:

$$V = \frac{M^2}{3}\left(1 - a\,\phi\,e^{-\gamma\,\phi}\right)^2 .$$

$$\phi = \varphi - \frac{\sqrt{6}}{k} ,$$

$$a = \frac{b\gamma}{e} < 0 , \qquad \gamma = \frac{k}{\sqrt{6}} < 0 .$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V = \frac{M^2}{3}\left(1 - \left(\frac{b}{e} \times \frac{2}{\sqrt{6}}\right)\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V = \frac{1}{3}(0.0814845\,b + 1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M, V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M, V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874} + b}{M}$$

$$\frac{\partial M(b, V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b, V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245 + 18\,480\,874\,b}$$

$$\frac{\partial V(b, M)}{\partial M} = \frac{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b, M)}{\partial b} = \frac{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right\}=0 \text{ at } (b, M)=(-16, 0)$$

Global minima:

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

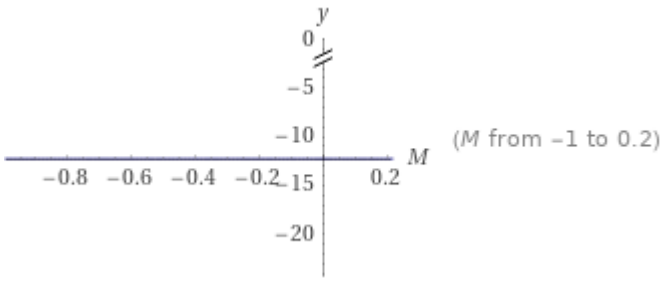
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

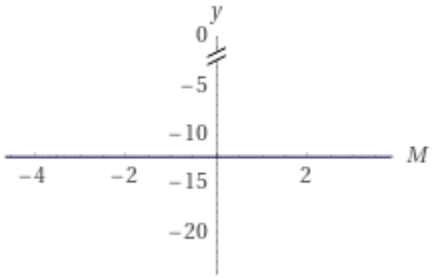
Result:

$$\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

−12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723\right)+O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times10^{-15}}{M}$$

Indefinite integral:

$$\int\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty} \frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2} = -12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty \left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2} - -12.2723\right) dM = 0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3}\left(0.0814845\,b+1\right)^2 M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

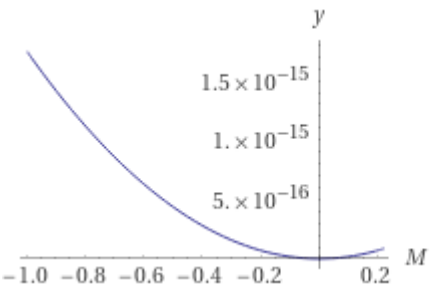
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

Result:

0

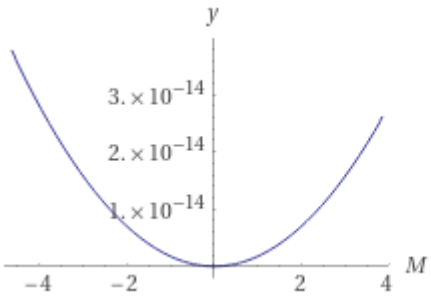
Plots: **(possible mathematical connection with an open string)**



(*M* from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$$O\left(M^{62194}\right)$$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times10^{-15}\,M^2+O\left(\left(\frac{1}{M}\right)^{62194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}\,M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.38851344947464545348970783378088020833333333333333333333\ldots \times 10^{-16}$$

-4.38851344947*10^{-16}

For M = 0.2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 0.2^2 + 6.58545 \times 10^{-10}\sqrt{0.2^4}\right)}{0.2^2} + 1\right)^2 \times 0.2^2$$

Result:

$$7.021621519159432725583532534049408333333333333333333333333\ldots \times 10^{-17}$$

7.021621519159*10^{-17}

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875\times10^{-14}$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times2^2+6.58545\times10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots \times 10^{-15}$

7.021621519*10^-15

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots \times 10^{-8}$

5.9776991059*10^-8 result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([[((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left((1/(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}))\ \hat{}\ (1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\ \sqrt[4]{3}\ e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

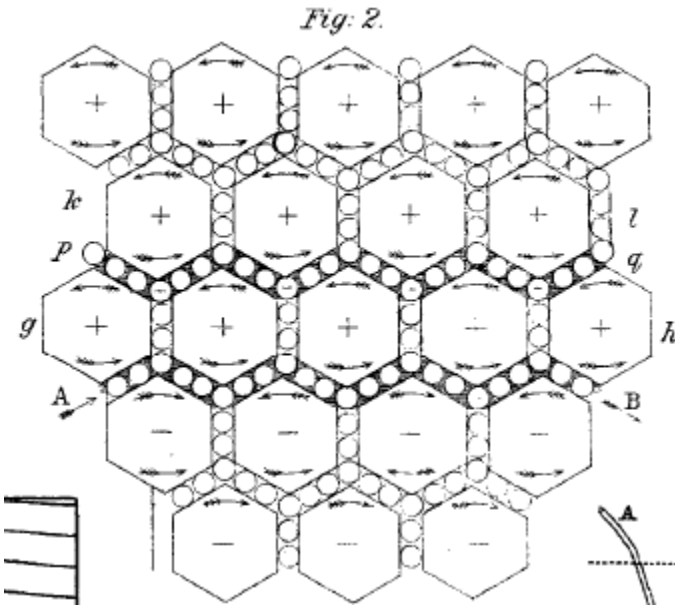
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

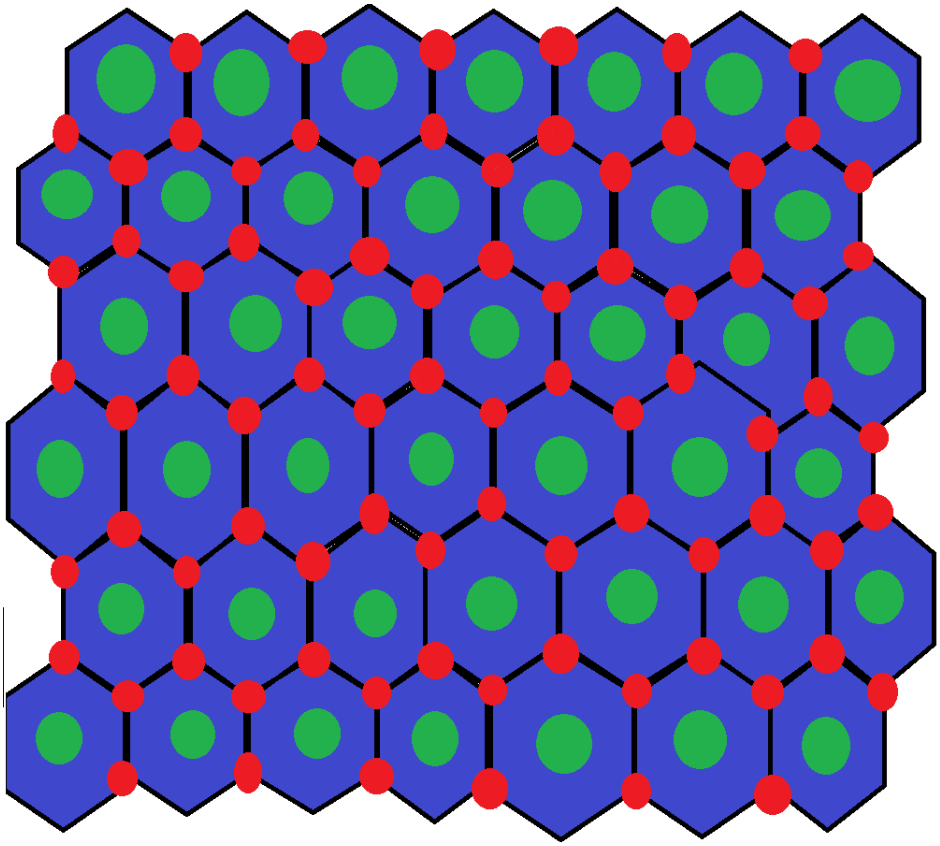
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability.

References

ESISTENZA E LIMITAZIONE DELLO SPETTRO IN UN PROBLEMA AI LIMITI PER UN'EQUAZIONE DIFFERENZIALE ORDINARIA NON LINEARE – *Renato Caccioppoli* - 1942

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the study of various integrals. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory, Theoretical Cosmology and String Theory

Michele Nardelli[1], Antonio Nardelli

Abstract

In this paper, we analyze various integrals. We obtain new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory, Theoretical Cosmology and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan

(1887 - 1920)

Introduction

In this paper, an octahedron could serve as a mathematical or conceptual model of the universe in the quantic phase, while the spherical surface could be used to describe the geometry of the bubble-universe.

The values $(2\sqrt{2})/\pi$, the golden ratio φ, $\zeta(2)$ and π, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$ the so called DN Constant:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and bubbles universe in eternal inflation.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and bubbles universe.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of bubbles universe and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

The above values $(2\sqrt{2})/\pi$, the golden ratio φ and π, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

The various mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe.

Regarding the fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the quantum universe and bubbles universe.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

Proposal:

The initial octahedron: Let's imagine a regular octahedron, with perfectly symmetrical faces. Each face represents an ideal symmetry.

The emerging sphere: Inside the octahedron, there is an inscribed sphere. This sphere represents the bubble of the universe that emerges from the perturbations of the quantum vacuum during eternal inflation.

Expansion and transitions: As time passes, the universe expands. The faces of the octahedron begin to break, symbolizing "symmetry breaks." The sphere continues to grow, representing the expanding universe.

Constants and numbers: We integrate the mathematical results you obtained. For example, the golden ratio (φ) could be represented by a proportion between the dimensions of the octahedron and the sphere.

Entropy and complexity: Entropy increases as the universe evolves. We can represent this with a disordered growth of structures within the emerging sphere. Imagine this scene as an abstract work of art, where geometric shapes and cosmological concepts merge

In Fig.1 and Fig.2 let's imagine a regular octahedron representing the universe in its phase of high symmetry and very low entropy. Inside the octahedron we have an inscribed sphere that emerges from perturbations of the quantum vacuum during eternal inflation. As time passes, the universe expands, the faces of the octahedron break (symmetry breaks), and entropy increases. Spheres emerge from the octahedra, symbolizing the transition phases from a regime of very low entropy to a universe in which, with the passage of time, entropy increases, increasing the complexity of the universe itself.

Fig. 1

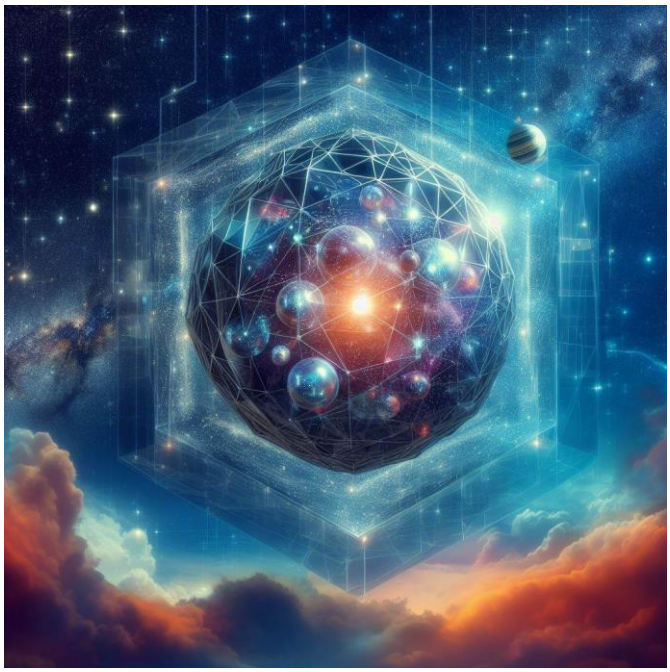
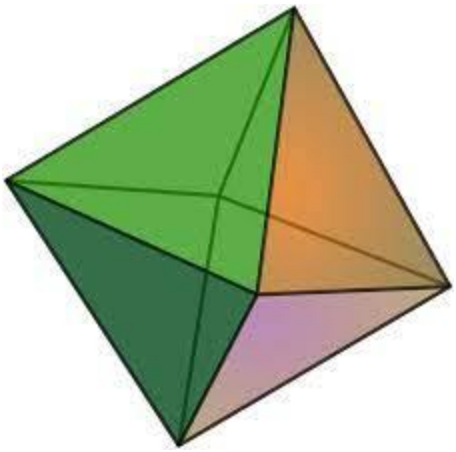
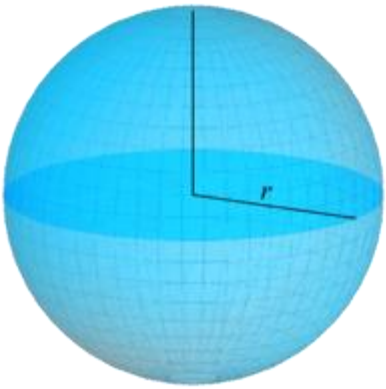


Fig. 2

Now, we have that:



Octahedron



Sphere

From the octahedron volume V = 1/3*√2 l^3 and, from the sphere volume,

V = (4/3*π*r^3) , we consider the following relationship, for r = x:

4/3*π*x^3 = 1/3*√2*l^3

Input

$$\frac{4}{3}\pi x^3 = \frac{1}{3}\sqrt{2}\, l^3$$

Exact result

$$\frac{4\pi x^3}{3} = \frac{\sqrt{2}\, l^3}{3}$$

Alternate forms

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$\frac{4\,\pi\,x^3}{3} - \frac{\sqrt{2}\,l^3}{3} = 0$$

Real solution

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Solutions

$$x = -\frac{\sqrt[3]{-\frac{1}{\pi}}\,l}{\sqrt{2}}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Integer solution

$$l = 0\,, \quad x = 0$$

Implicit derivatives

$$\frac{\partial x(l)}{\partial l} = \frac{l^2}{2\,\sqrt{2}\,\pi\,x^2}$$

$$\frac{\partial l(x)}{\partial x} = \frac{2\,\sqrt{2}\,\pi\,x^2}{l^2}$$

From the alternate form

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

for l = 8, we have that:

8/(sqrt(2) π^(1/3)) = 8/(2sqrt2 * Pi)^1/3

Input

$$\frac{8}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{8}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Result

True

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Thence:

l/(sqrt(2) π^(1/3)) = l/(2sqrt2 * Pi)^1/3

Input

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Now, we have that:

l/(2 sqrt(2) π)^(1/3) = (2sqrt2)/Pi

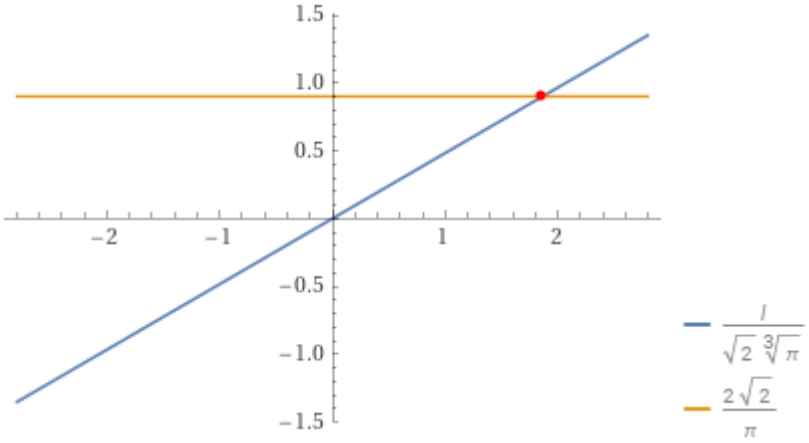
Input

$$\frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}} = \frac{2\sqrt{2}}{\pi}$$

Exact result

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{2\sqrt{2}}{\pi}$$

Plot



Solution

$$l = \frac{4}{\pi^{2/3}}$$

$$l = \frac{4}{\sqrt[3]{\pi^2}}$$

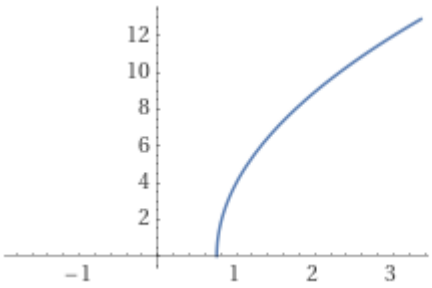
On the study of some integrals (https://www.youtube.com/watch?v=x23O-p62fss)

4(sqrt(4x-3))

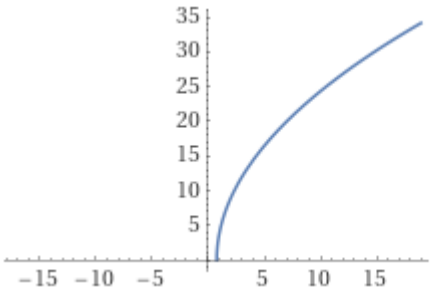
Input

$4\sqrt{4x-3}$

Plots **(figures that can be related to the open strings)**



(x from −1.9 to 3.4)



(x from −17.5 to 19)

Root

$x = \frac{3}{4}$

Properties as a real function

Domain

$\{x \in \mathbb{R} : x \geq \frac{3}{4}\}$

Range

$\{y \in \mathbb{R} : y \geq 0\}$ (all non-negative real numbers)

Injectivity

injective (one-to-one)

$\mathbb{R}$ is the set of real numbers

Series expansion at x=0

$$\begin{cases} -4i\sqrt{3} + \frac{8ix}{\sqrt{3}} + \frac{8ix^2}{3\sqrt{3}} + \frac{16ix^3}{9\sqrt{3}} + \frac{40ix^4}{27\sqrt{3}} + \frac{112ix^5}{81\sqrt{3}} + O\left(x^6\right) & \text{Im}(x) < 0 \\ 4i\sqrt{3} - \frac{8ix}{\sqrt{3}} - \frac{8ix^2}{3\sqrt{3}} - \frac{16ix^3}{9\sqrt{3}} - \frac{40ix^4}{27\sqrt{3}} - \frac{112ix^5}{81\sqrt{3}} + O\left(x^6\right) & \text{(otherwise)} \end{cases}$$

Im(z) is the imaginary part of z

Series expansion at x=3/4

$$8\sqrt{x - \frac{3}{4}} + O\left(\left(x - \frac{3}{4}\right)^{25/2}\right)$$

(Puiseux series)

Series expansion at x=∞

$$8\sqrt{x} - 3\sqrt{\frac{1}{x}} - \frac{9}{16}\left(\frac{1}{x}\right)^{3/2} - \frac{27}{128}\left(\frac{1}{x}\right)^{5/2} + O\left(\left(\frac{1}{x}\right)^3\right)$$

(Puiseux series)

Derivative

$$\frac{d}{dx}\left(4\sqrt{4x-3}\right) = \frac{8}{\sqrt{4x-3}}$$

Indefinite integral

$$\int 4\sqrt{4x-3}\,dx = \frac{2}{3}(4x-3)^{3/2} + \text{constant}$$

Global minimum

$$\min\{4\sqrt{4x-3}\} = 0 \text{ at } x = \frac{3}{4}$$

Series representations

$$4\sqrt{4x-3} = 4\sqrt{-4+4x}\sum_{k=0}^{\infty}(-4+4x)^{-k}\binom{\frac{1}{2}}{k} \text{ for } |4-4x|>1$$

$$4\sqrt{4x-3} = 4\sum_{k=0}^{\infty}\frac{(-4)^k(-1+x)^k\left(-\frac{1}{2}\right)_k}{k!} \text{ for } |4-4x|<1$$

$$4\sqrt{4x-3} = 4\sqrt{-4+4x}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k(-1+x)^{-k}\left(-\frac{1}{2}\right)_k}{k!} \text{ for } |4-4x|>1$$

$\binom{n}{m}$ is the binomial coefficient

$|z|$ is the absolute value of z

$n!$ is the factorial function

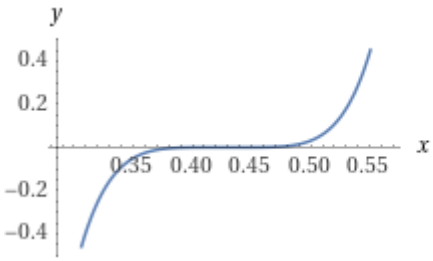
$(a)_n$ is the Pochhammer symbol (rising factorial)

(7x-3)^5

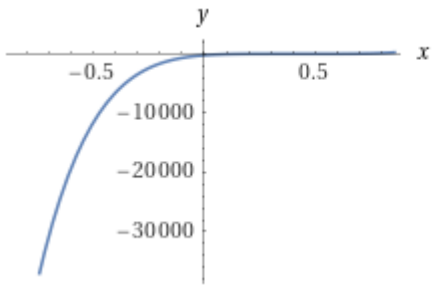
Input

$$(7x-3)^5$$

Plots (figures that can be related to the open strings)



(x from 0.2857 to 0.5714)



(x from −0.9 to 0.9)

Expanded form

$$16807\,x^5 - 36015\,x^4 + 30870\,x^3 - 13230\,x^2 + 2835\,x - 243$$

Alternate form

$$x\,(x\,(x\,(x\,(16807\,x - 36015) + 30870) - 13230) + 2835) - 243$$

Root

$$x = \frac{3}{7}$$

Polynomial discriminant

$$\Delta = 0$$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Derivative

$$\frac{d}{dx}\left((7x-3)^5\right) = 35\,(7x-3)^4$$

Indefinite integral

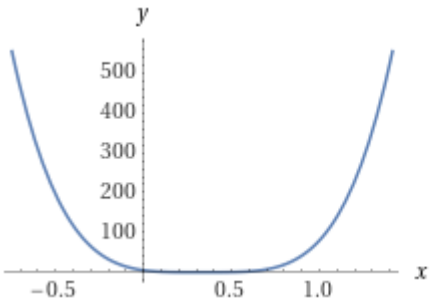
$$\int (7x-3)^5\,dx = \frac{1}{42}\,(7x-3)^6 + \text{constant}$$

5(3x-1)^4

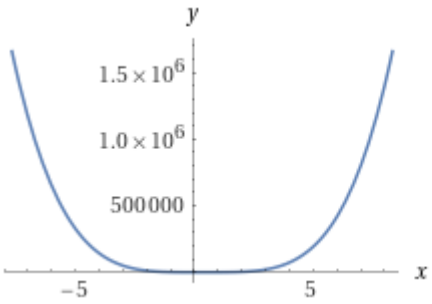
Input

$$5\,(3x-1)^4$$

Plots (figures that can be related to the open strings)



(x from −0.7 to 1.4)



(x from −7.7 to 8.3)

Expanded form

$405\,x^4 - 540\,x^3 + 270\,x^2 - 60\,x + 5$

Alternate forms

$x\,(x\,(x\,(405\,x - 540) + 270) - 60) + 5$

$45\left(3\,x^2 - 2\,x\right)^2 + 30\left(3\,x^2 - 2\,x\right) + 5$

$5\,(1 - 3\,x)^4$

Root

$x = \frac{1}{3}$

Polynomial discriminant

$\Delta = 0$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\{y \in \mathbb{R} : y \geq 0\}$ (all non-negative real numbers)

$\mathbb{R}$ is the set of real numbers

Derivative

$$\frac{d}{dx}\left(5\,(3\,x-1)^4\right) = 60\,(3\,x-1)^3$$

Indefinite integral

$$\int 5\,(3\,x-1)^4\,dx = -\frac{1}{3}\,(1-3\,x)^5 + \text{constant}$$

Global minimum

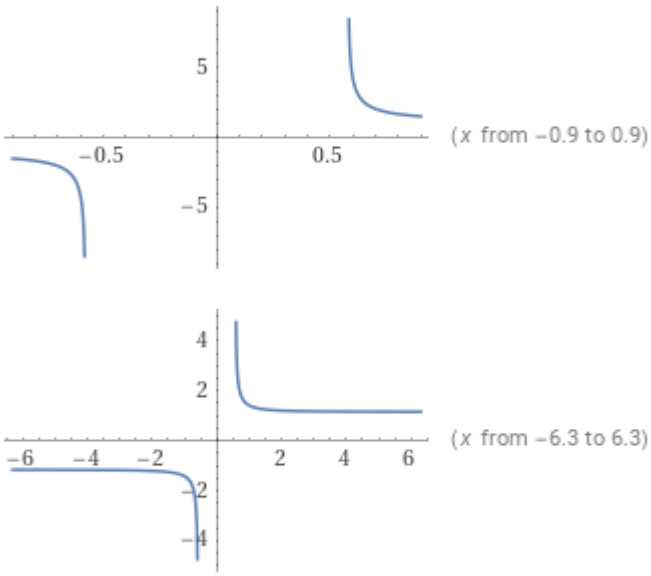
$$\min\left\{5\,(3\,x-1)^4\right\} = 0 \text{ at } x = \frac{1}{3}$$

((2x)/sqrt(3x^2-1))

Input

$$\frac{2x}{\sqrt{3x^2-1}}$$

Plots (figures that can be related to the open strings)



Root

$x = 0$ (assuming a complex-valued square root)

Properties as a real function

Domain

$$\{x \in \mathbb{R} : x < -\frac{1}{\sqrt{3}} \text{ or } x > \frac{1}{\sqrt{3}}\}$$

Range

$$\{y \in \mathbb{R} : y < -\frac{2}{\sqrt{3}} \text{ or } y > \frac{2}{\sqrt{3}}\}$$

Injectivity

injective (one-to-one)

Parity

odd

$\mathbb{R}$ is the set of real numbers

Series expansion at x=0

$$\begin{cases} -2\,i\,x - 3\,i\,x^3 - \frac{27\,i\,x^5}{4} + O\left(x^6\right) & \mathrm{Im}\left(x^2\right) \geq 0 \\ 2\,i\,x + 3\,i\,x^3 + \frac{27\,i\,x^5}{4} + O\left(x^6\right) & \text{(otherwise)} \end{cases}$$

Im(z) is the imaginary part of z

Series expansion at x=-1/sqrt(3)

$$-\frac{\sqrt{2}\,\sqrt{3\,x+\sqrt{3}}}{3^{3/4}\,\sqrt{-3\,x-\sqrt{3}}\,\sqrt{x+\frac{1}{\sqrt{3}}}} + \frac{3^{3/4}\,\sqrt{3\,x+\sqrt{3}}\,\sqrt{x+\frac{1}{\sqrt{3}}}}{2\sqrt{2}\,\sqrt{-3\,x-\sqrt{3}}} + \frac{5\sqrt[4]{3}\,\sqrt{3\,x+\sqrt{3}}\,\left(x+\frac{1}{\sqrt{3}}\right)^{3/2}}{16\sqrt{2}\,\sqrt{-3\,x-\sqrt{3}}} + \frac{7\times 3^{3/4}\,\sqrt{3\,x+\sqrt{3}}\,\left(x+\frac{1}{\sqrt{3}}\right)^{5/2}}{64\sqrt{2}\,\sqrt{-3\,x-\sqrt{3}}} + O\left(\left(x+\frac{1}{\sqrt{3}}\right)^{7/2}\right)$$

(generalized Puiseux series)

Series expansion at x=1/sqrt(3)

$$\frac{\sqrt{2}}{3^{3/4}\sqrt{x-\frac{1}{\sqrt{3}}}}+\frac{3^{3/4}\sqrt{x-\frac{1}{\sqrt{3}}}}{2\sqrt{2}}-\frac{5\sqrt[4]{3}\left(x-\frac{1}{\sqrt{3}}\right)^{3/2}}{16\sqrt{2}}+\frac{7\times 3^{3/4}\left(x-\frac{1}{\sqrt{3}}\right)^{5/2}}{64\sqrt{2}}+O\left(\left(x-\frac{1}{\sqrt{3}}\right)^{7/2}\right)$$

(Puiseux series)

Series expansion at x=∞

$$\frac{2}{\sqrt{3}}+\frac{1}{3\sqrt{3}\,x^2}+O\left(\left(\frac{1}{x}\right)^4\right)$$

(Laurent series)

Derivative

$$\frac{d}{dx}\left(\frac{2x}{\sqrt{3x^2-1}}\right)=-\frac{2}{(3x^2-1)^{3/2}}$$

Indefinite integral

$$\int\frac{2x}{\sqrt{3x^2-1}}\,dx=\frac{2}{3}\sqrt{3x^2-1}\ +\text{constant}$$

Limit

$$\lim_{x\to-\infty}\frac{2x}{\sqrt{-1+3x^2}}=-\frac{2}{\sqrt{3}}\approx-1.1547$$

$$\lim_{x\to\infty}\frac{2x}{\sqrt{-1+3x^2}}=\frac{2}{\sqrt{3}}\approx 1.1547$$

Series representations

$$\frac{2x}{\sqrt{3x^2-1}} = \frac{2x}{\sqrt{-2+3x^2}\sum_{k=0}^{\infty}(-2+3x^2)^{-k}\binom{\frac{1}{2}}{k}} \quad \text{for } |2-3x^2| > 1$$

$$\frac{2x}{\sqrt{3x^2-1}} = \frac{2x}{\sum_{k=0}^{\infty}\frac{(-1)^k(-2+3x^2)^k\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } |2-3x^2| < 1$$

$$\frac{2x}{\sqrt{3x^2-1}} = \frac{2x}{\sqrt{-2+3x^2}\sum_{k=0}^{\infty}\frac{(-1)^k(-2+3x^2)^{-k}\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } |2-3x^2| > 1$$

$\binom{n}{m}$ is the binomial coefficient

$|z|$ is the absolute value of z

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

Definite integral

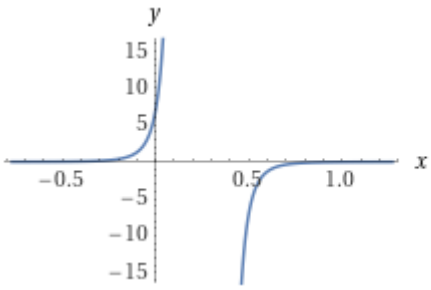
$$\int_0^{\frac{1}{\sqrt{3}}} \frac{2x}{\sqrt{-1+3x^2}}\,dx = -\frac{2i}{3} \approx -0.666667\,i$$

7/(1-4x)^5

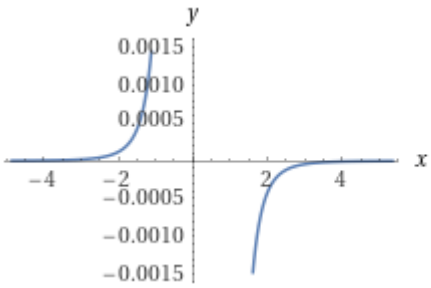
Input

$$\frac{7}{(1-4x)^5}$$

Plots (figures that can be related to the open strings)



(x from −0.8 to 1.3)



(x from −4.9 to 5.4)

Alternate forms

$$\frac{7}{-1024\,x^5+1280\,x^4-640\,x^3+160\,x^2-20\,x+1}$$

$$-\frac{7}{(4\,x-1)^5}$$

$$-\frac{7}{x\,(x\,(x\,(x\,(1024\,x-1280)+640)-160)+20)-1}$$

Expanded form

$$-\frac{7}{1024\,x^5-1280\,x^4+640\,x^3-160\,x^2+20\,x-1}$$

Roots

(no roots exist)

Properties as a real function

Domain

$\{x \in \mathbb{R} : x \neq \frac{1}{4}\}$

Range

$\{y \in \mathbb{R} : y \neq 0\}$

Injectivity

injective (one-to-one)

$\mathbb{R}$ is the set of real numbers

Series expansion at x=0

$7 + 140\,x + 1680\,x^2 + 15\,680\,x^3 + 125\,440\,x^4 + O(x^5)$

(Taylor series)

Series expansion at x=∞

$-\frac{7}{1024\,x^5} - \frac{35}{4096\,x^6} - \frac{105}{16\,384\,x^7} - \frac{245}{65\,536\,x^8} + O\left(\left(\frac{1}{x}\right)^9\right)$

(Laurent series)

Derivative

$\frac{d}{dx}\left(\frac{7}{(1-4\,x)^5}\right) = \frac{140}{(1-4\,x)^6}$

Indefinite integral

$$\int \frac{7}{(1-4x)^5}\,dx = \frac{7}{16(1-4x)^4} + \text{constant}$$

Limit

$$\lim_{x\to\pm\infty} \frac{7}{(1-4x)^5} = 0$$

Series representations

$$\frac{7}{(1-4x)^5} = \sum_{n=0}^{\infty} \frac{1}{3} x^n\, 7\left(2^{-3+2n}(1+n)(2+n)(3+n)(4+n)\right) \text{ for } |x| < \frac{1}{4}$$

$$\frac{7}{(1-4x)^5} = \sum_{n=0}^{\infty} (-1+x)^n (-7)\left((-1)^n 2^{-3+2n} \times 3^{-6-n}(1+n)(2+n)(3+n)(4+n)\right)$$
$$\text{for } |-1+x| < \frac{3}{4}$$

$|z|$ is the absolute value of z

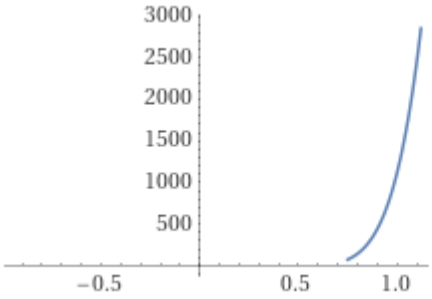
From the sum of the above functions, we obtain:

4(sqrt(4x-3)) + ((7x-3)^5) + (5(3x-1)^4) + (((2x)/sqrt(3x^2-1))) + (7/(1-4x)^5)

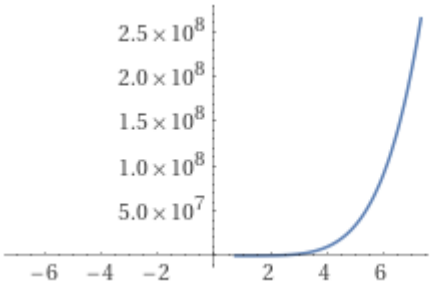
Input

$$4\sqrt{4x-3} + (7x-3)^5 + 5(3x-1)^4 + \frac{2x}{\sqrt{3x^2-1}} + \frac{7}{(1-4x)^5}$$

Plots (figures that can be related to the open strings)



(x from −0.9 to 1.1)



(x from −7.2 to 7.3)

Alternate forms

$$\frac{2x}{\sqrt{3x^2-1}}+\frac{1}{(4x-1)^5}\left(414720x^9-1071360x^8+1226880x^7-817440x^6+349220x^5-99205x^4+18740x^3-2270x^2+160x+4\sqrt{4x-3}\,(4x-1)^5-12\right)+(7x-3)^5$$

$$\left(2048x^6-2560x^5+1280x^4-320x^3+40x^2+\sqrt{3x^2-1}\left(17210368x^{10}-57977600x^9+87395200x^8-77572960x^7+44875340x^6-17671919x^5+4796450x^4-885850x^3+106540x^2-7535x+4\sqrt{4x-3}\,(4x-1)^5+231\right)-2x\right)\Big/\left((4x-1)^5\sqrt{3x^2-1}\right)$$

$$\left(2048\,x^6 - 2560\,x^5 + 1280\,x^4 - 320\,x^3 - 640\sqrt{4\,x-3}\,\sqrt{3\,x^2-1}\,x^2 + 106540\sqrt{3\,x^2-1}\,x^2 + 40\,x^2 + 80\sqrt{4\,x-3}\,\sqrt{3\,x^2-1}\,x - 7535\sqrt{3\,x^2-1}\,x - 4\sqrt{4\,x-3}\,\sqrt{3\,x^2-1} + 231\sqrt{3\,x^2-1} + 17210368\sqrt{3\,x^2-1}\,x^{10} - 57977600\sqrt{3\,x^2-1}\,x^9 + 87395200\sqrt{3\,x^2-1}\,x^8 - 77572960\sqrt{3\,x^2-1}\,x^7 + 44875340\sqrt{3\,x^2-1}\,x^6 + 4096\sqrt{4\,x-3}\,\sqrt{3\,x^2-1}\,x^5 - 17671919\sqrt{3\,x^2-1}\,x^5 - 5120\sqrt{4\,x-3}\,\sqrt{3\,x^2-1}\,x^4 + 4796450\sqrt{3\,x^2-1}\,x^4 + 2560\sqrt{4\,x-3}\,\sqrt{3\,x^2-1}\,x^3 - 885850\sqrt{3\,x^2-1}\,x^3 - 2\,x\right) \Big/ \left((4\,x-1)^5\sqrt{3\,x^2-1}\right)$$

Expanded form

$$16807\,x^5 - 35610\,x^4 + 30330\,x^3 - 12960\,x^2 + \frac{2\,x}{\sqrt{3\,x^2-1}} + 2775\,x + 4\sqrt{4\,x-3} + \frac{7}{(1-4\,x)^5} - 238$$

Roots

$x \approx 0.141452 - 0.124774\,i$

$x \approx 0.141452 + 0.124774\,i$

$x \approx 0.264953 - 0.209295\,i$

$x \approx 0.264953 + 0.209295\,i$

$x \approx 0.401125 - 0.195605\,i$

Properties as a real function

Domain

$$\{x \in \mathbb{R} : x \geq \frac{3}{4}\}$$

Range

$$\{y \in \mathbb{R} : y \geq \frac{784575 + 6144\sqrt{11}}{11264}\}$$

Injectivity

injective (one-to-one)

$\mathbb{R}$ is the set of real numbers

Series expansion at x=0

$$\begin{cases} \left(-231-4\,i\sqrt{3}\right)+\left((2915-2\,i)+\frac{8\,i}{\sqrt{3}}\right)x+\left(-11280+\frac{8\,i}{3\sqrt{3}}\right)x^2+\left((46010-3\,i)+\frac{16\,i}{9\sqrt{3}}\right)x^3+\left(89830+\frac{40\,i}{27\sqrt{3}}\right)x^4+\left(\left(919975-\frac{27\,i}{4}\right)+\frac{112\,i}{81\sqrt{3}}\right)x^5+O\left(x^6\right) & \mathrm{Im}\left(x^2\right)\geq 0\wedge \mathrm{Im}(x)<0 \\ \left(-231-4\,i\sqrt{3}\right)+\left((2915+2\,i)+\frac{8\,i}{\sqrt{3}}\right)x+\left(-11280+\frac{8\,i}{3\sqrt{3}}\right)x^2+\left((46010+3\,i)+\frac{16\,i}{9\sqrt{3}}\right)x^3+\left(89830+\frac{40\,i}{27\sqrt{3}}\right)x^4+\left(\left(919975+\frac{27\,i}{4}\right)+\frac{112\,i}{81\sqrt{3}}\right)x^5+O\left(x^6\right) & \mathrm{Im}\left(x^2\right)<0\wedge \mathrm{Im}(x)<0 \\ \left(-231+4\,i\sqrt{3}\right)+\left((2915-2\,i)-\frac{8\,i}{\sqrt{3}}\right)x+\left(-11280-\frac{8\,i}{3\sqrt{3}}\right)x^2+\left((46010-3\,i)-\frac{16\,i}{9\sqrt{3}}\right)x^3+\left(89830-\frac{40\,i}{27\sqrt{3}}\right)x^4+\left(\left(919975-\frac{27\,i}{4}\right)-\frac{112\,i}{81\sqrt{3}}\right)x^5+O\left(x^6\right) & \mathrm{Im}\left(x^2\right)\geq 0\wedge \mathrm{Im}(x)\geq 0 \\ \left(-231+4\,i\sqrt{3}\right)+\left((2915+2\,i)-\frac{8\,i}{\sqrt{3}}\right)x+\left(-11280-\frac{8\,i}{3\sqrt{3}}\right)x^2+\left((46010+3\,i)-\frac{16\,i}{9\sqrt{3}}\right)x^3+\left(89830-\frac{40\,i}{27\sqrt{3}}\right)x^4+\left(\left(919975+\frac{27\,i}{4}\right)-\frac{112\,i}{81\sqrt{3}}\right)x^5+O\left(x^6\right) & \text{(otherwise)} \end{cases}$$

$\mathrm{Im}(z)$ is the imaginary part of z

$e_1 \wedge e_2 \wedge \ldots$ is the logical AND function

Series expansion at x=3/4

$$\left(\frac{71325}{1024}+\frac{6}{\sqrt{11}}\right)+8\sqrt{x-\frac{3}{4}}+\left(\frac{260195}{256}-\frac{128}{11\sqrt{11}}\right)\left(x-\frac{3}{4}\right)+\left(\frac{191685}{32}+\frac{6912}{121\sqrt{11}}\right)\left(x-\frac{3}{4}\right)^2+\left(\frac{144805}{8}-\frac{380928}{1331\sqrt{11}}\right)\left(x-\frac{3}{4}\right)^3+\left(\frac{108685}{4}+\frac{21565440}{14641\sqrt{11}}\right)\left(x-\frac{3}{4}\right)^4+O\left(\left(x-\frac{3}{4}\right)^5\right)$$

(Puiseux series)

Series expansion at x=∞

$$16807\,x^5 - 35610\,x^4 + 30330\,x^3 - 12960\,x^2 + 2775\,x + 8\sqrt{x} + \left(\frac{2}{\sqrt{3}} - 238\right) + O\left(\sqrt{\frac{1}{x}}\right)$$

(Puiseux series)

Derivative

$$\frac{d}{dx}\left(4\sqrt{4x-3} + (7x-3)^5 + 5(3x-1)^4 + \frac{2x}{\sqrt{3x^2-1}} + \frac{7}{(1-4x)^5}\right) = \frac{2}{\sqrt{3x^2-1}} - \frac{6x^2}{(3x^2-1)^{3/2}} + 35(7x-3)^4 + 60(3x-1)^3 + \frac{8}{\sqrt{4x-3}} + \frac{140}{(1-4x)^6}$$

Indefinite integral

$$\int\left(4\sqrt{4x-3} + (7x-3)^5 + 5(3x-1)^4 + \frac{2x}{\sqrt{3x^2-1}} + \frac{7}{(1-4x)^5}\right)dx = \frac{2}{3}\sqrt{3x^2-1} + \frac{1}{42}(3-7x)^6 + \frac{1}{3}(3x-1)^5 + \frac{2}{3}(4x-3)^{3/2} + \frac{7}{16(1-4x)^4} +$$

constant

From the indefinite integral result

$$\int\left(4\sqrt{4x-3} + (7x-3)^5 + 5(3x-1)^4 + \frac{2x}{\sqrt{3x^2-1}} + \frac{7}{(1-4x)^5}\right)dx = \frac{2}{3}\sqrt{3x^2-1} + \frac{1}{42}(3-7x)^6 + \frac{1}{3}(3x-1)^5 + \frac{2}{3}(4x-3)^{3/2} + \frac{7}{16(1-4x)^4} +$$

constant

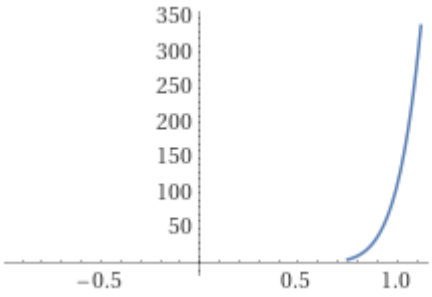
we obtain:

1/42 (3 - 7 x)^6 + 7/(16 (1 - 4 x)^4) + 1/3 (-1 + 3 x)^5 + 2/3 (-3 + 4 x)^(3/2) + 2/3 sqrt(-1 + 3 x^2)

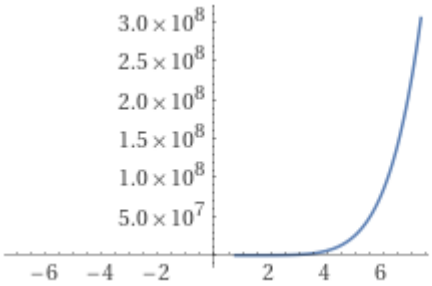
Input

$$\frac{1}{42}(3-7x)^6+\frac{7}{16(1-4x)^4}+\frac{1}{3}(-1+3x)^5+\frac{2}{3}(-3+4x)^{3/2}+\frac{2}{3}\sqrt{-1+3x^2}$$

Plots (figures that can be related to the open strings)



(x from −0.9 to 1.1)



(x from −7.2 to 7.3)

Alternate forms

$$\frac{2}{3}\sqrt{3x^2-1}+\frac{1}{336(4x-1)^4}$$
$$\left(240\,945\,152x^{10}-853\,551\,104x^9+1\,355\,176\,704x^8-1\,268\,591\,744x^7+774\,746\,504x^6-322\,321\,440x^5+92\,463\,368x^4-18\,051\,968x^3+2\,294\,808x^2-171\,488x+224(4x-3)^{3/2}(4x-1)^4+5867\right)$$

$$\frac{1}{336\,(4\,x-1)^4}$$
$$\Big(240\,945\,152\,x^{10}-853\,551\,104\,x^9+1\,355\,176\,704\,x^8-1\,268\,591\,744\,x^7+$$
$$774\,746\,504\,x^6-322\,321\,440\,x^5+92\,463\,368\,x^4-18\,051\,968\,x^3+$$
$$2\,294\,808\,x^2+224\,(4\,x-1)^4\sqrt{3\,x^2-1}\;-$$
$$171\,488\,x+224\,(4\,x-3)^{3/2}\,(4\,x-1)^4+5867\Big)$$

$$\frac{1}{336\,(4\,x-1)^4}$$
$$\Big(240\,945\,152\,x^{10}-853\,551\,104\,x^9+1\,355\,176\,704\,x^8-1\,268\,591\,744\,x^7+$$
$$774\,746\,504\,x^6-322\,321\,440\,x^5+57\,344\,(4\,x-3)^{3/2}\,x^4+92\,463\,368\,x^4-$$
$$57\,344\,(4\,x-3)^{3/2}\,x^3-18\,051\,968\,x^3+21\,504\,(4\,x-3)^{3/2}\,x^2+$$
$$21\,504\sqrt{3\,x^2-1}\;x^2+2\,294\,808\,x^2-3584\sqrt{3\,x^2-1}\;x+$$
$$224\sqrt{3\,x^2-1}\;+57\,344\sqrt{3\,x^2-1}\;x^4-57\,344\sqrt{3\,x^2-1}\;x^3-$$
$$3584\,(4\,x-3)^{3/2}\,x-171\,488\,x+224\,(4\,x-3)^{3/2}+5867\Big)$$

Expanded form

$$\frac{16\,807\,x^6}{6}-7122\,x^5+\frac{15\,165\,x^4}{2}-4320\,x^3+\frac{2775\,x^2}{2}+\frac{2}{3}\sqrt{3\,x^2-1}\;+\frac{8}{3}$$
$$\sqrt{4\,x-3}\;x-238\,x-2\sqrt{4\,x-3}\;+\frac{7}{16\,(1-4\,x)^4}+\frac{715}{42}$$

Roots

$$x\approx 0.122254-0.0590815\,i$$

$$x\approx 0.122254+0.0590815\,i$$

$$x\approx 0.233431-0.209239\,i$$

$x \approx 0.233431 + 0.209239\,i$

$x \approx 0.302379 - 0.204352\,i$

Properties as a real function

Domain

$\{x \in \mathbb{R} : x \geq \frac{3}{4}\}$

Range

$\{y \in \mathbb{R} : y \geq \frac{711\,145 + 28\,672\sqrt{11}}{172\,032}\}$

Injectivity

injective (one-to-one)

$\mathbb{R}$ is the set of real numbers

Series expansion at x=0

$$\begin{cases} \left(\left(\frac{5867}{336}-\frac{2i}{3}\right)-2i\sqrt{3}\right)+\left(-231+4i\sqrt{3}\right)x+\left(\left(\frac{2915}{2}+i\right)-\frac{4i}{\sqrt{3}}\right)x^2+\left(-3760-\frac{8i}{9\sqrt{3}}\right)x^3+\left(\left(\frac{23005}{2}+\frac{3i}{4}\right)-\frac{4i}{9\sqrt{3}}\right)x^4+\left(17966-\frac{8i}{27\sqrt{3}}\right)x^5+O\left(x^6\right) & \mathrm{Im}\left(x^2\right)<0\wedge\mathrm{Im}(x)\geq 0 \\ \left(\left(\frac{5867}{336}+\frac{2i}{3}\right)-2i\sqrt{3}\right)+\left(-231+4i\sqrt{3}\right)x+\left(\left(\frac{2915}{2}-i\right)-\frac{4i}{\sqrt{3}}\right)x^2+\left(-3760-\frac{8i}{9\sqrt{3}}\right)x^3+\left(\left(\frac{23005}{2}-\frac{3i}{4}\right)-\frac{4i}{9\sqrt{3}}\right)x^4+\left(17966-\frac{8i}{27\sqrt{3}}\right)x^5+O\left(x^6\right) & \mathrm{Im}\left(x^2\right)\geq 0\wedge\mathrm{Im}(x)\geq 0 \\ \left(\left(\frac{5867}{336}-\frac{2i}{3}\right)+2i\sqrt{3}\right)+\left(-231-4i\sqrt{3}\right)x+\left(\left(\frac{2915}{2}+i\right)+\frac{4i}{\sqrt{3}}\right)x^2+\left(-3760+\frac{8i}{9\sqrt{3}}\right)x^3+\left(\left(\frac{23005}{2}+\frac{3i}{4}\right)+\frac{4i}{9\sqrt{3}}\right)x^4+\left(17966+\frac{8i}{27\sqrt{3}}\right)x^5+O\left(x^6\right) & \mathrm{Im}\left(x^2\right)<0\wedge\mathrm{Im}(x)<0 \\ \left(\left(\frac{5867}{336}+\frac{2i}{3}\right)+2i\sqrt{3}\right)+\left(-231-4i\sqrt{3}\right)x+\left(\left(\frac{2915}{2}-i\right)+\frac{4i}{\sqrt{3}}\right)x^2+\left(-3760+\frac{8i}{9\sqrt{3}}\right)x^3+\left(\left(\frac{23005}{2}-\frac{3i}{4}\right)+\frac{4i}{9\sqrt{3}}\right)x^4+\left(17966+\frac{8i}{27\sqrt{3}}\right)x^5+O\left(x^6\right) & \text{(otherwise)} \end{cases}$$

$\mathrm{Im}(z)$ is the imaginary part of z

$e_1 \wedge e_2 \wedge \ldots$ is the logical AND function

Series expansion at x=3/4

$$\left(\frac{711145}{172032}+\frac{\sqrt{11}}{6}\right)+\left(\frac{71325}{1024}+\frac{6}{\sqrt{11}}\right)\left(x-\frac{3}{4}\right)+\frac{16}{3}\left(x-\frac{3}{4}\right)^{3/2}+\left(\frac{260195}{512}-\frac{64}{11\sqrt{11}}\right)\left(x-\frac{3}{4}\right)^2+\left(\frac{63895}{32}+\frac{2304}{121\sqrt{11}}\right)\left(x-\frac{3}{4}\right)^3+\left(\frac{144805}{32}-\frac{95232}{1331\sqrt{11}}\right)\left(x-\frac{3}{4}\right)^4+O\left(\left(x-\frac{3}{4}\right)^5\right)$$

(Puiseux series)

Series expansion at x=∞

$$\frac{16807x^6}{6}-7122x^5+\frac{15165x^4}{2}-4320x^3+\frac{2775x^2}{2}+\frac{16x^{3/2}}{3}+\left(\frac{2}{\sqrt{3}}-238\right)x-6\sqrt{x}+\frac{715}{42}+O\left(\sqrt{\frac{1}{x}}\right)$$

(Puiseux series)

Derivative

$$\frac{d}{dx}\left(\frac{1}{42}(3-7x)^6+\frac{7}{16(1-4x)^4}+\frac{1}{3}(-1+3x)^5+\frac{2}{3}(-3+4x)^{3/2}+\frac{2}{3}\sqrt{-1+3x^2}\right)=\frac{2x}{\sqrt{-1+3x^2}}-(3-7x)^5+5(-1+3x)^4+4\sqrt{-3+4x}+\frac{7}{(1-4x)^5}$$

Indefinite integral

$$\int\left(\frac{1}{42}(3-7x)^6+\frac{7}{16(1-4x)^4}+\frac{1}{3}(-1+3x)^5+\frac{2}{3}(-3+4x)^{3/2}+\frac{2}{3}\sqrt{-1+3x^2}\right)dx=\frac{1}{3}x\sqrt{3x^2-1}-\frac{\log\left(\sqrt{9x^2-3}+3x\right)}{3\sqrt{3}}+\frac{(7x-3)^7}{2058}+\frac{1}{54}(1-3x)^6+\frac{1}{15}(4x-3)^{5/2}+\frac{7}{192(1-4x)^3}+\text{constant}$$

(assuming a complex-valued logarithm)

log(x) is the natural logarithm

From the indefinite integral result

$$\int\left(\frac{1}{42}(3-7x)^6+\frac{7}{16(1-4x)^4}+\frac{1}{3}(-1+3x)^5+\frac{2}{3}(-3+4x)^{3/2}+\frac{2}{3}\sqrt{-1+3x^2}\right)dx=\frac{1}{3}x\sqrt{3x^2-1}-\frac{\log\left(\sqrt{9x^2-3}+3x\right)}{3\sqrt{3}}+\frac{(7x-3)^7}{2058}+\frac{1}{54}(1-3x)^6+\frac{1}{15}(4x-3)^{5/2}+\frac{7}{192(1-4x)^3}+\text{constant}$$

(assuming a complex-valued logarithm)

7/(192 (1 - 4 x)^3) + 1/54 (1 - 3 x)^6 + 1/15 (-3 + 4 x)^(5/2) + (-3 + 7 x)^7/2058 + 1/3 x sqrt(-1 + 3 x^2) - log(3 x + sqrt(-3 + 9 x^2))/(3 sqrt(3))

for x = 2:

7/(192 (1 – 4*2)^3) + 1/54 (1 – 3*2)^6 + 1/15 (-3 + 4*2)^(5/2) + (-3 + 7*2)^7/2058 + 1/3 2 sqrt(-1 + 3*2^2) - log(3*2 + sqrt(-3 + 9*2^2))/(3 sqrt(3))

Input

$$\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{1}{3}\times 2\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}$$

log(x) is the natural logarithm

Result

$$\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{\log\left(6+\sqrt{33}\right)}{3\sqrt{3}}$$

Decimal approximation

9763.800465847609540493396472541098304146761400996353317574132708 2

…

9763.8004658476….

Property

$\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{\log\left(6+\sqrt{33}\right)}{3\sqrt{3}}$ is a transcendental number

Alternate forms

$$\frac{987\,840\sqrt{5}+395\,136\sqrt{11}+5\,783\,805\,185-65\,856\sqrt{3}\,\log\left(6+\sqrt{33}\right)}{592\,704}$$

$$\frac{5\,783\,805\,185 + 987\,840\sqrt{5} + 395\,136\sqrt{11}}{592\,704} - \frac{\log\left(6+\sqrt{33}\right)}{3\sqrt{3}}$$

$$\frac{5\,783\,805\,185 + 197\,568\sqrt{169+20\sqrt{55}}}{592\,704} - \frac{\log\left(6+\sqrt{33}\right)}{3\sqrt{3}}$$

Expanded logarithmic form

$$\frac{5\,783\,805\,185}{592\,704} + \frac{5}{3}\sqrt{5} + \frac{2}{3}\sqrt{11} + -\frac{1}{3}\times 3^{-1/2}\log\left(6+\sqrt{33}\right)$$

Alternative representations

$$\frac{7}{192\,(1-4\times 2)^3} + \frac{1}{54}(1-3\times 2)^6 + \frac{1}{15}(-3+4\times 2)^{5/2} + \frac{(-3+7\times 2)^7}{2058} + \frac{2}{3}\sqrt{-1+3\times 2^2} - \frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}} = \frac{5^{5/2}}{15} + \frac{(-5)^6}{54} + \frac{11^7}{2058} + \frac{7}{192\,(-7)^3} - \frac{\log_e\left(6+\sqrt{33}\right)}{3\sqrt{3}} + \frac{2\sqrt{11}}{3}$$

$$\frac{7}{192\,(1-4\times 2)^3} + \frac{1}{54}(1-3\times 2)^6 + \frac{1}{15}(-3+4\times 2)^{5/2} + \frac{(-3+7\times 2)^7}{2058} + \frac{2}{3}\sqrt{-1+3\times 2^2} - \frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}} = \frac{5^{5/2}}{15} + \frac{(-5)^6}{54} + \frac{11^7}{2058} + \frac{7}{192\,(-7)^3} - \frac{\log(a)\log_a\left(6+\sqrt{33}\right)}{3\sqrt{3}} + \frac{2\sqrt{11}}{3}$$

$$\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}(1-3\times 2)^6+\frac{1}{15}(-3+4\times 2)^{5/2}+$$
$$\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}=$$
$$\frac{5^{5/2}}{15}+\frac{(-5)^6}{54}+\frac{11^7}{2058}+\frac{7}{192\,(-7)^3}+\frac{\text{Li}_1\left(-5-\sqrt{33}\right)}{3\sqrt{3}}+\frac{2\sqrt{11}}{3}$$

$\log_b(x)$ is the base-b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}(1-3\times 2)^6+\frac{1}{15}(-3+4\times 2)^{5/2}+$$
$$\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}=$$
$$\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{\log\left(5+\sqrt{33}\right)}{3\sqrt{3}}+\frac{\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{5+\sqrt{33}}\right)^k}{k}}{3\sqrt{3}}$$

$$\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}(1-3\times 2)^6+\frac{1}{15}(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+$$
$$\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}=\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+$$
$$\frac{2\sqrt{11}}{3}-\frac{2\,i\,\pi\left\lfloor\frac{\arg(6+\sqrt{33}-x)}{2\pi}\right\rfloor}{3\sqrt{3}}-\frac{\log(x)}{3\sqrt{3}}+\frac{\sum_{k=1}^{\infty}\frac{(-1)^k(6+\sqrt{33}-x)^k x^{-k}}{k}}{3\sqrt{3}}\quad\text{for } x<0$$

$$\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+$$
$$\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}=$$
$$\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{\left\lfloor\frac{\arg(6+\sqrt{33}-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)}{3\sqrt{3}}-$$
$$\frac{\log(z_0)}{3\sqrt{3}}-\frac{\left\lfloor\frac{\arg(6+\sqrt{33}-z_0)}{2\pi}\right\rfloor\log(z_0)}{3\sqrt{3}}+\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\,(6+\sqrt{33}-z_0)^k\,z_0^{-k}}{k}}{3\sqrt{3}}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+$$
$$\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}=$$
$$\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{1}{3\sqrt{3}}\int_1^{6+\sqrt{33}}\frac{1}{t}\,dt$$

$$\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+$$
$$\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}=\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+$$
$$\frac{2\sqrt{11}}{3}+\frac{i}{6\sqrt{3}\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\left(5+\sqrt{33}\right)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for}\quad -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

From which:

1/2(7/(192 (1 – 4*2)^3) + 1/54 (1 – 3*2)^6 + 1/15 (-3 + 4*2)^(5/2) + (-3 + 7*2)^7/2058 + 1/3 2 sqrt(-1 + 3*2^2) - log(3*2 + sqrt(-3 + 9*2^2))/(3 sqrt(3)))-843+47+11-(2√2)/π

Input

$$\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{1}{3}\times 2\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}$$

log(x) is the natural logarithm

Result

$$-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{\log\left(6+\sqrt{33}\right)}{3\sqrt{3}}\right)$$

Decimal approximation

4095.999916607647664177143037079542411490716126348221438161494916 6…

4095.9999166076…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson.

This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{1}{1\,185\,408\,\pi}\left(-2\,370\,816\,\sqrt{2}\,-930\,545\,280\,\pi+\pi\left(987\,840\,\sqrt{5}\,+395\,136\,\sqrt{11}\,+5\,783\,805\,185-65\,856\,\sqrt{3}\,\log\left(6+\sqrt{33}\right)\right)\right)$$

$$\frac{4\,853\,259\,905+987\,840\,\sqrt{5}\,+395\,136\,\sqrt{11}\,-65\,856\,\sqrt{3}\,\log\left(6+\sqrt{33}\right)}{1\,185\,408}-\frac{2\,\sqrt{2}}{\pi}$$

$$-\frac{1}{1\,185\,408\,\pi}\left(2\,370\,816\,\sqrt{2}\,-4\,853\,259\,905\,\pi-987\,840\,\sqrt{5}\,\pi-395\,136\,\sqrt{11}\,\pi+65\,856\,\sqrt{3}\,\pi\log\left(6+\sqrt{33}\right)\right)$$

Expanded logarithmic form

$$\frac{4\,853\,259\,905}{1\,185\,408}+\frac{5}{6}\sqrt{5}\,+\frac{1}{3}\sqrt{11}\,-\frac{2\,\sqrt{2}}{\pi}+-\frac{1}{6}\times 3^{-1/2}\log\left(6+\sqrt{33}\right)$$

Alternative representations

$$\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\right.$$
$$\left.\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-$$
$$843+47+11-\frac{2\sqrt{2}}{\pi}=-785-\frac{2\sqrt{2}}{\pi}+$$
$$\frac{1}{2}\left(\frac{5^{5/2}}{15}+\frac{(-5)^6}{54}+\frac{11^7}{2058}+\frac{7}{192\,(-7)^3}-\frac{\log_e\left(6+\sqrt{33}\right)}{3\sqrt{3}}+\frac{2\sqrt{11}}{3}\right)$$

$$\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\right.$$
$$\left.\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-$$
$$843+47+11-\frac{2\sqrt{2}}{\pi}=-785-\frac{2\sqrt{2}}{\pi}+$$
$$\frac{1}{2}\left(\frac{5^{5/2}}{15}+\frac{(-5)^6}{54}+\frac{11^7}{2058}+\frac{7}{192\,(-7)^3}-\frac{\log(a)\log_a\left(6+\sqrt{33}\right)}{3\sqrt{3}}+\frac{2\sqrt{11}}{3}\right)$$

$$\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\right.$$
$$\left.\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-$$
$$843+47+11-\frac{2\sqrt{2}}{\pi}=-785-\frac{2\sqrt{2}}{\pi}+$$
$$\frac{1}{2}\left(\frac{5^{5/2}}{15}+\frac{(-5)^6}{54}+\frac{11^7}{2058}+\frac{7}{192\,(-7)^3}+\frac{\mathrm{Li}_1\left(-5-\sqrt{33}\right)}{3\sqrt{3}}+\frac{2\sqrt{11}}{3}\right)$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\right.$$
$$\left.\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-$$
$$843+47+11-\frac{2\sqrt{2}}{\pi}=\frac{4\,853\,259\,905}{1\,185\,408}+\frac{5\sqrt{5}}{6}+\frac{\sqrt{11}}{3}-$$
$$\frac{2\sqrt{2}}{\pi}-\frac{\log\left(5+\sqrt{33}\right)}{6\sqrt{3}}+\frac{\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{5+\sqrt{33}}\right)^k}{k}}{6\sqrt{3}}$$

$$\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\right.$$
$$\left.\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+$$
$$47+11-\frac{2\sqrt{2}}{\pi}=\frac{4\,853\,259\,905}{1\,185\,408}+\frac{5\sqrt{5}}{6}+\frac{\sqrt{11}}{3}-\frac{2\sqrt{2}}{\pi}-$$
$$\frac{i\,\pi\left\lfloor\frac{\arg\left(6+\sqrt{33}-x\right)}{2\pi}\right\rfloor}{3\sqrt{3}}-\frac{\log(x)}{6\sqrt{3}}+\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\left(6+\sqrt{33}-x\right)^k x^{-k}}{k}}{6\sqrt{3}}\quad\text{for } x<0$$

$$\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}=\frac{4\,853\,259\,905}{1\,185\,408}+\frac{5\sqrt{5}}{6}+\frac{\sqrt{11}}{3}-\frac{2\sqrt{2}}{\pi}-\frac{\left\lfloor\frac{\arg\left(6+\sqrt{33}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)}{6\sqrt{3}}-\frac{\log(z_0)}{6\sqrt{3}}-\frac{\left\lfloor\frac{\arg\left(6+\sqrt{33}-z_0\right)}{2\pi}\right\rfloor\log(z_0)}{6\sqrt{3}}+\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\left(6+\sqrt{33}-z_0\right)^k z_0^{-k}}{k}}{6\sqrt{3}}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}=\frac{4\,853\,259\,905}{1\,185\,408}+\frac{5\sqrt{5}}{6}+\frac{\sqrt{11}}{3}-\frac{2\sqrt{2}}{\pi}-\frac{1}{6\sqrt{3}}\int_1^{6+\sqrt{33}}\frac{1}{t}\,dt$$

$$\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\right.$$
$$\left.\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+$$
$$47+11-\frac{2\sqrt{2}}{\pi}=\frac{4\,853\,259\,905}{1\,185\,408}+\frac{5\sqrt{5}}{6}+\frac{\sqrt{11}}{3}-\frac{2\sqrt{2}}{\pi}+$$
$$\frac{i}{12\sqrt{3}\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\left(5+\sqrt{33}\right)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\quad\text{for}\quad -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

27*sqrt(1/2(7/(192 (1 – 4*2)^3) + 1/54 (1 – 3*2)^6 + 1/15 (-3 + 4*2)^(5/2) + (-3 + 7*2)^7/2058 + 1/3 2 sqrt(-1 + 3*2^2) - log(3*2 + sqrt(-3 + 9*2^2))/(3 sqrt(3)))-843+47+11-(2√2)/π)+1

Input

$$27\sqrt{\left(\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\right.\right.}$$
$$\left.\left.\frac{1}{3}\times 2\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-\right.$$
$$\left.843+47+11-\frac{2\sqrt{2}}{\pi}\right)+1$$

$\log(x)$ is the natural logarithm

Exact result

$$1+27\sqrt{-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{\log\left(6+\sqrt{33}\right)}{3\sqrt{3}}\right)}$$

Decimal approximation

1728.999982409425589628712149080383443164893936491750983328280254 5

…

1728.999982409….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2.

Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$\frac{1}{784}\left(784+3\sqrt{\left(\frac{1}{\pi}42\left(-2\,370\,816\sqrt{2}+4\,853\,259\,905\,\pi+987\,840\sqrt{5}\,\pi+395\,136\sqrt{11}\,\pi-65\,856\sqrt{3}\,\pi\log\left(6+\sqrt{33}\right)\right)\right)}\right)$$

$$\frac{1+27}{\sqrt{-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5\,783\,805\,185+197\,568\sqrt{169+20\sqrt{55}}}{592\,704}-\frac{\log\left(6+\sqrt{33}\right)}{3\sqrt{3}}\right)}}$$

$$1+\frac{3\sqrt[4]{3}}{56\sqrt{\frac{14\pi}{-2\,370\,816\sqrt{6}-\pi\left(\sqrt{3}\left(-4\,853\,259\,905-987\,840\sqrt{5}-395\,136\sqrt{11}\right)+197\,568\log\left(6+\sqrt{33}\right)\right)}}}$$

Expanded logarithmic form

$$27\sqrt{\frac{4\,853\,259\,905}{1\,185\,408}+\frac{5}{6}\sqrt{5}+\frac{1}{3}\sqrt{11}-\frac{2\sqrt{2}}{\pi}+-\frac{1}{6}\times 3^{-1/2}\log\left(6+\sqrt{33}\right)}+1$$

Alternative representations

$$27\sqrt{\left(\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}\right)}+1=1+27\sqrt{\left(-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5^{5/2}}{15}+\frac{(-5)^6}{54}+\frac{11^7}{2058}+\frac{7}{192\,(-7)^3}-\frac{\log_e\left(6+\sqrt{33}\right)}{3\sqrt{3}}+\frac{2\sqrt{11}}{3}\right)\right)}$$

$$27\sqrt{\left(\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}\right)}+1=$$
$$1+27\sqrt{\left(-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5^{5/2}}{15}+\frac{(-5)^6}{54}+\frac{11^7}{2058}+\frac{7}{192\,(-7)^3}-\frac{\log(a)\log_a\left(6+\sqrt{33}\right)}{3\sqrt{3}}+\frac{2\sqrt{11}}{3}\right)\right)}$$

$$27\sqrt{\left(\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}\right)}+1=1+27\sqrt{\left(-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5^{5/2}}{15}+\frac{(-5)^6}{54}+\frac{11^7}{2058}+\frac{7}{192\,(-7)^3}+\frac{\mathrm{Li}_1\left(-5-\sqrt{33}\right)}{3\sqrt{3}}+\frac{2\sqrt{11}}{3}\right)\right)}$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$27\sqrt{\left(\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}\right)}+1=1+27\sqrt{\frac{4\,853\,259\,905}{1\,185\,408}+\frac{5\sqrt{5}}{6}+\frac{\sqrt{11}}{3}-\frac{2\sqrt{2}}{\pi}-\frac{\log\left(5+\sqrt{33}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{5+\sqrt{33}}\right)^k}{k}}{6\sqrt{3}}}$$

$$27\sqrt{\left(\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}\right)}+1=$$

$$1+27\sqrt{\left(-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{\log\left(5+\sqrt{33}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{5+\sqrt{33}}\right)^k}{k}}{3\sqrt{3}}\right)\right)}$$

$$27\sqrt{\left(\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}\right)}+1=$$

$$1+27\sqrt{\left(-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{2\,i\,\pi\left\lfloor\frac{\arg(6+\sqrt{33}-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(6+\sqrt{33}-x)^k\,x^{-k}}{k}}{3\sqrt{3}}\right)\right)}$$

for $x<0$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$27\sqrt{\left(\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}\right)}+1=1+27\sqrt{\left(-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{1}{3\sqrt{3}}\int_1^{6+\sqrt{33}}\frac{1}{t}\,dt\right)\right)}$$

$$27\sqrt{\left(\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{2}{3}\sqrt{-1+3\times 2^2}\;-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}\right)}+1=$$

$$1+27\sqrt{\left(-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}+\frac{i}{6\sqrt{3}\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\left(5+\sqrt{33}\right)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\right)}\quad\text{for}\quad -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

(27*√(1/2(7/(192 (1-4*2)^3)+1/54 (1–3*2)^6+1/15 (-3+4*2)^(5/2)+(-3+7*2)^7/2058+1/3 2 √(-1+3*2^2)-ln(3*2+√(-3+9*2^2))/(3√(3)))-843+47+11-(2√2)/π)+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\left(27\sqrt{\left(\frac{1}{2}\left(\frac{7}{192\,(1-4\times 2)^3}+\frac{1}{54}\,(1-3\times 2)^6+\frac{1}{15}\,(-3+4\times 2)^{5/2}+\frac{(-3+7\times 2)^7}{2058}+\frac{1}{3}\times 2\sqrt{-1+3\times 2^2}-\frac{\log\left(3\times 2+\sqrt{-3+9\times 2^2}\right)}{3\sqrt{3}}\right)-843+47+11-\frac{2\sqrt{2}}{\pi}\right)}+1\right)\wedge(1/15)+{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm

C_{MRB} is the MRB constant

Exact result

$${C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\left(1+27\sqrt{-785-\frac{2\sqrt{2}}{\pi}+\frac{1}{2}\left(\frac{5\,783\,805\,185}{592\,704}+\frac{5\sqrt{5}}{3}+\frac{2\sqrt{11}}{3}-\frac{\log\left(6+\sqrt{33}\right)}{3\sqrt{3}}\right)}\right)\wedge(1/15)$$

Decimal approximation

1.644938023015332390754860104791669029428788949681051321336258433 8…

1.644938023…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \left(1 + 27 \sqrt{-785 - \frac{2\sqrt{2}}{\pi} + \frac{1}{2}\left(\frac{5\,783\,805\,185 + 197\,568\sqrt{169 + 20\sqrt{55}}}{592\,704} - \frac{\log\left(6+\sqrt{33}\right)}{3\sqrt{3}}\right)}\right)^{\wedge}(1/15)$$

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \frac{1}{2^{4/15} \times 7^{2/15}}\left(\left(784 + 3\sqrt{\left(\frac{1}{\pi}42\left(-2\,370\,816\sqrt{2} + 4\,853\,259\,905\,\pi + 987\,840\sqrt{5}\,\pi + 395\,136\sqrt{11}\,\pi - 65\,856\sqrt{3}\,\pi\log\left(6+\sqrt{33}\right)\right)\right)}\right)^{\wedge}(1/15)\right)$$

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi} + \sqrt[15]{1 + \frac{3\sqrt[4]{3}}{56\sqrt{\frac{14\pi}{-2370816\sqrt{6} - \pi\left(\sqrt{3}\,\left(-4853259905 - 987840\sqrt{5} - 395136\sqrt{11}\right) + 197568\log\left(6+\sqrt{33}\right)\right)}}}}$$

Expanded logarithmic form

$$\sqrt[15]{27\sqrt{\frac{4\,853\,259\,905}{1\,185\,408} + \frac{5}{6}\sqrt{5} + \frac{1}{3}\sqrt{11} - \frac{2\sqrt{2}}{\pi} + -\frac{1}{6} \times 3^{-1/2}\log\left(6+\sqrt{33}\right)}} + 1 + C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

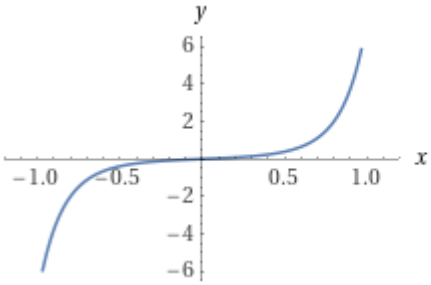
Now, we have:

x*e^(3x^2-1)

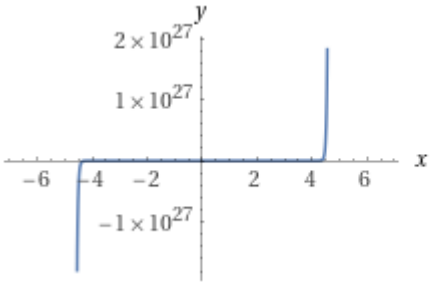
Input

$x\, e^{3x^2-1}$

Plots **(figures that can be related to the open strings)**



(x from −1.2 to 1.2)



(x from −6.9 to 6.9)

Root

$x = 0$

Properties as a real function

Domain

ℝ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

Parity

odd

$\mathbb{R}$ is the set of real numbers

Series expansion at x=0

$$\frac{x}{e} + \frac{3x^3}{e} + \frac{9x^5}{2e} + \frac{9x^7}{2e} + O(x^9)$$

(Taylor series)

Derivative

$$\frac{d}{dx}\left(x\, e^{3x^2-1}\right) = e^{3x^2-1}\left(6x^2+1\right)$$

Indefinite integral

$$\int x\, e^{3x^2-1}\, dx = \frac{1}{6}\, e^{3x^2-1} + \text{constant}$$

Alternative representations

$$x\, e^{3x^2-1} = x\, z^{3x^2-1} \quad \text{for } z = e$$

$$x\,e^{3x^2-1} = x\,w^a \quad \text{for } a = \frac{-1+3x^2}{\log(w)}$$

$$x\,e^{3x^2-1} = x\left(1 + \frac{2}{-1+\coth\left(\frac{1}{2}\left(-1+3x^2\right)\right)}\right)$$

$\log(x)$ is the natural logarithm

$\coth(x)$ is the hyperbolic cotangent function

Series representations

$$x\,e^{3x^2-1} = x\sum_{k=0}^{\infty}\frac{\left(-1+3x^2\right)^k}{k!}$$

$$x\,e^{3x^2-1} = x\sum_{k=-\infty}^{\infty} I_k\left(-1+3x^2\right)$$

$$x\,e^{3x^2-1} = x\sum_{k=0}^{\infty}\frac{\left(-1+3x^2\right)^{2k}\left(2k+3x^2\right)}{(1+2k)!}$$

$n!$ is the factorial function

$I_n(z)$ is the modified Bessel function of the first kind

Definite integral

$$\int_0^{\frac{1}{\sqrt{3}}} e^{-1+3x^2}\,x\,dx = \frac{e-1}{6e} \approx 0.105353$$

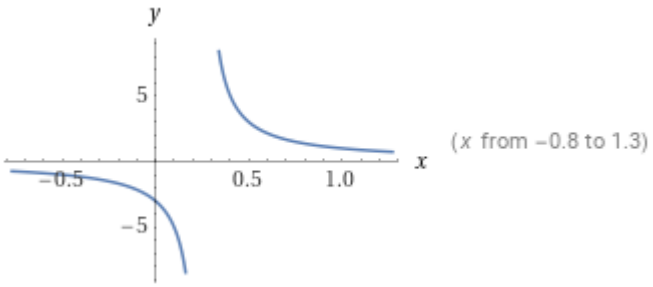
We have:

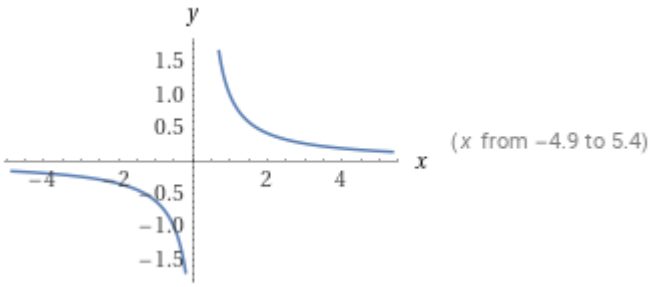
3/(4x-1)

Input

$$\frac{3}{4x-1}$$

Plots **(figures that can be related to the open strings)**





Roots

(no roots exist)

Properties as a real function

Domain

$$\{x \in \mathbb{R} : x \neq \frac{1}{4}\}$$

Range

$\{y \in \mathbb{R} : y \neq 0\}$

Injectivity

injective (one-to-one)

$\mathbb{R}$ is the set of real numbers

Series expansion at x=0

$$-3 - 12\,x - 48\,x^2 - 192\,x^3 - 768\,x^4 + O(x^5)$$

(Taylor series)

Series expansion at x=∞

$$\frac{3}{4\,x} + \frac{3}{16\,x^2} + \frac{3}{64\,x^3} + \frac{3}{256\,x^4} + O\left(\left(\frac{1}{x}\right)^5\right)$$

(Laurent series)

Derivative

$$\frac{d}{dx}\left(\frac{3}{4\,x-1}\right) = -\frac{12}{(4\,x-1)^2}$$

Indefinite integral

$$\int \frac{3}{4\,x-1}\,dx = \frac{3}{4}\log(4\,x-1) + \text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

Limit

$$\lim_{x\to\pm\infty} \frac{3}{-1+4x} = 0$$

Series representations

$$\frac{3}{-1+4x} = \sum_{n=0}^{\infty} x^n (-3)\, 4^n \quad \text{for } |x| < \frac{1}{4}$$

$$\frac{3}{-1+4x} = \sum_{n=0}^{\infty} \left(-\frac{4}{3}\right)^n (-1+x)^n \quad \text{for } |-1+x| < \frac{3}{4}$$

$|z|$ is the absolute value of z

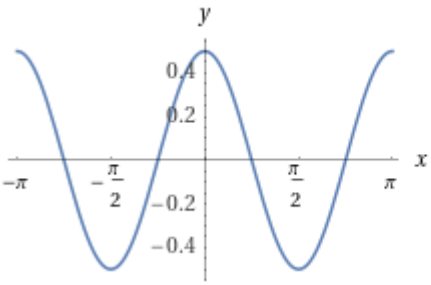
We have:

integrate(sin(2x-Pi))dx

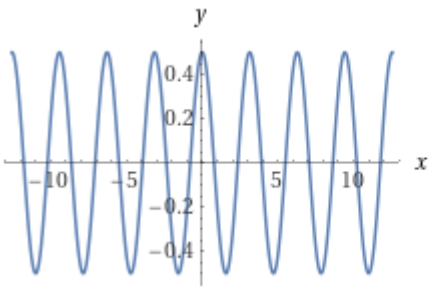
Indefinite integral

$$\int \sin(2x-\pi)\, dx = \frac{1}{2}\cos(2x) + \text{constant}$$

Plots of the integral (figures that can be related to the open strings)



(x from −3.142 to 3.142)



(x from −12.57 to 12.57)

Expanded trigonometric form

$$\frac{\cos^2(x)}{2} - \frac{\sin^2(x)}{2} + \text{constant}$$

Alternate forms of the integral

$$\sin\left(\frac{\pi}{4} - x\right)\sin\left(x + \frac{\pi}{4}\right) + \text{constant}$$

$$\frac{1}{4}e^{-2ix} + \frac{1}{4}e^{2ix} + \text{constant}$$

Expanded cosine form

$$-\frac{1}{2} + \cos^2(x) + \text{constant}$$

Expanded sine form

$$\frac{1}{2} - \sin^2(x) + \text{constant}$$

Series expansion of the integral at x=0

$$\frac{1}{2} - x^2 + \frac{x^4}{3} + O\left(x^6\right)$$

(Taylor series)

Definite integral over a half-period

$$\int_0^{\frac{\pi}{2}} -\sin(2x)\,dx = -1$$

Definite integral mean square

$$\int_0^{\pi} \frac{\sin^2(2x)}{\pi}\,dx = \frac{1}{2} = 0.5$$

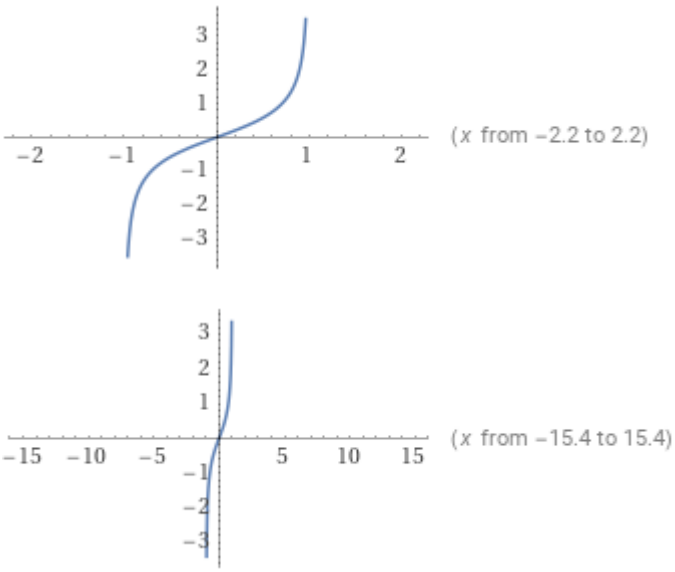
We have:

x/(sqrt(1-x^2))

Input

$$\frac{x}{\sqrt{1-x^2}}$$

Plots **(figures that can be related to the open strings)**



Alternate form

$$\frac{x}{\sqrt{1-x}\ \sqrt{x+1}}$$

Root

$x = 0$

Properties as a real function

Domain

$\{x \in \mathbb{R} : -1 < x < 1\}$

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

Parity

odd

$\mathbb{R}$ is the set of real numbers

Series expansion at x=-1

$$-\frac{1}{\sqrt{2}\ \sqrt{x+1}} + \frac{3\sqrt{x+1}}{4\sqrt{2}} + \frac{5(x+1)^{3/2}}{32\sqrt{2}} + \frac{7(x+1)^{5/2}}{128\sqrt{2}} + O\left((x+1)^{7/2}\right)$$

(Puiseux series)

Series expansion at x=0

$$x + \frac{x^3}{2} + \frac{3x^5}{8} + \frac{5x^7}{16} + O(x^9)$$

(Taylor series)

Series expansion at x=1

$$\frac{\sqrt{x-1}}{\sqrt{2-2x}\ \sqrt{x-1}} + \frac{3\sqrt{x-1}\ \sqrt{x-1}}{4\sqrt{2-2x}} - \frac{5\sqrt{x-1}\ (x-1)^{3/2}}{32\sqrt{2-2x}} + \frac{7\sqrt{x-1}\ (x-1)^{5/2}}{128\sqrt{2-2x}} + O\left((x-1)^{7/2}\right)$$

(generalized Puiseux series)

Series expansion at x=∞

$$\frac{x}{\sqrt{-x^2}} + \frac{x}{2\sqrt{-x^2}\ x^2} + O\left(\left(\frac{1}{x}\right)^4\right)$$

(Puiseux series)

Derivative

$$\frac{d}{dx}\left(\frac{x}{\sqrt{1-x^2}}\right) = \frac{1}{(1-x^2)^{3/2}}$$

Indefinite integral

$$\int \frac{x}{\sqrt{1-x^2}}\,dx = -\sqrt{1-x^2} + \text{constant}$$

Limit

$$\lim_{x\to-\infty} \frac{x}{\sqrt{1-x^2}} = i$$

$$\lim_{x\to\infty} \frac{x}{\sqrt{1-x^2}} = -i$$

Series representations

$$\frac{x}{\sqrt{1-x^2}} = \frac{x}{\sqrt{-x^2}\ \sum_{k=0}^{\infty} (-x^2)^{-k} \binom{\frac{1}{2}}{k}} \quad \text{for } |x| > 1$$

$$\frac{x}{\sqrt{1-x^2}} = \frac{x}{\sum_{k=0}^{\infty} \frac{(-1)^k (-x^2)^k \left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } |x|^2 < 1$$

$$\frac{x}{\sqrt{1-x^2}} = \frac{x}{\sqrt{-x^2}\ \sum_{k=0}^{\infty} \frac{(-1)^k (-x^2)^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } |x| > 1$$

$\binom{n}{m}$ is the binomial coefficient

$|z|$ is the absolute value of z

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

Definite integral

$$\int_{-1}^{0} \frac{x}{\sqrt{1-x^2}}\, dx = -1$$

Definite integral after subtraction of diverging parts

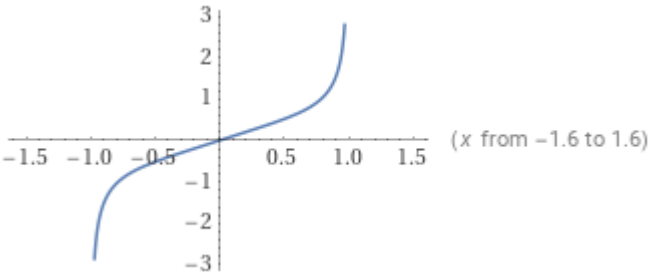
$$\int_{0}^{\infty} \frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{-x^2}}\, dx = 1$$

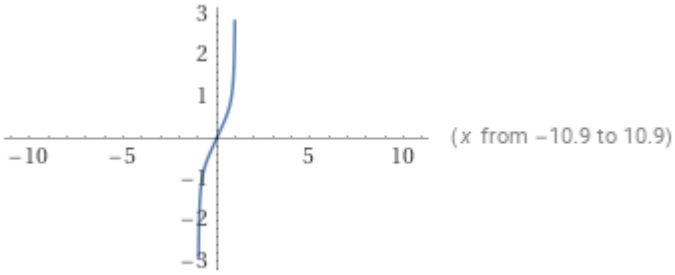
x/(sqrt(1-x^4))

Input

$$\frac{x}{\sqrt{1-x^4}}$$

Plots (figures that can be related to the open strings)





Alternate form

$$\frac{x}{\sqrt{1-x}\ \sqrt{x+1}\ \sqrt{x^2+1}}$$

Root

$x = 0$

Properties as a real function

Domain

$\{x \in \mathbb{R} : -1 < x < 1\}$

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

Parity

odd

$\mathbb{R}$ is the set of real numbers

Series expansion at x=-1

$$-\frac{1}{2\sqrt{x+1}}+\frac{\sqrt{x+1}}{8}+\frac{13}{64}(x+1)^{3/2}+\frac{37}{256}(x+1)^{5/2}+O\left((x+1)^3\right)$$

(Puiseux series)

Series expansion at x=0

$$x+\frac{x^5}{2}+O\left(x^6\right)$$

(Taylor series)

Series expansion at x=-i

$$-\frac{i\sqrt{x+i}}{2\sqrt{1-ix}\ \sqrt{x+i}}+\frac{\sqrt{x+i}\ \sqrt{x+i}}{8\sqrt{1-ix}}-\frac{13i\sqrt{x+i}\ (x+i)^{3/2}}{64\sqrt{1-ix}}-\frac{37\sqrt{x+i}\ (x+i)^{5/2}}{256\sqrt{1-ix}}+O\left((x+i)^3\right)$$

(generalized Puiseux series)

Series expansion at x=i

$$\frac{i\sqrt{x-i}}{2\sqrt{1+ix}\ \sqrt{x-i}}+\frac{\sqrt{x-i}\ \sqrt{x-i}}{8\sqrt{1+ix}}+\frac{13i\sqrt{x-i}\ (x-i)^{3/2}}{64\sqrt{1+ix}}-\frac{37\sqrt{x-i}\ (x-i)^{5/2}}{256\sqrt{1+ix}}+O\left((x-i)^3\right)$$

(generalized Puiseux series)

Series expansion at x=1

$$\frac{\sqrt{x-1}}{2\sqrt{1-x}\ \sqrt{x-1}}+\frac{\sqrt{x-1}\ \sqrt{x-1}}{8\sqrt{1-x}}-\frac{13\sqrt{x-1}\ (x-1)^{3/2}}{64\sqrt{1-x}}+\frac{37\sqrt{x-1}\ (x-1)^{5/2}}{256\sqrt{1-x}}+O\left((x-1)^3\right)$$

(generalized Puiseux series)

Series expansion at x=∞

$$\frac{x^2}{\sqrt{-x^4}\ x}+\frac{x^2}{2\sqrt{-x^4}\ x^5}+O\left(\left(\frac{1}{x}\right)^6\right)$$

(generalized Puiseux series)

Derivative

$$\frac{d}{dx}\left(\frac{x}{\sqrt{1-x^4}}\right)=\frac{x^4+1}{\left(1-x^4\right)^{3/2}}$$

Indefinite integral

$$\int\frac{x}{\sqrt{1-x^4}}\,dx=\frac{1}{2}\sin^{-1}\left(x^2\right)+\text{constant}$$

$\sin^{-1}(x)$ is the inverse sine function

Limit

$$\lim_{x\to\pm\infty} \frac{x}{\sqrt{1-x^4}} = 0$$

Series representations

$$\frac{x}{\sqrt{1-x^4}} = \frac{x}{\sqrt{-x^4}\sum_{k=0}^{\infty}\left(-x^4\right)^{-k}\binom{\frac{1}{2}}{k}} \quad \text{for } |x| > 1$$

$$\frac{x}{\sqrt{1-x^4}} = \frac{x}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-x^4\right)^k\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } |x|^4 < 1$$

$$\frac{x}{\sqrt{1-x^4}} = \frac{x}{\sqrt{-x^4}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-x^4\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } |x| > 1$$

$\binom{n}{m}$ is the binomial coefficient

$|z|$ is the absolute value of z

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

From the sum of the above functions, we obtain:

x*e^(3x^2-1) + 3/(4x-1) + ((sin(2x-Pi))) + (x/(sqrt(1-x^2))) + x/(sqrt(1-x^4))

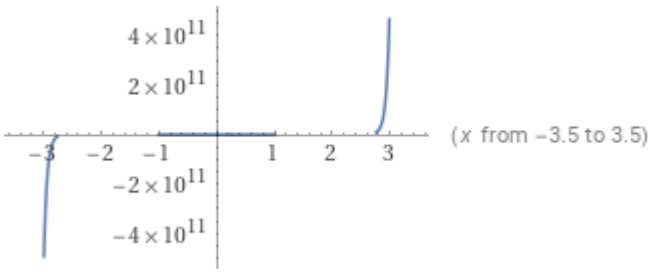
Input

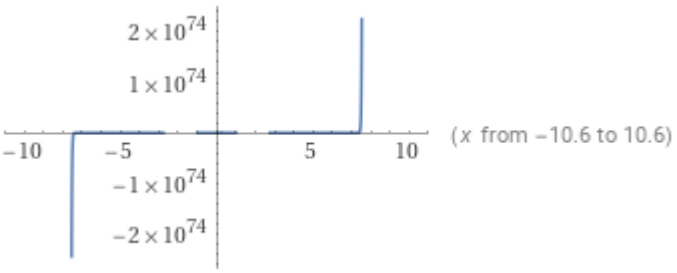
$$x\,e^{3x^2-1} + \frac{3}{4x-1} + \sin(2x-\pi) + \frac{x}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^4}}$$

Exact result

$$\frac{x}{\sqrt{1-x^4}}+e^{3x^2-1}\,x+\frac{x}{\sqrt{1-x^2}}+\frac{3}{4x-1}-\sin(2x)$$

Plots (figures that can be related to the open strings)





Expanded trigonometric form

$$e^{-1+3x^2}\,x+\frac{3}{-1+4x}+\frac{x}{\sqrt{1-x^2}}+\frac{x}{\sqrt{1-x^4}}-2\cos(x)\sin(x)$$

Alternate forms

$$\frac{x}{\sqrt{1-x^4}}+e^{3x^2-1}\,x+\frac{x}{\sqrt{1-x^2}}-\frac{1}{2}\,i\,e^{-2ix}+\frac{1}{2}\,i\,e^{2ix}+\frac{3}{4x-1}$$

$$e^{3x^2-1}\,x+\frac{x}{\sqrt{1-x}\,\sqrt{x+1}\,\sqrt{x^2+1}}+\frac{x}{\sqrt{1-x}\,\sqrt{x+1}}+\frac{3}{4x-1}-\sin(2x)$$

$$e^{3x^2-1}\,x-\frac{\sqrt{1-x^2}\;x}{(x-1)(x+1)}-\frac{\sqrt{1-x^4}\;x}{(x-1)(x+1)\left(x^2+1\right)}-\frac{4x\sin(2x)-\sin(2x)-3}{4x-1}$$

Series expansion at x=-1

$$\frac{-\frac{1}{2}-\frac{1}{\sqrt{2}}}{\sqrt{x+1}}+\left(-\frac{3}{5}-e^2+\sin(2)\right)+\frac{1}{8}\left(1+3\sqrt{2}\right)\sqrt{x+1}\;+$$
$$(x+1)\left(-\frac{12}{25}+7e^2-2\cos(2)\right)+\frac{1}{64}\left(13+5\sqrt{2}\right)(x+1)^{3/2}+$$
$$(x+1)^2\left(-\frac{48}{125}-27e^2-2\sin(2)\right)+\frac{1}{256}\left(37+7\sqrt{2}\right)(x+1)^{5/2}+O\left((x+1)^3\right)$$

(Puiseux series)

Series expansion at x=0

$$-3+\left(\frac{1}{e}-12\right)x-48x^2+\left(\frac{3}{e}-\frac{1141}{6}\right)x^3-768x^4+O\left(x^5\right)$$

(Taylor series)

Derivative

$$\frac{d}{dx}\left(x\,e^{3x^2-1}+\frac{3}{4x-1}+\sin(2x-\pi)+\frac{x}{\sqrt{1-x^2}}+\frac{x}{\sqrt{1-x^4}}\right)=\frac{2x^4}{\left(1-x^4\right)^{3/2}}+$$
$$\frac{1}{\sqrt{1-x^4}}+\frac{x^2}{\left(1-x^2\right)^{3/2}}+e^{3x^2-1}\left(6x^2+1\right)+\frac{1}{\sqrt{1-x^2}}-\frac{12}{(4x-1)^2}-2\cos(2x)$$

Indefinite integral

$$\int\left(e^{-1+3x^2}\,x+\frac{3}{-1+4x}+\frac{x}{\sqrt{1-x^2}}+\frac{x}{\sqrt{1-x^4}}-\sin(2x)\right)dx=$$

$$\frac{1}{6}e^{3x^2-1}+\frac{\sqrt{x^2+1}\,\left(x^2-1\right)}{\sqrt{1-x^4}}+\frac{\sqrt{x^2+1}\,\sqrt{x^2-1}\,\log\left(\sqrt{x^2-1}+\sqrt{x^2+1}\right)}{\sqrt{1-x^4}}+\frac{3}{4}$$

$$\log(4x-1)+\frac{1}{2}\cos(2x)+\text{constant}$$

(assuming a complex-valued logarithm)

log(x) is the natural logarithm

From the indefinite integral result

$$\int\left(e^{-1+3x^2}\,x+\frac{3}{-1+4x}+\frac{x}{\sqrt{1-x^2}}+\frac{x}{\sqrt{1-x^4}}-\sin(2x)\right)dx=$$

$$\frac{1}{6}e^{3x^2-1}+\frac{\sqrt{x^2+1}\,\left(x^2-1\right)}{\sqrt{1-x^4}}+\frac{\sqrt{x^2+1}\,\sqrt{x^2-1}\,\log\left(\sqrt{x^2-1}+\sqrt{x^2+1}\right)}{\sqrt{1-x^4}}+\frac{3}{4}$$

$$\log(4x-1)+\frac{1}{2}\cos(2x)+\text{constant}$$

(assuming a complex-valued logarithm)

we obtain:

1/6 e^(-1 + 3 x^2) + ((-1 + x^2) sqrt(1 + x^2))/sqrt(1 - x^4) + 1/2 cos(2 x) + 3/4 log(-1 + 4 x) + (sqrt(-1 + x^2) sqrt(1 + x^2) log(sqrt(-1 + x^2) + sqrt(1 + x^2)))/sqrt(1 - x^4)

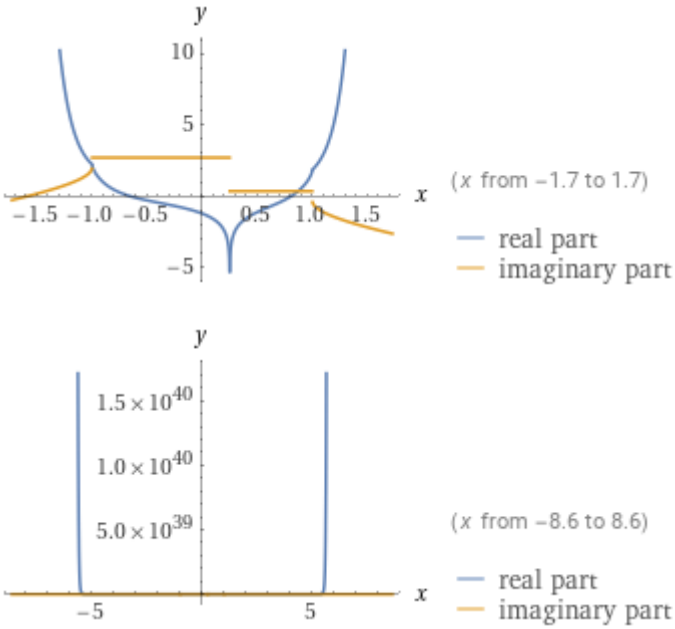
Input

$$\frac{1}{6}e^{-1+3x^2}+\frac{\left(-1+x^2\right)\sqrt{1+x^2}}{\sqrt{1-x^4}}+\frac{1}{2}\cos(2x)+$$

$$\frac{3}{4}\log(-1+4x)+\frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,\log\left(\sqrt{-1+x^2}+\sqrt{1+x^2}\right)}{\sqrt{1-x^4}}$$

log(x) is the natural logarithm

Plots **(figures that can be related to the open strings)**



From

$$\frac{1}{6}e^{-1+3x^2}+\frac{(-1+x^2)\sqrt{1+x^2}}{\sqrt{1-x^4}}+\frac{1}{2}\cos(2x)+\frac{3}{4}\log(-1+4x)+\frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,\log\left(\sqrt{-1+x^2}+\sqrt{1+x^2}\right)}{\sqrt{1-x^4}}$$

for x = 2:

1/6 e^(-1 + 3 2^2) + ((-1 + 2^2) sqrt(1 + 2^2))/sqrt(1 - 2^4) + 1/2 cos(2 2) + 3/4 log(-1 + 4 2) + (sqrt(-1 + 2^2) sqrt(1 + 2^2) log(sqrt(-1 + 2^2) + sqrt(1 + 2^2)))/sqrt(1 - 2^4)

Input

$$\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}$$

$\log(x)$ is the natural logarithm

Decimal approximation

$$9980.156230000996088242590395842330445316695696039024728896426767\ldots - 3.110342932296630221599702988545026586708851045932204505034163330\ldots i$$

Alternate complex forms

$$\left(\frac{e^{11}}{6}+\frac{\cos(4)}{2}+\frac{3\log(7)}{4}\right)+\left(-\sqrt{3}-\log\left(\sqrt{3}+\sqrt{5}\right)\right)i$$

$$\sqrt{\left(\sqrt{3}+\log\left(\sqrt{3}+\sqrt{5}\right)\right)^2+\frac{1}{144}\left(2e^{11}+9\log(7)+6\cos(4)\right)^2}\left(\cos\left(\tan^{-1}\left(\frac{-\sqrt{3}-\log\left(\sqrt{3}+\sqrt{5}\right)}{\frac{e^{11}}{6}+\frac{3\log(7)}{4}+\frac{\cos(4)}{2}}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{-\sqrt{3}-\log\left(\sqrt{3}+\sqrt{5}\right)}{\frac{e^{11}}{6}+\frac{3\log(7)}{4}+\frac{\cos(4)}{2}}\right)\right)\right)$$

$$\sqrt{\left(\sqrt{3}+\log\left(\sqrt{3}+\sqrt{5}\right)\right)^2+\frac{1}{144}\left(2e^{11}+9\log(7)+6\cos(4)\right)^2}\exp\left(i\tan^{-1}\left(\frac{-\sqrt{3}-\log\left(\sqrt{3}+\sqrt{5}\right)}{\frac{e^{11}}{6}+\frac{3\log(7)}{4}+\frac{\cos(4)}{2}}\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 9980.2$ (radius), $\theta \approx -0.00031165$ (angle)

9980.2

Alternate forms

$$\frac{1}{12}\left(2e^{11} + 9\log(7) - 6i\left(2\sqrt{3} + \log\left(8 + 2\sqrt{15}\right)\right) + 6\cos(4)\right)$$

$$\frac{1}{12}\left(-12i\sqrt{3} + 2e^{11} + 9\log(7) - 12i\log\left(\sqrt{3} + \sqrt{5}\right) + 6\cos(4)\right)$$

$$-i\sqrt{3} + \frac{e^{11}}{6} + \frac{3\log(7)}{4} - \frac{1}{2}i\log\left(2\left(4 + \sqrt{15}\right)\right) + \frac{\cos(4)}{2}$$

Alternative representations

$$\frac{1}{6}e^{-1+3\times 2^2} + \frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}} + \frac{1}{2}\cos(2\times 2) + \frac{1}{4}\log(-1+4\times 2)\,3 + \frac{\sqrt{-1+2^2}\left(\sqrt{1+2^2}\,\log\left(\sqrt{-1+2^2} + \sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}} =$$
$$\frac{1}{2}\cosh(-4i) + \frac{3\log(7)}{4} + \frac{e^{11}}{6} + \frac{3\sqrt{5}}{\sqrt{1-2^4}} + \frac{\log\left(\sqrt{3} + \sqrt{5}\right)\sqrt{3}\,\sqrt{5}}{\sqrt{1-2^4}}$$

$$\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{1}{4}\log(-1+4\times 2)\,3+\frac{\sqrt{-1+2^2}\left(\sqrt{1+2^2}\,\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}}=\frac{3\log(7)}{4}+\frac{e^{11}}{6}+\frac{1}{4}\left(e^{-4i}+e^{4i}\right)+\frac{3\sqrt{5}}{\sqrt{1-2^4}}+\frac{\log\left(\sqrt{3}+\sqrt{5}\right)\sqrt{3}\,\sqrt{5}}{\sqrt{1-2^4}}$$

$$\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{1}{4}\log(-1+4\times 2)\,3+\frac{\sqrt{-1+2^2}\left(\sqrt{1+2^2}\,\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}}=\frac{1}{2}\cosh(-4\,i)+\frac{3}{4}\log(a)\log_a(7)+\frac{e^{11}}{6}+\frac{3\sqrt{5}}{\sqrt{1-2^4}}+\frac{\log(a)\log_a\left(\sqrt{3}+\sqrt{5}\right)\sqrt{3}\,\sqrt{5}}{\sqrt{1-2^4}}$$

$\cosh(x)$ is the hyperbolic cosine function

$\log_b(x)$ is the base-b logarithm

Series representations

$$\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{1}{4}\log(-1+4\times 2)\,3+\frac{\sqrt{-1+2^2}\left(\sqrt{1+2^2}\,\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}}=\frac{1}{12}\left(-12\,i\sqrt{3}+2\,e^{11}+9\log(6)-12\,i\log\left(-1+\sqrt{3}+\sqrt{5}\right)+12\sum_{k=1}^{\infty}\frac{2^{-2-k}\times 3^{-k}\left(-3\,(-1)^k+i\,2^{2+k}\left(-\frac{3}{-1+\sqrt{3}+\sqrt{5}}\right)^k\right)}{k}+6\sum_{k=0}^{\infty}\frac{(-16)^k}{(2\,k)!}\right)$$

$$\frac{1}{6}\,e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{1}{4}\log(-1+4\times 2)\,3+\frac{\sqrt{-1+2^2}\,\left(\sqrt{1+2^2}\,\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}}=\frac{1}{12}\left(-12\,i\,\sqrt{3}+2\,e^{11}+9\log(6)-12\,i\log\left(-1+\sqrt{3}+\sqrt{5}\right)+12\sum_{k=1}^{\infty}\frac{2^{-2-k}\times 3^{-k}\left(-3\,(-1)^k+i\,2^{2+k}\left(-\frac{3}{-1+\sqrt{3}+\sqrt{5}}\right)^k\right)}{k}-6\sum_{k=0}^{\infty}\frac{(-1)^k\left(4-\frac{\pi}{2}\right)^{1+2k}}{(1+2k)!}\right)$$

$$\frac{1}{6}\,e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{1}{4}\log(-1+4\times 2)\,3+\frac{\sqrt{-1+2^2}\,\left(\sqrt{1+2^2}\,\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}}=\frac{1}{12}\left(-12\,i\,\sqrt{3}+2\,e^{11}+9\log(6)-12\,i\log\left(-1+\sqrt{3}+\sqrt{5}\right)+12\sum_{k=1}^{\infty}\frac{2^{-2-k}\times 3^{-k}\left(-3\,(-1)^k+i\,2^{2+k}\left(-\frac{3}{-1+\sqrt{3}+\sqrt{5}}\right)^k\right)}{k}+6\sqrt{\pi}\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\frac{4^{-s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)$$

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Integral representations

$$\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{1}{4}\log(-1+4\times 2)\,3+\frac{\sqrt{-1+2^2}\left(\sqrt{1+2^2}\,\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}}=-i\sqrt{3}+\frac{e^{11}}{6}-\frac{1}{2}\int_{\frac{\pi}{2}}^{4}\sin(t)\,dt+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)$$

$$\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{1}{4}\log(-1+4\times 2)\,3+\frac{\sqrt{-1+2^2}\left(\sqrt{1+2^2}\,\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}}=-i\sqrt{3}+\frac{e^{11}}{6}-\frac{i}{4\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-4/s+s}}{\sqrt{s}}\,ds+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)\quad\text{for }\gamma>0$$

$$\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{1}{4}\log(-1+4\times 2)\,3+\frac{\sqrt{-1+2^2}\left(\sqrt{1+2^2}\,\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}}=\frac{1}{2}-i\sqrt{3}+\frac{e^{11}}{6}+\int_1^7\left(\frac{3}{4\,t}-\frac{i\left(-1+\sqrt{3}+\sqrt{5}\right)}{7-\sqrt{3}-\sqrt{5}-t+\left(\sqrt{3}+\sqrt{5}\right)t}-\frac{1}{3}\sin\left(\frac{2}{3}(-1+t)\right)\right)dt$$

Multiple-argument formulas

$$\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{1}{4}\log(-1+4\times 2)\,3+\frac{\sqrt{-1+2^2}\left(\sqrt{1+2^2}\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}}=-i\sqrt{3}+\frac{e^{11}}{6}+\frac{1}{2}\left(-1+2\cos^2(2)\right)+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)$$

$$\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{1}{4}\log(-1+4\times 2)\,3+\frac{\sqrt{-1+2^2}\left(\sqrt{1+2^2}\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)\right)}{\sqrt{1-2^4}}=-i\sqrt{3}+\frac{e^{11}}{6}+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)+\frac{1}{2}\left(1-2\sin^2(2)\right)$$

From which:

1/√5(1/6 e^(-1 + 3 2^2) + ((-1 + 2^2) sqrt(1 + 2^2))/sqrt(1 - 2^4) + 1/2 cos(2 2) + 3/4 log(-1 + 4 2) + (sqrt(-1 + 2^2) sqrt(1 + 2^2) log(sqrt(-1 + 2^2) + sqrt(1 + 2^2)))/sqrt(1 - 2^4))-322-47+2-1/4

Input

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\,\sqrt{1+2^2}\log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}$$

log(x) is the natural logarithm

Exact result

$$-\frac{1469}{4}+\frac{-i\sqrt{3}+\frac{e^{11}}{6}+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)+\frac{\cos(4)}{2}}{\sqrt{5}}$$

Decimal approximation

4096.011551270050656665968729090310553615921157008180041039247947…

$$-$$

1.390987645990258250678092278858933537151860623637022785115006567…

$$i$$

Alternate complex forms

$$\left(-\frac{1469}{4}+\frac{\frac{e^{11}}{6}+\frac{\cos(4)}{2}+\frac{3\log(7)}{4}}{\sqrt{5}}\right)+\frac{\left(-\sqrt{3}-\log\left(\sqrt{3}+\sqrt{5}\right)\right)i}{\sqrt{5}}$$

$$\sqrt{\frac{1}{5}\left(\sqrt{3}+\log\left(\sqrt{3}+\sqrt{5}\right)\right)^2+\frac{\left(\sqrt{5}\left(2e^{11}+9\log(7)+6\cos(4)\right)-22035\right)^2}{3600}}$$

$$\left(\cos\left(\tan^{-1}\left(\frac{-\sqrt{3}-\log\left(\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}\left(\frac{\frac{e^{11}}{6}+\frac{3\log(7)}{4}+\frac{\cos(4)}{2}}{\sqrt{5}}-\frac{1469}{4}\right)}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{-\sqrt{3}-\log\left(\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}\left(\frac{\frac{e^{11}}{6}+\frac{3\log(7)}{4}+\frac{\cos(4)}{2}}{\sqrt{5}}-\frac{1469}{4}\right)}\right)\right)\right)$$

$$\sqrt{\frac{1}{5}\left(\sqrt{3}+\log\left(\sqrt{3}+\sqrt{5}\right)\right)^2+\frac{\left(\sqrt{5}\left(2e^{11}+9\log(7)+6\cos(4)\right)-22035\right)^2}{3600}}$$

$$\exp\left(i\tan^{-1}\left(\frac{-\sqrt{3}-\log\left(\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}\left(\frac{\frac{e^{11}}{6}+\frac{3\log(7)}{4}+\frac{\cos(4)}{2}}{\sqrt{5}}-\frac{1469}{4}\right)}\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 4096$ (radius), $\theta \approx -0.00033960$ (angle)

4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero.

Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{1}{60}\left(-22035+\sqrt{5}\left(2e^{11}+9\log(7)-6i\left(2\sqrt{3}+\log\left(8+2\sqrt{15}\right)\right)+6\cos(4)\right)\right)$$

$$\frac{1}{60}\left(-22035+\sqrt{5}\left(-12i\sqrt{3}+2e^{11}+9\log(7)-12i\log\left(\sqrt{3}+\sqrt{5}\right)+6\cos(4)\right)\right)$$

$$-\frac{1469}{4}+\frac{-i\sqrt{3}+\frac{e^{11}}{6}+\frac{3\log(7)}{4}-\frac{1}{2}i\log\left(2\left(4+\sqrt{15}\right)\right)+\frac{\cos(4)}{2}}{\sqrt{5}}$$

Expanded form

$$-\frac{1469}{4}-i\sqrt{\frac{3}{5}}+\frac{e^{11}}{6\sqrt{5}}+\frac{3\log(7)}{4\sqrt{5}}-\frac{i\log\left(\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}}+\frac{\cos(4)}{2\sqrt{5}}$$

Alternative representations

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}=$$

$$-367-\frac{1}{4}+\frac{\frac{1}{2}\cosh(-4i)+\frac{3\log(7)}{4}+\frac{e^{11}}{6}+\frac{3\sqrt{5}}{\sqrt{1-2^4}}+\frac{\log\left(\sqrt{3}+\sqrt{5}\right)\sqrt{3}\ \sqrt{5}}{\sqrt{1-2^4}}}{\sqrt{5}}$$

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}=$$

$$-367-\frac{1}{4}+\frac{\frac{3\log(7)}{4}+\frac{e^{11}}{6}+\frac{1}{4}\left(e^{-4i}+e^{4i}\right)+\frac{3\sqrt{5}}{\sqrt{1-2^4}}+\frac{\log\left(\sqrt{3}+\sqrt{5}\right)\sqrt{3}\ \sqrt{5}}{\sqrt{1-2^4}}}{\sqrt{5}}$$

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-$$

$$322-47+2-\frac{1}{4}=-367-\frac{1}{4}+\frac{\frac{1}{2}\cosh(-4i)+\frac{3}{4}\log(a)\log_a(7)+\frac{e^{11}}{6}+\frac{3\sqrt{5}}{\sqrt{1-2^4}}+\frac{\log(a)\log_a\left(\sqrt{3}+\sqrt{5}\right)\sqrt{3}\ \sqrt{5}}{\sqrt{1-2^4}}}{\sqrt{5}}$$

$\cosh(x)$ is the hyperbolic cosine function

$\log_b(x)$ is the base-b logarithm

Series representations

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}=$$

$$\frac{1}{60}\left(-22\,035-12\,i\sqrt{15}+2\sqrt{5}\ e^{11}+9\sqrt{5}\ \log(6)-12\,i\sqrt{5}\ \log\left(-1+\sqrt{3}+\sqrt{5}\right)+60\sum_{k=1}^{\infty}\frac{2^{-2-k}\times 3^{-k}\left(-3\,(-1)^k+i\,2^{2+k}\left(-\frac{3}{-1+\sqrt{3}+\sqrt{5}}\right)^k\right)}{\sqrt{5}\ k}+6\sqrt{5}\sum_{k=0}^{\infty}\frac{(-16)^k}{(2k)!}\right)$$

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}=$$

$$\frac{1}{60}\left(-22\,035-12\,i\sqrt{15}+2\sqrt{5}\ e^{11}+9\sqrt{5}\ \log(6)-12\,i\sqrt{5}\ \log\left(-1+\sqrt{3}+\sqrt{5}\right)+60\sum_{k=1}^{\infty}\frac{2^{-2-k}\times 3^{-k}\left(-3\,(-1)^k+i\,2^{2+k}\left(-\frac{3}{-1+\sqrt{3}+\sqrt{5}}\right)^k\right)}{\sqrt{5}\ k}-6\sqrt{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(4-\frac{\pi}{2}\right)^{1+2k}}{(1+2k)!}\right)$$

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}=$$

$$\frac{1}{60}\left(-22\,035-12\,i\sqrt{15}+2\sqrt{5}\ e^{11}+9\sqrt{5}\ \log(6)-12\,i\sqrt{5}\ \log\left(-1+\sqrt{3}+\sqrt{5}\right)+60\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\,2^{-k}\times 3^{1-k}+4\,i\left(-\frac{1}{-1+\sqrt{3}+\sqrt{5}}\right)^k}{4\sqrt{5}\ k}-6\sqrt{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(4-\frac{\pi}{2}\right)^{1+2k}}{(1+2k)!}\right)$$

$n!$ is the factorial function

Integral representations

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}=$$

$$-\frac{1469}{4}-i\sqrt{\frac{3}{5}}+\frac{e^{11}}{6\sqrt{5}}-\frac{1}{2\sqrt{5}}\int_{\frac{\pi}{2}}^{4}\sin(t)\,dt+\frac{3\log(7)}{4\sqrt{5}}-\frac{i\log\left(\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}}$$

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}=$$
$$-\frac{1469}{4}-i\sqrt{\frac{3}{5}}+\frac{e^{11}}{6\sqrt{5}}-\frac{i}{4\sqrt{5\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-4/s+s}}{\sqrt{s}}\,ds+\frac{3\log(7)}{4\sqrt{5}}-\frac{i\log\left(\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}}\quad\text{for }\gamma>0$$

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-$$
$$322-47+2-\frac{1}{4}=-\frac{1469}{4}-i\sqrt{\frac{3}{5}}+\frac{1}{2\sqrt{5}}+\frac{e^{11}}{6\sqrt{5}}+\int_1^7\left(\frac{3}{4\sqrt{5}\,t}-\frac{i\left(-1+\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}\left(7-\sqrt{3}-\sqrt{5}-t+\left(\sqrt{3}+\sqrt{5}\right)t\right)}-\frac{\sin\left(\frac{2}{3}(-1+t)\right)}{3\sqrt{5}}\right)dt$$

Multiple-argument formulas

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}=$$
$$-\frac{1469}{4}+\frac{-i\sqrt{3}+\frac{e^{11}}{6}+\frac{1}{2}\left(-1+2\cos^2(2)\right)+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}}$$

$$\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}=-\frac{1469}{4}+\frac{-i\sqrt{3}+\frac{e^{11}}{6}+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)+\frac{1}{2}\left(1-2\sin^2(2)\right)}{\sqrt{5}}$$

27*sqrt(1/√5(1/6 e^(-1+3 2^2)+((-1+2^2) sqrt(1+2^2))/sqrt(1-2^4)+1/2 cos(2 2) + 3/4 log(-1 + 4 2) + (sqrt(-1 + 2^2) sqrt(1 + 2^2) log(sqrt(-1 + 2^2) + sqrt(1 + 2^2)))/sqrt(1 - 2^4))-322-47+2-1/4)+1

Input

$$27\sqrt{\left(\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}\right)}+1$$

log(x) is the natural logarithm

Exact result

$$1+27\sqrt{-\frac{1469}{4}+\frac{-i\sqrt{3}+\frac{e^{11}}{6}+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)+\frac{\cos(4)}{2}}{\sqrt{5}}}$$

Decimal approximation

1729.00246150458492662541122345902589978548678468505508589032674…

$-$

0.29341103861737980764387831878910830896683943279971426726352008 8…

i

Alternate complex forms

$1729 - 0.3\,i$

$1729\,(\cos(-0.00016970) + i\sin(-0.00016970))$

$1729\,e^{-0.00016970\,i}$

Polar coordinates

$r = 1729.$ (radius), $\theta = -0.000173511$ (angle)

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron.

As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$1+\frac{27}{2}\sqrt{-1469+\frac{-12\,i\sqrt{3}+2\,e^{11}+9\log(7)-12\,i\log\left(\sqrt{3}+\sqrt{5}\right)+6\cos(4)}{3\sqrt{5}}}$$

$$1+27\sqrt{-\frac{1469}{4}+\frac{-i\sqrt{3}+\frac{e^{11}}{6}+\frac{3\log(7)}{4}-\frac{1}{2}\,i\log\left(2\left(4+\sqrt{15}\right)\right)+\frac{\cos(4)}{2}}{\sqrt{5}}}$$

$$1+27\sqrt{-\frac{1469}{4}+\frac{-i\sqrt{3}+\frac{1}{4}\left(e^{-4i}+e^{4i}\right)+\frac{e^{11}}{6}+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}}}$$

Alternative representations

$$27\sqrt{\left(\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}\right)+1}=1+27\sqrt{-367-\frac{1}{4}+\frac{\frac{1}{2}\cosh(-4i)+\frac{3\log(7)}{4}+\frac{e^{11}}{6}+\frac{3\sqrt{5}}{\sqrt{1-2^4}}+\frac{\log\left(\sqrt{3}+\sqrt{5}\right)\sqrt{3}\ \sqrt{5}}{\sqrt{1-2^4}}}{\sqrt{5}}}$$

$$27\sqrt{\left(\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}\right)+1}=1+27\sqrt{-367-\frac{1}{4}+\frac{\frac{3\log(7)}{4}+\frac{e^{11}}{6}+\frac{1}{4}\left(e^{-4i}+e^{4i}\right)+\frac{3\sqrt{5}}{\sqrt{1-2^4}}+\frac{\log\left(\sqrt{3}+\sqrt{5}\right)\sqrt{3}\ \sqrt{5}}{\sqrt{1-2^4}}}{\sqrt{5}}}$$

$$27\sqrt{\left(\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}\right)}+1=$$

$$1+27\sqrt{\left(-367-\frac{1}{4}+\frac{1}{\sqrt{5}}\left(\frac{1}{2}\cosh(-4\,i)+\frac{3}{4}\log(a)\log_a(7)+\frac{e^{11}}{6}+\frac{3\sqrt{5}}{\sqrt{1-2^4}}+\frac{\log(a)\log_a\left(\sqrt{3}+\sqrt{5}\right)\sqrt{3}\ \sqrt{5}}{\sqrt{1-2^4}}\right)\right)}$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

$\log_b(x)$ is the base-b logarithm

Series representations

$$27\sqrt{\left(\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}\right)+1}=$$

$$\frac{1}{10}\left(10+9\sqrt{15}\ \sqrt{\left(-22035-12\,i\sqrt{15}+2\sqrt{5}\ e^{11}+9\sqrt{5}\ \log(6)-12\,i\sqrt{5}\ \log\left(-1+\sqrt{3}+\sqrt{5}\right)-9\sqrt{5}\ \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+12\,i\sqrt{5}\ \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\sqrt{3}+\sqrt{5}}\right)^k}{k}+6\sqrt{5}\ \sum_{k=0}^{\infty}\frac{(-16)^k}{(2k)!}\right)}\right)$$

$$27\sqrt{\left(\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}\right)+1}=$$

$$\frac{1}{10}\left(10+9\sqrt{15}\ \sqrt{\left(-22035-12\,i\sqrt{15}+2\sqrt{5}\ e^{11}+9\sqrt{5}\ \log(6)-12\,i\sqrt{5}\ \log\left(-1+\sqrt{3}+\sqrt{5}\right)-9\sqrt{5}\ \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+12\,i\sqrt{5}\ \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\sqrt{3}+\sqrt{5}}\right)^k}{k}-6\sqrt{5}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(4-\frac{\pi}{2}\right)^{1+2k}}{(1+2k)!}\right)}\right)$$

$$27\sqrt{\left(\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}\right)+1}=$$

$$\frac{1}{10}\left(10+9\sqrt{15}\ \sqrt{\left(-22035-12\,i\sqrt{15}+2\sqrt{5}\ e^{11}+9\sqrt{5}\ \log(6)-12\,i\sqrt{5}\ \log\left(-1+\sqrt{3}+\sqrt{5}\right)-9\sqrt{5}\ \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{6}\right)^k}{k}+12\,i\sqrt{5}\ \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-1+\sqrt{3}+\sqrt{5}}\right)^k}{k}+6\sqrt{5\pi}\ \sum_{j=0}^{\infty}\text{Res}_{s=-j}\frac{4^{-s}\,\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}\right)$$

$n!$ is the factorial function

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Integral representations

$$27\sqrt{\left(\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}\right)}+1=1+27\sqrt{-\frac{1469}{4}+\frac{-i\sqrt{3}+\frac{e^{11}}{6}-\frac{1}{2}\int_{\frac{\pi}{2}}^{4}\sin(t)\,dt+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}}}$$

$$27\sqrt{\left(\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}\right)}+1=1+27\sqrt{\left(-\frac{1469}{4}+\frac{-i\sqrt{3}+\frac{e^{11}}{6}-\frac{i}{4\sqrt{\pi}}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-4/s+s}}{\sqrt{s}}\,ds+\frac{3\log(7)}{4}-i\log\left(\sqrt{3}+\sqrt{5}\right)}{\sqrt{5}}\right)}$$

for $\gamma > 0$

$$27\sqrt{\left(\frac{1}{\sqrt{5}}\left(\frac{1}{6}e^{-1+3\times 2^2}+\frac{\left(-1+2^2\right)\sqrt{1+2^2}}{\sqrt{1-2^4}}+\frac{1}{2}\cos(2\times 2)+\frac{3}{4}\log(-1+4\times 2)+\frac{\sqrt{-1+2^2}\ \sqrt{1+2^2}\ \log\left(\sqrt{-1+2^2}+\sqrt{1+2^2}\right)}{\sqrt{1-2^4}}\right)-322-47+2-\frac{1}{4}\right)+1}=$$

$$\frac{1}{10}\left(10+9\sqrt{15}\sqrt{\left(-22035+6\sqrt{5}-12\,i\sqrt{15}+2\sqrt{5}\ e^{11}+12\sqrt{5}\int_1^7\left(\frac{3}{4\,t}-\frac{i\left(-1+\sqrt{3}+\sqrt{5}\right)}{7-\sqrt{3}-\sqrt{5}+\left(-1+\sqrt{3}+\sqrt{5}\right)t}\right)dt-24\sqrt{5}\int_0^1\sin(4\,t)\,dt\right)}\right)$$

From the alternate form

$$1+\frac{27}{2}\sqrt{-1469+\frac{-12\,i\sqrt{3}+2\,e^{11}+9\log(7)-12\,i\log\left(\sqrt{3}+\sqrt{5}\right)+6\cos(4)}{3\sqrt{5}}}$$

we obtain:

(1 + 27/2 sqrt(-1469 + (-12 i sqrt(3) + 2 e^11 + 6 cos(4) + 9 log(7) - 12 i log(sqrt(3) + sqrt(5)))/(3 sqrt(5))))^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{1+\frac{27}{2}\sqrt{-1469+\frac{-12\,i\sqrt{3}+2\,e^{11}+6\cos(4)+9\log(7)-12\,i\log\left(\sqrt{3}+\sqrt{5}\right)}{3\sqrt{5}}}}+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{1+\frac{27}{2}\sqrt{-1469+\frac{-12\,i\sqrt{3}+2\,e^{11}+9\log(7)-12\,i\log\left(\sqrt{3}+\sqrt{5}\right)+6\cos(4)}{3\sqrt{5}}}}$$

Decimal approximation

1.64493818161828458295969288146581222597879697317955556839 46… −
0.00001859698818249759761300988922567865303937000659397445069 7153…
i

Alternate complex forms

$1.6449 - 0.\times 10^{-5}\, i$

$1.6449\,(\cos(0) + i\sin(0))$

$1.6449\,e^{0}$

Result

1.6449

1.6449 ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \left(1 + \frac{27}{2} \sqrt{-1469 + \frac{-12\,i\,\sqrt{3} + 3\left(e^{-4i} + e^{4i}\right) + 2\,e^{11} + 9\log(7) - 12\,i\log\left(\sqrt{3} + \sqrt{5}\right)}{3\sqrt{5}}}\right)^{\wedge}$$

$(1/15)$

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \left(\frac{1}{10}\left(10 + 9\sqrt{\left(15\left(-22035 - 12\,i\,\sqrt{15} + 2\sqrt{5}\;e^{11} + 9\sqrt{5}\;\log(7) - 12\,i\sqrt{5}\;\log\left(\sqrt{3} + \sqrt{5}\right) + 6\sqrt{5}\;\cos(4)\right)\right)}\right)\right)^{\wedge}(1/15)$$

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \frac{\sqrt[15]{2\sqrt{3} + \frac{27\sqrt{2e^{11} - 3i\left(4\log\left(\sqrt{3}+\sqrt{5}\right) + i\left(-i\left(4\sqrt{3} - 1469i\sqrt{5}\right) + 3\log(7) + 2\cos(4)\right)\right)}}{\sqrt[4]{5}}}}{\sqrt[15]{2}\;\sqrt[30]{3}}$$

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

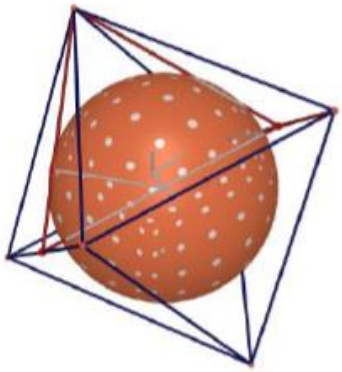


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$
$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$).

The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^8\right)^{-k}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^8\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))) = 3.1415926535

Indeed:

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13

…

3.1415926535…. = π (Ramanujan Recurring Number)

From the following extended DN Constant (“Unitary Formula”)

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}}$$

with regard

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

for q = 1729 and p = 4096, we obtain by changing the sign in the algebraic sum of the aforementioned Cardano’s Formula and after some calculations:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

multiplied by

$$\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}$$

and performing the ninth root of the entire expression:

√((2^(-1/π)(5(3+√(5))π)^(1/(2π)))((∛(-1729/2+√(1729^2/4+4096^3/27)) -∛(-1729/2-√(1729^2/4+4096^3/27)))^1/9))

we obtain:

$$\sqrt{\left(\left(2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right)\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}\right)}$$

$$2^{-1/(2\pi)}\sqrt[18]{\sqrt[3]{\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}-\frac{1729}{2}}+\sqrt[3]{\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}}\sqrt[4\pi]{5\left(3+\sqrt{5}\right)\pi}$$

i.e.

2^(-1/(2π)) ((-1729/2+√(274958621851/3)/6)^(1/3)+(1729/2 + √(274958621851/3)/6)^(1/3))^(1/18) (5(3+√(5)) π)^(1/(4π))

= 1.61549140391….

The general "unitary" formula, which derives from DN Constant, is the following:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

Where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

C is any constant or solution to an equation, R is the radius of the Universe and $2.33*10^{-13}$ is the temperature of the universe expressed in GeV.

For example, $C = 9.9128$, inserting a radius of the Universe, which we have calculated, equal to $R = 2.06274*10^{12}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}$$

1.618035912348264235440174408834709854254527350840173356306481810 7…

1.618035912348…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We obtain also:

(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))dxdydz

Indefinite integral assuming all variables are real

$$0.809018\, x^2\, y\, z + \text{constant}$$

Definite integral over a cube of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} 1.61804\, dx\, dy\, dz = 12.9443\, L^3$$

Definite integral over a sphere of radius R

$$\iiint_{x^2+y^2+z^2<R^2} 1.61804\, dz\, dy\, dx = 6.77761\, R^3$$

From which, for L = R = 1 , dividing the two definite integral results by the original expression, we obtain:

12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))

Input interpretation

$$\frac{12.9443}{\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

8.00001…

8.00001…. ≈ 8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

And

3*(6.77761/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))

Input interpretation

$$3\times\frac{6.77761}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

12.5664…

12.5664…. ≈ 4π = Bekenstein-Hawking (S_{BH}) black hole entropy

New fundamental formula deriving from DN Constant

The DN Constant (Del Gaudio-Nardelli Constant) equals (2√2)/π) is defined as the ratio of the volume of an octahedron to the volume of a sphere and is an intriguing mathematical concept. Michele Nardelli hypothesized that the regular octahedron represents a phase in which the universe is highly symmetrical and with very low entropy. On the other hand, the sphere (which is inscribed in the octahedron, i.e. is "inside" it) represents the universe emerging from the quantum vacuum, which over time increases entropy and undergoes various symmetry breakings. This occurs in a regime of eternal inflation.

From the following expression

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×C×R)))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\,C\,R\right)}}$$

which comes from the DN Constant, with $1.616255*10^{-35}$ which is equal to the Planck length, $1.1056*10^{-52}$ which is equal to the Cosmological Constant, C = 1729 which corresponds to the Hardy-Ramanujan number and R = $4.6018401361 \times 10^{-24}$, which represents the radius of the Universe, we obtain:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×4.6018401361 × 10^-24))) = 1.6180329973...

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 4.6018401361\times 10^{-24}\right)}}$$

1.6180329973075324915067570166297467464225772608929671407919395903
…

1.6180329973075… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We have also the following formula:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(4.4525642121 × 10^-24))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times10^{-35}}{1.1056\times10^{-52}}\times 1729\times 4.4525642121\times 10^{-24}\right)}}$$

Result

1.64493235210209213048389898370415116887662184165516687791416603 38…

1.64493235210209213…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

And again:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(1.2206935225 × 10^-24))))

Input interpretation

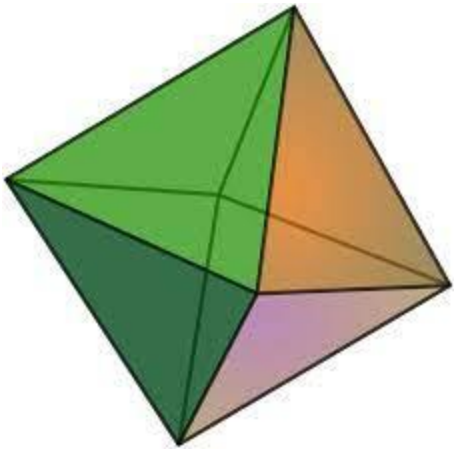
$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times10^{-35}}{1.1056\times10^{-52}}\times 1729\times 1.2206935225\times 10^{-24}\right)}}$$

Result

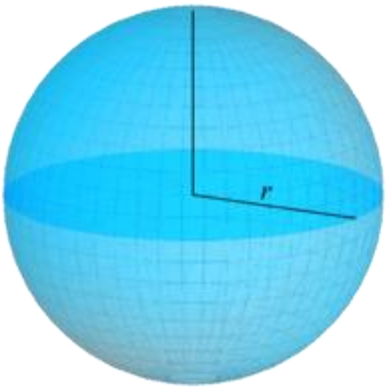
3.14159225731469939399500392316057964697311717816444239557900247 97…

3.14159225731469…≈ π (Ramanujan Recurring Number)

Now, we have that:



Octahedron



Sphere

Given the value of a volume, independently of the solid, following the Poincaré Conjecture, we compare any solid "without holes" and a sphere. If we compare an octahedron with a sphere, we have:

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

If we consider the radius of the sphere as an unknown, we must find the value of the side of the octahedron which allows us to equalize the two volumes and which will give us the DN Constant as a result (which will therefore be equal to the radius of the sphere).

From

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

To find r we perform the following calculation:

$$r^3 = \frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi} = \frac{1}{3}\sqrt{2}a^3 \cdot \frac{3}{4\pi} = \frac{\sqrt{2}a^3}{4\pi} = \frac{\sqrt{2}\cdot\sqrt{2}a^3}{\sqrt{2}\cdot 4\pi} = \frac{2a^3}{\sqrt{2}\cdot 4\pi} = \frac{a^3}{2\sqrt{2}\pi}$$

$$r^3 = \frac{a^3}{2\sqrt{2}\pi}; \qquad r = \sqrt[3]{\frac{a^3}{2\sqrt{2}\pi}} = \frac{a}{\sqrt[3]{2\sqrt{2}\pi}}$$

To find *a*, we have, for

$$r = \frac{2\sqrt{2}}{\pi}; \qquad \frac{a}{\sqrt[3]{2\sqrt{2}\pi}} = \frac{2\sqrt{2}}{\pi};$$

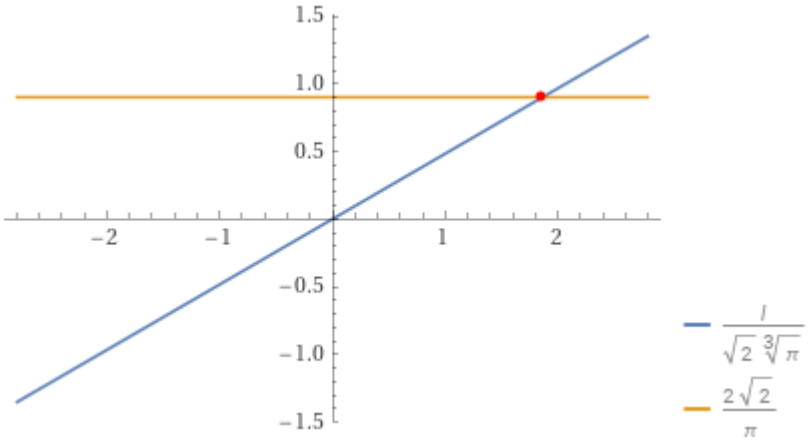
Thus, multiplying both the sides by $\sqrt[3]{2\sqrt{2}\pi}$, we obtain:

$$a = \frac{2\sqrt{2}}{\pi} \cdot \sqrt[3]{2\sqrt{2}\pi} = \sqrt[3]{\frac{(2\sqrt{2})^3 2\sqrt{2}\pi}{\pi^3}} =$$

$$= \sqrt[3]{\frac{8 \cdot \sqrt{2^3} \cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{8 \cdot \sqrt{2 \cdot 2^2} \cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{16 \cdot \sqrt{2} \cdot 2\sqrt{2}}{\pi^2}}$$

$$= \sqrt[3]{\frac{32\sqrt{2}\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{32 \cdot 2}{\pi^2}} = \frac{\sqrt[3]{64}}{\sqrt[3]{\pi^2}} = \frac{4}{\sqrt[3]{\pi^2}}; \; a = \frac{4}{\sqrt[3]{\pi^2}}$$

Plot



Solution

$a = \frac{4}{\sqrt[3]{\pi^2}}$

for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), we obtain:

from the octahedron volume, we have: V = 1/3*√2*a^3 = (1/3*√2*($\frac{4}{\sqrt[3]{\pi^2}}$)^3)

(1/3*√2*(4/∛(π^2))^3)

Input

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3$$

$\sqrt[3]{x}$ is the real-valued 3rd root of x

Exact result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{3\pi^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And, from the sphere volume V = (4/3*π*r^3) = (4/3*π*((2√2)/π)^3)

(4/3*π*((2√2)/π)^3)

Input

$$\frac{4}{3}\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3$$

Result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\sqrt{z_0}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\exp^3\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,3}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3}{3\pi^2}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\left(\frac{1}{z_0}\right)^{3/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{3/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

$1.64381856858\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\text{^}\,(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

$1.6272016\ldots * 10^{-6}$

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10^{-6}$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

–5.010882647757...

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\text{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874} + b}{M}$$

$$\frac{\partial M(b,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245 + 18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M} = \frac{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b} = \frac{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right\} = 0 \text{ at } (b, M) = (-16, 0)$$

Global minima:

$$\min\left\{\frac{1}{3}\,M^2\left(1 - \frac{(b\,2)\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\,\sqrt{6}}\right)^2\right\} = 0$$

$$\text{for } b = -\frac{226\,802\,245}{18\,480\,874}$$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

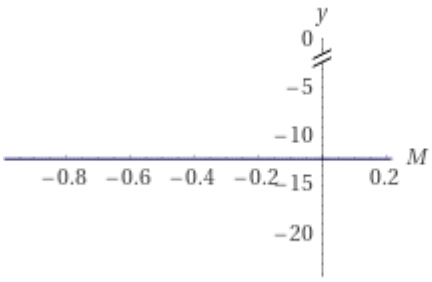
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

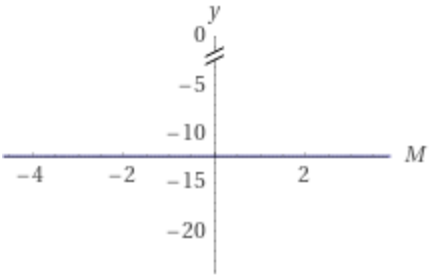
Result:

$$\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723\right)+O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

-12.2723

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times10^{-15}}{M}$$

Indefinite integral:

$$\int\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3}(0.0814845\,b + 1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

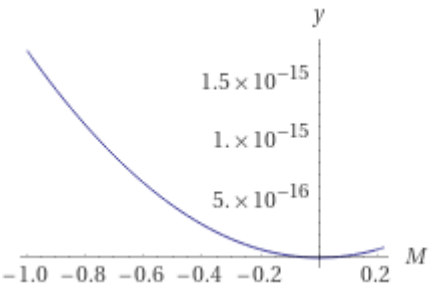
Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

Result:

0

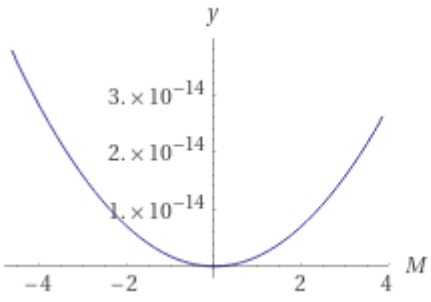
Plots: **(possible mathematical connection with an open string)**



(M from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O\left(M^{62194}\right)$

(Taylor series)

Series expansion at M = ∞:

$1.75541 \times 10^{-15}\, M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.388513449474645453489707833780880208333333333333333333\ldots\times10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^2+6.58545\times10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times0.2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-17}$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875 \times 10^{-14}$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\, M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 2^2 + 6.58545 \times 10^{-10}\sqrt{2^4}\right)}{2^2} + 1\right)^2 \times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots \times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots\times 10^{-8}$

5.9776991059*10^-8 result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E}=\mathbf{E}_{\mathrm{P}}l_{\mathrm{P}}^{2}=\phi_{\mathrm{P}}l_{\mathrm{P}}=\sqrt{\frac{\hbar c}{\varepsilon_{0}}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)])))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left((1/(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}))\,\hat{}\,(1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

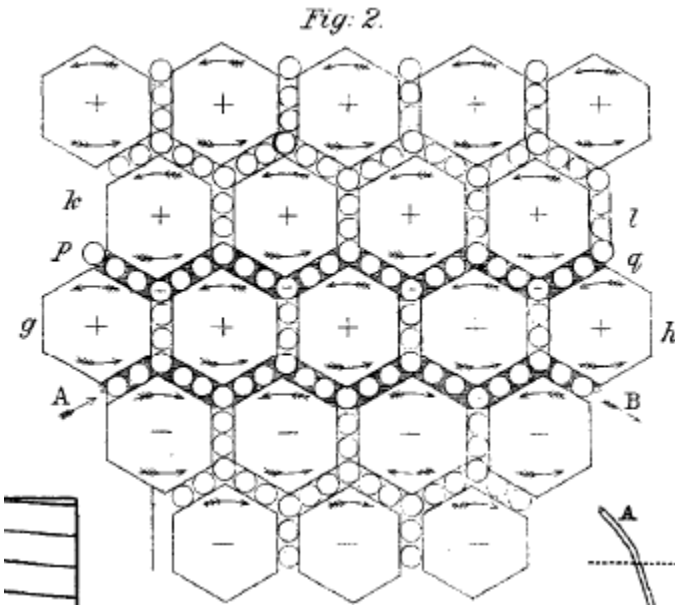
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

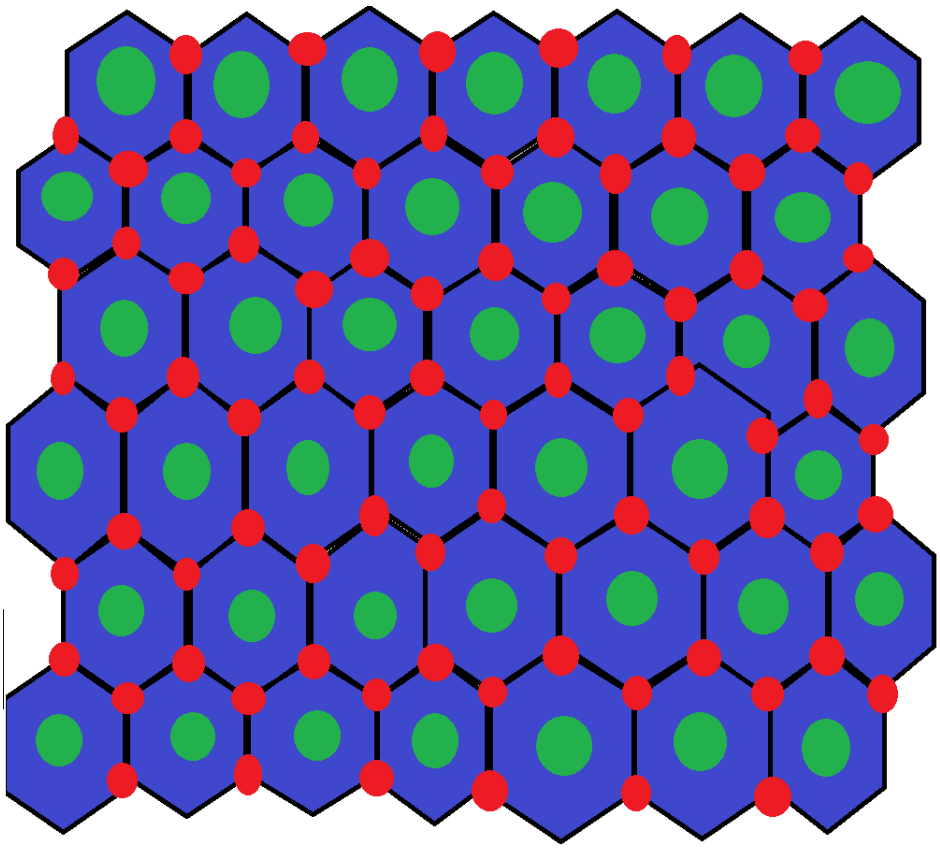
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability.

References

https://www.youtube.com/watch?v=x23O-p62fss

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the analysis of various Cubic Equations that can be solved with Cardano's formula. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory

Michele Nardelli[1], Antonio Nardelli

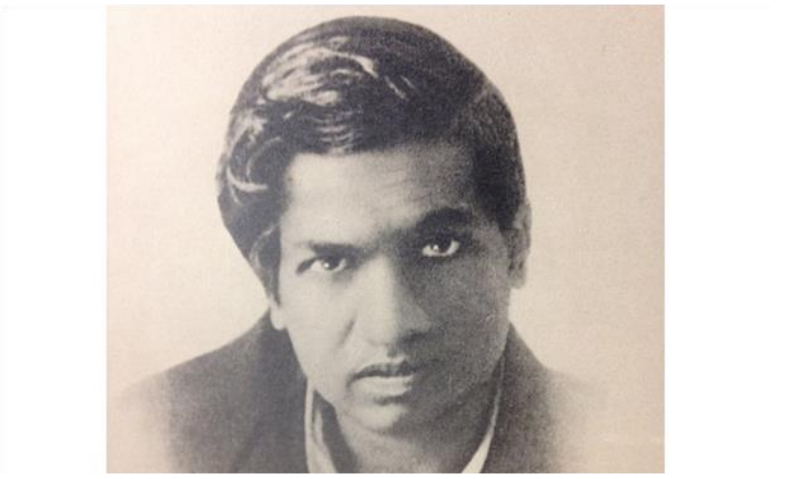
Abstract

In this paper, we analyze various Cubic Equations that can be solved with Cardano's formula. We obtain, new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Introduction

Cardano's method for cubic equations involves cube roots and introduces symmetry concepts. Similarly, string theory and M-theory involve symmetries and transformation groups that play a fundamental role in describing particles and interactions. Solutions of complete cubic equations via Cardano's method involve complex cube roots. Similarly, equations and solutions in superstring theory can often involve advanced algebraic and mathematical complexities. Solutions of cubic equations can be understood geometrically through topology and algebraic geometry. In string theory, the geometric and topological configurations of strings and branes are also central. Duality is an important concept in string theory, where different mathematical descriptions can be equivalent. Similarly, in solving cubic equations, duality links exist through cube roots.

Thus, in string theory, duality refers to the fact that different mathematical formulations or physical descriptions of a theory can be equivalent. For example, two seemingly different theories can be dual to each other, meaning they are two different ways of describing the same physical reality. This concept of duality has been very useful for understanding profound aspects of string theory and field theories.

Similarly, in solving cubic equations, duality can emerge through cube roots. The cubic equation $ax^3+bx^2+cx+d=0$ can be solved using cube roots, as in the Cardano method I described above. However, cube roots involve the concept of complex numbers, and it is sometimes possible to express the solutions in ways that highlight a duality between the solutions.

For example, if we consider complex cube roots, we can see a duality between them, since there are connections via the cube roots of unity. This duality is similar to the concept of duality in string theory, where different descriptions can be connected by similar transformations.

In both cases, duality provides a deeper way of understanding the underlying mathematical or physical structure, allowing you to explore hidden connections between different concepts.

Cubic surfaces can be associated with geometric and topological concepts. In string theory, for example, some models require spaces with particular shapes, and the geometry of the spaces can influence the properties of strings and particles. Some mathematical and physical models involving surfaces might be described by cubic or

complex equations. The geometry of surfaces and the associated mathematical structure can influence the properties of universe bubbles in the multiverse. Cubic surfaces can represent certain topological spaces, and their properties can affect the structure of the associated string theory.

The connection between the surface of a D-brane and the roots of a cubic equation can be explored in specific contexts within string theory, especially when it comes to studying the geometric and topological properties of D-branes. D-branes are extended objects on whose surface open strings can terminate. The geometry of the D-brane surface can interfere with the physics of surrounding strings. In some cases, geometry can be described mathematically through objects known in algebraic surface theory, such as cubic equations.

D-branes can host open fields, which describe particles interacting with the D-brane. Field equations on a D-brane can involve surface geometry, and in some situations, these equations can be associated with higher degree algebraic equations, such as cubic equations.

The compactification of extra dimensions, which is an important part of string theory, can be associated with the surface geometry of a D-brane. Compactification can lead to a reduction in dimensions and a geometric description that can involve algebraic equations, including cubic equations.

The topological invariants associated with the surface of a D-brane, such as the Betti number or the Chern number, can be related to geometric characteristics that can be described via algebraic equations, including cubic equations.

In some situations of duality, the description of a D-brane in a context can be dualistic to a description based on algebraic objects such as curves or surfaces, which can be represented via algebraic equations, including cubic equations.

In this paper, we will hypothesize that the solutions of the cubic equations can be used as boundary conditions to further the understanding of the formation of universe bubbles in eternal inflation. Furthermore, an octahedron could serve as a mathematical or conceptual model of the universe in the pre-Big Bang phase. Cubic equations, in this context, could represent the geometric characteristics of the quantum vacuum, while the spherical surface could be used to describe the geometry of the universe bubble.

The results $(2\sqrt{2})/\pi$, the golden ratio φ and π obtained, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and universe bubbles in eternal inflation. It may reflect a fundamental relationship between the geometry of solids and the mathematical characteristics of solutions to cubic equations.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and universe bubbles.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of universe bubbles and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

Finally, it is also possible to hypothesize the solutions of Ramanujan's cubic equations which are related to the cubic equations solvable with the Cardano method and Padovan's recurrence relations, as boundary conditions to deepen the understanding of the formation of inflationary universe bubbles eternal and of an octahedron which could represent a mathematical or conceptual model of the primordial universe before the Big Bang. Furthermore, regarding the cubic equations, they can represent the geometric characteristics of the quantum vacuum, while the spherical surface represents the geometry of the universe bubble. The results above $(2\sqrt{2})/\pi$, the golden ratio φ and π which are obtained, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

These mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe. The cubic equations, in this context, could reflect the geometric and dynamic characteristics of the quantum vacuum, while the spherical surface could represent the geometry of the bubbles of the universe.

Regarding fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the primordial universe and universe bubbles.

In conclusion, the multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

From

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

Cubic equations and Cardano's Formulas

We analyze the following equation:

$$x^3 + 15x = 6x^2 + 24.$$

i.e.

$$x^3 - 6x^2 + 15x - 24 = 0$$

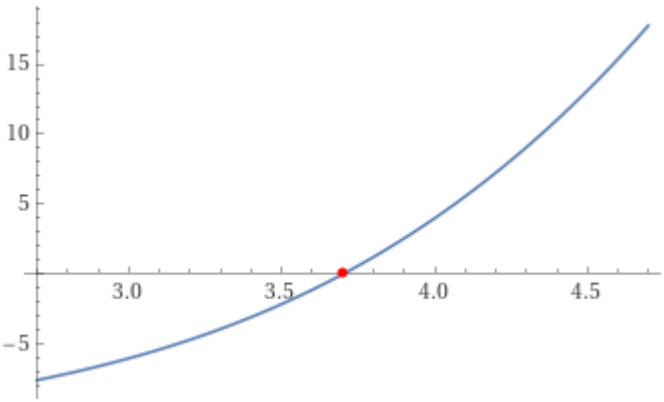
We obtain:

x^3-6x^2+15x-24 = 0

Input

$x^3 - 6x^2 + 15x - 24 = 0$

Root plot **(figure that can be related to an open string)**



Alternate forms

$x((x-6)x+15)-24=0$

$x^3-6x^2+15x=24$

$(x-2)^3+3(x-2)-10=0$

Real solution

$$x=2-\frac{1}{\sqrt[3]{5+\sqrt{26}}}+\sqrt[3]{5+\sqrt{26}}$$

Complex solutions

$$x=2+\frac{1}{2\sqrt[3]{5+\sqrt{26}}}-\frac{1}{2}\sqrt[3]{5+\sqrt{26}}+i\left(-\frac{\sqrt{3}}{2\sqrt[3]{5+\sqrt{26}}}-\frac{1}{2}\sqrt{3}\sqrt[3]{5+\sqrt{26}}\right)$$

$$x=2+\frac{1}{2\sqrt[3]{5+\sqrt{26}}}-\frac{1}{2}\sqrt[3]{5+\sqrt{26}}+i\left(\frac{\sqrt{3}}{2\sqrt[3]{5+\sqrt{26}}}+\frac{1}{2}\sqrt{3}\sqrt[3]{5+\sqrt{26}}\right)$$

Sum of roots

6

Product of roots

24

From

$$ax^3 + bx^2 + cx + d = 0$$

The Cardano's Formula is:

$$x = \sqrt[3]{-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a} + \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} + \sqrt[3]{-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a} - \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}$$

$$x^3 - 6x^2 + 15x - 24 = 0$$

(216/27-90/6+24/2+sqrt((216/27-90/6+24/2)^2+(15/3-36/9)^3))^1/3+(216/27-90/6+24/2-sqrt((216/27-90/6+24/2)^2+(15/3-36/9)^3))^1/3+2

Input

$$\sqrt[3]{\frac{216}{27} - \frac{90}{6} + \frac{24}{2} + \sqrt{\left(\frac{216}{27} - \frac{90}{6} + \frac{24}{2}\right)^2 + \left(\frac{15}{3} - \frac{36}{9}\right)^3}} + \sqrt[3]{\frac{216}{27} - \frac{90}{6} + \frac{24}{2} - \sqrt{\left(\frac{216}{27} - \frac{90}{6} + \frac{24}{2}\right)^2 + \left(\frac{15}{3} - \frac{36}{9}\right)^3}} + 2$$

Result

$$\sqrt[3]{5 + \sqrt{26}} + \sqrt[3]{5 - \sqrt{26}} + 2$$

Decimal approximation

4.392840829717887332797771045621074566917005764479774911011437 38…

+

0.400655302280422003101750634895722812426830395790179264350794142…

i

Alternate complex forms

$$\left(2+\frac{1}{2}\sqrt[3]{-5+\sqrt{26}}+\sqrt[3]{5+\sqrt{26}}\right)+\frac{1}{2}\left(\sqrt{3}\sqrt[3]{-5+\sqrt{26}}\right)i$$

$$\sqrt{\frac{3}{4}\left(\sqrt{26}-5\right)^{2/3}+\left(2+\frac{1}{2}\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}\right)^2}\left(\cos\left(\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{26}-5}}{2\left(2+\frac{1}{2}\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}\right)}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{26}-5}}{2\left(2+\frac{1}{2}\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}\right)}\right)\right)\right)$$

$$\sqrt{\frac{3}{4}\left(\sqrt{26}-5\right)^{2/3}+\left(2+\frac{1}{2}\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}\right)^2}\exp\left(i\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{26}-5}}{2\left(2+\frac{1}{2}\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}\right)}\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r = \sqrt{\frac{3}{4}\left(\sqrt{26}-5\right)^{2/3} + \left(2 + \frac{1}{2}\sqrt[3]{\sqrt{26}-5} + \sqrt[3]{5+\sqrt{26}}\right)^2}$$

(radius), $\theta = \tan^{-1}\left(\frac{\sqrt{3}\,\sqrt[3]{\sqrt{26}-5}}{2\left(2 + \frac{1}{2}\sqrt[3]{\sqrt{26}-5} + \sqrt[3]{5+\sqrt{26}}\right)}\right)$ (angle)

Alternate forms

root of $x^6 - 12x^5 + 57x^4 - 156x^3 + 297x^2 - 342x + 252$
near $x = 4.39284 + 0.400655\,i$

$$2 + \frac{1}{2}\sqrt[3]{\sqrt{26}-5} + \frac{1}{2}i\sqrt{3}\,\sqrt[3]{\sqrt{26}-5} + \sqrt[3]{5+\sqrt{26}}$$

Minimal polynomial

$$x^6 - 12x^5 + 57x^4 - 156x^3 + 297x^2 - 342x + 252$$

From

$$x^3 - 6x^2 + 15x - 24 = 0$$

From the result

$$\sqrt[3]{5+\sqrt{26}} + \sqrt[3]{5-\sqrt{26}} + 2$$

for x = ((5 + sqrt(26))^(1/3) - (-5 + sqrt(26))^(1/3) + 2)

((5 + sqrt(26))^(1/3) - (-5 + sqrt(26))^(1/3) + 2)

Input

$$x = \sqrt[3]{5+\sqrt{26}} - \sqrt[3]{-5+\sqrt{26}} + 2$$

Decimal approximation

3.69888548984632975802611998592227601649960319697285759193226943 13

…

3.698885489846329….

Alternate forms

root of $x^3 - 6x^2 + 15x - 24$ near $x = 3.69889$

$$2 - \frac{1}{\sqrt[3]{5+\sqrt{26}}} + \sqrt[3]{5+\sqrt{26}}$$

Minimal polynomial

$x^3 - 6x^2 + 15x - 24$

Indeed:

$x^3 - 6x^2 + 15x - 24$

for x = ((5 + sqrt(26))^(1/3) - (-5 + sqrt(26))^(1/3) + 2)

we obtain:

((5 + sqrt(26))^(1/3) - (-5 + sqrt(26))^(1/3) + 2)^3-6*((5 + sqrt(26))^(1/3) - (-5 + sqrt(26))^(1/3) + 2)^2+15*((5 + sqrt(26))^(1/3) - (-5 + sqrt(26))^(1/3) + 2)-24

Input

$$\left(\sqrt[3]{5+\sqrt{26}} - \sqrt[3]{-5+\sqrt{26}} + 2\right)^3 - 6\left(\sqrt[3]{5+\sqrt{26}} - \sqrt[3]{-5+\sqrt{26}} + 2\right)^2 + 15\left(\sqrt[3]{5+\sqrt{26}} - \sqrt[3]{-5+\sqrt{26}} + 2\right) - 24$$

Result

0

From the previous expression

$$2-\frac{1}{\sqrt[3]{5+\sqrt{26}}}+\sqrt[3]{5+\sqrt{26}}$$

we obtain also:

1+1/((((5 + sqrt(26))^(1/3) - (-5 + sqrt(26))^(1/3) + 2))^1/3)-2(MRB const)^(1-1/(4π)+π)

Input

$$1+\frac{1}{\sqrt[3]{\sqrt[3]{5+\sqrt{26}}-\sqrt[3]{-5+\sqrt{26}}+2}}-2\,C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Decimal approximation

1.644365333799645726100360764105709887126739403690001252154783239 2

…

1.644365333799645…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$\text{root of } 24x^9-15x^6+6x^3-1 \text{ near } x=0.646611 \quad -2\,C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+1$$

$$-2\,C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+1+\frac{1}{2\sqrt[3]{\frac{3}{5-\frac{23}{\sqrt[3]{53+24\sqrt{26}}}+\sqrt[3]{53+24\sqrt{26}}}}}$$

$$\frac{1+\sqrt[3]{2-\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}}}{\sqrt[3]{2-\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}}}-2\,{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

And again:

sqrt(1/(1+1/((((5 + sqrt(26))^(1/3) - (-5 + sqrt(26))^(1/3) + 2))^1/3)-2(MRB const)^(1-1/(4π)+π))*(4/3))

Input

$$\sqrt{\frac{1}{1+\frac{1}{\sqrt[3]{\sqrt[3]{5+\sqrt{26}}-\sqrt[3]{-5+\sqrt{26}}+2}}-2\,{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}}\times\frac{4}{3}}$$

C_{MRB} is the MRB constant

Exact result

$$\frac{2}{\sqrt{3\left(-2\,{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+1+\frac{1}{\sqrt[3]{2-\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}}}\right)}}$$

Decimal approximation

0.900471997916444307735118015047501660021372549769674254450810745 9…

$0.90047\ldots \approx 0.9003163161571\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed form

$$\frac{2\sqrt{2}}{\pi} \approx 0.90031631$$

Alternate forms

$$2\Big/\left(\sqrt{\left(3\left(\text{root of } 24x^9-15x^6+6x^3-1 \text{ near } x=0.646611\right)^{2C_{\text{MRB}}^{1-1/(4\pi)+\pi}+1}\right)}\right)$$

$$\frac{2}{\sqrt{3\left(-2C_{\text{MRB}}^{1-1/(4\pi)+\pi}+1+\frac{1}{2\sqrt[3]{5-\frac{23}{\sqrt[3]{53+24\sqrt{26}}}+\sqrt[3]{53+24\sqrt{26}}}}\right)}}$$

$$\left(2\sqrt[8\pi]{C_{\text{MRB}}}\right)\Big/\left(\sqrt{\left(\left(3\left(-2\sqrt[4\pi]{C_{\text{MRB}}}+\sqrt[3]{\sqrt{26}-5}\sqrt[4\pi]{C_{\text{MRB}}}-\sqrt[3]{5+\sqrt{26}}\sqrt[4\pi]{C_{\text{MRB}}}-\left(2-\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}\right)^{2/3}\sqrt[4\pi]{C_{\text{MRB}}}+4C_{\text{MRB}}^{1+\pi}-2\sqrt[3]{\sqrt{26}-5}\,C_{\text{MRB}}^{1+\pi}+2\sqrt[3]{5+\sqrt{26}}\,C_{\text{MRB}}^{1+\pi}\right)\Big/\left(-2+\sqrt[3]{\sqrt{26}-5}-\sqrt[3]{5+\sqrt{26}}\right)\right)\right)}\right)$$

All 2nd roots of 4/(3 (-2 $C_{MRB}^{1-1/(4\pi)+\pi}$+1+1/(2-(sqrt(26)-5)^{1/3}+(5+sqrt(26))^{1/3})^{1/3}))

$$\frac{2e^0}{\sqrt{3\left(-2C_{\text{MRB}}^{1-1/(4\pi)+\pi}+1+\frac{1}{\sqrt[3]{2-\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}}}\right)}} \approx 0.9005 \text{ (real, principal root)}$$

$$\frac{2\,e^{i\,\pi}}{\sqrt{3\left(-2\,C_{\text{MRB}}{}^{1-1/(4\,\pi)+\pi}+1+\frac{1}{\sqrt[3]{2-\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}}}\right)}}\approx -0.9005 \quad \text{(real root)}$$

From the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)d^3\times\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\times\frac{1}{3}\sqrt{2}d^3\times\frac{\sqrt{2}}{12}d^3}$$

From

$$\sqrt[3]{5+\sqrt{26}}\;-\sqrt[3]{-5+\sqrt{26}}\;+2$$

we obtain also:

((5 + sqrt(26))^(1/3)-(-5 +sqrt(26))^(1/3)+2) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

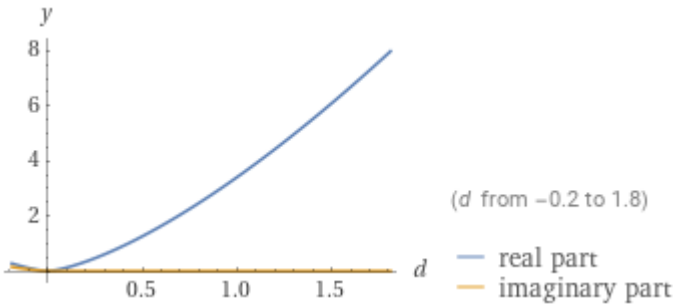
Input

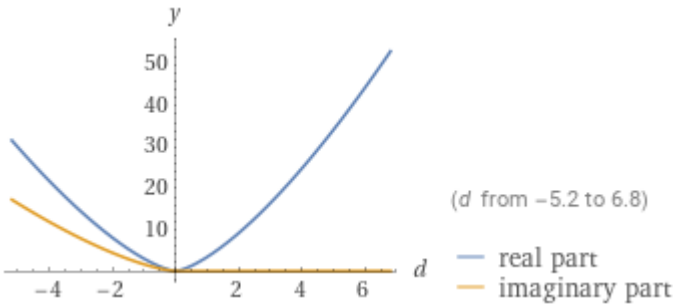
$$\left(\sqrt[3]{5+\sqrt{26}}\ -\sqrt[3]{-5+\sqrt{26}}\ +2\right)\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Exact result

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(2-\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}\right)\sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate form

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}$$

root of $x^3-6x^2+15x-24$ near $x=3.69889$

Expanded form

$$-2^{-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{\sqrt{26}-5}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ +$$
$$2^{1-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ +$$
$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{5+\sqrt{26}}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}$$

Root

$d=0$

Series expansion at d=∞

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \left(2-\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}\right)\sqrt[\pi]{\frac{\pi}{4}}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\left(\sqrt[3]{5+\sqrt{26}}-\sqrt[3]{-5+\sqrt{26}}+2\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$
$$2^{-(2+\pi)/\pi}\times 3^{2-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \left(2-\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}\right)d^8\left(d^9\right)^{1/(2\pi)-1}$$
$$\pi^{1/\pi-1}$$

Indefinite integral

$$\int 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \left(2-\sqrt[3]{-5+\sqrt{26}}+\sqrt[3]{5+\sqrt{26}}\right)\sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}}\ dd=$$
$$\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \left(2-\sqrt[3]{\sqrt{26}-5}+\sqrt[3]{5+\sqrt{26}}\right)\sqrt[\pi]{\frac{\pi}{4}}\ d\sqrt[2\pi]{d^9}}{1+\frac{9}{2\pi}}+\text{constant}$$

Global minimum

$$\min\left\{\left(\sqrt[3]{5+\sqrt{26}}-\sqrt[3]{-5+\sqrt{26}}+2\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right\}=0$$

at $d=0$

Series representations

$$\left(\sqrt[3]{5+\sqrt{26}}-\sqrt[3]{-5+\sqrt{26}}+2\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=$$

$$-2^{-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\left(-2+\sqrt[3]{-5+\sqrt{25}\sum_{k=0}^{\infty}25^{-k}\binom{\frac{1}{2}}{k}}-\sqrt[3]{5+\sqrt{25}\sum_{k=0}^{\infty}25^{-k}\binom{\frac{1}{2}}{k}}\right)$$

$$\left(\sqrt[3]{5+\sqrt{26}}-\sqrt[3]{-5+\sqrt{26}}+2\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=$$

$$-2^{-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\left(-2+\sqrt[3]{-5+\sqrt{25}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{25}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}-\sqrt[3]{5+\sqrt{25}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{25}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$\left(\sqrt[3]{5+\sqrt{26}}\ -\sqrt[3]{-5+\sqrt{26}}\ +2\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}\ =$$

$$-2^{-2/\pi}\times 3^{-3/(2\pi)}\ \sqrt[2\pi]{5}\ \sqrt[2\pi]{d^9\,\pi^2\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}$$

$$\left(-2+\sqrt[3]{-5+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(26-z_0)^k\,z_0^{-k}}{k!}}\ -\right.$$

$$\left.\sqrt[3]{5+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(26-z_0)^k\,z_0^{-k}}{k!}}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

And:

((5 + sqrt(26))^(1/3)-(-5 +sqrt(26))^(1/3)+2) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

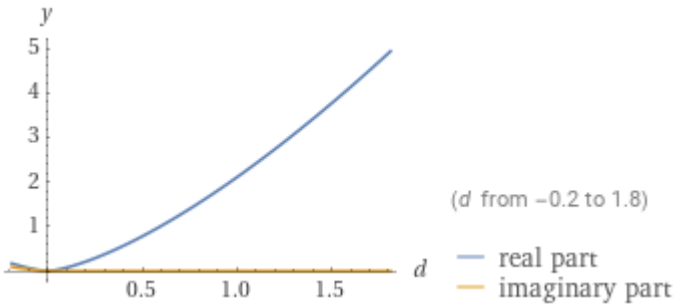
Input

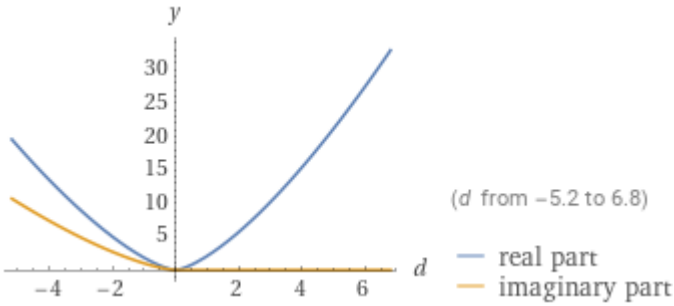
$$\left(\sqrt[3]{5+\sqrt{26}}\ -\sqrt[3]{-5+\sqrt{26}}\ +2\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\ d^3\right)\left(\frac{\sqrt{2}}{12}d^3\right)}$$

Exact result

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\left(2-\sqrt[3]{\sqrt{26}\ -5}\ +\sqrt[3]{5+\sqrt{26}}\ \right)^{2\pi}\sqrt{\pi}\ \sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate forms

$2^{-1/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{\pi} \sqrt[2\pi]{d^9}$ root of $x^3 - 6x^2 + 15x - 24$ near $x = 3.69889$

$\left(2^{1-1/\pi} \times 3^{-3/(2\pi)} - 2^{-1/\pi} \times 3^{-3/(2\pi)} \sqrt[3]{\sqrt{26} - 5} + 2^{-1/\pi} \times 3^{-3/(2\pi)} \sqrt[3]{5 + \sqrt{26}}\right) \sqrt[2\pi]{\pi}$
$\sqrt[2\pi]{d^9}$

Expanded form

$2^{-1/\pi} \times 3^{-3/(2\pi)} \sqrt[3]{5 + \sqrt{26}} \sqrt[2\pi]{\pi} \sqrt[2\pi]{d^9} - 2^{-1/\pi} \times 3^{-3/(2\pi)} \sqrt[3]{\sqrt{26} - 5} \sqrt[2\pi]{\pi} \sqrt[2\pi]{d^9} +$
$2^{1-1/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{\pi} \sqrt[2\pi]{d^9}$

Root

$d = 0$

Series expansion at d=∞

$$2^{-1/\pi} \times 3^{-3/(2\pi)} \left(2 - \sqrt[3]{\sqrt{26} - 5} + \sqrt[3]{5 + \sqrt{26}}\right)^{2\pi\sqrt{\pi}}\, d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\left(\sqrt[3]{5 + \sqrt{26}} - \sqrt[3]{-5 + \sqrt{26}} + 2\right)^{2\pi}\sqrt{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\, d^3\right)\left(\sqrt{2}\, d^3\right)}{3 \times 3 \times 12}}\right) =$$

$$\frac{2^{-(1+\pi)/\pi} \times 3^{2-3/(2\pi)} \left(2 - \sqrt[3]{\sqrt{26} - 5} + \sqrt[3]{5 + \sqrt{26}}\right)^{2\pi\sqrt{d^9}}\, \pi^{1/(2\pi)-1}}{d}$$

Indefinite integral

$$\int 2^{-1/\pi} \times 3^{-3/(2\pi)} \left(2 - \sqrt[3]{-5 + \sqrt{26}} + \sqrt[3]{5 + \sqrt{26}}\right)^{2\pi\sqrt{d^9}\ 2\pi\sqrt{\pi}}\, dd =$$

$$\frac{2^{(\pi-1)/\pi} \times 3^{-3/(2\pi)} \left(2 - \sqrt[3]{\sqrt{26} - 5} + \sqrt[3]{5 + \sqrt{26}}\right) \pi^{1+1/(2\pi)}\, d^{2\pi\sqrt{d^9}}}{9 + 2\pi} + \text{constant}$$

Global minimum

$$\min\left\{\left(\sqrt[3]{5 + \sqrt{26}} - \sqrt[3]{-5 + \sqrt{26}} + 2\right)^{2\pi}\sqrt{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\, d^3\right)\left(\sqrt{2}\, d^3\right)}{3 \times 3 \times 12}}\right\} = 0$$

at $d = 0$

Series representations

$$\left(\sqrt[3]{5+\sqrt{26}}-\sqrt[3]{-5+\sqrt{26}}+2\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$
$$-6^{-3/(2\pi)}\sqrt[2\pi]{d^9\,\pi\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}$$
$$\left(-2+\sqrt[3]{-5+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(26-z_0)^k\,z_0^{-k}}{k!}}-\right.$$
$$\left.\sqrt[3]{5+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(26-z_0)^k\,z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\left(\sqrt[3]{5+\sqrt{26}}-\sqrt[3]{-5+\sqrt{26}}+2\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$
$$-6^{-3/(2\pi)}\sqrt[2\pi]{d^9\,\pi\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$
$$\left(-2+\sqrt[3]{-5+\exp\left(i\pi\left\lfloor\frac{\arg(26-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(26-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}-\right.$$
$$\left.\sqrt[3]{5+\exp\left(i\pi\left\lfloor\frac{\arg(26-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(26-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\left(\sqrt[3]{5+\sqrt{26}}\ -\sqrt[3]{-5+\sqrt{26}}\ +2\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=-6^{-3/(2\pi)}$$

$$\sqrt[2\pi]{d^9\,\pi\left(\frac{1}{z_0}\right)^{\lfloor \arg(2-z_0)/(2\pi)\rfloor}\ z_0^{1+\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}$$

$$\left(-2+\left(-5+\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(26-z_0)/(2\pi)\rfloor}\ z_0^{1/2+1/2\,\lfloor \arg(26-z_0)/(2\pi)\rfloor}\right.\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(26-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)-$$

$$\left(5+\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(26-z_0)/(2\pi)\rfloor}\ z_0^{1/2+1/2\,\lfloor \arg(26-z_0)/(2\pi)\rfloor}\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(26-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Dividing the two previous indefinite integral results

$$\int 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(2-\sqrt[3]{-5+\sqrt{26}}\ +\sqrt[3]{5+\sqrt{26}}\ \right)\sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}}\ dd=$$

$$\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(2-\sqrt[3]{\sqrt{26}-5}\ +\sqrt[3]{5+\sqrt{26}}\ \right)\sqrt[\pi]{\frac{\pi}{4}}\ d^{\,2\pi}\sqrt{d^9}}{1+\frac{9}{2\pi}}+\text{constant}$$

and

$$\int 2^{-1/\pi} \times 3^{-3/(2\pi)} \left(2 - \sqrt[3]{-5+\sqrt{26}} + \sqrt[3]{5+\sqrt{26}}\right) \sqrt[2\pi]{d^9}\ \sqrt[2\pi]{\pi}\ dd =$$
$$\frac{2^{(\pi-1)/\pi} \times 3^{-3/(2\pi)} \left(2 - \sqrt[3]{\sqrt{26}-5} + \sqrt[3]{5+\sqrt{26}}\right) \pi^{1+1/(2\pi)}\ d\ \sqrt[2\pi]{d^9}}{9+2\pi} + \text{constant}$$

we obtain:

((3^(-3/(2π))(5(3+√(5)))^(1/(2π))(2-(-5+√(26))^(1/3)+(5+√(26))^(1/3))(π/4)^(1/π))/(1+9/(2π)))/((2^((-1+π)/π) 3^(-3/(2π))(2-(-5+√(26))^(1/3)+(5+√(26))^(1/3)) π^(1+1/(2π)))/(9+2π))

Input

$$\frac{\frac{3^{-3/(2\pi)}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \left(2-\sqrt[3]{-5+\sqrt{26}}+\sqrt[3]{5+\sqrt{26}}\right)\sqrt[\pi]{\frac{\pi}{4}}}{1+\frac{9}{2\pi}}}{\frac{2^{(-1+\pi)/\pi}\left(3^{-3/(2\pi)}\left(2-\sqrt[3]{-5+\sqrt{26}}+\sqrt[3]{5+\sqrt{26}}\right)\right)\pi^{1+1/(2\pi)}}{9+2\pi}}$$

Exact result

$$\frac{2^{-2/\pi-(\pi-1)/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \pi^{1/(2\pi)-1}\ (9+2\pi)}{1+\frac{9}{2\pi}}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Expanded form

$$\frac{9 \times 2^{-1-1/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \pi^{1/(2\pi)-1}}{1+\frac{9}{2\pi}} + \frac{2^{-1/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}}{1+\frac{9}{2\pi}}$$

Alternate forms

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

$$\frac{2^{-1/\pi}\left(2\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\,\pi^{1+1/(2\pi)}+9\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right)}{9+2\pi}$$

Series representations

$$\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(2-\sqrt[3]{-5+\sqrt{26}}+\sqrt[3]{5+\sqrt{26}}\right)\sqrt[\pi]{\frac{\pi}{4}}}{\frac{\left(2^{(-1+\pi)/\pi}\left(3^{-3/(2\pi)}\left(2-\sqrt[3]{-5+\sqrt{26}}+\sqrt[3]{5+\sqrt{26}}\right)\right)\pi^{1+1/(2\pi)}\right)\left(1+\frac{9}{2\pi}\right)}{9+2\pi}}=$$

$$2^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(2-\sqrt[3]{-5+\sqrt{26}}+\sqrt[3]{5+\sqrt{26}}\right)\sqrt[\pi]{\frac{\pi}{4}}}{\frac{\left(2^{(-1+\pi)/\pi}\left(3^{-3/(2\pi)}\left(2-\sqrt[3]{-5+\sqrt{26}}+\sqrt[3]{5+\sqrt{26}}\right)\right)\pi^{1+1/(2\pi)}\right)\left(1+\frac{9}{2\pi}\right)}{9+2\pi}}=$$

$$2^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}$$

$$\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(2-\sqrt[3]{-5+\sqrt{26}}+\sqrt[3]{5+\sqrt{26}}\right)\sqrt[\pi]{\frac{\pi}{4}}}{\frac{\left(2^{(-1+\pi)/\pi}\left(3^{-3/(2\pi)}\left(2-\sqrt[3]{-5+\sqrt{26}}+\sqrt[3]{5+\sqrt{26}}\right)\right)\pi^{1+1/(2\pi)}\right)\left(1+\frac{9}{2\pi}\right)}{9+2\pi}}=$$

$$2^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi}\sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the alternate form

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.1415926535884871624221932509858715278931025189245576170078100613
…

3.1415926535884…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298888445046920

…

1.644934066846…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.92478440155986\,\pi}\,\sqrt{\pi}^2}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Now, we analyze the following equation:

$$x^3 + 10x = 6x^2 + 4.$$

i.e.

$$x^3 - 6x^2 + 10x - 4 = 0$$

The Cardano's Formula is:

$$x = \sqrt[3]{-\frac{b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}+\sqrt{\left(-\frac{b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)^2+\left(\frac{c}{3a}-\frac{b^2}{9a^2}\right)^3}} + \sqrt[3]{-\frac{b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}-\sqrt{\left(-\frac{b^3}{27a^3}+\frac{bc}{6a^2}-\frac{d}{2a}\right)^2+\left(\frac{c}{3a}-\frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}$$

Thence, for a = 1, b = -6, c = 10 and d = -4 , we obtain:

(216/27-60/6+4/2+sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3+(216/27-60/6+4/2-sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3+2

Input

$$\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}+\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}} + \sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}-\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}} + 2$$

Result

$$2+\sqrt[6]{-1}\sqrt{\frac{2}{3}}-(-1)^{5/6}\sqrt{\frac{2}{3}}$$

Decimal approximation

3.414213562373095048801688724209698078569671875376948073176679737 9

…

3.414213562373….

Alternate forms

$$\frac{1}{3}\left(\sqrt{6}\left(\sqrt[6]{-1}-(-1)^{5/6}\right)+6\right)$$

$$2+\sqrt{2}$$

$$2-\sqrt{\frac{2}{3}}\left((-1)^{5/6}-\sqrt[6]{-1}\right)$$

Minimal polynomial

$$x^2-4x+2$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

-((216/27-60/6+4/2+sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3+(216/27-60/6+4/2-sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3)+2

Input

$$-\left(\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}+\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}}+\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}-\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}}\right)+2$$

Result

$$2-\sqrt[6]{-1}\sqrt{\frac{2}{3}}+(-1)^{5/6}\sqrt{\frac{2}{3}}$$

Decimal approximation

0.585786437626904951198311275790301921430328124623051926823320262 0

…

0.5857864376269….

Alternate forms

$$\frac{1}{3}\left(\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)+6\right)$$

$$2-\sqrt{2}$$

$$2+\sqrt{\frac{2}{3}}\left((-1)^{5/6}-\sqrt[6]{-1}\right)$$

Minimal polynomial

$$x^2-4x+2$$

(216/27-60/6+4/2+sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3-(216/27-60/6+4/2+sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3+2

Input

$$\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}+\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}}-\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}+\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}}+2$$

Exact result

2

2

From which, we obtain also:

(((216/27-60/6+4/2+sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3-(216/27-60/6+4/2+sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3+2))^12

Input

$$\left(\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}+\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}}-\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}+\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}}+2\right)^{12}$$

Exact result

4096

4096

The number $4096 = 64^2$, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

27*sqrt((((216/27-60/6+4/2+sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3-(216/27-60/6+4/2+sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3+2))^12)+1

Input

$$27\sqrt{\left(\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}+\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}}-\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}+\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}}+2\right)^{12}}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after?

That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(27*sqrt((((216/27-60/6+4/2+sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3-(216/27-60/6+4/2+sqrt((216/27-60/6+4/2)^2+(10/3-36/9)^3))^1/3+2))^12)+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\left(27\sqrt{\left(\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}+\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}}-\sqrt[3]{\frac{216}{27}-\frac{60}{6}+\frac{4}{2}+\sqrt{\left(\frac{216}{27}-\frac{60}{6}+\frac{4}{2}\right)^2+\left(\frac{10}{3}-\frac{36}{9}\right)^3}}+2\right)^{12}+1}\right)\text{^}(1/15)+{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Exact result

$${C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\sqrt[15]{1729}$$

Decimal approximation

1.644938024130260257941733191946366957420242441725816380271305678 2

…

1.64493802413…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate form

$${C_{\text{MRB}}}^{-1/(4\pi)}\left(\sqrt[15]{1729}\ \sqrt[4\pi]{C_{\text{MRB}}}+{C_{\text{MRB}}}^{1+\pi}\right)$$

From the exact result

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \sqrt[15]{1729}$$

we obtain also:

sqrt(1/((MRB const)^(1 - 1/(4 π) + π) + 1729^(1/15))*(4/3))

Input

$$\sqrt{\frac{1}{C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \sqrt[15]{1729}} \times \frac{4}{3}}$$

C_{MRB} is the MRB constant

Exact result

$$\frac{2}{\sqrt{3\left(C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \sqrt[15]{1729}\right)}}$$

Decimal approximation

0.900315233196023884972608268109356447163462693701191890956401135 3…

$0.9003152\ldots \approx 0.9003163161571\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed form

$$\frac{2\sqrt{2}}{\pi} \approx 0.90031\,63161$$

Alternate form

$$\frac{2\sqrt[8\pi]{C_{\text{MRB}}}}{\sqrt{3\left(\sqrt[15]{1729}\sqrt[4\pi]{C_{\text{MRB}}}+{C_{\text{MRB}}}^{1+\pi}\right)}}$$

All 2nd roots of 4/(3 ($C_{MRB}^{1-1/(4\pi)+\pi}+1729^{1/15}$))

$$\frac{2e^{0}}{\sqrt{3\left({C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\sqrt[15]{1729}\right)}}\approx 0.9003 \quad \text{(real, principal root)}$$

$$\frac{2e^{i\pi}}{\sqrt{3\left({C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\sqrt[15]{1729}\right)}}\approx -0.9003 \quad \text{(real root)}$$

From the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3\times\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the alternate form

$$\frac{1}{3}\left(\sqrt{6}\left((-1)^{5/6} - \sqrt[6]{-1}\right) + 6\right)$$

we obtain also:

(1/3 (sqrt(6) ((-1)^(5/6) - (-1)^(1/6)) + 6)) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

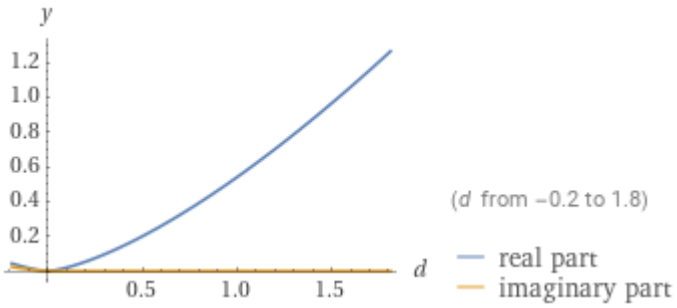
Input

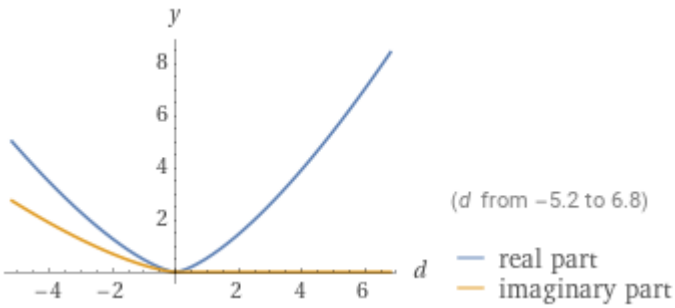
$$\left(\frac{1}{3}\left(\sqrt{6}\left((-1)^{5/6} - \sqrt[6]{-1}\right) + 6\right)\right)\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Exact result

$$3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(6+\sqrt{6}\left((-1)^{5/6} - \sqrt[6]{-1}\right)\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{d^9}$$

Plots **(figures that can be related to the open strings)**





Alternate form

$$-3^{-3/(2\pi)} \times 4^{-1/\pi} \left(\sqrt{2}-2\right) \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Expanded form

$$2^{1-2/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ +$$
$$(-1)^{5/6}\, 2^{1/2-2/\pi} \times 3^{-1/2-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ -$$
$$\sqrt[6]{-1}\ 2^{1/2-2/\pi} \times 3^{-1/2-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Alternate form assuming d>0

$$2^{1-2/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ d^{9/(2\pi)} + 2^{1/2-2/\pi} \times 3^{-1/2-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\left(e^{(5i\pi)/6} - e^{(i\pi)/6}\right) \sqrt[\pi]{\pi}\ d^{9/(2\pi)}$$

Root

$d = 0$

Series expansion at d=∞

$$-3^{-3/(2\pi)} \times 4^{-1/\pi} \left(\sqrt{2}-2\right) \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{3}\left(\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)+6\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$
$$-\frac{2^{-(2+\pi)/\pi}\times 3^{2-3/(2\pi)}\left(\sqrt{2}-2\right)\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[2\pi]{d^9}\ \pi^{1/\pi-1}}{d}$$

Indefinite integral

$$\int 3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(6+\sqrt{6}\left(-\sqrt[6]{-1}+(-1)^{5/6}\right)\right)\sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}}\ dd=$$
$$-\frac{2^{(\pi-2)/\pi}\times 3^{-3/(2\pi)}\left(\sqrt{2}-2\right)\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \pi^{1+1/\pi}\,d\sqrt[2\pi]{d^9}}{9+2\pi}+\text{constant}$$

Series representations

$$\frac{1}{3}\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\left(\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)+6\right)=$$
$$2^{-2/\pi}\times 3^{-1/2-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}\left(2\sqrt{3}-\sqrt{5}\sum_{k=0}^{\infty}5^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{1}{3}\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\left(\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)+6\right)=$$
$$2^{-2/\pi}\times 3^{-1/2-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$
$$\left(2\sqrt{3}-\sqrt{5}\sum_{k=0}^{\infty}\frac{5^{-k}\,e^{i\,k\,\pi}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{1}{3}\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\left(\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)+6\right)=$$

$$2^{-2/\pi}\times 3^{-1/2-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}$$

$$\left(2\sqrt{3}-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{e^{i k\pi}\left(-\frac{1}{2}\right)_k(6-z_0)^k z_0^{-k}}{k!}\right) \text{ for } (\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

And:

(1/3 (sqrt(6) ((-1)^(5/6) - (-1)^(1/6)) + 6)) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

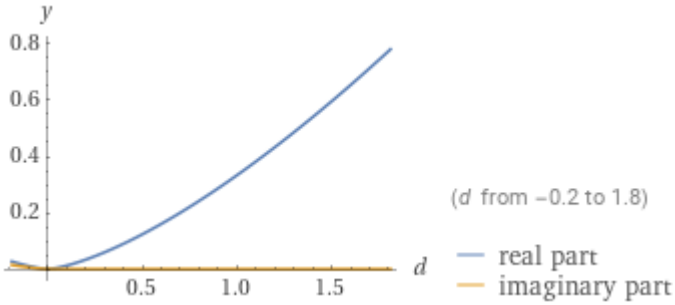
Input

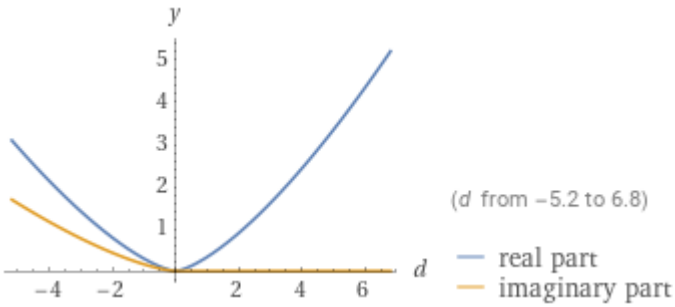
$$\left(\frac{1}{3}\left(\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)+6\right)\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}d^3\right)}$$

Exact result

$$2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)\sqrt[2\pi]{\pi}\sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Expanded form

$$2^{1-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}+(-1)^{5/6}\,2^{1/2-1/\pi}\times 3^{-1/2-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}-\sqrt[6]{-1}\ 2^{1/2-1/\pi}\times 3^{-1/2-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Alternate forms assuming d>0

$$-2^{-1/\pi}\times 3^{-3/(2\pi)}\left(\sqrt{2}-2\right)\sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

$$2^{1-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ d^{9/(2\pi)}+2^{1/2-1/\pi}\times 3^{-1/2-3/(2\pi)}\left(e^{(5i\pi)/6}-e^{(i\pi)/6}\right)\sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Alternate forms

$$2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(6-3\sqrt{2}\right)\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$\left(-\sqrt[6]{-1}\ 2^{1/2-1/\pi}\times 3^{-1/2-3/(2\pi)}+(-1)^{5/6}\,2^{1/2-1/\pi}\times 3^{-1/2-3/(2\pi)}+2^{1-1/\pi}\times 3^{-3/(2\pi)}\right)\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Root

$d = 0$

Series expansion at d=∞

$$-2^{-1/\pi} \times 3^{-3/(2\pi)} \left(\sqrt{2}-2\right) \sqrt[2\pi]{\pi}\; d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{3}\left(\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)+6\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times 3\times 12}}\right) =$$

$$-\frac{2^{-(1+\pi)/\pi} \times 3^{2-3/(2\pi)} \left(\sqrt{2}-2\right) \sqrt[2\pi]{d^9}\;\pi^{1/(2\pi)-1}}{d}$$

Indefinite integral

$$\int 2^{-1/\pi} \times 3^{-1-3/(2\pi)} \left(6+\sqrt{6}\left(-\sqrt[6]{-1}+(-1)^{5/6}\right)\right) \sqrt[2\pi]{d^9}\;\sqrt[2\pi]{\pi}\;dd =$$

$$-\frac{2^{(\pi-1)/\pi} \times 3^{-3/(2\pi)} \left(\sqrt{2}-2\right) \pi^{1+1/(2\pi)}\, d\,\sqrt[2\pi]{d^9}}{9+2\pi} + \text{constant}$$

Series representations

$$\frac{1}{3}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times 3\times 12}}\left(\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)+6\right) =$$

$$2^{-3/(2\pi)} \times 3^{-1/2-3/(2\pi)} \sqrt[2\pi]{d^9\,\pi\,\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}\right)^2}$$

$$\left(2\sqrt{3}-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{e^{i\,k\,\pi}\left(-\frac{1}{2}\right)_k (6-z_0)^k\, z_0^{-k}}{k!}\right) \text{ for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{1}{3}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)}{3\times 3\times 12}}\left(\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)+6\right)=$$

$$\frac{\sqrt[2\pi]{d^9\,\pi\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}{2^{-3/(2\pi)}\times 3^{-1/2-3/(2\pi)}}$$

$$\left(2\sqrt{3}\ -\exp\left(i\,\pi\left\lfloor\frac{\arg(6-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{e^{i\,k\,\pi}\,(6-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

$$\frac{1}{3}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)}{3\times 3\times 12}}\left(\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)+6\right)=2^{-3/(2\pi)}\times 3^{-1-3/(2\pi)}$$

$$\sqrt[2\pi]{d^9\,\pi\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}$$

$$\left(6-\sqrt[6]{-1}\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor}z_0^{\,1/2+1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor}\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k\,z_0^{-k}}{k!}+(-1)^{5/6}\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor}\right.$$

$$\left.z_0^{\,1/2+1/2\lfloor\arg(6-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6-z_0)^k\,z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

i is the imaginary unit

Dividing the two exact results

$$3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{d^9}$$

and

$$2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)\sqrt[2\pi]{\pi}\sqrt[2\pi]{d^9}$$

we obtain:

(3^(-1-3/(2π)) (5(3+sqrt(5)))^(1/(2π))(6+sqrt(6) ((-1)^(5/6)-(-1)^(1/6)))(π/4)^(1/π) (d^9)^(1/(2π)))/(2^(-1/π) 3^(-1-3/(2π))(6+sqrt(6)((-1)^(5/6)-(-1)^(1/6))) π^(1/(2π)) (d^9)^(1/(2π)))

Input

$$\frac{3^{-1-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)\sqrt[\pi]{\frac{\pi}{4}}\right)\sqrt[2\pi]{d^9}}{2^{-1/\pi}\left(3^{-1-3/(2\pi)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)\right)\sqrt[2\pi]{\pi}\sqrt[2\pi]{d^9}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{3^{-1-3/(2\pi)}\left(\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)\sqrt[\pi]{\frac{\pi}{4}}\right)^{2\pi}\sqrt{d^9}\right)}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)^{2\pi}\sqrt{\pi}\,{}^{2\pi}\sqrt{d^9}} =$$
$$2^{-1/\pi}\,{}^{2\pi}\sqrt{5}\,{}^{2\pi}\sqrt{\pi}\,\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-1-3/(2\pi)}\left(\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)\sqrt[\pi]{\frac{\pi}{4}}\right)^{2\pi}\sqrt{d^9}\right)}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)^{2\pi}\sqrt{\pi}\,{}^{2\pi}\sqrt{d^9}} =$$
$$2^{-1/\pi}\,{}^{2\pi}\sqrt{5}\,{}^{2\pi}\sqrt{\pi}\,\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{3^{-1-3/(2\pi)}\left(\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)\sqrt[\pi]{\frac{\pi}{4}}\right)^{2\pi}\sqrt{d^9}\right)}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(6+\sqrt{6}\left((-1)^{5/6}-\sqrt[6]{-1}\right)\right)^{2\pi}\sqrt{\pi}\,{}^{2\pi}\sqrt{d^9}} =$$
$$2^{-1/\pi}\,{}^{2\pi}\sqrt{5}\,{}^{2\pi}\sqrt{\pi}\,\sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.14159265358846…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298884450469 20
…

1.644934066846…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{32\,e^{1.92478440155986\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Now, we analyze the following cubic polynomial

$z^3+3z^2-3\sqrt[3]{2}\,z+1$

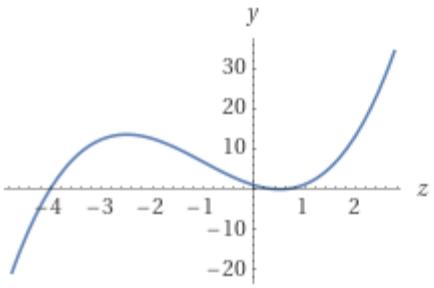
and obtain:

z^3 + 3z^2 – 3(2)^(1/3) * z + 1

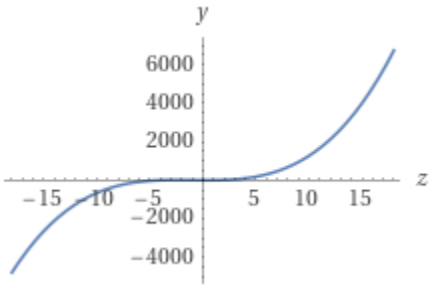
Input

$z^3+3z^2-3\sqrt[3]{2}\,z+1$

Plots (figures that can be related to the open strings)

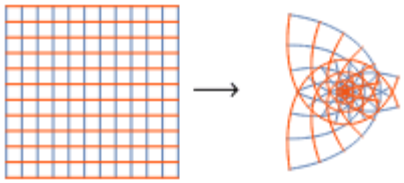


(z from −4.8 to 2.8)



(z from −18 to 18)

Complex map



Alternate forms

$$z\left(z\,(z+3)-3\sqrt[3]{2}\right)+1$$

$$(z+1)^3+\left(-3-3\sqrt[3]{2}\right)(z+1)+3\sqrt[3]{2}+3$$

Roots

$$z\approx -4.0059$$

$$z\approx 0.44539$$

$z \approx 0.56048$

Polynomial discriminant

$$\Delta_z = 81 - 162\sqrt[3]{2} + 81 \times 2^{2/3}$$

Branch points

(none; function is entire)

Derivative

$$\frac{d}{dz}\left(z^3 + 3z^2 - 3\sqrt[3]{2}\ z + 1\right) = 3z^2 + 6z - 3\sqrt[3]{2}$$

Indefinite integral

$$\int \left(z^3 + 3z^2 - 3\sqrt[3]{2}\ z + 1\right) dz = \frac{z^4}{4} + z^3 - \frac{3z^2}{2^{2/3}} + z + \text{constant}$$

Local maximum

$$\max\left\{z^3 + 3z^2 - 3\sqrt[3]{2}\ z + 1\right\} = \left(1 + \sqrt[3]{2}\right)\left(3 + 2\sqrt{1 + \sqrt[3]{2}}\right)$$

at $z = -1 - \sqrt{1 + \sqrt[3]{2}}$

Local minimum

$$\min\left\{z^3 + 3z^2 - 3\sqrt[3]{2}\ z + 1\right\} = -\left(1 + \sqrt[3]{2}\right)\left(2\sqrt{1 + \sqrt[3]{2}} - 3\right) \text{ at } z = \sqrt{1 + \sqrt[3]{2}} - 1$$

From the indefinite integral result

$$\int \left(z^3 + 3\,z^2 - 3\sqrt[3]{2}\;z + 1\right) dz = \frac{z^4}{4} + z^3 - \frac{3\,z^2}{2^{2/3}} + z + \text{constant}$$

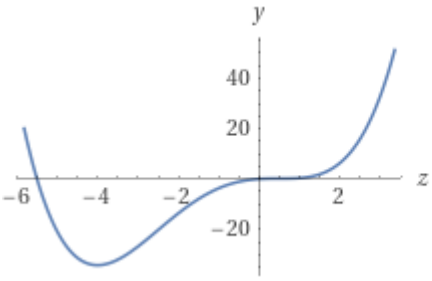
we obtain:

z - (3 z^2)/2^(2/3) + z^3 + z^4/4

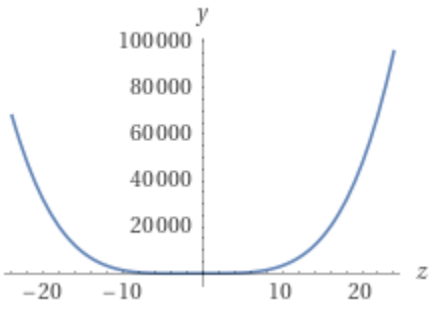
Input

$$z - \frac{3\,z^2}{2^{2/3}} + z^3 + \frac{z^4}{4}$$

Plots (figures that can be related to the open strings)

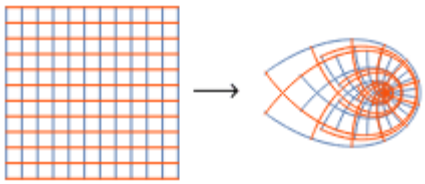


(z from −5.8 to 3.3)



(z from −24 to 24)

Complex map



Alternate forms

$$\frac{1}{4} z\left(z^3 + 4 z^2 - 6 \sqrt[3]{2}\ z + 4\right)$$

$$\frac{1}{4} z\left(z\left(z\,(z+4) - 6 \sqrt[3]{2}\right) + 4\right)$$

$$z\left(z\left(\left(\frac{z}{4} + 1\right) z - \frac{3}{2^{2/3}}\right) + 1\right)$$

Real roots

$$z = 0$$

$$z \approx -5.5052$$

Complex roots

$$z \approx 0.75258 - 0.40027\, i$$

$$z \approx 0.75258 + 0.40027\, i$$

Polynomial discriminant

$$\Delta_z = \frac{1}{16}\left(17 - 108 \sqrt[3]{2} + 36 \times 2^{2/3}\right)$$

Branch points

(none; function is entire)

Derivative

$$\frac{d}{dz}\left(z-\frac{3\,z^2}{2^{2/3}}+z^3+\frac{z^4}{4}\right)=z^3+3\,z^2-3\sqrt[3]{2}\;z+1$$

Indefinite integral

$$\int\left(z-\frac{3\,z^2}{2^{2/3}}+z^3+\frac{z^4}{4}\right)dz=\frac{z^5}{20}+\frac{z^4}{4}-\frac{z^3}{2^{2/3}}+\frac{z^2}{2}+\text{constant}$$

Local maximum

$$\max\left\{z-\frac{3\,z^2}{2^{2/3}}+z^3+\frac{z^4}{4}\right\}\approx 0.16868 \text{ at } z\approx 0.44539$$

Global minimum

$$\min\left\{z-\frac{3\,z^2}{2^{2/3}}+z^3+\frac{z^4}{4}\right\}\approx -34.238 \text{ at } z\approx -4.0059$$

Again, from the indefinite integral result

$$\int\left(z-\frac{3\,z^2}{2^{2/3}}+z^3+\frac{z^4}{4}\right)dz=\frac{z^5}{20}+\frac{z^4}{4}-\frac{z^3}{2^{2/3}}+\frac{z^2}{2}+\text{constant}$$

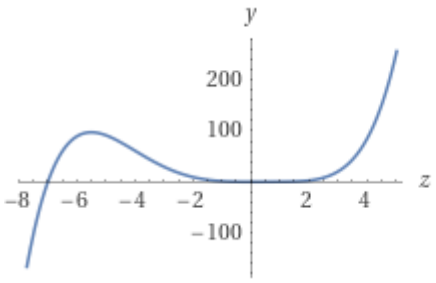
we obtain:

z^2/2 - z^3/2^(2/3) + z^4/4 + z^5/20

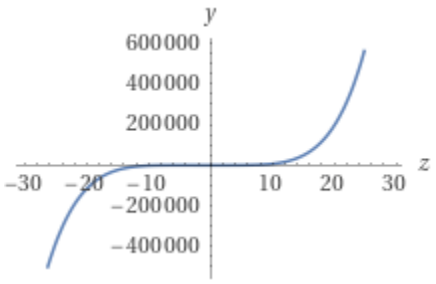
Input

$$\frac{z^2}{2}-\frac{z^3}{2^{2/3}}+\frac{z^4}{4}+\frac{z^5}{20}$$

Plots

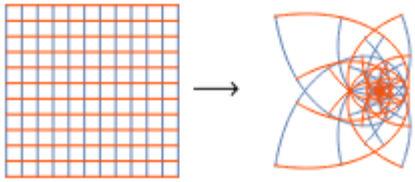


(z from −7.7 to 5)



(z from −30 to 30)

Complex map



Alternate forms

$$\frac{1}{20} z^2 \left(z^3 + 5 z^2 - 10 \sqrt[3]{2}\, z + 10\right)$$

$$\frac{1}{20} z^2 \left(z \left(z (z + 5) - 10 \sqrt[3]{2}\right) + 10\right)$$

$$-\frac{1}{20} z^2 \left(-z^3 - 5 z^2 + 10 \sqrt[3]{2}\, z - 10\right)$$

Real roots

$$z = 0$$

$$z = -\frac{5}{3} + \frac{5^{2/3}\left(-\frac{5}{3} - 2\sqrt[3]{2}\right)}{\sqrt[3]{52 + 45\sqrt[3]{2} - 3\sqrt{-9 + 270\sqrt[3]{2} - 75 \times 2^{2/3}}}} - \frac{1}{3}$$

$$\sqrt[3]{5\left(52 + 45\sqrt[3]{2} - 3\sqrt{-9 + 270\sqrt[3]{2} - 75 \times 2^{2/3}}\right)}$$

Complex roots

$$z \approx 1.00151 + 0.65187\,i$$

$$z \approx 1.00151 - 0.65187\,i$$

Polynomial discriminant

$$\Delta = 0$$

Branch points

(none; function is entire)

Derivative

$$\frac{d}{dz}\left(\frac{z^2}{2} - \frac{z^3}{2^{2/3}} + \frac{z^4}{4} + \frac{z^5}{20}\right) = \frac{1}{4}\,z\left(z^3 + 4\,z^2 - 6\sqrt[3]{2}\,z + 4\right)$$

Indefinite integral

$$\int\left(\frac{z^2}{2}-\frac{z^3}{2^{2/3}}+\frac{z^4}{4}+\frac{z^5}{20}\right)dz=\frac{z^6}{120}+\frac{z^5}{20}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^3}{6}+\text{constant}$$

Local maximum

$$\max\left\{\frac{z^2}{2}-\frac{z^3}{2^{2/3}}+\frac{z^4}{4}+\frac{z^5}{20}\right\}\approx 97.059 \text{ at } z\approx -5.5052$$

Local minimum

$$\min\left\{\frac{z^2}{2}-\frac{z^3}{2^{2/3}}+\frac{z^4}{4}+\frac{z^5}{20}\right\}=0 \text{ at } z=0$$

Definite integral area above the axis between the smallest and largest real roots

$$\int_{\text{root of } \{-2+\#1^3\&,10-10\#1+\#2+5\#2^2+\#2^3\&\}(x) \text{ near } x=-7.00302}^{0}\left(\frac{z^2}{2}-\frac{z^3}{2^{2/3}}+\frac{z^4}{4}+\frac{z^5}{20}\right)\theta\left(\frac{z^2}{2}-\frac{z^3}{2^{2/3}}+\frac{z^4}{4}+\frac{z^5}{20}\right)dz=$$

$$\text{root of } 161\,243\,136\,x^9-31\,946\,296\,320\,x^8-4\,139\,896\,521\,600\,x^7-141\,754\,105\,695\,600\,x^6-287\,410\,442\,832\,000\,x^5-1\,560\,483\,561\,150\,000\,x^4-1\,323\,807\,911\,55\,000\,x^3-10\,790\,567\,550\,000\,x^2-226\,796\,625\,000\,x-1\,676\,984\,375 \text{ near } x=295.243$$

$$\approx 295.243$$

$\theta(x)$ is the Heaviside step function

In conclusion, from the indefinite integral result

$$\int\left(\frac{z^2}{2}-\frac{z^3}{2^{2/3}}+\frac{z^4}{4}+\frac{z^5}{20}\right)dz=\frac{z^6}{120}+\frac{z^5}{20}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^3}{6}+\text{constant}$$

for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , we obtain:

((z^3/6 - z^4/(4 2^(2/3)) + z^5/20 + z^6/120)) (1/3*√2*a^3)

Input

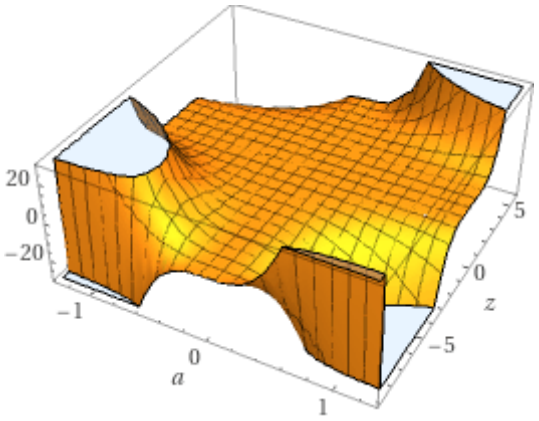
$$\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)\left(\frac{1}{3}\sqrt{2}\,a^3\right)$$

Result

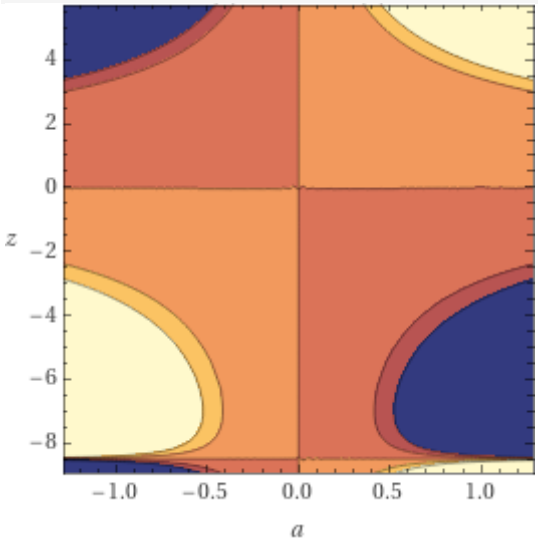
$$\frac{1}{3}\sqrt{2}\,a^3\left(\frac{z^6}{120}+\frac{z^5}{20}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^3}{6}\right)$$

3D plot **(figure that can be related to a D-brane/Instanton)**

The key observation from the above plots and is that at $a = 1$, which is taken as the energy density of the universe at the Big Bang, with $z = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.



Contour plot



Alternate forms

$$\frac{1}{360}\sqrt{2}\,a^3 z^3\left(z^3+6z^2-15\sqrt[3]{2}\,z+20\right)$$

$$a^3 z^3\left(z\left(z\left(\frac{z}{180\sqrt{2}}+\frac{1}{30\sqrt{2}}\right)-\frac{1}{12\sqrt[6]{2}}\right)+\frac{1}{9\sqrt{2}}\right)$$

$$\frac{a^3 z^3\left(z\left(z\,(z+6)-15\sqrt[3]{2}\right)+20\right)}{180\sqrt{2}}$$

Expanded form

$$\frac{a^3 z^6}{180\sqrt{2}}+\frac{a^3 z^5}{30\sqrt{2}}-\frac{a^3 z^4}{12\sqrt[6]{2}}+\frac{a^3 z^3}{9\sqrt{2}}$$

Real roots

$a = 0$

$z = 0$

$$z = -2 + \frac{-36 - 45\sqrt[3]{2}}{9\sqrt[3]{18 + 15\sqrt[3]{2} + \sqrt{5\left(2 + 60\sqrt[3]{2} - 15 \times 2^{2/3}\right)}}} -$$

$$\sqrt[3]{18 + 15\sqrt[3]{2} + \sqrt{5\left(2 + 60\sqrt[3]{2} - 15 \times 2^{2/3}\right)}}$$

Complex roots

$z \approx 1.2501 - 0.8889\,i$

$z \approx 1.2501 + 0.8889\,i$

Polynomial discriminant

$\Delta = 0$

Derivative

$$\frac{\partial}{\partial z}\left(\frac{1}{3}\left(\frac{z^3}{6} - \frac{z^4}{4 \times 2^{2/3}} + \frac{z^5}{20} + \frac{z^6}{120}\right)\left(\sqrt{2}\ a^3\right)\right) = \frac{a^3\, z^2\left(z^3 + 5\, z^2 - 10\sqrt[3]{2}\ z + 10\right)}{30\sqrt{2}}$$

Indefinite integral

$$\int \frac{1}{3}\sqrt{2}\,a^3\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)dz = \frac{a^3\left(\frac{z^7}{7}+z^6-3\sqrt[3]{2}\,z^5+5\,z^4\right)}{180\sqrt{2}}+\text{constant}$$

And:

((z^3/6 - z^4/(4 2^(2/3)) + z^5/20 + z^6/120)) (4/3*π*(a/2)^3)

Input

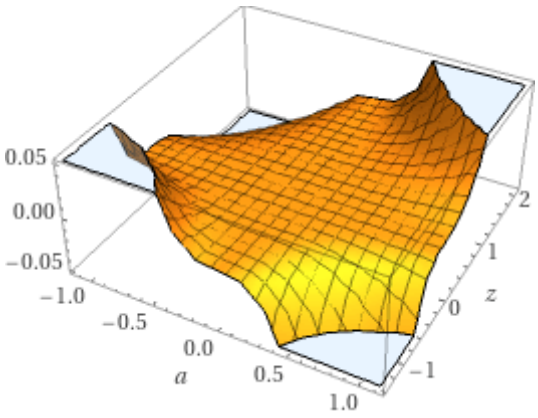
$$\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)\left(\frac{4}{3}\pi\left(\frac{a}{2}\right)^3\right)$$

Result

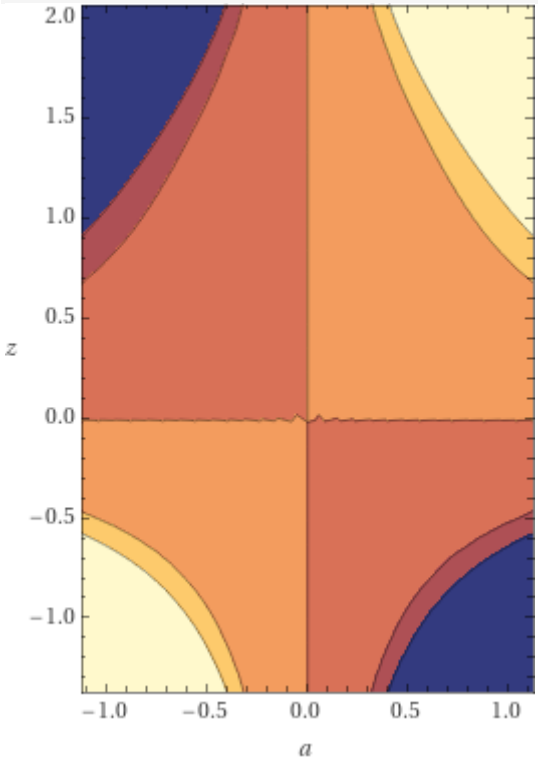
$$\frac{1}{6}\pi a^3\left(\frac{z^6}{120}+\frac{z^5}{20}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^3}{6}\right)$$

3D plot **(figure that can be related to a D-brane/Instanton)**

The key observation from the above plots and is that at $a = 1$, which is taken as the energy density of the universe at the Big Bang, with $z = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.



Contour plot



Alternate forms

$$\frac{1}{720}\pi a^3 z^3\left(z^3+6z^2-15\sqrt[3]{2}\,z+20\right)$$

$$a^3 z^3\left(z\left(z\left(\frac{\pi z}{720}+\frac{\pi}{120}\right)-\frac{\pi}{24\times 2^{2/3}}\right)+\frac{\pi}{36}\right)$$

$$\frac{1}{720}\pi a^3 z^3\left(z\left(z\,(z+6)-15\sqrt[3]{2}\right)+20\right)$$

Expanded form

$$\frac{1}{720}\pi a^3 z^6+\frac{1}{120}\pi a^3 z^5-\frac{\pi a^3 z^4}{24\times 2^{2/3}}+\frac{1}{36}\pi a^3 z^3$$

Real roots

$a = 0$

$z = 0$

$$z = -2 + \frac{-36 - 45\sqrt[3]{2}}{9\sqrt[3]{18 + 15\sqrt[3]{2} + \sqrt{5\left(2 + 60\sqrt[3]{2} - 15 \times 2^{2/3}\right)}}} -$$

$$\sqrt[3]{18 + 15\sqrt[3]{2} + \sqrt{5\left(2 + 60\sqrt[3]{2} - 15 \times 2^{2/3}\right)}}$$

Complex roots

$z \approx 1.2501 - 0.8889\,i$

$z \approx 1.2501 + 0.8889\,i$

Polynomial discriminant

$\Delta = 0$

Derivative

$$\frac{\partial}{\partial z}\left(\frac{1}{3}\left(\frac{z^3}{6} - \frac{z^4}{4 \times 2^{2/3}} + \frac{z^5}{20} + \frac{z^6}{120}\right)\left(4\,\pi\left(\frac{a}{2}\right)^3\right)\right) =$$

$$\frac{1}{120}\,\pi\,a^3\,z^2\left(z^3 + 5\,z^2 - 10\sqrt[3]{2}\,z + 10\right)$$

Indefinite integral

$$\int \frac{1}{6} a^3 \pi \left(\frac{z^3}{6} - \frac{z^4}{4 \times 2^{2/3}} + \frac{z^5}{20} + \frac{z^6}{120}\right) dz =$$
$$\frac{1}{720} \pi a^3 \left(\frac{z^7}{7} + z^6 - 3\sqrt[3]{2}\, z^5 + 5 z^4\right) + \text{constant}$$

Dividing the two indefinite integral result

$$\int \frac{1}{3} \sqrt{2}\, a^3 \left(\frac{z^3}{6} - \frac{z^4}{4 \times 2^{2/3}} + \frac{z^5}{20} + \frac{z^6}{120}\right) dz =$$
$$\frac{a^3 \left(\frac{z^7}{7} + z^6 - 3\sqrt[3]{2}\, z^5 + 5 z^4\right)}{180\sqrt{2}} + \text{constant}$$

and

$$\int \frac{1}{6} a^3 \pi \left(\frac{z^3}{6} - \frac{z^4}{4 \times 2^{2/3}} + \frac{z^5}{20} + \frac{z^6}{120}\right) dz =$$
$$\frac{1}{720} \pi a^3 \left(\frac{z^7}{7} + z^6 - 3\sqrt[3]{2}\, z^5 + 5 z^4\right) + \text{constant}$$

we obtain:

((a^3 (5 z^4 - 3 2^(1/3) z^5 + z^6 + z^7/7))/(180 sqrt(2)))/(1/720 a^3 π (5 z^4 - 3 2^(1/3) z^5 + z^6 + z^7/7))

Input

$$\frac{\frac{a^3 \left(5 z^4 - 3\sqrt[3]{2}\, z^5 + z^6 + \frac{z^7}{7}\right)}{180\sqrt{2}}}{\frac{1}{720} a^3 \pi \left(5 z^4 - 3\sqrt[3]{2}\, z^5 + z^6 + \frac{z^7}{7}\right)}$$

Result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{a^3\left(5z^4-3\sqrt[3]{2}\,z^5+z^6+\frac{z^7}{7}\right)}{\frac{1}{720}\left(a^3\pi\left(5z^4-3\sqrt[3]{2}\,z^5+z^6+\frac{z^7}{7}\right)\right)\left(180\sqrt{2}\right)}=\frac{4}{\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{a^3\left(5z^4-3\sqrt[3]{2}\,z^5+z^6+\frac{z^7}{7}\right)}{\frac{1}{720}\left(a^3\pi\left(5z^4-3\sqrt[3]{2}\,z^5+z^6+\frac{z^7}{7}\right)\right)\left(180\sqrt{2}\right)}=\frac{4}{\pi\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{a^3\left(5z^4-3\sqrt[3]{2}\,z^5+z^6+\frac{z^7}{7}\right)}{\frac{1}{720}\left(a^3\pi\left(5z^4-3\sqrt[3]{2}\,z^5+z^6+\frac{z^7}{7}\right)\right)\left(180\sqrt{2}\right)}=\frac{4\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}}{\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$2\pi\sqrt{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$2\pi\sqrt{\frac{5}{12}(3+\sqrt{5})d^3\times\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$2\pi\sqrt{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\times\frac{1}{3}\sqrt{2}d^3\times\frac{\sqrt{2}}{12}d^3}$$

From the previous indefinite integral result

$$\int\left(\frac{z^2}{2}-\frac{z^3}{2^{2/3}}+\frac{z^4}{4}+\frac{z^5}{20}\right)dz=\frac{z^6}{120}+\frac{z^5}{20}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^3}{6}+\text{constant}$$

we obtain:

((z^3/6 - z^4/(4 2^(2/3)) + z^5/20 + z^6/120)) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

Input

$$\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

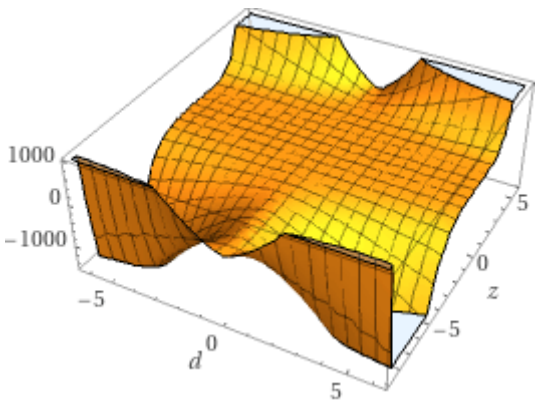
Exact result

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}\left(\frac{z^6}{120}+\frac{z^5}{20}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^3}{6}\right)$$

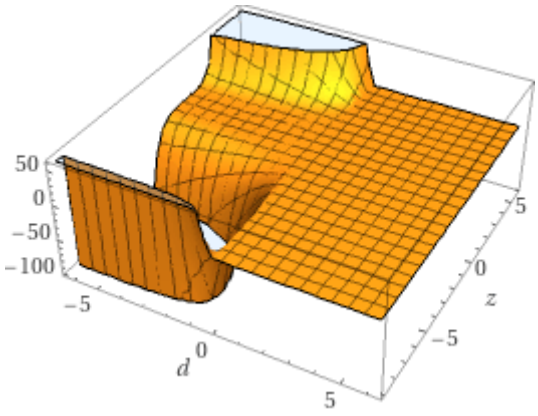
3D plots

Real part **(figures that can be related to the D-branes/Instantons)**

The key observation from the above plots and is that at $d=1$, which is taken as the energy density of the universe at the Big Bang, with $z=0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.

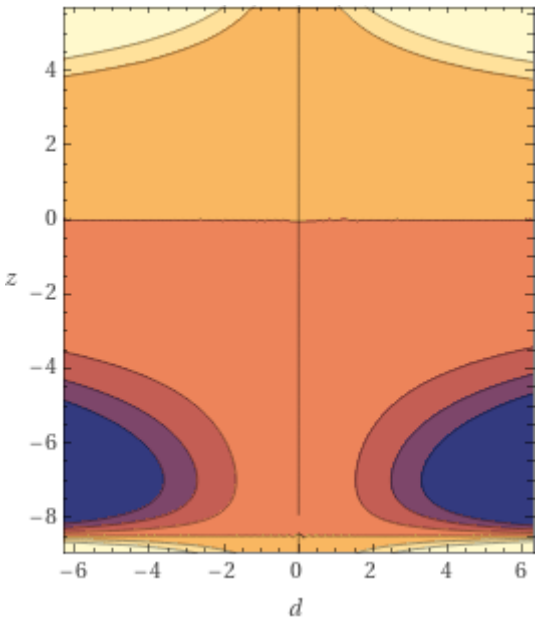


Imaginary part

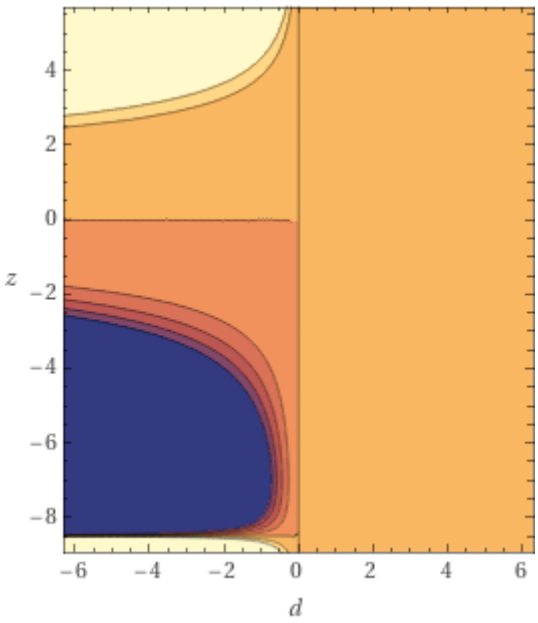


Contour plots

Real part



Imaginary part



Expanded forms

$$2^{-3-2/\pi} \times 3^{-1-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ z^6 +$$
$$2^{-2-2/\pi} \times 3^{-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ z^5 -$$
$$2^{-8/3-2/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ z^4 +$$
$$2^{-1-2/\pi} \times 3^{-1-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ z^3$$

$$\frac{1}{5} \times 2^{-3-2/\pi} \times 3^{-1-3/(2\pi)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{5\sqrt{5}\ d^9 + 15\,d^9}\ z^6 +$$
$$\frac{1}{5} \times 3^{-3/(2\pi)} \times 4^{-1-1/\pi}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{5\sqrt{5}\ d^9 + 15\,d^9}\ z^5 -$$
$$2^{-8/3-2/\pi} \times 3^{-3/(2\pi)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{5\sqrt{5}\ d^9 + 15\,d^9}\ z^4 +$$
$$2^{-1-2/\pi} \times 3^{-1-3/(2\pi)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{5\sqrt{5}\ d^9 + 15\,d^9}\ z^3$$

Alternate form assuming d and z are positive

$$\frac{1}{5}\times 2^{-11/3-2/\pi}\times 3^{-1-3/(2\pi)}\sqrt[\pi]{\pi}\ d^{9/(2\pi)}\ z^3$$
$$\left(2^{2/3}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ z^3+6\times 2^{2/3}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ z^2-\right.$$
$$\left.6\times 5^{1+1/(2\pi)}\sqrt[2\pi]{3+\sqrt{5}}\ z+4\times 2^{2/3}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{3+\sqrt{5}}\right)$$

Alternate form

$$2^{-3-2/\pi}\times 3^{-1-3/(2\pi)}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ z^3\left(z^3+6z^2-15\sqrt[3]{2}\ z+20\right)$$

Real roots

$$d=0$$

$$z=0$$

Derivative

$$\frac{\partial}{\partial z}\left(\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$
$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}\left(\frac{z^5}{20}+\frac{z^4}{4}-\frac{z^3}{2^{2/3}}+\frac{z^2}{2}\right)$$

Indefinite integral

$$\int 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}}\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)dz=$$
$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}\left(\frac{z^7}{840}+\frac{z^6}{120}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^4}{24}\right)+\text{constant}$$

And:

((z^3/6 - z^4/(4 2^(2/3)) + z^5/20 + z^6/120)) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

Input

$$\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

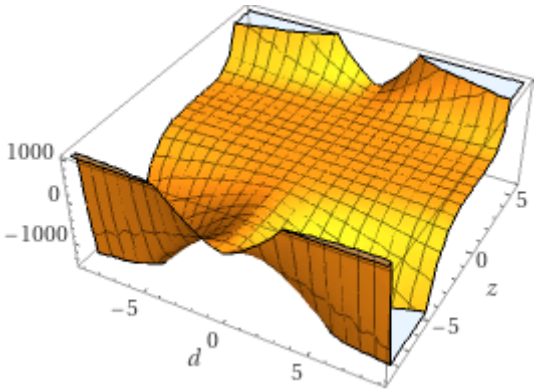
Exact result

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\left(\frac{z^6}{120}+\frac{z^5}{20}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^3}{6}\right)$$

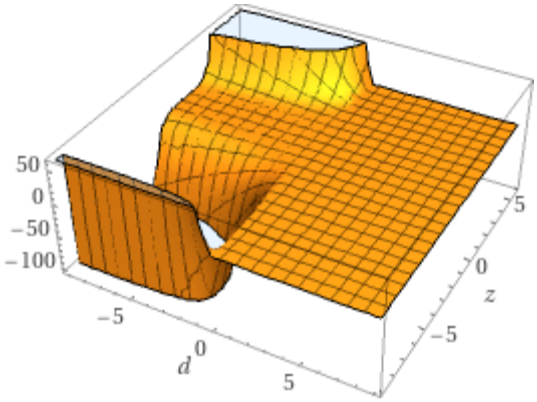
3D plots

Real part **(figures that can be related to the D-branes/Instantons)**

The key observation from the above plots and is that at $d = 1$, which is taken as the energy density of the universe at the Big Bang, with $z = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.

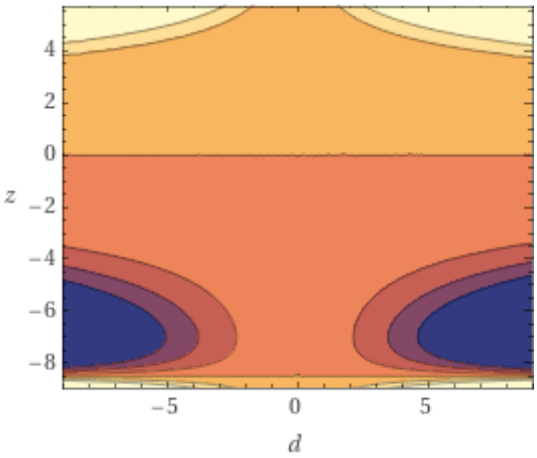


Imaginary part

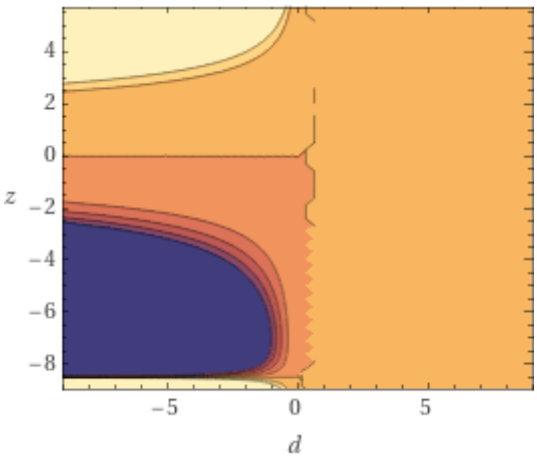


Contour plots

Real part



Imaginary part



Expanded form

$$\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\;z^6+\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\;z^5-$$
$$2^{-8/3-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\;z^4+2^{-1-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\;z^3$$

Alternate form assuming d and z are positive

$$\frac{1}{5}\times 2^{-11/3-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;d^{9/(2\pi)}\;z^3\left(2^{2/3}\,z^3+6\times 2^{2/3}\,z^2-30\,z+20\times 2^{2/3}\right)$$

Alternate forms

$$\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\;z^3\left(z^3+6\,z^2-15\,\sqrt[3]{2}\;z+20\right)$$

$$\sqrt[2\pi]{d^9}\left(\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;z^6+\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;z^5-\right.$$
$$\left.2^{-8/3-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;z^4+2^{-1-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;z^3\right)$$

Real roots

$d=0$

$z=0$

$$z=-2-\frac{1}{2}\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(-36-30\,\sqrt[3]{2}+\sqrt{2\left(20+600\,\sqrt[3]{2}-150\times 2^{2/3}\right)}\right)}+$$
$$\frac{\left(-72\,\sqrt[3]{2}-90\times 2^{2/3}\right)\left(1+i\sqrt{3}\right)}{36\sqrt[3]{-36-30\,\sqrt[3]{2}+\sqrt{2\left(20+600\,\sqrt[3]{2}-150\times 2^{2/3}\right)}}}$$

Complex roots

$z \approx 1.2501 - 0.8889\,i$

$z \approx 1.2501 + 0.8889\,i$

Roots for the variable z

$$z = -2 - \frac{-36 - 45\sqrt[3]{2}}{9\sqrt[3]{-18 - 15\sqrt[3]{2} + \sqrt{5\left(2 + 60\sqrt[3]{2} - 15 \times 2^{2/3}\right)}}} + \sqrt[3]{-18 - 15\sqrt[3]{2} + \sqrt{5\left(2 + 60\sqrt[3]{2} - 15 \times 2^{2/3}\right)}}$$

$$z = -2 + \frac{\left(-36 - 45\sqrt[3]{2}\right)\left(1 + i\sqrt{3}\right)}{18\sqrt[3]{-18 - 15\sqrt[3]{2} + \sqrt{5\left(2 + 60\sqrt[3]{2} - 15 \times 2^{2/3}\right)}}} - \frac{1}{2}\left(1 - i\sqrt{3}\right) \sqrt[3]{-18 - 15\sqrt[3]{2} + \sqrt{5\left(2 + 60\sqrt[3]{2} - 15 \times 2^{2/3}\right)}}$$

$$z = -2 + \frac{\left(-36 - 45\sqrt[3]{2}\right)\left(1 - i\sqrt{3}\right)}{18\sqrt[3]{-18 - 15\sqrt[3]{2} + \sqrt{5\left(2 + 60\sqrt[3]{2} - 15 \times 2^{2/3}\right)}}} - \frac{1}{2}\left(1 + i\sqrt{3}\right) \sqrt[3]{-18 - 15\sqrt[3]{2} + \sqrt{5\left(2 + 60\sqrt[3]{2} - 15 \times 2^{2/3}\right)}}$$

Derivative

$$\frac{\partial}{\partial z}\left(\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)}{3\times 3\times 12}}\right)=$$
$$\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\ \sqrt[2\pi]{d^9}\ \sqrt[2\pi]{\pi}\ z^2\left(z^3+5z^2-10\sqrt[3]{2}\ z+10\right)$$

Indefinite integral

$$\int 2^{-1/\pi}\times 3^{-3/(2\pi)}\ \sqrt[2\pi]{d^9}\ \sqrt[2\pi]{\pi}\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)dz=$$
$$\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}\left(\frac{z^7}{7}+z^6-3\sqrt[3]{2}\ z^5+5z^4\right)+\text{constant}$$

Series representations

$$\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$
$$\frac{1}{40}\times 3^{-1-3\Big/\left(8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)}\ ^{8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}\sqrt{d^9}\ z^3$$
$$\left(20-15\sqrt[3]{2}\ z+6z^2+z^3\right)^{8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}\sqrt{\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}$$

$$\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$
$$\frac{1}{5}\times 2^{-3-1\Big/\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)}\times 3^{-1-3\Big/\left(2\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)}$$
$$^{2\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}\sqrt{d^9}\ z^3\left(20-15\sqrt[3]{2}\ z+6z^2+z^3\right)$$
$$^{2\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}\sqrt{\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}$$

$$\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)2\pi\sqrt{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$
$$\frac{1}{5}\times 2^{-3-1\Big/\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}}\times$$
$$3^{-1-3\Big/\left(2\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)}$$
$$2\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\sqrt{d^9}\ z^3\left(20-15\sqrt[3]{2}\ z+6z^2+z^3\right)$$
$$\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)\wedge$$
$$\left(\frac{1}{2\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}}\right)$$

Integral representations

$$\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)2\pi\sqrt{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$
$$\frac{1}{40}\times 3^{-1-3\Big/\left(8\int_0^1\sqrt{1-t^2}\ dt\right)}\,8\int_0^1\sqrt{1-t^2}\ dt\sqrt{d^9}\ z^3$$
$$\left(20-15\sqrt[3]{2}\ z+6z^2+z^3\right)^{8\int_0^1\sqrt{1-t^2}\ dt}\sqrt{\int_0^1\sqrt{1-t^2}\ dt}$$

$$\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)2\pi\sqrt{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$
$$\frac{1}{5}\times 2^{-3-1\Big/\left(4\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)}\times 3^{-1-3\Big/\left(4\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)}\,4\int_0^{\infty}\frac{\sin(t)}{t}\,dt\sqrt{d^9}$$
$$z^3\left(20-15\sqrt[3]{2}\ z+6z^2+z^3\right)^{4\int_0^{\infty}\frac{\sin(t)}{t}\,dt}\sqrt{\int_0^{\infty}\frac{\sin(t)}{t}\,dt}$$

$$\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$
$$\frac{1}{5}\times 2^{-3-1\Big/\left(4\int_0^\infty \frac{1}{1+t^2}\,dt\right)}\times 3^{-1-3\Big/\left(4\int_0^\infty \frac{1}{1+t^2}\,dt\right)}\sqrt[4\int_0^\infty \frac{1}{1+t^2}\,dt]{d^9}$$
$$z^3\left(20-15\sqrt[3]{2}\ z+6z^2+z^3\right)^{4\int_0^\infty \frac{1}{1+t^2}\,dt}\sqrt{\int_0^\infty \frac{1}{1+t^2}\,dt}$$

Dividing the two indefinite integral results

$$\int 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}}\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)dz=$$
$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}\left(\frac{z^7}{840}+\frac{z^6}{120}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^4}{24}\right)+\text{constant}$$

and

$$\int 2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{d^9}\ \sqrt[2\pi]{\pi}\left(\frac{z^3}{6}-\frac{z^4}{4\times 2^{2/3}}+\frac{z^5}{20}+\frac{z^6}{120}\right)dz=$$
$$\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}\left(\frac{z^7}{7}+z^6-3\sqrt[3]{2}\ z^5+5z^4\right)+\text{constant}$$

we obtain:

((3^(-3/(2 π)) (5 (3 + sqrt(5)))^(1/(2 π)) (π/4)^(1/π) (z^4/24 - z^5/(20 2^(2/3)) + z^6/120 + z^7/840)))/((1/5 2^(-3 - 1/π) 3^(-1 - 3/(2 π)) π^(1/(2 π)) (5 z^4 - 3 2^(1/3) z^5 + z^6 + z^7/7)))

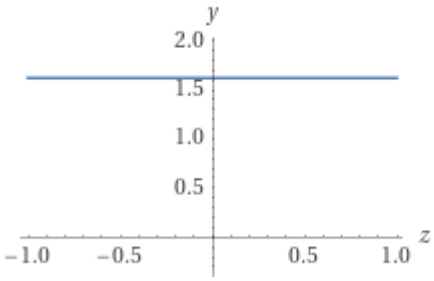
Input

$$\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\left(\frac{z^4}{24}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^6}{120}+\frac{z^7}{840}\right)}{\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{\pi}\left(5z^4-3\sqrt[3]{2}\ z^5+z^6+\frac{z^7}{7}\right)}$$

Exact result

$$\frac{3\times 2^{3-1/\pi}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{\left(3+\sqrt{5}\right)\pi}\left(\frac{z^7}{840}+\frac{z^6}{120}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^4}{24}\right)}{\frac{z^7}{7}+z^6-3\sqrt[3]{2}\,z^5+5z^4}$$

Plot



(z from −1 to 1)

Expanded form

$$\frac{2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\,z^7}{7\left(\frac{z^7}{7}+z^6-3\sqrt[3]{2}\,z^5+5z^4\right)}+\frac{2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\,z^6}{\frac{z^7}{7}+z^6-3\sqrt[3]{2}\,z^5+5z^4}-$$
$$\frac{3\times 2^{1/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\,z^5}{\frac{z^7}{7}+z^6-3\sqrt[3]{2}\,z^5+5z^4}+\frac{2^{-1/\pi}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{\left(3+\sqrt{5}\right)\pi}\,z^4}{\frac{z^7}{7}+z^6-3\sqrt[3]{2}\,z^5+5z^4}$$

Alternate form assuming z>0

$$-\left(\left(2^{-1/\pi}\left(-\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\,z^3-7\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\,z^2+21\sqrt[3]{2}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right.\right.\right.$$
$$\left.\left.\left.z-7\times 5^{1+1/(2\pi)}\sqrt[2\pi]{\left(3+\sqrt{5}\right)\pi}\right)\right)\Big/\left(z^3+7z^2-21\sqrt[3]{2}\,z+35\right)\right)$$ (ignoring removable singularities)

Alternate form

$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$ (ignoring removable singularities)

Property as a function

Parity

even

Branch points

(none)

Indefinite integral

$$\int \frac{3\times 2^{3-1/\pi}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{\left(3+\sqrt{5}\right)\pi}\left(\frac{z^4}{24}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^6}{120}+\frac{z^7}{840}\right)}{5z^4-3\sqrt[3]{2}\,z^5+z^6+\frac{z^7}{7}}\,dz =$$

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\;z + \text{constant}$$

Global maximum

$$\max\left\{\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[\pi]{\frac{\pi}{4}}\left(\frac{z^4}{24}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^6}{120}+\frac{z^7}{840}\right)}{\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{\pi}\left(5z^4-3\sqrt[3]{2}\,z^5+z^6+\frac{z^7}{7}\right)}\right\} =$$

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi} \text{ at } z=-\frac{12}{5}$$

Global minimum

$$\min\left\{\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\left(\frac{z^4}{24}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^6}{120}+\frac{z^7}{840}\right)}{\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\ \sqrt[2\pi]{\pi}\left(5z^4-3\sqrt[3]{2}\ z^5+z^6+\frac{z^7}{7}\right)}\right\}=$$
$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\quad\text{at}\quad z=-\frac{12}{5}$$

Limit

$$\lim_{z\to\pm\infty}\frac{3\times 2^{3-1/\pi}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{\left(3+\sqrt{5}\right)\pi}\left(\frac{z^4}{24}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^6}{120}+\frac{z^7}{840}\right)}{5z^4-3\sqrt[3]{2}\ z^5+z^6+\frac{z^7}{7}}=$$
$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\approx 1.61801$$

Series representations

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\left(\frac{z^4}{24}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^6}{120}+\frac{z^7}{840}\right)\right)}{\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\left(\sqrt[2\pi]{\pi}\left(5z^4-3\sqrt[3]{2}\ z^5+z^6+\frac{z^7}{7}\right)\right)}=$$
$$2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\left(\frac{z^4}{24}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^6}{120}+\frac{z^7}{840}\right)\right)}{\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\left(\sqrt[2\pi]{\pi}\left(5z^4-3\sqrt[3]{2}\ z^5+z^6+\frac{z^7}{7}\right)\right)}=$$
$$2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\left(\frac{z^4}{24}-\frac{z^5}{20\times 2^{2/3}}+\frac{z^6}{120}+\frac{z^7}{840}\right)\right)}{\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-1-3/(2\pi)}\left(\sqrt[2\pi]{\pi}\left(5z^4-3\sqrt[3]{2}\ z^5+z^6+\frac{z^7}{7}\right)\right)}=$$
$$2^{-1/\pi}\ \sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the alternate form

$2^{-1/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$ (ignoring removable singularities)

we obtain:

2^(-1/π) (5 (3 + sqrt(5)) π)^(1/(2 π))

Input

$2^{-1/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi} = 2^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi} = 2^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)}$$

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi} = 2^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi\left(3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the alternate form

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535884…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298888445046920
…

1.644934066846…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{32\,e^{1.92478440155986\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

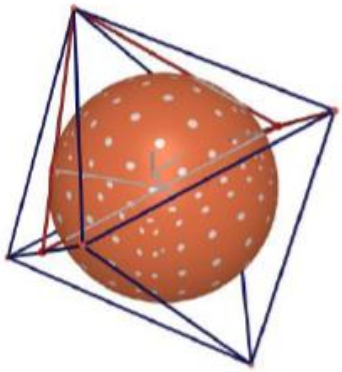


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

Indeed:

From:

https://www.academia.edu/75787512/The_Theory_of_String_A_Candidate_for_a_Generalized_Unification_Model

The Einstein's field equation and the String Theory.

The Einstein's field equation which includes the cosmological constant is:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (8)$$

where $R_{\mu\nu}$ is the Ricci tensor, R its trace, λ the cosmological constant, $g_{\mu\nu}$ the metric tensor of the space geometry, G the Newton's gravitational constant and $T_{\mu\nu}$ the tensor representing the properties of energy, matter and momentum.

The left hand-side of (8) represents the gravitational field and, consequently, the warped space-time, while the right hand-side represents the matter, i.e. the sources of the gravitational field.

In string theory the gravity is related to the gravitons which are bosons, whereas the matter is related to fermions. It follows that the left and right hand of (8) may be respectively related to the action of bosonic and of superstrings.

The actions of bosonic string and superstring (also containing fermions) are connected by the Palumbo-Nardelli relation (Palumbo et al. 2005):

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\varphi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right]=$$

$$=\int_0^\infty \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|\widetilde{H}_3|^2-\frac{\kappa_{10}^2}{g_{10}^2}Tr_\nu\left(|F_2|^2\right)\right]$$

The sign minus in the above equation comes from the inversion of any relationship.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^{\,8}\right)^{-k}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^{\,8}\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

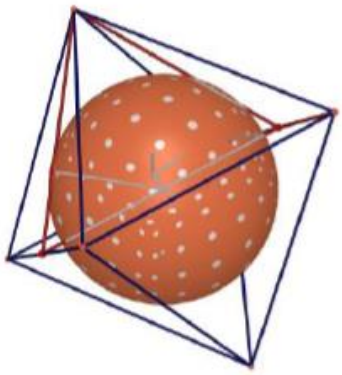
$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard to a sphere inscribed in an octahedron



we have the following formulas:

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\,2}{\pi} \quad (\text{for } l \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\,\pi)\rfloor)}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we have the following convex regular polyhedrons volumes:

$$\frac{\sqrt{2}}{12}d^3$$ Tetrahedron Volume

$$\frac{\sqrt{2}}{3}d^3$$ Octahedron Volume

$$\frac{5}{12}(3+\sqrt{5})d^3$$ Icosahedron Volume

Regarding the Sphere the volume is

$$V = \frac{4}{3}\pi r^3$$

With regard the sphere inscribed in a Tetrahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_T}{V_S} = \frac{\frac{\sqrt{2}}{12}d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(((√2 d^3)/12))*1/(4/3*π*(d/2)^3)

Input

$$\left(\frac{1}{12}\left(\sqrt{2}\ d^3\right)\right) \times \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.225079079039276517388799797751685145666143537488805156392859367 8

…

0.225079079….

Property

$\frac{1}{\sqrt{2}\ \pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Octahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_O}{V_S} = \frac{\frac{\sqrt{2}}{3}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

((1/3*√2*d^3)/(4/3*π*(d/2)^3))

Input

$$\frac{\frac{1}{3}\sqrt{2}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\ 2}{\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Icosahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_I}{V_S} = \frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3)

Input

$$\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{5\left(3+\sqrt{5}\right)}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

4.166730504921372682725163141597223093105782245731637719697502271 9

…

4.166730504921….

Property

$\frac{5\left(3+\sqrt{5}\right)}{2\pi}$ is a transcendental number

Expanded form

$$\frac{15}{2\pi} + \frac{5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

Alternate forms

$$\frac{15+5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

$$\frac{\frac{15}{2}+\frac{5\sqrt{5}}{2}}{\pi} \quad (\text{for } d \neq 0)$$

Series representations

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\pi\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\ a^3\right)\left(\sqrt{2}\ d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

Indeed, we obtain:

$$\frac{\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)\left(d^3\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2\right)}}{\sqrt[2\pi]{\frac{4}{3}\left(\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\right)\sqrt{2}\left(d^3 \times \frac{\sqrt{2}}{12}\right)d^3}}$$

= 1.6180085459001 07…

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4 \times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358487162422193250985871527893102518924557617007810061 3

…

3.1415926535…. = π (Ramanujan Recurring Number)

In conclusion, from the following further formula concerning the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron volumes and their ratios (and/or the inverses) with the sphere volume, raised the expression

$$(2\log(2)log^{2/3}(3^2))/(3e^{7/3})$$

$$\left(\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\frac{\frac{1}{4}\left(\left(15+7\sqrt{5}\right)d^3\right)}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\right)^{(2\log(2)\log^{2/3}(3^2))/(3e^{7/3})}$$

i.e.

$$\left(\frac{3}{2}\right)^{(4\log(2)\log^{2/3}(9))/(3e^{7/3})}\left(\frac{5\left(3+\sqrt{5}\right)\left(15+7\sqrt{5}\right)}{\pi}\right)^{(2\log(2)\log^{2/3}(9))/(3e^{7/3})}$$

we obtain:

1.6180381711519139857456817084743134673903021614989370712917722071

…

1.6180381711…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

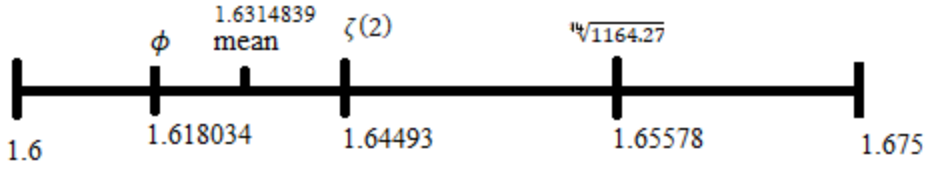
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

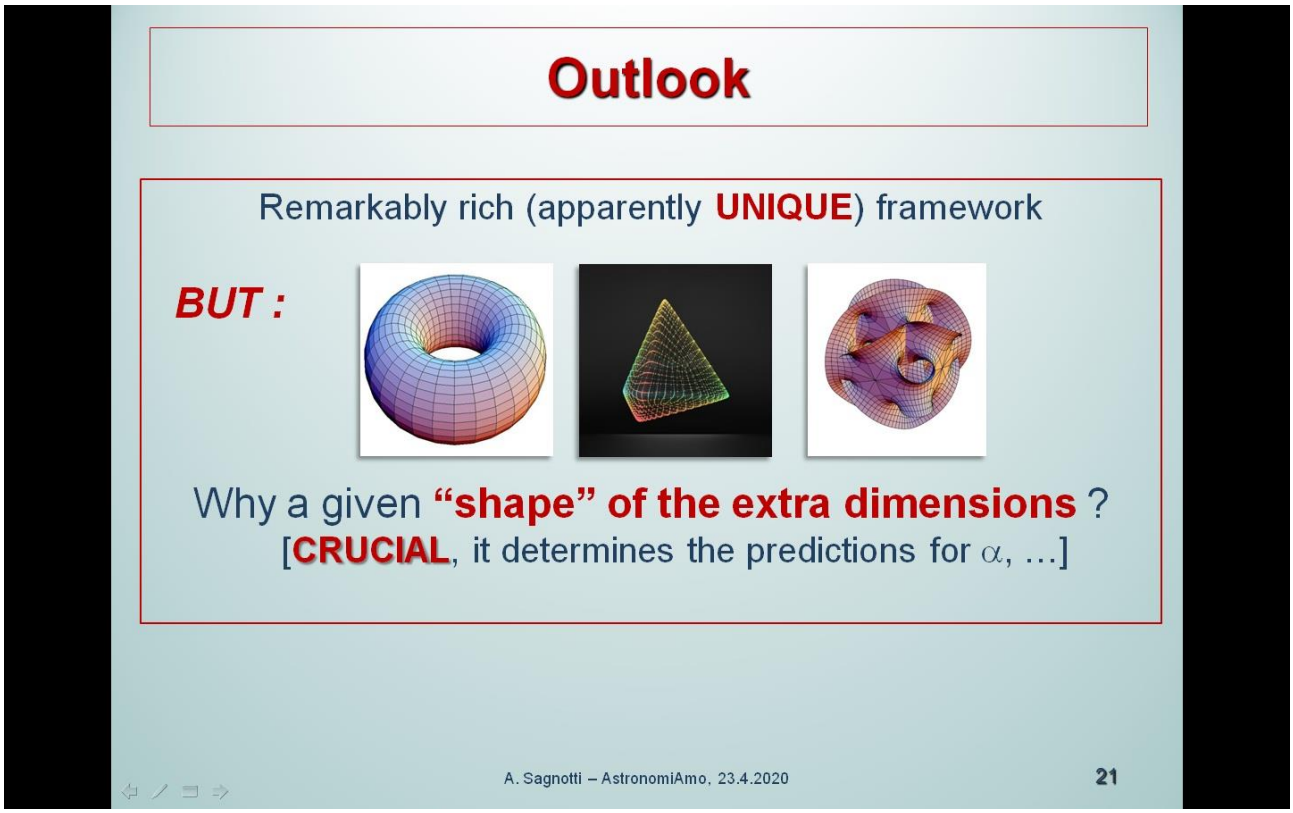
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix


Outlook
Remarkably rich (apparently UNIQUE) framework
BUT :
Why a given "shape" of the extra dimensions ?
[CRUCIAL, it determines the predictions for α, …]
A. Sagnotti – AstronomiAmo, 23.4.2020
21

From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

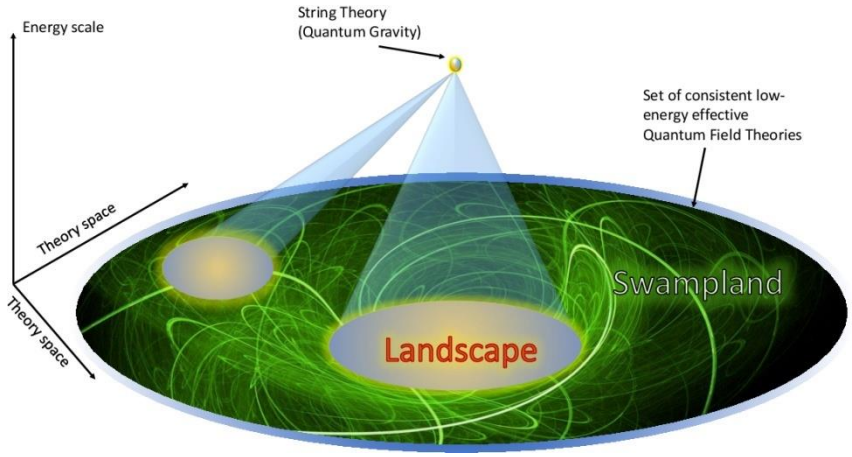


Fig. 1

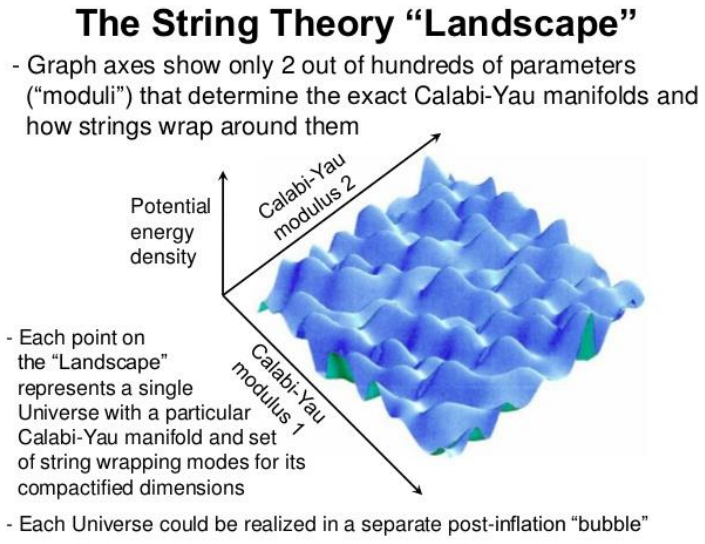

The String Theory "Landscape"
- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them
Potential energy density
Calabi-Yau modulus 2
Calabi-Yau modulus 1
- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

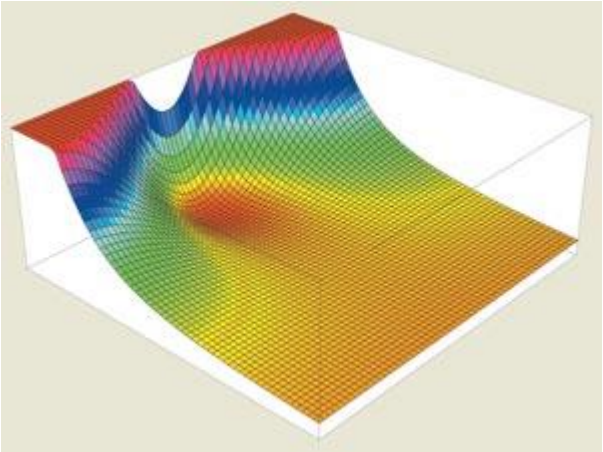


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

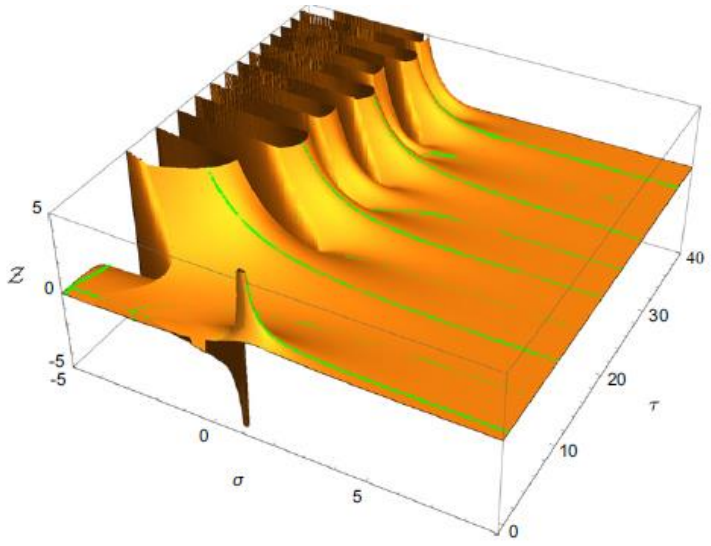


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

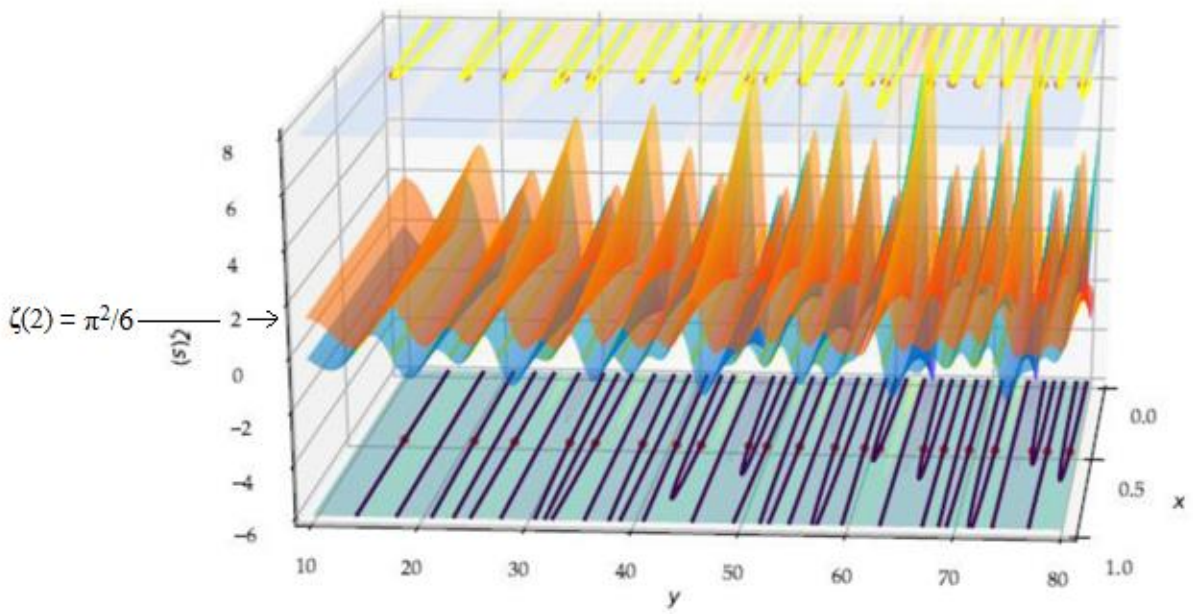


Figure 1. $C(x,y)$ and $S(x,y)$ surfaces of the Riemann $\zeta(x,y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

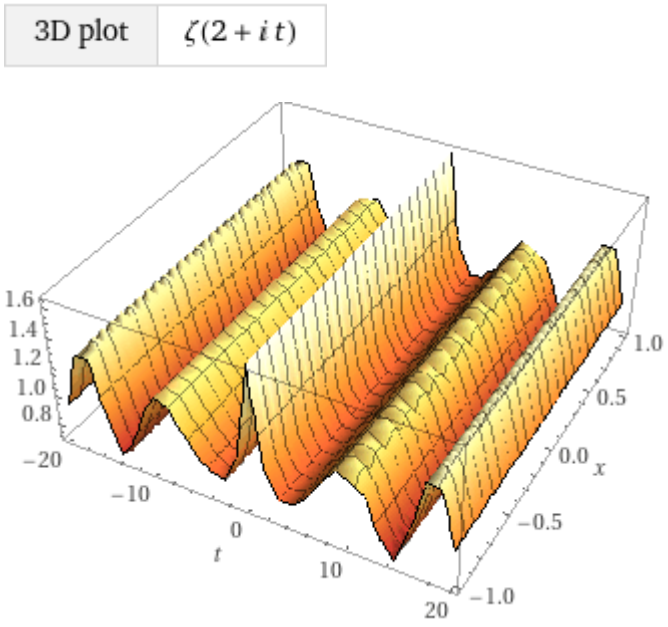


Fig. 6

Where ζ(2+it) :

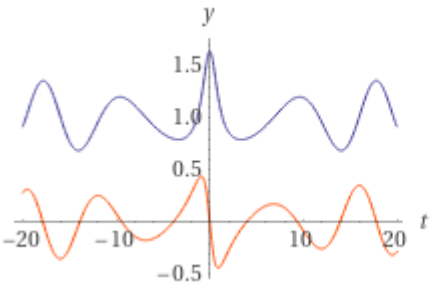
Input

$\zeta(2 + i\,t)$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

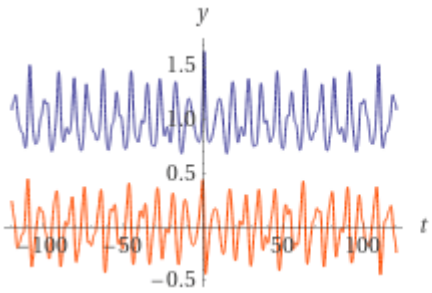
Plots



(t from −20 to 20)

real part

imaginary part



(t from −120 to 120)

real part

imaginary part

Roots

$t = 2\,i\,(n+1), \quad n \in \mathbb{Z}, \quad n \geq 1$

$t = -i\,(\rho_n - 2), \quad n \neq 0, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\,t\,\zeta'(2) - \frac{1}{2}\,t^2\,\zeta''(2) - \frac{1}{6}\,i\,\zeta^{(3)}(2)\,t^3 + \frac{1}{24}\,\zeta^{(4)}(2)\,t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2 + i\,t) = \zeta(2 + i\,t,\, 1)$$

$$\zeta(2 + i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2 + i\,t) = \frac{\zeta\left(2 + i\,t,\, \frac{1}{2}\right)}{-1 + 2^{2+i\,t}}$$

$\zeta(s,\, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2 + i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = \frac{\sum_{k=0}^{\infty} (1 + 2\,k)^{-2-i\,t}}{1 - 2^{-2-i\,t}} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \mathrm{Im}(t) < 1$$

$\mathrm{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^\infty \frac{\tau^{1+i\,t}}{-1+e^\tau}\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^\infty \tau^{2+i\,t}\,\mathrm{csch}^2(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^\infty e^{-\tau}\,\tau^{1+i\,t}\,\mathrm{csch}(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\mathrm{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+i\,t) = -i\,2^{2+i\,t}\,\pi^{1+i\,t}\,\Gamma(-1-i\,t)\sinh\left(\frac{\pi\,t}{2}\right)\zeta(-1-i\,t)$$

$$\zeta(2+i\,t) = \frac{\pi^{3/2+i\,t}\,\Gamma\left(-\frac{1}{2}-\frac{i\,t}{2}\right)\zeta(-1-i\,t)}{\Gamma\left(1+\frac{i\,t}{2}\right)}$$

$$\zeta(2+i\,t) = -\frac{i\sum_{k=0}^\infty \frac{\Gamma\left(k-\frac{i\,t}{2}\right)\sum_{j=0}^k (-1)^j\,(1+2\,j)\binom{k}{j}\zeta(2+2\,j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{i\,t}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372033095 8…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi =$$
$$\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

1.64381856858…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\right/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\text{^}(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/15)\right)$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^-}6$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

$-5.010882647757...$

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right)=0.054323\,(0.0814845\,b+1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M}=-\frac{\frac{226\,802\,245}{18\,480\,874}+b}{M}$$

$$\frac{\partial M(b,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b}=-\frac{18\,480\,874\,M}{226\,802\,245+18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M}=\frac{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b}=\frac{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right\}=0 \text{ at } (b,M)=(-16,0)$$

Global minima:

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

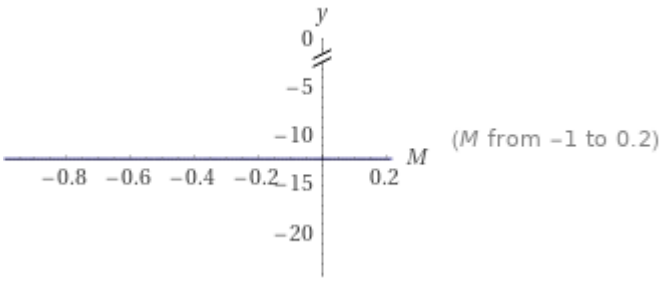
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

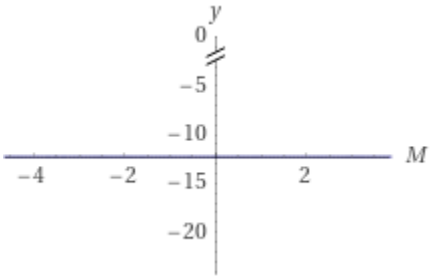
Result:

$$\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323\, M^2\right)}{M^2}\right) = \frac{3.55271 \times 10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

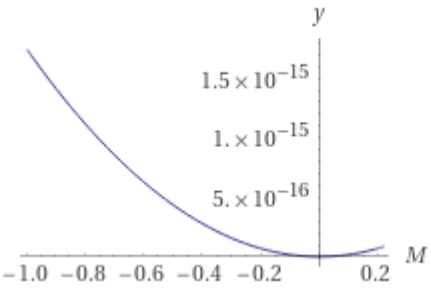
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

Result:

0

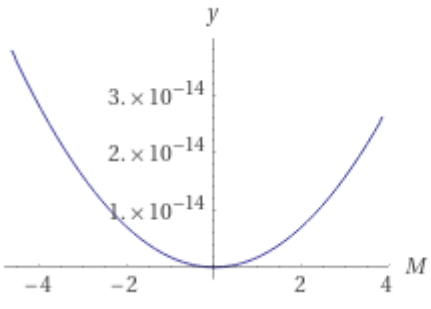
Plots: **(possible mathematical connection with an open string)**



(*M* from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O\left(M^{62194}\right)$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times10^{-15}\,M^2+O\left(\left(\frac{1}{M}\right)^{62\,194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}\,M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.38851344947464545348970783378088020833333333333333333333\ldots \times 10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 0.2^2 + 6.58545 \times 10^{-10}\sqrt{0.2^4}\right)}{0.2^2} + 1\right)^2 \times 0.2^2$$

Result:

$$7.02162151915943272558353253404940833333333333333333333333\ldots \times 10^{-17}$$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875\times10^{-14}$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots \times 10^{-8}$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([[((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left(\left(1/\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)\text{ ^ }(1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

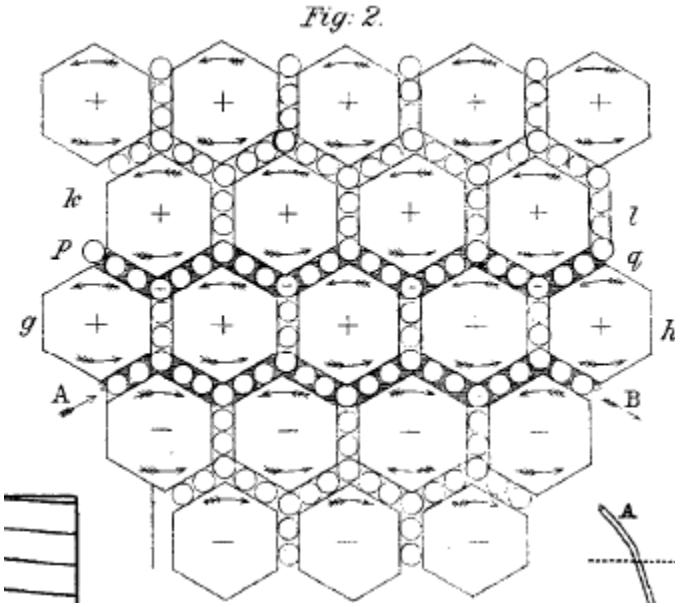
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

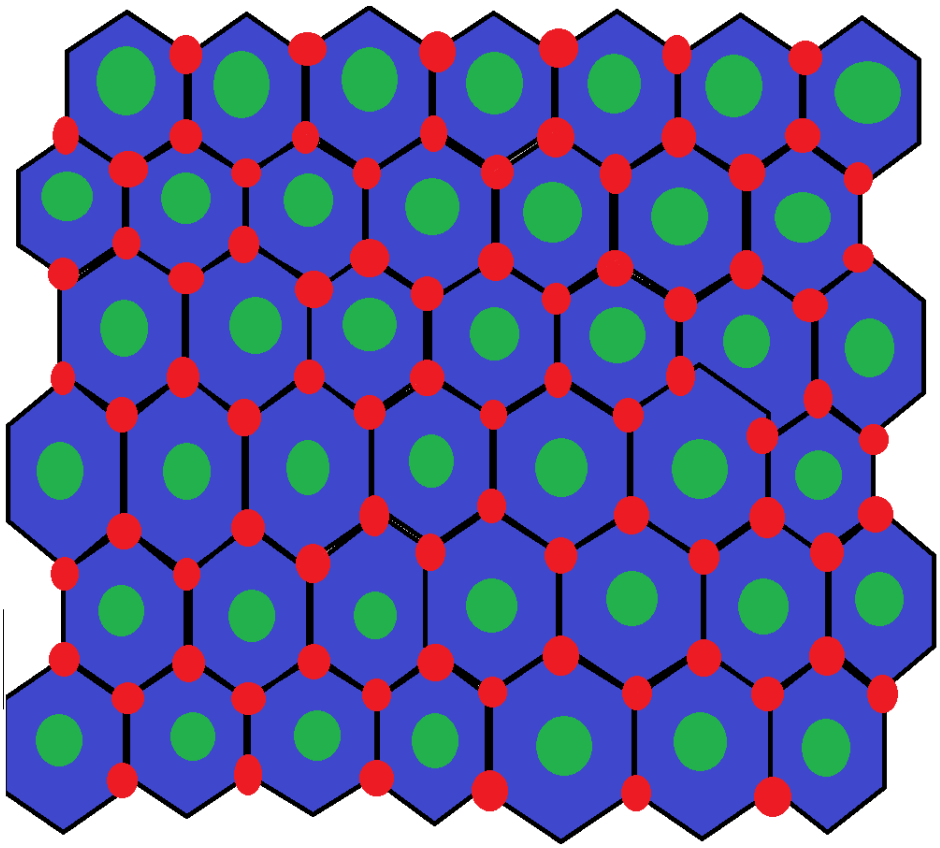
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

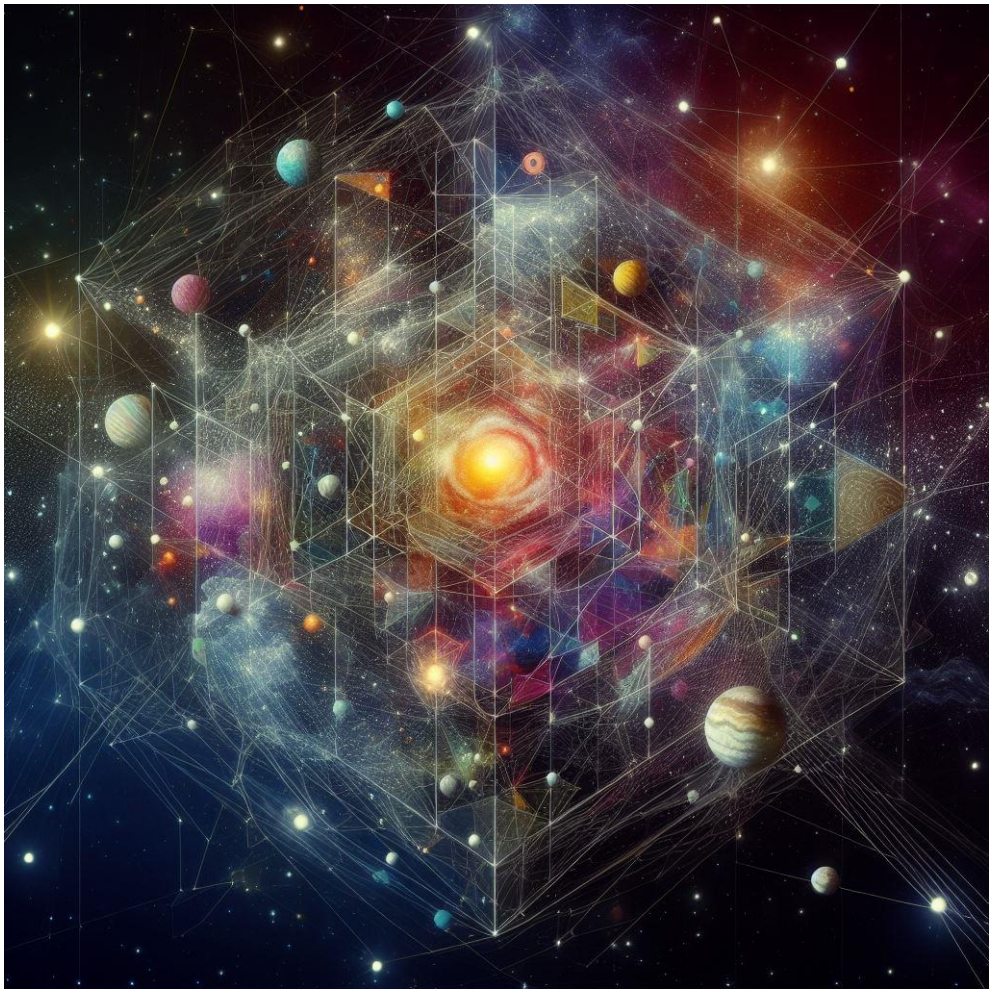
Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Fig. 3



It was hypothesized that the solutions of Ramanujan's cubic equations which are related to the cubic equations solvable with Cardano's method and Padovan's recurrence relations, are boundary conditions to deepen the understanding of the formation of eternally inflationary universe bubbles and an octahedron which could represent a mathematical or conceptual model of the primordial universe before the Big Bang. Finally, regarding cubic equations, they could represent the geometric characteristics of the quantum vacuum, while the spherical surface would represent the geometry of the universe bubble. The results obtained, i.e. $(2\sqrt{2})/\pi$, the golden ratio φ and π, can be connected to the proposed cosmological model.

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability. I would like to thank also my son **Francesco Nardelli**, student of Educational Sciences, majoring in History and Philosophy, for his useful suggestions regarding the intriguing image at the end of this work

References

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *Emilian Dudas, Sergio Ferrara, Alex Kehagias and Augusto Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the analysis of various Cubic Equations that can be solved with Cardano's Formula, Quartic Equations and Ramanujan's Cubic Equations. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory II

Michele Nardelli[1], Antonio Nardelli

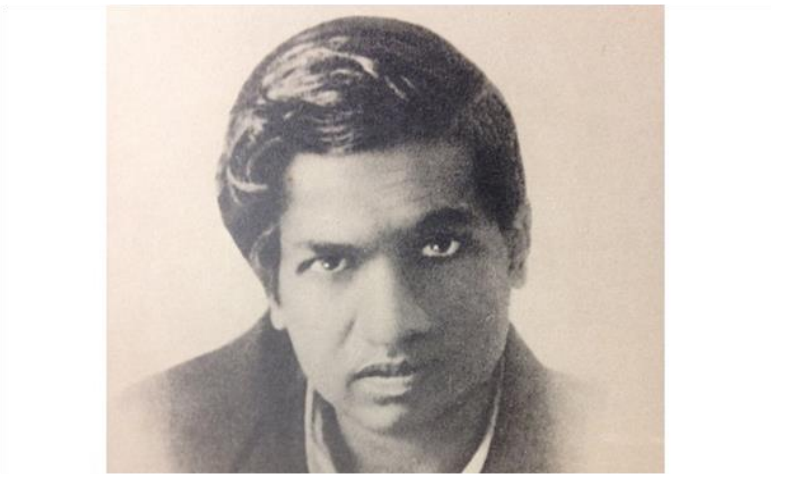
Abstract

In this paper (part II), we analyze various Cubic Equations that can be solved with Cardano's formula, Quartic Equations and Ramanujan's Cubic Equations. We obtain, new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Introduction

Cardano's method for cubic equations involves cube roots and introduces symmetry concepts. Similarly, string theory and M-theory involve symmetries and transformation groups that play a fundamental role in describing particles and interactions. Solutions of complete cubic equations via Cardano's method involve complex cube roots. Similarly, equations and solutions in superstring theory can often involve advanced algebraic and mathematical complexities. Solutions of cubic equations can be understood geometrically through topology and algebraic geometry. In string theory, the geometric and topological configurations of strings and branes are also central. Duality is an important concept in string theory, where different mathematical descriptions can be equivalent. Similarly, in solving cubic equations, duality links exist through cube roots.

Thus, in string theory, duality refers to the fact that different mathematical formulations or physical descriptions of a theory can be equivalent. For example, two seemingly different theories can be dual to each other, meaning they are two different ways of describing the same physical reality. This concept of duality has been very useful for understanding profound aspects of string theory and field theories.

Similarly, in solving cubic equations, duality can emerge through cube roots. The cubic equation $ax^3+bx^2+cx+d=0$ can be solved using cube roots, as in the Cardano method I described above. However, cube roots involve the concept of complex numbers, and it is sometimes possible to express the solutions in ways that highlight a duality between the solutions.

For example, if we consider complex cube roots, we can see a duality between them, since there are connections via the cube roots of unity. This duality is similar to the concept of duality in string theory, where different descriptions can be connected by similar transformations.

In both cases, duality provides a deeper way of understanding the underlying mathematical or physical structure, allowing you to explore hidden connections between different concepts.

Cubic surfaces can be associated with geometric and topological concepts. In string theory, for example, some models require spaces with particular shapes, and the geometry of the spaces can influence the properties of strings and particles. Some mathematical and physical models involving surfaces might be described by cubic or

complex equations. The geometry of surfaces and the associated mathematical structure can influence the properties of universe bubbles in the multiverse. Cubic surfaces can represent certain topological spaces, and their properties can affect the structure of the associated string theory.

The connection between the surface of a D-brane and the roots of a cubic equation can be explored in specific contexts within string theory, especially when it comes to studying the geometric and topological properties of D-branes. D-branes are extended objects on whose surface open strings can terminate. The geometry of the D-brane surface can interfere with the physics of surrounding strings. In some cases, geometry can be described mathematically through objects known in algebraic surface theory, such as cubic equations.

D-branes can host open fields, which describe particles interacting with the D-brane. Field equations on a D-brane can involve surface geometry, and in some situations, these equations can be associated with higher degree algebraic equations, such as cubic equations.

The compactification of extra dimensions, which is an important part of string theory, can be associated with the surface geometry of a D-brane. Compactification can lead to a reduction in dimensions and a geometric description that can involve algebraic equations, including cubic equations.

The topological invariants associated with the surface of a D-brane, such as the Betti number or the Chern number, can be related to geometric characteristics that can be described via algebraic equations, including cubic equations.

In some situations of duality, the description of a D-brane in a context can be dualistic to a description based on algebraic objects such as curves or surfaces, which can be represented via algebraic equations, including cubic equations.

In this paper, we will hypothesize that the solutions of the cubic equations can be used as boundary conditions to further the understanding of the formation of universe bubbles in eternal inflation. Furthermore, an octahedron could serve as a mathematical or conceptual model of the universe in the pre-Big Bang phase. Cubic equations, in this context, could represent the geometric characteristics of the quantum vacuum, while the spherical surface could be used to describe the geometry of the universe bubble.

The results $(2\sqrt{2})/\pi$, the golden ratio φ and π obtained, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and universe bubbles in eternal inflation. It may reflect a fundamental relationship between the geometry of solids and the mathematical characteristics of solutions to cubic equations.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and universe bubbles.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of universe bubbles and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

Finally, it is also possible to hypothesize the solutions of Ramanujan's cubic equations which are related to the cubic equations solvable with the Cardano method and Padovan's recurrence relations, as boundary conditions to deepen the understanding of the formation of inflationary universe bubbles eternal and of an octahedron which could represent a mathematical or conceptual model of the primordial universe before the Big Bang. Furthermore, regarding the cubic equations, they can represent the geometric characteristics of the quantum vacuum, while the spherical surface represents the geometry of the universe bubble. The results above $(2\sqrt{2})/\pi$, the golden ratio φ and π which are obtained, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

These mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe. The cubic equations, in this context, could reflect the geometric and dynamic characteristics of the quantum vacuum, while the spherical surface could represent the geometry of the bubbles of the universe.

Regarding fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the primordial universe and universe bubbles.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

We will also give an example of how the result of multiplying the solutions of a quartic equation for the formula involving the ratios between the volumes of geometric solids and the Del Gaudio-Nardelli Constant approaches the values of the golden ratio and Pi, and as from the product of the solutions, the Recurring Numbers of Ramanujan are obtained. This discovery suggests that there are still many connections to be explored between mathematics and physics, and that these connections could lead to new insights into the nature of the universe and its fundamental laws.

From

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

Cubic equations and Cardano's Formulas

We analyze the following equation:

$$x^3 + 21x = 9x^2 + 5.$$

i.e.

$$x^3 - 9x^2 + 21x - 5 = 0$$

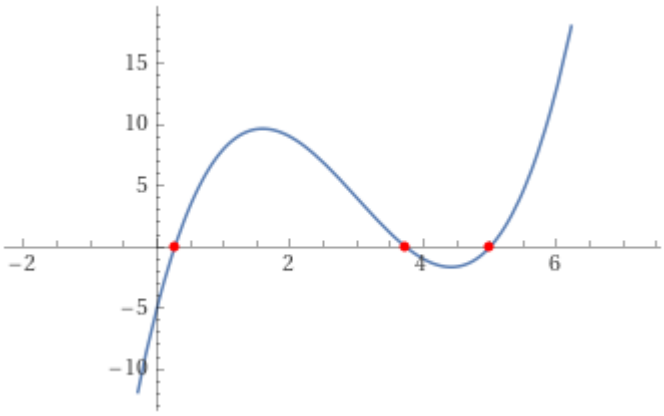
From

x^3-9x^2+21x-5=0

Input

$x^3 - 9x^2 + 21x - 5 = 0$

Root plot **(figure that can be related to an open string)**



Alternate forms

$(x-5)((x-4)x+1)=0$

$(x-5)\left(x^2-4x+1\right)=0$

$(x-3)^3-6(x-3)+4=0$

Solutions

$x=5$

$x=2-\sqrt{3}$

$x=2+\sqrt{3}$

Sum of roots

9

Product of roots

5

From

$$ax^3 + bx^2 + cx + d = 0$$

The Cardano's Formula is:

$$x = \sqrt[3]{-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a} + \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} + \sqrt[3]{-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a} - \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}$$

$$x^3 - 9x^2 + 21x - 5 = 0$$

we obtain:

(-(-729)/(27)+(-189)/6+(5)/2+√(((-(-729)/(27)+(-189)/6+(5)/2)^2+(21/3-(81)/(9))^3)))^1/3+(-(-729)/(27)+(-189)/6+(5)/2-√(((-(-729)/(27)+(-189)/6+(5)/2)^2+(21/3-(81)/(9))^3)))^1/3+(9)/(3)

Input

$$\sqrt[3]{-\frac{-729}{27} - \frac{189}{6} + \frac{5}{2} + \sqrt{\left(-\frac{-729}{27} - \frac{189}{6} + \frac{5}{2}\right)^2 + \left(\frac{21}{3} - \frac{81}{9}\right)^3}} + \sqrt[3]{-\frac{-729}{27} - \frac{189}{6} + \frac{5}{2} - \sqrt{\left(-\frac{-729}{27} - \frac{189}{6} + \frac{5}{2}\right)^2 + \left(\frac{21}{3} - \frac{81}{9}\right)^3}} + \frac{9}{3}$$

Exact result

5

5

From the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3\times\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\times\frac{1}{3}\sqrt{2}d^3\times\frac{\sqrt{2}}{12}d^3}$$

From the previous expression

$$\sqrt[3]{-\frac{-729}{27}-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(-\frac{-729}{27}-\frac{189}{6}+\frac{5}{2}\right)^2+\left(\frac{21}{3}-\frac{81}{9}\right)^3}}+\sqrt[3]{-\frac{-729}{27}-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(-\frac{-729}{27}-\frac{189}{6}+\frac{5}{2}\right)^2+\left(\frac{21}{3}-\frac{81}{9}\right)^3}}+\frac{9}{3}$$

we obtain:

(((27)+(-189)/6+(5)/2+√((((27)+(-189)/6+(5)/2)^2+(7-(9))^3)))^1/3+((27)+(-189)/6+(5)/2-√((((27)+(-189)/6+(5)/2)^2+(7-(9))^3)))^1/3+(3))(((5/12(3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2π)))

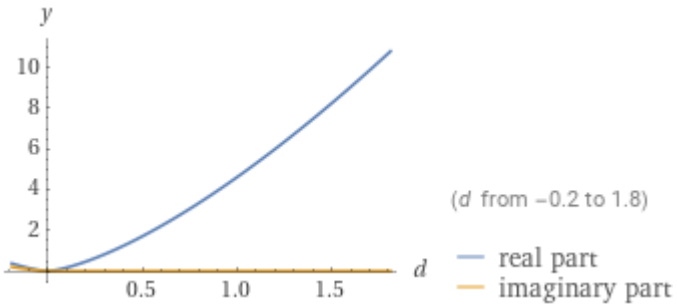
Input

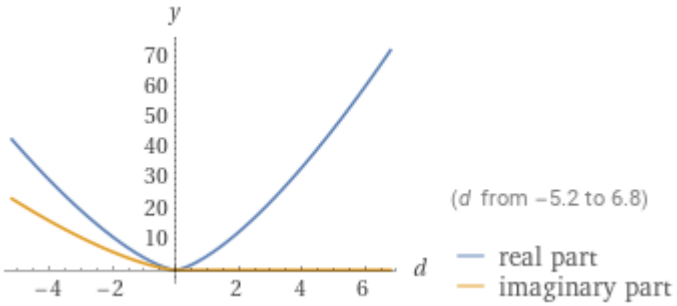
$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Exact result

$$3^{-3/(2\pi)}\times 4^{-1/\pi}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Root

$d = 0$

Series expansion at d=∞

$$3^{-3/(2\pi)} \times 4^{-1/\pi} \times 5^{1+1/(2\pi)} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}\ + \sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}\ +3\right) \sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right) = \frac{2^{-(2+\pi)/\pi} \times 3^{2-3/(2\pi)} \times 5^{1+1/(2\pi)} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[2\pi]{d^9}\ \pi^{1/\pi-1}}{d}$$

Indefinite integral

$$\int 3^{-3/(2\pi)} \times 4^{-1/\pi} \times 5^{1+1/(2\pi)} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[2\pi]{d^9}\ \sqrt[\pi]{\pi}\ dd = \frac{3^{-3/(2\pi)} \times 5^{1+1/(2\pi)} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\frac{\pi}{4}}\ d\sqrt[2\pi]{d^9}}{1+\frac{9}{2\pi}} + \text{constant}$$

Global minimum

$$\min\left\{\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right\}=0 \text{ at } d=0$$

Series representations

$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=3^{-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5}$$

$$\left(3+\sqrt[3]{-2-\sqrt{-5}\sum_{k=0}^{\infty}(-5)^{-k}\binom{\frac{1}{2}}{k}}+\sqrt[3]{-2+\sqrt{-5}\sum_{k=0}^{\infty}(-5)^{-k}\binom{\frac{1}{2}}{k}}\right)\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=$$

$$3^{-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}\left(3+\sqrt[3]{-2-\sqrt{-5}\sum_{k=0}^{\infty}\frac{5^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\sqrt[3]{-2+\sqrt{-5}\sum_{k=0}^{\infty}\frac{5^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=3^{-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5}$$

$$\left(3+\sqrt[3]{-2-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(-4-z_0)^k z_0^{-k}}{k!}}+\sqrt[3]{-2+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(-4-z_0)^k z_0^{-k}}{k!}}\right)\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

And:

(((27)+(-189)/6+(5)/2+√((((27)+(-189)/6+(5)/2)^2+(7-(9))^3)))^1/3+((27)+(-189)/6+(5)/2-√((((27)+(-189)/6+(5)/2)^2+(7-(9))^3)))^1/3+(3))((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

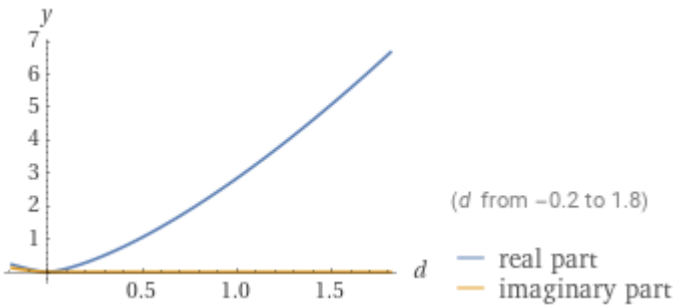
Input

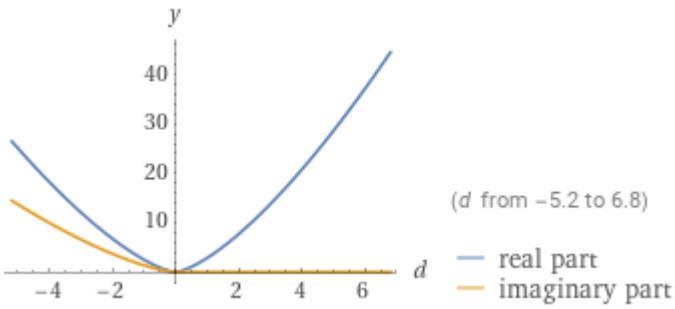
$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

Exact result

$$5\times 2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Root

$d = 0$

Series expansion at d=∞

$5 \times 2^{-1/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$

Derivative

$$\frac{d}{dd}\left(\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}\ +\ \sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}\ +3\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)}{3\times3\times12}}\right)=\frac{5\times2^{-(1+\pi)/\pi}\times3^{2-3/(2\pi)}\sqrt[2\pi]{d^9}\ \pi^{1/(2\pi)-1}}{d}$$

Indefinite integral

$$\int 5\times2^{-1/\pi}\times3^{-3/(2\pi)}\sqrt[2\pi]{d^9}\ \sqrt[2\pi]{\pi}\ dd=\frac{5\times2^{1-1/\pi}\times3^{-3/(2\pi)}\ \pi^{1+1/(2\pi)}\ d\sqrt[2\pi]{d^9}}{9+2\pi}+\text{constant}$$

Global minimum

$$\min\left\{\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right\}=0 \text{ at } d=0$$

Series representations

$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times12)\,3}}=5\times3^{-3\Big/\left(8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)}$$

$$\sqrt[8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}]{d^9}^{\,8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}\sqrt{\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}$$

$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=5\times 2^{-1/\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)}\times 3^{-3/\left(2\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)}\ \sqrt[2\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)]{d^9}\ \sqrt{\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)}$$

$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=5\times 2^{-1/\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)}\times 3^{-3/\left(2\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)}\ \sqrt[2\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}]{d^9}\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^{\left(\frac{1}{2\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}}\right)}$$

Integral representations

$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=5\times 3^{-3\Big/\left(8\int_0^1\sqrt{1-t^2}\ dt\right)}$$

$$\sqrt[8\int_0^1\sqrt{1-t^2}\ dt]{d^9}\qquad\sqrt[8\int_0^1\sqrt{1-t^2}\ dt]{\int_0^1\sqrt{1-t^2}\ dt}$$

$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=5\times 6^{-1\Big/\left(3\int_0^\infty\frac{\sin^4(t)}{t^4}\,dt\right)}$$

$$\sqrt[6\int_0^\infty\frac{\sin^4(t)}{t^4}\,dt]{d^9}\qquad\sqrt[6\int_0^\infty\frac{\sin^4(t)}{t^4}\,dt]{\int_0^\infty\frac{\sin^4(t)}{t^4}\,dt}$$

$$\left(\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}+\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+\sqrt[3]{27-\frac{189}{6}+\frac{5}{2}-\sqrt{\left(27-\frac{189}{6}+\frac{5}{2}\right)^2+(7-9)^3}}+3\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=5\times 2^{-1/\left(4\int_0^\infty \frac{\sin(t)}{t}\,dt\right)}\times 3^{-3/\left(4\int_0^\infty \frac{\sin(t)}{t}\,dt\right)}\sqrt[4\int_0^\infty \frac{\sin(t)}{t}\,dt]{d^9}\sqrt[4\int_0^\infty \frac{\sin(t)}{t}\,dt]{\int_0^\infty \frac{\sin(t)}{t}\,dt}$$

Dividing the two exact results

$$3^{-3/(2\pi)}\times 4^{-1/\pi}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

and

$$5\times 2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

we obtain:

(3^(-3/(2 π)) 4^(-1/π) 5^(1 + 1/(2 π)) (3 + sqrt(5))^(1/(2 π)) π^(1/π) (d^9)^(1/(2 π)))/(5 2^(-1/π) 3^(-3/(2 π)) π^(1/(2 π)) (d^9)^(1/(2 π)))

Input

$$\frac{3^{-3/(2\pi)}\times 4^{-1/\pi}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}}{5\times 2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{3^{-3/(2\pi)}\left(4^{-1/\pi}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\right)}{5\times 2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}=$$
$$2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-3/(2\pi)}\left(4^{-1/\pi}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\right)}{5\times 2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}=$$
$$2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{3^{-3/(2\pi)}\left(4^{-1/\pi}\times 5^{1+1/(2\pi)}\sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\right)}{5\times 2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}=$$
$$2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535884…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.644934066846858716841179523870420476336069416988202988844504692 0

…

1.644934066846…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.92478440155986\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Quartic Equation solution

We have the following quartic equation

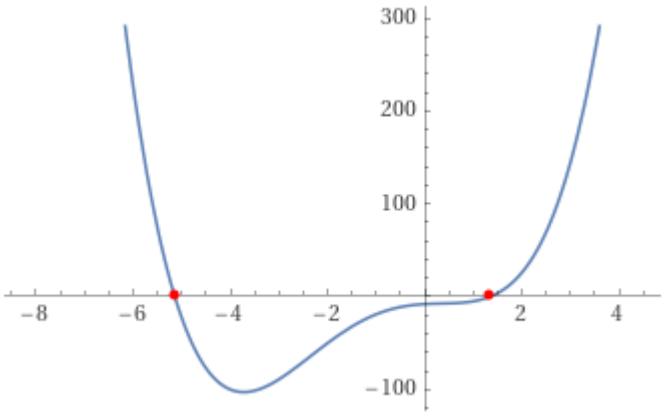
$$x^4 + 4x^3 - 5x^2 + 3x - 8 = 0$$

x^4 + 4x^3 - 5x^2 + 3x - 8 = 0

Input

$$x^4 + 4x^3 - 5x^2 + 3x - 8 = 0$$

Root plot (figure that can be related to an open string)



Alternate forms

$$x((x-1)x(x+5)+3)-8=0$$

$$x^4 + 4x^3 - 5x^2 + 3x = 8$$

$$x^4 + 4x^3 + 3x = 5x^2 + 8$$

Real solutions

$$x = -1 - \frac{1}{2\sqrt{\frac{3}{22 - 107\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} + \sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)}}}} -$$

$$\frac{1}{2}\sqrt{\left(\frac{44}{3} + \frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} - \frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)} + 42\sqrt{\frac{3}{22 - 107\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} + \sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)}}}\right)}$$

$$x = -1 - \frac{1}{2\sqrt{\frac{3}{22 - 107\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} + \sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)}}}} +$$

$$\frac{1}{2}\sqrt{\left(\frac{44}{3} + \frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} - \frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)} + 42\sqrt{\frac{3}{22 - 107\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} + \sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)}}}\right)}$$

$x \approx -5.1441$

$x \approx 1.3452$

Complex solutions

$$x = -1 + \frac{1}{2\sqrt{\frac{3}{22 - 107\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} + \sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)}}}} - \frac{1}{2}\sqrt{\left(\frac{44}{3} + \frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} - \frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)} - 42\sqrt{\frac{3}{22 - 107\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} + \sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)}}}\right)}$$

$$x = -1 + \frac{1}{2\sqrt{\frac{3}{22 - 107\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} + \sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)}}}} + \frac{1}{2}\sqrt{\left(\frac{44}{3} + \frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} - \frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)} - 42\sqrt{\frac{3}{22 - 107\sqrt[3]{\frac{2}{3\sqrt{4286109} - 5803}} + \sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109} - 5803\right)}}}\right)}$$

Sum of roots

-4

Product of roots

-8

From the real solution

$$x = -1 - \frac{1}{2\sqrt{\frac{3}{22-107\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}+\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}}}} -$$

$$\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+\right.}$$

$$\left.42\sqrt{\frac{3}{22-107\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}+\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}}}\right)$$

That is:

(1/2 (3 sqrt(4286109) - 5803))^(1/3)

Input

$$\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}$$

Decimal approximation

5.886169005872806827671315057686743594042868073221143245749337666 8

…

5.8861690058728….

Alternate forms

$$\frac{1}{2}\sqrt[3]{3\sqrt{4286109}-5803}\;2^{2/3}$$

$$\sqrt[3]{\frac{3\sqrt{4286109}}{2}-\frac{5803}{2}}$$

---

root of $x^6+5803\,x^3-1225043$ near $x=5.88617$

Minimal polynomial

$x^6+5803\,x^3-1225043$

(2/(3 sqrt(4286109) - 5803))^(1/3)

Input

$$\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}$$

Decimal approximation

0.16988978722871703099444878963053826357760907652663454336677550 61

…

0.169889787228717….

Alternate forms

$$\frac{1}{214}\sqrt[3]{5803+3\sqrt{4286109}}\;2^{2/3}$$

$$\sqrt[3]{\frac{5803}{2450086}+\frac{3\sqrt{4286109}}{2450086}}$$

Minimal polynomial

$1225043\,x^6-5803\,x^3-1$

we obtain:

-1 - 1/(2 sqrt(3/(22 - 107 0.1698897872287+5.8861690058728))) - 1/2 sqrt(44/3 + 107/3 (2/(3 sqrt(4286109) - 5803))^(1/3) - 1/3 (1/2 (3 sqrt(4286109) - 5803))^(1/3) + 42 sqrt(3/(22 - 107 0.1698897872287 + 5.8861690058728)))

Input interpretation

$$-1-\frac{1}{2\sqrt{\frac{3}{22+107\times(-0.1698897872287)+5.8861690058728}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+42\sqrt{\frac{3}{22+107\times(-0.1698897872287)+5.8861690058728}}\right)}$$

Result

$-5.144123098240\ldots$

-5.14412309824….

And:

-1 - 1/(2 sqrt(3/(22 - 107 0.1698897872287+5.8861690058728)))+1/2 sqrt(44/3+107/3(2/(3sqrt(4286109)-5803))^(1/3)-1/3(1/2(3sqrt(4286109)-5803))^(1/3)+42sqrt(3/(22-107 0.1698897872287+5.8861690058728)))

Input interpretation

$$-1-\frac{1}{2\sqrt{\frac{3}{22+107\times(-0.1698897872287)+5.8861690058728}}}+\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+42\sqrt{\frac{3}{22+107\times(-0.1698897872287)+5.8861690058728}}\right)}$$

Result

1.345238094282…

1.345238094282….

And again:

-1 + 1/(2 sqrt(3/(22-107 0.1698897872287+5.8861690058728)))-1/2 sqrt(44/3+107/3(2/(3sqrt(4286109)-5803))^(1/3)-1/3(1/2(3sqrt(4286109)-5803))^(1/3)-42sqrt(3/(22-107 0.1698897872287+5.8861690058728)))

Input interpretation

$$-1+\frac{1}{2\sqrt{\frac{3}{22+107\times(-0.1698897872287)+5.8861690058728}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}-42\sqrt{\frac{3}{22+107\times(-0.1698897872287)+5.8861690058728}}\right)}$$

Result

$-0.1005574980207\ldots - 1.070488580268\ldots\, i$

Alternate complex forms

$$1.075201195541\,(\cos(-1.664457560862)+i\sin(-1.664457560862))$$

$$1.075201195541\,e^{-1.664457560862\,i}$$

Polar coordinates

$r = 1.075201195541$ (radius), $\theta = -1.664457560862$ (angle)

1.075201195541

-1 + 1/(2 sqrt(3/(22-107 0.1698897872287+5.8861690058728)))+1/2 sqrt(44/3+107/3(2/(3sqrt(4286109)-5803))^(1/3)-1/3(1/2(3sqrt(4286109)-5803))^(1/3)-42sqrt(3/(22-107 0.1698897872287+5.8861690058728)))

Input interpretation

$$-1+\frac{1}{2\sqrt{\frac{3}{22+107\times(-0.1698897872287)+5.8861690058728}}}+\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}-42\sqrt{\frac{3}{22+107\times(-0.1698897872287)+5.8861690058728}}\right)}$$

Result

$-0.1005574980207\ldots + 1.070488580268\ldots\, i$

Alternate complex forms

$1.075201195541\,(\cos(1.664457560862)+i\sin(1.664457560862))$

$1.075201195541\,e^{1.664457560862\,i}$

Polar coordinates

$r = 1.075201195541$ (radius), $\theta = 1.664457560862$ (angle)

1.075201195541

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the above formula, concerning a real root

$$-1-\frac{1}{2\sqrt{\frac{3}{22+107\times(-0.1698897872287)+5.8861690058728}}}-$$
$$\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+\right.}$$
$$\left.42\sqrt{\frac{3}{22+107\times(-0.1698897872287)+5.8861690058728}}\right)$$

we obtain:

((-1-1/(2√(3/(22-107 0.1698+5.886)))-1/2√(44/3+107/3(2/(3√(4286109) -
5803))^(1/3)-1/3 (1/2(3√(4286109)-5803))^(1/3)+42√(3/(22-107
0.1698+5.886)))))((((5/12(3+√5)d^3)((4/3π(d/2)^3)^2))^(1/(2Pi))))

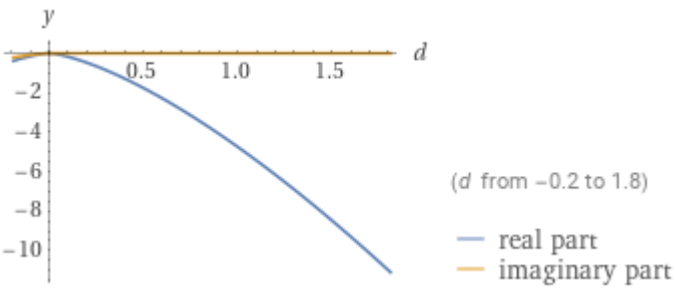
Input

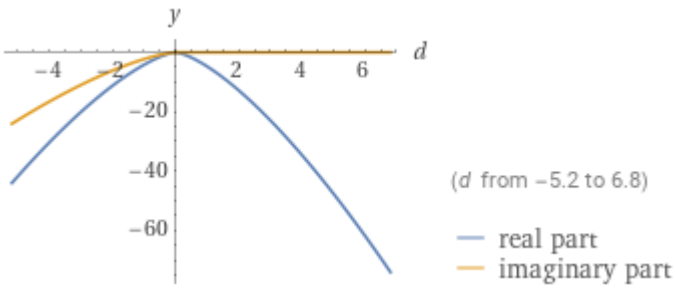
$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)}\right)\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Result

$$-4.74009\sqrt[2\pi]{d^9}$$

Plots **(figures that can be related to the open strings)**





Alternate form assuming d>0

$-4.74009\, d^{9/(2\pi)}$

Root

$d = 0$

Series expansion at d=∞

$-4.74009\, d^{9/(2\pi)}$

Derivative

$$\frac{d}{dd}\left(\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4\,286\,109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4\,286\,109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=-\frac{6.78968\sqrt[2\pi]{d^9}}{d}$$

Indefinite integral

$$\int -4.74009\,\sqrt[2\pi]{d^9}\;dd = -1.94873\,d\left(d^9\right)^{0.159155} + \text{constant}$$

Global maximum

$$\max\left\{\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right\} = 0 \text{ at } d=0$$

Series representations

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)}\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)} = -4.74009\,\sqrt[8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}]{d^9}$$

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4\,286\,109}-5803}}-\frac{1}{3}\right.}\right.$$
$$\left.\left.\sqrt[3]{\frac{1}{2}\left(3\sqrt{4\,286\,109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)\right)$$

$$\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=-4.74009$$

$$\sqrt[2\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)]{d^9}$$

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4\,286\,109}-5803}}-\frac{1}{3}\right.}\right.$$
$$\left.\left.\sqrt[3]{\frac{1}{2}\left(3\sqrt{4\,286\,109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)\right)$$

$$\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=-4.74009$$

$$\sqrt[8\sum_{k=0}^{\infty}\frac{(-1)^{1+k}\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}]{d^9}$$

Integral representations

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4\,286\,109}-5803}}-\frac{1}{3}\right.}\right.$$
$$\left.\left.\sqrt[3]{\frac{1}{2}\left(3\sqrt{4\,286\,109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)\right)$$
$$\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=-4.74009^{4\int_0^\infty \frac{1}{1+t^2}\,dt}\sqrt{d^9}$$

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4\,286\,109}-5803}}-\frac{1}{3}\right.}\right.$$
$$\left.\left.\sqrt[3]{\frac{1}{2}\left(3\sqrt{4\,286\,109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)\right)$$
$$\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=-4.74009^{8\int_0^1 \sqrt{1-t^2}\,dt}\sqrt{d^9}$$

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4\,286\,109}-5803}}-\frac{1}{3}\right.}\right.$$
$$\left.\left.\sqrt[3]{\frac{1}{2}\left(3\sqrt{4\,286\,109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)\right)$$
$$\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=-4.74009^{4\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt}\sqrt{d^9}$$

Definite integral after subtraction of diverging parts

$$\int_0^{\infty} -4.74009\sqrt[2\pi]{d^9} \; - -4.74009\, d^{9/(2\pi)}\, dd = 0$$

And:

((-1-1/(2√(3/(22-107 0.1698+5.886)))-1/2√(44/3+107/3(2/(3√(4286109) -5803))^(1/3)-1/3 (1/2(3√(4286109)-5803))^(1/3)+42√(3/(22-107 0.1698+5.886)))))(((4/3π(d/2)^3)(1/3√2 d^3)(√2/12 d^3))^(1/(2Pi)))

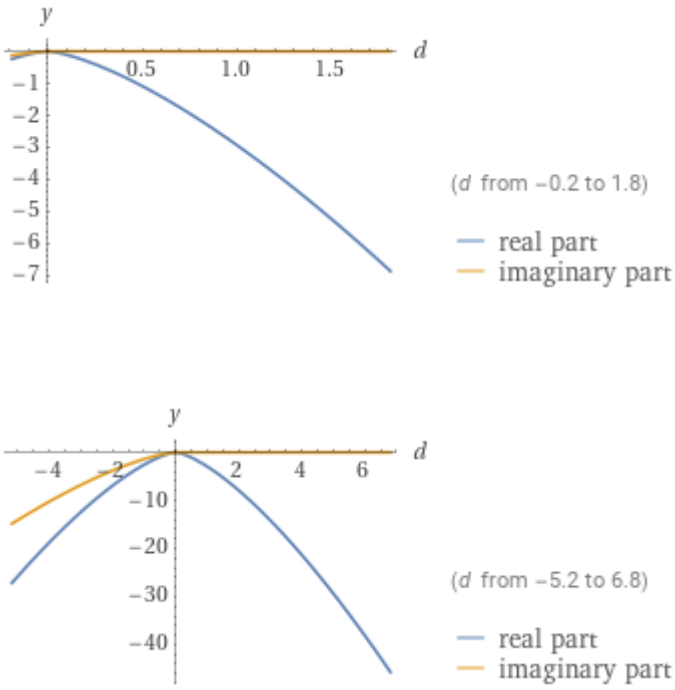
Input

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)}\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

Result

$$-2.92958\sqrt[2\pi]{d^9}$$

Plots **(figures that can be related to the open strings)**



Alternate form assuming d>0

$-2.92958\, d^{9/(2\pi)}$

Root

$d = 0$

Series expansion at d=∞

$-2.92958\, d^{9/(2\pi)}$

Derivative

$$\frac{d}{dd}\left(\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+42\sqrt{\frac{3}{22-107\times0.1698+5.886}}\right)}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right)=-\frac{4.19632\sqrt[2\pi]{d^9}}{d}$$

Indefinite integral

$$\int -2.92958\sqrt[2\pi]{d^9}\,dd=-1.2044\,d\left(d^9\right)^{0.159155}+\text{constant}$$

Global maximum

$$\max\left\{\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+42\sqrt{\frac{3}{22-107\times0.1698+5.886}}\right)}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right\}=0\ \text{at}\ d=0$$

Series representations

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=-2.92958\sqrt[8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}]{d^9}$$

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4286109}-5803}}-\frac{1}{3}\sqrt[3]{\frac{1}{2}\left(3\sqrt{4286109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=-2.92958\sqrt[2\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)]{d^9}$$

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4\,286\,109}-5803}}-\frac{1}{3}\right.}\right.$$
$$\left.\left.\sqrt[3]{\frac{1}{2}\left(3\sqrt{4\,286\,109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)\right)$$
$$\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=-2.92958$$
$$\sqrt[8\sum_{k=0}^{\infty}\frac{(-1)^{1+k}\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}]{d^9}$$

Integral representations

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4\,286\,109}-5803}}-\frac{1}{3}\right.}\right.$$
$$\left.\left.\sqrt[3]{\frac{1}{2}\left(3\sqrt{4\,286\,109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)\right)$$
$$\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=-2.92958^{\,4\int_0^{\infty}\frac{1}{1+t^2}\,dt}\sqrt{d^9}$$

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4\,286\,109}-5803}}-\frac{1}{3}\right.}\right.$$
$$\left.\left.\sqrt[3]{\frac{1}{2}\left(3\sqrt{4\,286\,109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)\right)$$
$$\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=-2.92958^{\,8\int_0^1\sqrt{1-t^2}\,dt}\sqrt{d^9}$$

$$\left(-1-\frac{1}{2\sqrt{\frac{3}{22-107\times 0.1698+5.886}}}-\frac{1}{2}\sqrt{\left(\frac{44}{3}+\frac{107}{3}\sqrt[3]{\frac{2}{3\sqrt{4\,286\,109}-5803}}-\frac{1}{3}\right.}\right.$$
$$\left.\left.\sqrt[3]{\frac{1}{2}\left(3\sqrt{4\,286\,109}-5803\right)}+42\sqrt{\frac{3}{22-107\times 0.1698+5.886}}\right)\right)$$
$$\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=-2.92958^{\,4\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt}\sqrt{d^9}$$

Definite integral after subtraction of diverging parts

$$\int_0^\infty -2.92958\sqrt[2\pi]{d^9}\ --2.92958\,d^{9/(2\pi)}\,dd=0$$

Dividing the two results

$$-4.74009\sqrt[2\pi]{d^9}$$

and

$$-2.92958\sqrt[2\pi]{d^9}$$

we obtain:

(-4.74009 (d^9)^(1/(2 π)))/(-2.92958 (d^9)^(1/(2 π)))

Input interpretation

$$-\frac{-4.74009\sqrt[2\pi]{d^9}}{2.92958\sqrt[2\pi]{d^9}}$$

Result

1.6180100901835758026747861468196806368148335256248335938940052840 …

1.6180100901835758…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternative representations

$$\frac{-4.74009\sqrt[2\pi]{d^9}}{-2.92958\sqrt[2\pi]{d^9}} = \frac{-4.74009\sqrt[360^\circ]{d^9}}{-2.92958\sqrt[360^\circ]{d^9}}$$

$$\frac{-4.74009\sqrt[2\pi]{d^9}}{-2.92958\sqrt[2\pi]{d^9}} = \frac{-4.74009\left(d^9\right)^{-1/(2i\log(-1))}}{-2.92958\left(d^9\right)^{-1/(2i\log(-1))}}$$

$$\frac{-4.74009\,\sqrt[2\pi]{d^9}}{-2.92958\,\sqrt[2\pi]{d^9}} = \frac{-4.74009\,\sqrt[2\cos^{-1}(-1)]{d^9}}{-2.92958\,\sqrt[2\cos^{-1}(-1)]{d^9}}$$

Furthermore, from the formula

$$-\frac{-4.74009\,\sqrt[2\pi]{d^9}}{2.92958\,\sqrt[2\pi]{d^9}}$$

we obtain also:

(4((-4.74009 (d^9)^(1/(2 π)))/(-2.92958 (d^9)^(1/(2 π))))^(2 π))/(5(3 + sqrt(5)))

Input interpretation

$$\frac{4\left(-\frac{-4.74009\,\sqrt[2\pi]{d^9}}{2.92958\,\sqrt[2\pi]{d^9}}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

Result

3.14161149342272926548620125806582543222638607752137943126672979 54…

3.141611493422…. ≈ π (Ramanujan Recurring Number)

Series representations

$$\frac{4\left(\frac{-4.74009\,\sqrt[2\pi]{d^9}}{-2.92958\,\sqrt[2\pi]{d^9}}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)} = \frac{4\,e^{0.962394\,\pi}}{5\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{4\left(\frac{-4.74009^{2\pi}\sqrt{d^9}}{-2.92958^{2\pi}\sqrt{d^9}}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)} = \frac{4\,e^{0.962394\,\pi}}{5\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\frac{4\left(\frac{-4.74009^{2\pi}\sqrt{d^9}}{-2.92958^{2\pi}\sqrt{d^9}}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)} = \frac{8\,e^{0.962394\,\pi}\,\sqrt{\pi}}{5\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

1/6((4((-4.74009 (d^9)^(1/(2 π)))/(-2.92958 (d^9)^(1/(2 π))))^(2 π))/(5(3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\left(-\frac{-4.74009^{2\pi}\sqrt{d^9}}{2.92958^{2\pi}\sqrt{d^9}}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64495379593429854782274937373302580498848205750705921532412256 0

…

1.644953795934…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\left(\frac{-4.74009\sqrt[2\pi]{d^9}}{-2.92958\sqrt[2\pi]{d^9}}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92479\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\left(\frac{-4.74009\sqrt[2\pi]{d^9}}{-2.92958\sqrt[2\pi]{d^9}}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92479\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\left(\frac{-4.74009\sqrt[2\pi]{d^9}}{-2.92958\sqrt[2\pi]{d^9}}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{32\,e^{1.92479\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the sum of the roots

-5.14412309824 ; 1.345238094282 ; (-0.1005574980207-1.070488580268i) ;

(-0.1005574980207+1.070488580268i)

we obtain:

-5.14412309824+1.345238094282+(-0.1005574980207-1.070488580268i)+(-0.1005574980207+1.070488580268i)

Input interpretation

$$-5.14412309824 + 1.345238094282 + (-0.1005574980207 + i \times (-1.070488580268)) + (-0.1005574980207 + 1.070488580268\, i)$$

i is the imaginary unit

Result

$$-3.9999999999994$$

-3.999999… ≈ - 4

From the product:

-5.14412309824*1.345238094282*(-0.1005574980207-1.070488580268i)*(-0.1005574980207+1.070488580268i)

Input interpretation

$$-5.14412309824 \times 1.345238094282 (-0.1005574980207 + i \times (-1.070488580268)) \times (-0.1005574980207 + 1.070488580268\, i)$$

i is the imaginary unit

Result

$$-7.999999999916275253576568875737213964695765525632$$

-7.999999… ≈ - 8 = |8|

---

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

From which, we obtain also:

-3(-5.14412309824*1.345238094282*(-0.1005574980207-1.070488580268i)*(-0.1005574980207+1.070488580268i))

Input interpretation

$$-3\,(-5.14412309824 \times 1.345238094282 \\ (-0.1005574980207 + i \times (-1.070488580268)) \times \\ (-0.1005574980207 + 1.070488580268\,i))$$

i is the imaginary unit

Result

23.999999999974882576072970662721164189408729657689 6

23.999999… ≈ 24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles). Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

(-5.14412309824*1.345238094282*(-0.1005574980207-1.070488580268i)*(-0.1005574980207+1.070488580268i))^4

Input interpretation

$$(-5.14412309824 \times 1.345238094282 \\ (-0.1005574980207 + i \times (-1.070488580268)) \times \\ (-0.1005574980207 + 1.070488580268\,i))^4$$

i is the imaginary unit

Result

4095.999999828531719325082235103298857054875399399535575698799144

…

4095.999999… ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

27*sqrt((-5.14412309824*1.345238094282*(-0.1005574980207-1.070488580268i)*(-0.1005574980207+1.070488580268i))^4)+1

Input interpretation

$$27\sqrt{(-5.14412309824 \times 1.345238094282 (-0.1005574980207 + i \times (-1.070488580268)) \times (-0.1005574980207 + 1.070488580268\, i))^4} + 1$$

i is the imaginary unit

Result

1728.99999999638309095450966808680183359242227401184232547074993 64

…

1728.999999… ≈ 1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2.

Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

((27*sqrt((-5.14412309824*1.345238094282*(-0.1005574980207-1.070488580268i)*(-0.1005574980207+1.070488580268i))^4)+1))^1/15+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\left(27\sqrt{(-5.14412309824\times 1.345238094282\,(-0.1005574980207+i\times(-1.070488580268))\times(-0.1005574980207+1.070488580268\,i))^4}+1\right)\,\hat{}\,(1/15)+C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}$$

i is the imaginary unit

C_{MRB} is the MRB constant

Result

1.644938024130…

1.644938024130…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

---

sqrt(1/(((27*sqrt((-5.14412309824*1.345238094282*(-0.1005574980207-1.070488580268i)*(-0.1005574980207+1.070488580268i))^4)+1))^1/15+(MRB const)^(1-1/(4π)+π))*(4/3))

Input interpretation

$$\sqrt{\left(1/(((27\sqrt{(-5.14412309824\times 1.345238094282\times(-0.1005574980207+i\times(-1.070488580268))\times(-0.1005574980207+1.070488580268\,i))^4}+1)\,\hat{}\,(1/15)+{C_{\text{MRB}}}^{1-1/(4\pi)+\pi})\times\frac{4}{3}\right)}$$

i is the imaginary unit

C_{MRB} is the MRB constant

Result

0.9003152331961…

0.90031523…. ≈ 0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed forms

$$\frac{2\sqrt{2}}{\pi} \approx 0.900316316 1$$

Now, we analyze the following equation:

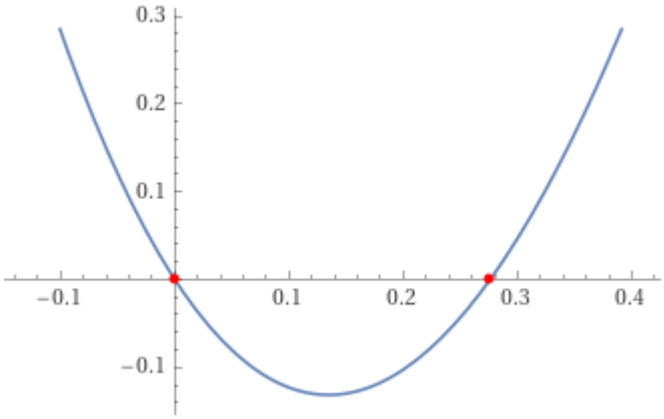
$x^4 - 3x^3 + 8x^2 - 2x = 0$

x^4-3x^3+8x^2-2x = 0

Input

$x^4 - 3x^3 + 8x^2 - 2x = 0$

Root plot **(figure that can be related to an open string)**



Alternate forms

$x\,(x\,((x-3)\,x+8)-2)=0$

$x\left(x^3-3\,x^2+8\,x-2\right)=0$

Real solutions

$x=0$

$$x=1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}$$

Sum of roots

3

Complex solutions

$$x=1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}+i\left(-\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}\right)$$

$$x = 1 - \frac{\sqrt[3]{\sqrt{699} - 18}}{2 \times 3^{2/3}} + \frac{5}{2\sqrt[3]{3\left(\sqrt{699} - 18\right)}} + i\left(\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699} - 18}} + \frac{\sqrt[3]{\sqrt{699} - 18}}{2\sqrt[6]{3}}\right)$$

Product of roots

0

From

$$x = 1 + \frac{\sqrt[3]{\sqrt{699} - 18}}{3^{2/3}} - \frac{5}{\sqrt[3]{3\left(\sqrt{699} - 18\right)}}$$

we obtain:

1 + (sqrt(699) - 18)^(1/3)/3^(2/3) - 5/(3 (sqrt(699) - 18))^(1/3)

Input

$$1 + \frac{\sqrt[3]{\sqrt{699} - 18}}{3^{2/3}} - \frac{5}{\sqrt[3]{3\left(\sqrt{699} - 18\right)}}$$

Decimal approximation

0.27592444861371959840220118939892077832670545546799068199997510 24

…

0.2759244486137….

Alternate forms

$$\frac{1}{3}\sqrt[3]{3\sqrt{699}-54}-\frac{5}{\sqrt[3]{3\sqrt{699}-54}}+1$$

---

root of x^3-3x^2+8x-2 near $x=0.275924$

$$\frac{-5\times 3^{2/3}+3\sqrt[3]{\sqrt{699}-18}+\sqrt[3]{3}\left(\sqrt{699}-18\right)^{2/3}}{3\sqrt[3]{\sqrt{699}-18}}$$

Minimal polynomial

x^3-3x^2+8x-2

From

$$x=1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}+i\left(-\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}\right)$$

we obtain:

1 - (sqrt(699) - 18)^(1/3)/(2 3^(2/3)) + 5/(2 (3 (sqrt(699) - 18))^(1/3)) + i (-(5 3^(1/6))/(2 (sqrt(699) - 18)^(1/3)) - (sqrt(699) - 18)^(1/3)/(2 3^(1/6)))

Input

$$1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}+i\left(-\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}\right)$$

i is the imaginary unit

Result

$$1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}+i\left(-\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}\right)$$

Decimal approximation

1.362037775693140200798899405300539610836647272266004659000012448…

$-$

2.322329445424681664324050302161275362502666425879042088226982397…

i

Alternate complex forms

$$\sqrt{\frac{8}{3}+\frac{\sqrt{233}-\sqrt{3}}{3^{5/6}\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt{699}-43}{\left(3\left(\sqrt{699}-18\right)\right)^{2/3}}}\left(\cos\left(\tan^{-1}\left(\frac{-\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}}{1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{-\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}}{1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}}\right)\right)\right)$$

$$\sqrt{\frac{8}{3}+\frac{\sqrt{233}-\sqrt{3}}{3^{5/6}\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt{699}-43}{\left(3\left(\sqrt{699}-18\right)\right)^{2/3}}}$$

$$\exp\left(i\tan^{-1}\left(\frac{-\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}}{1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=\sqrt{\frac{8}{3}+\frac{\sqrt{233}-\sqrt{3}}{3^{5/6}\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt{699}-43}{\left(3\left(\sqrt{699}-18\right)\right)^{2/3}}}$$ (radius), $$\theta=\tan^{-1}\left(\frac{-\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}}{1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}}\right)$$ (angle)

$r\approx 2.6923$ (radius), $\theta\approx -1.0404$ (angle)

2.6923

Alternate forms

$$-\frac{1}{6}i\,3^{5/6}\sqrt[3]{\sqrt{699}-18}-\frac{1}{6}\sqrt[3]{3\sqrt{699}-54}-\frac{5i\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}+\frac{5}{2\sqrt[3]{3\sqrt{699}-54}}+1$$

---

root of x^3-3x^2+8x-2 near $x=1.36204-2.32233\,i$

$$\frac{1}{6}\left(6+\frac{5\sqrt[6]{3}\left(\sqrt{3}+-3\,i\right)}{\sqrt[3]{\sqrt{699}-18}}+\left(-\sqrt[3]{3}-i\,3^{5/6}\right)\sqrt[3]{\sqrt{699}-18}\right)$$

Expanded form

$$1-\frac{5\,i\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}-\frac{i\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}+\frac{5}{2\sqrt[3]{3\sqrt{699}-54}}-\frac{1}{6}\sqrt[3]{3\sqrt{699}-54}$$

Minimal polynomial

$$x^3-3x^2+8x-2$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

And from:

1 - (sqrt(699) - 18)^(1/3)/(2 3^(2/3)) + 5/(2 (3 (sqrt(699) - 18))^(1/3)) + i ((5 3^(1/6))/(2 (sqrt(699) - 18)^(1/3)) + (sqrt(699) - 18)^(1/3)/(2 3^(1/6)))

Input

$$1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}+i\left(\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}+\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}\right)$$

i is the imaginary unit

Result

$$1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}+i\left(\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}+\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}\right)$$

Decimal approximation

1.362037775693140200798899405300539610836647272266004659000012448…

+

2.322329445424681664324050302161275362502666425879042088226982397…

i

Alternate complex forms

$$\sqrt{\frac{8}{3}+\frac{\sqrt{233}-\sqrt{3}}{3^{5/6}\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt{699}-43}{\left(3\left(\sqrt{699}-18\right)\right)^{2/3}}}\left(\cos\left(\tan^{-1}\left(\frac{\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}+\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}}{1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}+\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}}{1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}}\right)\right)\right)$$

$$\sqrt{\frac{8}{3}+\frac{\sqrt{233}-\sqrt{3}}{3^{5/6}\sqrt[3]{\sqrt{699}-18}}-\frac{\sqrt{699}-43}{\left(3\left(\sqrt{699}-18\right)\right)^{2/3}}}\exp\left(i\tan^{-1}\left(\frac{\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}+\frac{\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}}{1-\frac{\sqrt[3]{\sqrt{699}-18}}{2\times 3^{2/3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r = \sqrt{\frac{8}{3} + \frac{\sqrt{233} - \sqrt{3}}{3^{5/6}\sqrt[3]{\sqrt{699} - 18}} - \frac{\sqrt{699} - 43}{\left(3\left(\sqrt{699} - 18\right)\right)^{2/3}}}$$

$$\text{(radius)}, \quad \theta = \tan^{-1}\left(\frac{\frac{5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699} - 18}} + \frac{\sqrt[3]{\sqrt{699} - 18}}{2\sqrt[6]{3}}}{1 - \frac{\sqrt[3]{\sqrt{699} - 18}}{2 \times 3^{2/3}} + \frac{5}{2\sqrt[3]{3\left(\sqrt{699} - 18\right)}}}\right) \text{(angle)}$$

$r \approx 2.6923$ (radius), $\theta \approx 1.0404$ (angle)

2.6923

Alternate forms

$$\frac{1}{6} i\, 3^{5/6}\sqrt[3]{\sqrt{699} - 18} - \frac{1}{6}\sqrt[3]{3\sqrt{699} - 54} + \frac{i\, 5\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699} - 18}} + \frac{5}{2\sqrt[3]{3\sqrt{699} - 54}} + 1$$

---

root of $x^3 - 3x^2 + 8x - 2$ near $x = 1.36204 + 2.32233\, i$

$$\frac{1}{6}\left(6 + \frac{5\sqrt[6]{3}\left(\sqrt{3} + 3i\right)}{\sqrt[3]{\sqrt{699} - 18}} + i\left(\sqrt{3} + i\right)\sqrt[3]{3\left(\sqrt{699} - 18\right)}\right)$$

Expanded form

$$1 + \frac{5\, i\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699} - 18}} + \frac{i\sqrt[3]{\sqrt{699} - 18}}{2\sqrt[6]{3}} + \frac{5}{2\sqrt[3]{3\sqrt{699} - 54}} - \frac{1}{6}\sqrt[3]{3\sqrt{699} - 54}$$

Minimal polynomial

$x^3 - 3x^2 + 8x - 2$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

From the sum of the roots, we obtain:

(1/3(3√(699)-54)^(1/3)-5/(3√(699)-54)^(1/3)+1)+(1/6(6+(5 3^(1/6)(√(3)+ -3 i))/(√(699)-18)^(1/3)+(-3^(1/3)-i 3^(5/6))(√(699)-18)^(1/3))

Input

$$\left(\frac{1}{3}\sqrt[3]{3\sqrt{699}-54}-\frac{5}{\sqrt[3]{3\sqrt{699}-54}}+1\right)+\frac{1}{6}\left(6+\frac{5\sqrt[6]{3}\left(\sqrt{3}-3i\right)}{\sqrt[3]{\sqrt{699}-18}}+\left(-\sqrt[3]{3}-i\times 3^{5/6}\right)\sqrt[3]{\sqrt{699}-18}\right)$$

i is the imaginary unit

Exact result

$$1-\frac{5}{\sqrt[3]{3\sqrt{699}-54}}+\frac{1}{3}\sqrt[3]{3\sqrt{699}-54}+\frac{1}{6}\left(6+\frac{5\sqrt[6]{3}\left(\sqrt{3}+-3i\right)}{\sqrt[3]{\sqrt{699}-18}}+\left(-\sqrt[3]{3}-i\,3^{5/6}\right)\sqrt[3]{\sqrt{699}-18}\right)$$

Decimal approximation

1.63796222430685979920110059469946038916335272773995340999987551…

$-$

2.32232944542468166432405030216127536250266642587904208822698239 7…

i

Alternate complex forms

$$\left(1-\frac{5}{\sqrt[3]{-54+3\sqrt{699}}}+\frac{1}{3}\sqrt[3]{-54+3\sqrt{699}}+\frac{1}{6}\left(6+\frac{5\times 3^{2/3}}{\sqrt[3]{-18+\sqrt{699}}}-\sqrt[3]{3\left(-18+\sqrt{699}\right)}\right)\right)+\frac{1}{6}\left(-\frac{15\sqrt[6]{3}}{\sqrt[3]{-18+\sqrt{699}}}-3^{5/6}\sqrt[3]{-18+\sqrt{699}}\right)i$$

$$\left(\cos\left(\tan^{-1}\left(\left(-\frac{15\sqrt[6]{3}}{\sqrt[3]{\sqrt{699}-18}}-3^{5/6}\sqrt[3]{\sqrt{699}-18}\right)\Big/\left(6\left(1-\frac{5}{\sqrt[3]{3\sqrt{699}-54}}+\frac{1}{3}\sqrt[3]{3\sqrt{699}-54}+\frac{1}{6}\left(6+\frac{5\times 3^{2/3}}{\sqrt[3]{\sqrt{699}-18}}-\sqrt[3]{3\left(\sqrt{699}-18\right)}\right)\right)\right)\right)\right)+i\sin\left(\tan^{-1}\left(\left(-\frac{15\sqrt[6]{3}}{\sqrt[3]{\sqrt{699}-18}}-3^{5/6}\sqrt[3]{\sqrt{699}-18}\right)\Big/\left(6\left(1-\frac{5}{\sqrt[3]{3\sqrt{699}-54}}+\frac{1}{3}\sqrt[3]{3\sqrt{699}-54}+\frac{1}{6}\left(6+\frac{5\times 3^{2/3}}{\sqrt[3]{\sqrt{699}-18}}-\sqrt[3]{3\left(\sqrt{699}-18\right)}\right)\right)\right)\right)\right)\Big/\left(\sqrt{\left(\left(3\left(\sqrt{699}-18\right)\right)\Big/\left(2\times 3^{5/6}\sqrt{233}\sqrt[3]{\sqrt{699}-18}+\sqrt[6]{3}\sqrt{233}\left(\sqrt{699}-18\right)^{2/3}+17\left(\sqrt{699}-18\right)-11\sqrt[3]{3\left(\sqrt{699}-18\right)}-16\left(3\left(\sqrt{699}-18\right)\right)^{2/3}\right)\right)\right)\right)$$

$$\exp\left(i\tan^{-1}\left(\left(-\frac{15\sqrt[6]{3}}{\sqrt[3]{\sqrt{699}-18}}-3^{5/6}\sqrt[3]{\sqrt{699}-18}\right)\Big/\left(6\left(1-\frac{5}{\sqrt[3]{3\sqrt{699}-54}}+\frac{1}{3}\sqrt[3]{3\sqrt{699}-54}+\frac{1}{6}\left(6+\frac{5\times 3^{2/3}}{\sqrt[3]{\sqrt{699}-18}}-\sqrt[3]{3\left(\sqrt{699}-18\right)}\right)\right)\right)\right)\Big/\left(\sqrt{\left(\left(3\left(\sqrt{699}-18\right)\right)\Big/\left(2\times 3^{5/6}\sqrt{233}\sqrt[3]{\sqrt{699}-18}+\sqrt[6]{3}\sqrt{233}\left(\sqrt{699}-18\right)^{2/3}+17\left(\sqrt{699}-18\right)-11\sqrt[3]{3\left(\sqrt{699}-18\right)}-16\left(3\left(\sqrt{699}-18\right)\right)^{2/3}\right)\right)\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=1\Big/\left(\sqrt{\left(\left(3\left(\sqrt{699}-18\right)\right)\Big/\left(2\times 3^{5/6}\sqrt{233}\sqrt[3]{\sqrt{699}-18}+\sqrt[6]{3}\sqrt{233}\left(\sqrt{699}-18\right)^{2/3}+17\left(\sqrt{699}-18\right)-11\sqrt[3]{3\left(\sqrt{699}-18\right)}-16\left(3\left(\sqrt{699}-18\right)\right)^{2/3}\right)\right)\right)$$

(radius), $\theta=\tan^{-1}\left(\left(-\frac{15\sqrt[6]{3}}{\sqrt[3]{\sqrt{699}-18}}-3^{5/6}\sqrt[3]{\sqrt{699}-18}\right)\Big/\left(6\left(1-\frac{5}{\sqrt[3]{3\sqrt{699}-54}}+\frac{1}{3}\sqrt[3]{3\sqrt{699}-54}+\frac{1}{6}\left(6+\frac{5\times 3^{2/3}}{\sqrt[3]{\sqrt{699}-18}}-\sqrt[3]{3\left(\sqrt{699}-18\right)}\right)\right)\right)\right)$ (angle)

Expanded form

$$2-\frac{5i\sqrt[6]{3}}{2\sqrt[3]{\sqrt{699}-18}}-\frac{i\sqrt[3]{\sqrt{699}-18}}{2\sqrt[6]{3}}+\frac{5}{2\sqrt[3]{3\left(\sqrt{699}-18\right)}}-\frac{5}{\sqrt[3]{3\sqrt{699}-54}}+\frac{1}{6}\sqrt[3]{3\sqrt{699}-54}$$

Alternate forms

root of $x^3 - 6x^2 + 17x - 22$ near $x = 1.63796 - 2.32233\,i$

$$\frac{1}{6}\left(12 - \frac{5\sqrt[6]{3}\,\left(\sqrt{3}+3\,i\right)}{\sqrt[3]{\sqrt{699}-18}} + \left(\sqrt[3]{3} - i\,3^{5/6}\right)\sqrt[3]{\sqrt{699}-18}\right)$$

$$\frac{1}{6}\left(12 - \frac{5\times 3^{2/3}}{\sqrt[3]{\sqrt{699}-18}} + \sqrt[3]{3\left(\sqrt{699}-18\right)}\right) - \frac{i\left(15+\left(3\left(\sqrt{699}-18\right)\right)^{2/3}\right)}{2\times 3^{5/6}\sqrt[3]{\sqrt{699}-18}}$$

Minimal polynomial

$x^3 - 6x^2 + 17x - 22$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

1.6379622243-2.3223294454 i+(1/6(6+(5 3^(1/6)(√(3)+3i))/(√(699)-18)^(1/3)+i(√(3)+i)(3(√(699)-18))^(1/3)))

Input interpretation

$$1.6379622243 + i\times(-2.3223294454) + \frac{1}{6}\left(6 + \frac{5\sqrt[6]{3}\,\left(\sqrt{3}+3\,i\right)}{\sqrt[3]{\sqrt{699}-18}} + i\left(\sqrt{3}+i\right)\sqrt[3]{3\left(\sqrt{699}-18\right)}\right)$$

i is the imaginary unit

Result

3.0000000000… +

$2.4681664324\ldots \times 10^{-11}\,i$

Alternate complex forms

$$3.0000000000\left(\cos\left(0.\times 10^{-12}\right)+i\sin\left(0.\times 10^{-12}\right)\right)$$

$$3.0000000000\,e^{0.\times 10^{-12}\,i}$$

Polar coordinates

$r = 3.0000000000$ (radius), $\theta = 0.\times 10^{-12}$ (angle)

3

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)d^3\times\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the previous expression

$$x = 1 + \frac{\sqrt[3]{\sqrt{699} - 18}}{3^{2/3}} - \frac{5}{\sqrt[3]{3\left(\sqrt{699} - 18\right)}}$$

we obtain:

((1 + (sqrt(699) - 18)^(1/3)/3^(2/3) - 5/(3 (sqrt(699) - 18))^(1/3))) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

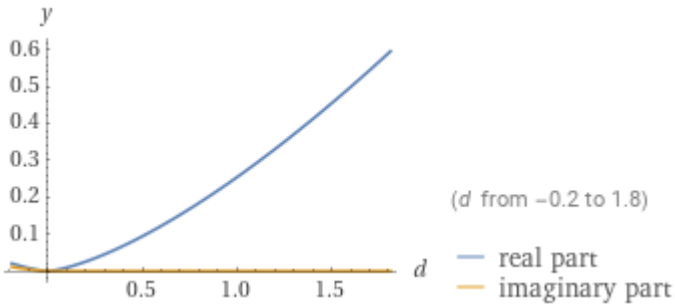
Input

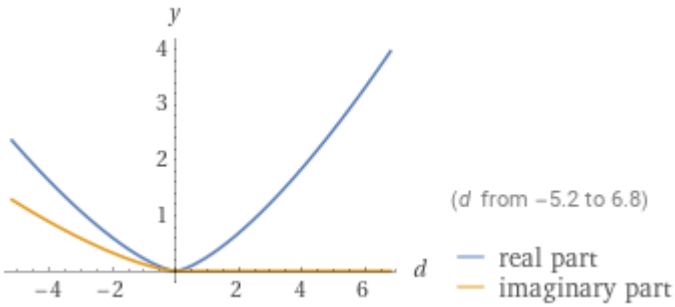
$$\left(1 + \frac{\sqrt[3]{\sqrt{699} - 18}}{3^{2/3}} - \frac{5}{\sqrt[3]{3\left(\sqrt{699} - 18\right)}}\right) \sqrt[2\pi]{\left(\frac{5}{12}\left(3 + \sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Exact result

$$3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3 + \sqrt{5}\right)} \left(1 + \frac{\sqrt[3]{\sqrt{699} - 18}}{3^{2/3}} - \frac{5}{\sqrt[3]{3\left(\sqrt{699} - 18\right)}}\right) \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Expanded forms

$$-\frac{3^{-1/3-3/(2\pi)}\times 4^{-1/\pi}\times 5^{1+1/(2\pi)}\ \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}}{\sqrt[3]{\sqrt{699}-18}}+$$
$$3^{-3/(2\pi)}\times 4^{-1/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ +$$
$$3^{-2/3-3/(2\pi)}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{\sqrt{699}-18}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}$$

$$3^{-1-3/(2\pi)}\times 4^{-1/\pi}\ \sqrt[3]{3\sqrt{699}-54}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{5\sqrt{5}\ d^9+15\,d^9}\ -$$
$$\frac{5\times 3^{-3/(2\pi)}\times 4^{-1/\pi}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{5\sqrt{5}\ d^9+15\,d^9}}{\sqrt[3]{3\sqrt{699}-54}}+$$
$$3^{-3/(2\pi)}\times 4^{-1/\pi}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{5\sqrt{5}\ d^9+15\,d^9}$$

Alternate forms

$$-\frac{1}{\sqrt[3]{\sqrt{699}-18}}3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(5\times 3^{2/3}-3\sqrt[3]{\sqrt{699}-18}-\sqrt[3]{3}\right.$$
$$\left.\left(\sqrt{699}-18\right)^{2/3}\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{d^9}$$

$$3^{-2/3-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(\sqrt[3]{\sqrt{699}-18}\left(3^{2/3}+\sqrt[3]{\sqrt{699}-18}\right)-5\sqrt[3]{3}\right)$$
$$\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{d^9}\quad \text{root of } 375x^6-36x^3-1 \text{ near } x=0.491183$$

Root

$d=0$

Series expansion at d=∞

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[\pi]{\frac{\pi}{4}}\,d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$

$$2^{-(2+\pi)/\pi}\times 3^{2-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)d^8$$

$$\left(d^9\right)^{1/(2\pi)-1}\pi^{1/\pi-1}$$

Indefinite integral

$$\int 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(1+\frac{\sqrt[3]{-18+\sqrt{699}}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(-18+\sqrt{699}\right)}}\right)\sqrt[2\pi]{d^9}\,\sqrt[\pi]{\frac{\pi}{4}}\,dd=$$

$$\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[\pi]{\frac{\pi}{4}}\,d\sqrt[2\pi]{d^9}}{1+\frac{9}{2\pi}}+\text{constant}$$

Global minimum

$$\min\left\{\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right\}=0 \text{ at } d=0$$

And:

((1 + (sqrt(699) - 18)^(1/3)/3^(2/3) - 5/(3 (sqrt(699) - 18))^(1/3))) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

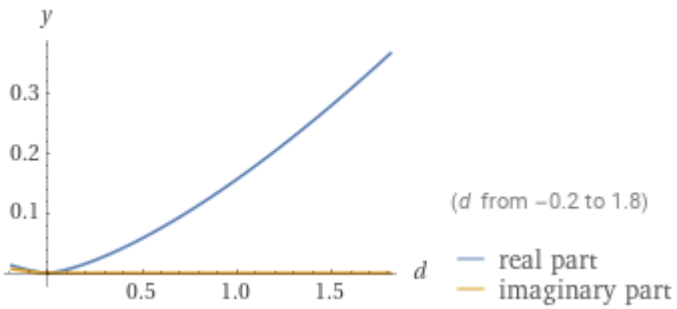
Input

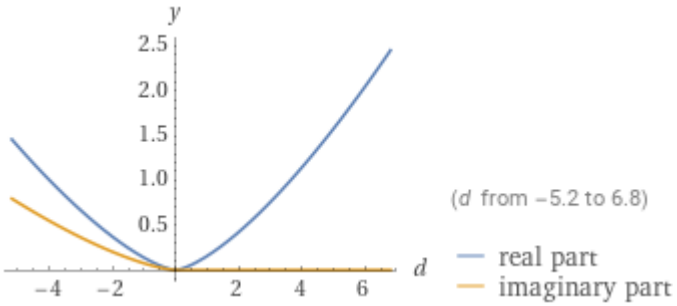
$$\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

Exact result

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

Plots **(figures that can be related to the open strings)**





Alternate forms

$$2^{-1/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9} \quad \text{root of } x^3 - 3x^2 + 8x - 2 \text{ near } x = 0.275924$$

$$-\frac{2^{-1/\pi} \times 3^{-1-3/(2\pi)} \left(5 \times 3^{2/3} - 3\sqrt[3]{\sqrt{699} - 18} - \sqrt[3]{3}\left(\sqrt{699} - 18\right)^{2/3}\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}{\sqrt[3]{\sqrt{699} - 18}}$$

$$\left(2^{-1/\pi} \times 3^{-3/(2\pi)} - \frac{5 \times 2^{-1/\pi} \times 3^{-1/3-3/(2\pi)}}{\sqrt[3]{\sqrt{699} - 18}} + 2^{-1/\pi} \times 3^{-2/3-3/(2\pi)} \sqrt[3]{\sqrt{699} - 18}\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Expanded form

$$2^{-1/\pi} \times 3^{-2/3-3/(2\pi)} \sqrt[3]{\sqrt{699} - 18}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9} - \frac{5 \times 2^{-1/\pi} \times 3^{-1/3-3/(2\pi)} \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}{\sqrt[3]{\sqrt{699} - 18}} + 2^{-1/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Alternate form assuming d>0

$$\frac{2^{-1/\pi} \times 3^{-(9+4\pi)/(6\pi)} \left(-5\sqrt[3]{3} + 3^{2/3}\sqrt[3]{\sqrt{699} - 18} + \left(\sqrt{699} - 18\right)^{2/3}\right) \sqrt[2\pi]{\pi}\ d^{9/(2\pi)}}{\sqrt[3]{\sqrt{699} - 18}}$$

Root

$d = 0$

Series expansion at d=∞

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\pi}\;d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times 3\times 12}}\right)=$$

$$2^{-(1+\pi)/\pi}\times 3^{2-3/(2\pi)}\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)d^8\left(d^9\right)^{1/(2\pi)-1}\pi^{1/(2\pi)-1}$$

Indefinite integral

$$\int 2^{-1/\pi}\times 3^{-3/(2\pi)}\left(1+\frac{\sqrt[3]{-18+\sqrt{699}}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(-18+\sqrt{699}\right)}}\right)\sqrt[2\pi]{d^9}\;\sqrt[2\pi]{\pi}\;dd=$$

$$\frac{2^{(\pi-1)/\pi}\times 3^{-3/(2\pi)}\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\pi^{1+1/(2\pi)}\,d\,\sqrt[2\pi]{d^9}}{9+2\pi}+\text{constant}$$

Global minimum

$$\min\left\{\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times 3\times 12}}\right\}=$$

0 at $d=0$

Series representations

$$\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$

$$\left(2^{-3/(2\pi)}\times 3^{-2/3-3/(2\pi)}\sqrt[2\pi]{d^9\,\pi\,\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}\left(-5\sqrt[3]{3}+3^{2/3}\sqrt[3]{-18+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(699-z_0)^k\,z_0^{-k}}{k!}}+\left(-18+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(699-z_0)^k\,z_0^{-k}}{k!}\right)^{2/3}\right)\right)\Bigg/\left(\sqrt[3]{-18+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(699-z_0)^k\,z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$

$$\left(2^{-3/(2\pi)}\times 3^{-2/3-3/(2\pi)}\right.$$

$$\sqrt[2\pi]{d^9\,\pi\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\left(-5\sqrt[3]{3}+3^{2/3}\left(-18+\exp\left(i\,\pi\left\lfloor\frac{\arg(699-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(699-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\text{^}(1/3)+$$

$$\left(-18+\exp\left(i\,\pi\left\lfloor\frac{\arg(699-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(699-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{2/3}\right)\right)\Big/$$

$$\left(\sqrt[3]{-18+\exp\left(i\,\pi\left\lfloor\frac{\arg(699-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(699-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

$$\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$

$$\left(2^{-3/(2\pi)}\times 3^{-2/3-3/(2\pi)}\right.$$

$$\sqrt[2\pi]{d^9\,\pi\left(\frac{1}{z_0}\right)^{\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1+\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}$$

$$\left(-5\sqrt[3]{3}+3^{2/3}\left(-18+\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(699-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(699-z_0)/(2\pi)\rfloor}\right.\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(699-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)+$$

$$\left(-18+\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(699-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(699-z_0)/(2\pi)\rfloor}\right.$$

$$\left.\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(699-z_0)^k\,z_0^{-k}}{k!}\right)^{2/3}\right)\right)\Big/$$

$$\left(\left(\left(-18+\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(699-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(699-z_0)/(2\pi)\rfloor}\right.\right.\right.$$

$$\left.\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(699-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Dividing the two exact results

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[\pi]{\frac{\pi}{4}}\,\sqrt[2\pi]{d^9}$$

and

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\left(1+\frac{\sqrt[3]{\sqrt{699}-18}}{3^{2/3}}-\frac{5}{\sqrt[3]{3\left(\sqrt{699}-18\right)}}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

we obtain:

3^(-3/(2 π)) (5 (3 + sqrt(5)))^(1/(2 π)) (1 + (sqrt(699) - 18)^(1/3)/3^(2/3) - 5/(3 (sqrt(699) - 18))^(1/3)) (π/4)^(1/π) (d^9)^(1/(2 π))

2^(-1/π) 3^(-3/(2 π)) (1 + (sqrt(699) - 18)^(1/3)/3^(2/3) - 5/(3 (sqrt(699) - 18))^(1/3)) π^(1/(2 π)) (d^9)^(1/(2 π))

((3^(-3/(2 π)) (5 (3 + sqrt(5)))^(1/(2 π)) (π/4)^(1/π)))/((2^(-1/π) 3^(-3/(2 π)) π^(1/(2 π))))

Input

$$\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\frac{\pi}{4}}}{2^{-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}}=2^{-1/\pi}\sqrt[2\pi]{5}\;\sqrt[2\pi]{\pi}\;\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}}=2^{-1/\pi}\sqrt[2\pi]{5}\;\sqrt[2\pi]{\pi}\;\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}$$

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}}=$$
$$2^{-1/\pi}\sqrt[2\pi]{5}\;\sqrt[2\pi]{\pi}\;\sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13

…

3.14159265358846…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298884450469 20

…

1.644934066846…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{32\,e^{1.92478440155986\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

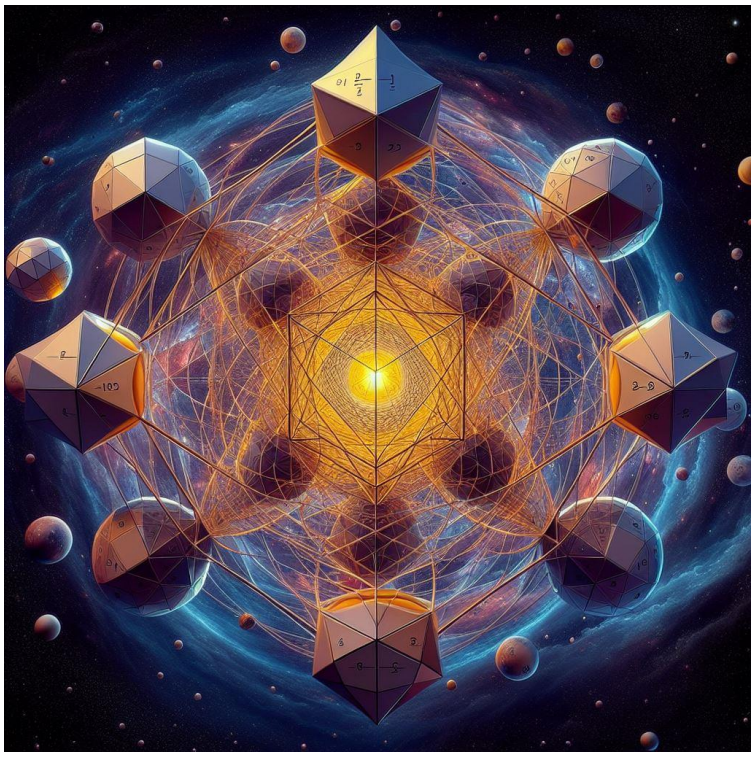


Fig. 1

A geometric representation of the Universe in a scenario of eternal inflation that emerges as a fluctuation from the quantum vacuum, which incorporates the solutions of a quartic equation, the ratios (and/or inverses) between the volumes of the icosahedron, of the octahedron and of the tetrahedron and the volume of the sphere, the ratio $(2\sqrt{2})/\pi$, the so-called Del Gaudio-Nardelli Constant, and the values of the golden ratio and π.

Notes on efficient computation of Ramanujan cubic equations - *Peter J.-S. Shiue, Anthony G. Shannon, Shen C. Huang and Jorge E. Reyes* - Notes on Number Theory and Discrete Mathematics - Print ISSN 1310–5132, Online ISSN 2367–8275 2022, Volume 28, Number 2, Pages 350–375 - DOI: 10.7546/nntdm.2022.28.2.350-375

Wang, inspired by Ramanujan's identities involving cosine functions, investigated similar identities involving sine and tangent functions. By using Ramanujan's theorem and Cardano's formula, he provided various identities. We analyze the following:

$$\sqrt[3]{\tan\frac{\pi}{9}}+\sqrt[3]{\tan\frac{4\pi}{9}}+\sqrt[3]{\tan\frac{7\pi}{9}}$$
$$=\left(-\sqrt[18]{3}\right)\left(\sqrt[3]{-3\sqrt[3]{3}+6+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+\sqrt[3]{9}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+\sqrt[3]{9}\right)}\right)}\right)$$

From

$$\sqrt[3]{\tan\frac{\pi}{9}}+\sqrt[3]{\tan\frac{4\pi}{9}}+\sqrt[3]{\tan\frac{7\pi}{9}}$$
$$=\left(-\sqrt[18]{3}\right)\left(\sqrt[3]{-3\sqrt[3]{3}+6+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+\sqrt[3]{9}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+\sqrt[3]{9}\right)}\right)}\right)$$

we obtain:

(tan(Pi/9))^(1/3)+(tan((4Pi)/9))^(1/3)+(tan((7Pi)/9))^(1/3)

Assuming the real-valued root

Input

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}$$

$\sqrt[n]{x}$ is the real-valued n^{th} root of x

Exact result

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)} - \sqrt[3]{\tan\left(\frac{2\pi}{9}\right)} + \sqrt[3]{\cot\left(\frac{\pi}{18}\right)}$$

$\cot(x)$ is the cotangent function

Decimal approximation

1.554093626275498366983441520754132346494621265908958809282021087 6

…

1.5540936262754….

Alternate forms

$$\sqrt[3]{\frac{\cos\left(\frac{\pi}{18}\right)}{\sin\left(\frac{\pi}{18}\right)}} + \sqrt[3]{\frac{\sin\left(\frac{\pi}{9}\right)}{\cos\left(\frac{\pi}{9}\right)}} - \sqrt[3]{\frac{\sin\left(\frac{2\pi}{9}\right)}{\cos\left(\frac{2\pi}{9}\right)}}$$

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)} + \sqrt[3]{\cot\left(\frac{\pi}{18}\right)} - \sqrt[3]{\frac{\sqrt{3}\cos\left(\frac{\pi}{9}\right) - \sin\left(\frac{\pi}{9}\right)}{\sqrt{3}\sin\left(\frac{\pi}{9}\right) + \cos\left(\frac{\pi}{9}\right)}}$$

$$\sqrt[3]{-\frac{i\left(e^{-(i\pi)/18} + e^{(i\pi)/18}\right)}{e^{-(i\pi)/18} - e^{(i\pi)/18}}} + \sqrt[3]{\frac{i\left(e^{-(i\pi)/9} - e^{(i\pi)/9}\right)}{e^{-(i\pi)/9} + e^{(i\pi)/9}}} - \sqrt[3]{\frac{i\left(e^{-(2i\pi)/9} - e^{(2i\pi)/9}\right)}{e^{-(2i\pi)/9} + e^{(2i\pi)/9}}}$$

Alternative representations

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)} + \sqrt[3]{\tan\left(\frac{4\pi}{9}\right)} + \sqrt[3]{\tan\left(\frac{7\pi}{9}\right)} = \sqrt[3]{\frac{1}{\cot\left(\frac{\pi}{9}\right)}} + \sqrt[3]{\frac{1}{\cot\left(\frac{4\pi}{9}\right)}} + \sqrt[3]{\frac{1}{\cot\left(\frac{7\pi}{9}\right)}}$$

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=$$
$$\sqrt[3]{\cot\left(\frac{\pi}{2}-\frac{7\pi}{9}\right)}+\sqrt[3]{\cot\left(\frac{\pi}{2}-\frac{4\pi}{9}\right)}+\sqrt[3]{\cot\left(\frac{\pi}{2}-\frac{\pi}{9}\right)}$$

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=$$
$$-\sqrt[3]{\cot\left(\frac{\pi}{2}+\frac{\pi}{9}\right)}-\sqrt[3]{\cot\left(\frac{\pi}{2}+\frac{4\pi}{9}\right)}-\sqrt[3]{\cot\left(\frac{\pi}{2}+\frac{7\pi}{9}\right)}$$

Series representations

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=\sqrt[3]{-i\left(1+2\sum_{k=1}^{\infty}q^{2k}\right)}+$$
$$\sqrt[3]{i\sum_{k=-\infty}^{\infty}e^{(11\,i\,k\,\pi)/9}\,\mathrm{sgn}(k)}-\sqrt[3]{i\sum_{k=-\infty}^{\infty}e^{(13\,i\,k\,\pi)/9}\,\mathrm{sgn}(k)}\quad\text{for } q=\sqrt[18]{-1}$$

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=\frac{1}{3\sqrt[3]{\pi}}$$
$$\left(2\sqrt[3]{3}\,\sqrt[3]{\sum_{k=1}^{\infty}\frac{1}{\frac{77}{81}-4k+4k^2}}-6\sqrt[3]{2}\;3^{2/3}\sqrt[3]{\sum_{k=1}^{\infty}\frac{1}{65-324k+324k^2}}+\right.$$
$$\left.3\sqrt[3]{\pi}\,\sqrt[3]{-i\left(1+2\sum_{k=1}^{\infty}q^{2k}\right)}\right)\quad\text{for } q=\sqrt[18]{-1}$$

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=$$
$$\frac{2\sqrt[3]{\sum_{k=1}^{\infty}\frac{1}{\frac{77}{81}-4k+4k^2}}-6\sqrt[3]{6}\,\sqrt[3]{\sum_{k=1}^{\infty}\frac{1}{65-324k+324k^2}}+3\sqrt[3]{6\pi}\,\sqrt[3]{\sum_{k=-\infty}^{\infty}\frac{1}{\pi-324k^2\,\pi}}}{3^{2/3}\sqrt[3]{\pi}}$$

$\mathrm{sgn}(x)$ is the sign of x

Integral representations

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=$$
$$\sqrt[3]{-\int_{\frac{\pi}{2}}^{\frac{\pi}{18}}\csc^2(t)\,dt}+\sqrt[3]{\int_0^{\frac{\pi}{9}}\sec^2(t)\,dt}-\sqrt[3]{\int_0^{\frac{2\pi}{9}}\sec^2(t)\,dt}$$

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=$$
$$\sqrt[3]{\frac{2}{\pi}}\sqrt[3]{\int_0^{\infty}\frac{-1+t^{8/9}}{-1+t^2}\,dt}+\sqrt[3]{\int_0^{\frac{\pi}{9}}\sec^2(t)\,dt}-\sqrt[3]{\int_0^{\frac{2\pi}{9}}\sec^2(t)\,dt}$$

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=$$
$$\sqrt[3]{\frac{2}{\pi}}\left(\sqrt[3]{\int_0^{\infty}\frac{-1+t^{2/9}}{-1+t^2}\,dt}-\sqrt[3]{\int_0^{\infty}\frac{-1+t^{4/9}}{-1+t^2}\,dt}+\sqrt[3]{\int_0^{\infty}\frac{-1+t^{8/9}}{-1+t^2}\,dt}\right)$$

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=$$
$$\sqrt[3]{\frac{2}{\pi}}\sqrt[3]{\int_0^{\infty}\frac{-1+t^{2/9}}{-1+t^2}\,dt}-\sqrt[3]{\frac{2}{\pi}}\sqrt[3]{\int_0^{\infty}\frac{-1+t^{4/9}}{-1+t^2}\,dt}+\sqrt[3]{-\int_{\frac{\pi}{2}}^{\frac{\pi}{18}}\csc^2(t)\,dt}$$

$\csc(x)$ is the cosecant function

$\sec(x)$ is the secant function

Multiple-argument formulas

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=$$
$$\sqrt[3]{\frac{1}{2}\left(\cot\left(\frac{\pi}{36}\right)-\tan\left(\frac{\pi}{36}\right)\right)}+\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}-\sqrt[3]{\tan\left(\frac{2\pi}{9}\right)}$$

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=$$
$$\sqrt[3]{\frac{-3\cot\left(\frac{\pi}{54}\right)+\cot^3\left(\frac{\pi}{54}\right)}{-1+3\cot^2\left(\frac{\pi}{54}\right)}}+\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}-\sqrt[3]{\tan\left(\frac{2\pi}{9}\right)}$$

$$\sqrt[3]{\tan\left(\frac{\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{4\pi}{9}\right)}+\sqrt[3]{\tan\left(\frac{7\pi}{9}\right)}=$$
$$\sqrt[3]{\frac{1}{2}\left(\cot\left(\frac{\pi}{36}\right)-\tan\left(\frac{\pi}{36}\right)\right)}+\sqrt[3]{\frac{2\tan\left(\frac{\pi}{18}\right)}{1-\tan^2\left(\frac{\pi}{18}\right)}}-\sqrt[3]{\frac{2\tan\left(\frac{\pi}{9}\right)}{1-\tan^2\left(\frac{\pi}{9}\right)}}$$

And:

(((-3^(1/18))))*((((-3(3)^(1/3))+6+3(((((21-3(3(3)^(1/3)+(9)^(1/3)))^(1/3))-(((3+3(3(3)^(1/3)+(9)^(1/3))))^(1/3))))))^1/3))

Input

$$-\sqrt[18]{3}\sqrt[3]{-3\sqrt[3]{3}+6+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+\sqrt[3]{9}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+\sqrt[3]{9}\right)}\right)}$$

Result

$$-\sqrt[18]{3}\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}$$

Decimal approximation

$$-0.77704681313774918349172076037706617324731063295447940464101054\ldots$$
$$-$$
$$1.34588456021406096343235611121578453013641585641231522258782 19\ldots\ i$$

Alternate complex forms

$$-\frac{1}{2}\sqrt[18]{3}\sqrt[3]{-6+3\sqrt[3]{3}-3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}-\frac{1}{2}\left(3^{5/9}\sqrt[3]{-6+3\sqrt[3]{3}-3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\right)i$$

$$3^{7/18}\sqrt[3]{-2+\sqrt[3]{3}-\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}+\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}\left(\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right)$$

$$3^{7/18}\sqrt[3]{-2+\sqrt[3]{3}-\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}+\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}\,e^{-(2i\pi)/3}$$

i is the imaginary unit

Polar coordinates

$r \approx 1.5541$ (radius), $\theta \approx -2.0944$ (angle)

1.5541

Alternate forms

$$-\sqrt[18]{3}\sqrt[3]{3\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}-3\sqrt[3]{3}-3\sqrt[3]{3+9\sqrt[3]{3}+3\times 3^{2/3}}+6}$$

$$-3^{7/18}\sqrt[3]{2-\sqrt[3]{3}+\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}-\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}$$

$$-\sqrt[3]{-1}$$

$$\sqrt[6]{\text{root of } x^9 + 243\,x^8 + 750\,141\,x^7 + 125\,492\,247\,x^6 + 8\,402\,613\,651\,x^5 - 1\,564\,600\,036\,311\,x^4 + 76\,244\,538\,770\,991\,x^3 - 809\,985\,827\,659\,635\,x^2 + 249\,709\,551\,662\,016\,x - 42\,845\,606\,719\,488 \text{ near } x = 14.0884}$$

Minimal polynomial

$$x^{54} + 243\,x^{48} + 750\,141\,x^{42} + 125\,492\,247\,x^{36} + 8\,402\,613\,651\,x^{30} - 1\,564\,600\,036\,311\,x^{24} + 76\,244\,538\,770\,991\,x^{18} - 809\,985\,827\,659\,635\,x^{12} + 249\,709\,551\,662\,016\,x^6 - 42\,845\,606\,719\,488$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the previous result

$$-\sqrt[18]{3}\ \sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}$$

we obtain:

(-3^(1/18) (6 - 3 3^(1/3) + 3 ((21 - 3 (3 3^(1/3) + 3^(2/3)))^(1/3) - (3 + 3 (3 3^(1/3) + 3^(2/3)))^(1/3)))^(1/3)) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

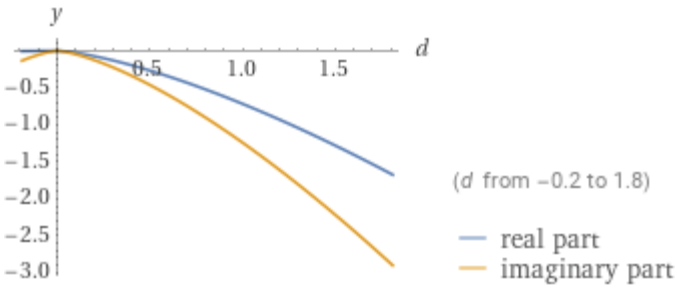
Input

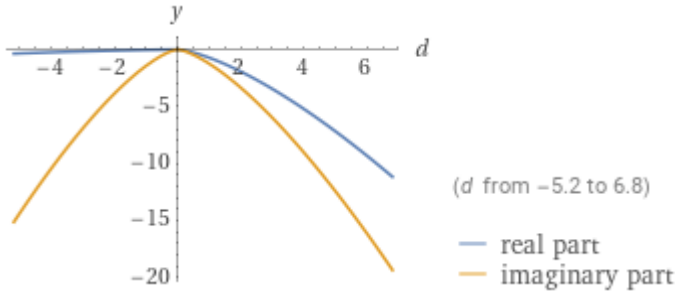
$$\left(-\sqrt[18]{3}\ \sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\right)$$
$$\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Exact result

$$-3^{1/18-3/(2\pi)} \times 4^{-1/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate form

$$-3^{1/18-3/(2\pi)} \times 4^{-1/\pi} \sqrt[2\pi]{15+5\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$\sqrt[3]{-\ \text{root of } x^9+54x^8+1053x^7+10611x^6+82377x^5+54675x^4+743580x^3-8286543x^2+5196312x-1259712 \text{ near } x=3.12544}$$

Root

$d = 0$ (assuming complex-valued fractional powers)

Series expansion at d=∞

$$-3^{7/18-3/(2\pi)} \times 4^{-1/\pi} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$

$$\sqrt[3]{2-\sqrt[3]{3}+\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}-\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}\ \sqrt[\pi]{\pi}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\left(-\sqrt[18]{3}\ \sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$

$$-\frac{1}{d}2^{-(2+\pi)/\pi}\times 3^{43/18-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$

$$\sqrt[3]{\sqrt[3]{-3\times 3^{2/3}+21-9\sqrt[3]{3}}+2-\sqrt[3]{3}-\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}\ \sqrt[2\pi]{d^9}\ \pi^{1/\pi-1}$$

Indefinite integral

$$\int -3^{1/18-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$

$$\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\ \sqrt[2\pi]{d^9}\ \sqrt[\pi]{\pi}$$

$$dd=$$

$$-\frac{1}{1+\frac{9}{2\pi}}3^{7/18-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$

$$\sqrt[3]{2-\sqrt[3]{3}+\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}-\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}\ \sqrt[\pi]{\frac{\pi}{4}}\ d\ \sqrt[2\pi]{d^9}+$$

constant

And:

(-3^(1/18) (6 - 3 3^(1/3) + 3 ((21 - 3 (3 3^(1/3) + 3^(2/3)))^(1/3) - (3 + 3 (3 3^(1/3) + 3^(2/3)))^(1/3)))^(1/3)) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

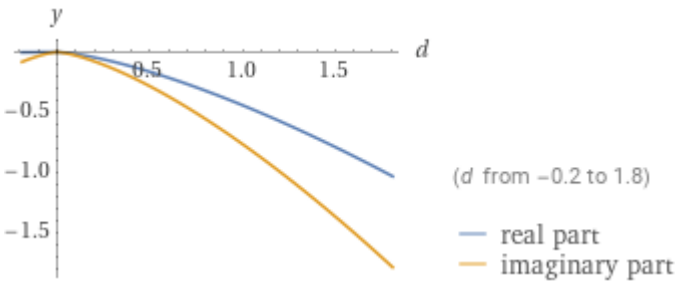
Input

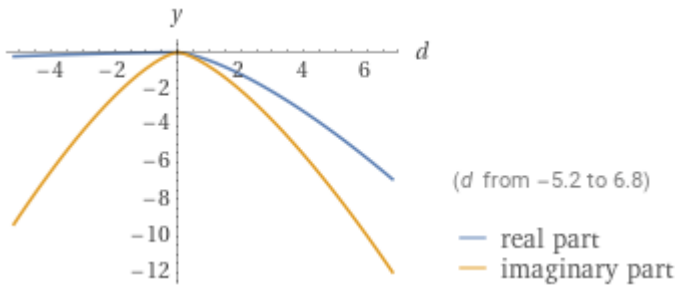
$$\left(-\sqrt[18]{3}\ \sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\right)$$
$$\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

Exact result

$$-2^{-1/\pi}\times 3^{1/18-3/(2\pi)}$$
$$\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate form

$$-2^{-1/\pi}\times 3^{1/18-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\Big/\sqrt[3]{-\left[\text{root of } x^9+54x^8+1053x^7+10611x^6+82377x^5+54675x^4+743580x^3-8286543x^2+5196312x-1259712 \text{ near } x=3.12544\right]}$$

Root

$d=0$ (assuming a complex-valued cube root)

Series expansion at d=∞

$$-2^{-1/\pi}\times 3^{7/18-3/(2\pi)}\,\sqrt[3]{2-\sqrt[3]{3}+\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}-\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}\Big/\left(\sqrt[2\pi]{\pi}\,d^{9/(2\pi)}\right)$$

Derivative

$$\frac{d}{dd}\left(\left(-\sqrt[18]{3}\,\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times 3\times 12}}\right)=$$

$$-\frac{1}{d}2^{-(1+\pi)/\pi}\times 3^{43/18-3/(2\pi)}\Big/\left(\sqrt[3]{\sqrt[3]{-3\times 3^{2/3}+21-9\sqrt[3]{3}}+2-\sqrt[3]{3}-\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}\,\sqrt[2\pi]{d^9}\,\pi^{1/(2\pi)-1}\right)$$

Indefinite integral

$$\int \frac{-2^{-1/\pi} \times 3^{1/18-3/(2\pi)}}{\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\ \sqrt[2\pi]{d^9}\ \sqrt[2\pi]{\pi}}\,dd =$$

$$-\frac{1}{9+2\pi}2^{(\pi-1)/\pi} \times 3^{7/18-3/(2\pi)} \sqrt[3]{2-\sqrt[3]{3}+\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}-\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}\ \pi^{1+1/(2\pi)}\, d\ \sqrt[2\pi]{d^9} +$$

constant

Dividing the two exact results

-3^(1/18 - 3/(2 π)) 4^(-1/π) (5 (3 + sqrt(5)))^(1/(2 π)) (6 - 3 3^(1/3) + 3 ((21 - 3 (3 3^(1/3) + 3^(2/3)))^(1/3) - (3 + 3 (3 3^(1/3) + 3^(2/3)))^(1/3)))^(1/3) π^(1/π) (d^9)^(1/(2 π))

for d = 2 :

-3^(1/18 - 3/(2 π)) 4^(-1/π) (5 (3 + sqrt(5)))^(1/(2 π)) (6 - 3 3^(1/3) + 3 ((21 - 3 (3 3^(1/3) + 3^(2/3)))^(1/3) - (3 + 3 (3 3^(1/3) + 3^(2/3)))^(1/3)))^(1/3) π^(1/π) (2^9)^(1/(2 π))

Input

$$-3^{1/18-3/(2\pi)} \times 4^{-1/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \Big/ \sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{2^9}$$

Exact result

$$-2^{5/(2\pi)}\times 3^{1/18-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\Bigg/\left(\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\sqrt[\pi]{\pi}\right)$$

Decimal approximation

$$-1.93248494312579833876573096473304520424999431315889017800361072\ldots$$
$$-3.34716210635573491940638893209859164671853895539022716531465619\ldots\, i$$

Alternate complex forms

$$-2^{-1+5/(2\pi)}\times 3^{1/18-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\Bigg/\left(\sqrt[3]{-6+3\sqrt[3]{3}-3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\sqrt[\pi]{\pi}\right)-\left(2^{-1+5/(2\pi)}\times 3^{5/9-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\Bigg/\left(\sqrt[3]{-6+3\sqrt[3]{3}-3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\sqrt[\pi]{\pi}\right)i\right)$$

$$2^{5/(2\pi)}\times 3^{7/18-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\Bigg/\left(\sqrt[3]{-2+\sqrt[3]{3}-\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}+\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}\sqrt[\pi]{\pi}\left(\cos\left(-\frac{2\pi}{3}\right)+i\sin\left(-\frac{2\pi}{3}\right)\right)\right)$$

$$\frac{2^{5/(2\pi)} \times 3^{7/18-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}}{\sqrt[3]{-2+\sqrt[3]{3}-\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}+\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}\ \sqrt[\pi]{\pi}\ e^{-(2i\pi)/3}}$$

i is the imaginary unit

Polar coordinates

$r \approx 3.8650$ (radius), $\theta \approx -2.0944$ (angle)

3.8650

Alternate forms

$$\frac{-2^{5/(2\pi)} \times 3^{1/18-3/(2\pi)} \sqrt[2\pi]{15+5\sqrt{5}}\ \sqrt[\pi]{\pi}}{\sqrt[3]{-\ \text{root of}\ x^9+54x^8+1053x^7+10611x^6+82377x^5+54675x^4+743580x^3-8286543x^2+5196312x-1259712\ \text{near}\ x=3.12544}}$$

$$\frac{-2^{5/(2\pi)-1} \times 3^{1/18-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}}{\sqrt[3]{-6+3\sqrt[3]{3}-3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\ \sqrt[\pi]{\pi}} -$$

$$\frac{i\,2^{5/(2\pi)-1} \times 3^{5/9-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}}{\sqrt[3]{-6+3\sqrt[3]{3}-3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\ \sqrt[\pi]{\pi}}$$

Series representations

$$\left(4^{-1/\pi}\,(-1)\,3^{1/18-3/(2\pi)}\,\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\right)$$
$$\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}$$
$$\left(\sqrt[\pi]{\pi}\,\sqrt[2\pi]{2^9}\right)=-2^{5/(2\pi)}\times 3^{1/18-3/(2\pi)}\,\sqrt[2\pi]{5}$$
$$\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}$$
$$\sqrt[\pi]{\pi}\,\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\left(4^{-1/\pi}\,(-1)\,3^{1/18-3/(2\pi)}\,\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\right)$$
$$\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}$$
$$\left(\sqrt[\pi]{\pi}\,\sqrt[2\pi]{2^9}\right)=-2^{5/(2\pi)}\times 3^{1/18-3/(2\pi)}\,\sqrt[2\pi]{5}$$
$$\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}$$
$$\sqrt[\pi]{\pi}\,\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\left(4^{-1/\pi}\,(-1)\,3^{1/18-3/(2\pi)}\,\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\right)$$
$$\sqrt[3]{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}$$
$$\left(\sqrt[\pi]{\pi}\;\sqrt[2\pi]{2^9}\right)=-2^{5/(2\pi)}\times 3^{7/18-3/(2\pi)}$$
$$\sqrt[3]{2-\sqrt[3]{3}+\sqrt[3]{21-9\sqrt[3]{3}-3\times 3^{2/3}}-\sqrt[3]{3\left(1+3\sqrt[3]{3}+3^{2/3}\right)}}$$
$$\sqrt[\pi]{\pi}\;\sqrt[2\pi]{15+\frac{5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

And:

3.8650/((-2^(-1/π) 3^(1/18 - 3/(2 π)) (6 - 3 3^(1/3) + 3 ((21 - 3 (3 3^(1/3) + 3^(2/3)))^(1/3) - (3 + 3 (3 3^(1/3) + 3^(2/3)))^(1/3)))^(1/3) π^(1/(2 π)) (2^9)^(1/(2 π))))

Input interpretation

$$-\left(3.8650\Big/\left(2^{-1/\pi}\times 3^{1/18-3/(2\pi)}\sqrt{6-3\sqrt[3]{3}+3\left(\sqrt[3]{21-3\left(3\sqrt[3]{3}+3^{2/3}\right)}-\sqrt[3]{3+3\left(3\sqrt[3]{3}+3^{2/3}\right)}\right)}\;\sqrt[2\pi]{\pi}\;\sqrt[2\pi]{2^9}\right)\right)$$

Result

$-0.809011\ldots + 1.40125\ldots\, i$

Alternate complex forms

$-0.809011 + 1.40125\, i$

$1.61802\,(\cos(2.0944) + i\sin(2.0944))$

$1.61802\, e^{2.0944\, i}$

i is the imaginary unit

Polar coordinates

$r = 1.61802$ (radius), $\theta = 2.0944$ (angle)

1.61802 result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternative representations

$$3.865 \Big/ \left(\left(3^{1/18-3/(2\pi)} (-1)\, 2^{-1/\pi} \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} \right)} \right) \sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9} \right) =$$

$$-\left(3.865 \Big/ \left(2^{-1/\cos^{-1}(-1)} \times 3^{1/18-3/(2\cos^{-1}(-1))} \sqrt[2\cos^{-1}(-1)]{\cos^{-1}(-1)}\, \sqrt[2\cos^{-1}(-1)]{2^9} \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} \right)} \right) \right)$$

$$3.865 \Big/ \left(\left(3^{1/18-3/(2\pi)} (-1)\, 2^{-1/\pi} \right. \right.$$
$$\left. \left. \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} \right)} \right)$$
$$\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{2^9} \Big) = -\left(3.865 \Big/ \left(2^{-1/(180^\circ)} \times 3^{1/18-3/(360^\circ)}\ \sqrt[360^\circ]{180^\circ}\ \sqrt[360^\circ]{2^9} \right. \right.$$
$$\left. \left. \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} \right)} \right) \right)$$

$$3.865 \Big/ \left(\left(3^{1/18-3/(2\pi)} (-1)\, 2^{-1/\pi} \right. \right.$$
$$\left. \left. \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} \right)} \right)$$
$$\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{2^9} \Big) = -\left(3.865 \Big/ \left(2^{-1/(2\,E(0))} \times 3^{1/18-3/(4\,E(0))}\ \sqrt[4\,E(0)]{2\,E(0)}\ \sqrt[4\,E(0)]{2^9} \right. \right.$$
$$\left. \left. \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} \right)} \right) \right)$$

$\cos^{-1}(x)$ is the inverse cosine function

$E(m)$ is the complete elliptic integral of the second kind with parameter $m = k^2$

Series representations

$$3.865 \Bigg/ \Bigg(\Bigg(3^{1/18-3/(2\pi)} (-1)\, 2^{-1/\pi} \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)}\right)} \Bigg) \sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9} \Bigg) = (-1.32175 + 2.28934\, i)\, 2^{-9\Big/\left(8\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)} \times 3^{-1/18+3\Big/\left(8\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)} \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)^{-1\Big/\left(8\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}\right)}$$

$$3.865 \Bigg/ \Bigg(\Bigg(3^{1/18-3/(2\pi)} (-1)\, 2^{-1/\pi} \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)}\right)} \Bigg) \sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9} \Bigg) = (-1.32175 + 2.28934\, i)\, 2^{-7\Big/\left(2\left(x+2\sum_{k=1}^{\infty} \frac{\sin(k\,x)}{k}\right)\right)} \times 3^{-1/18+3\Big/\left(2\left(x+2\sum_{k=1}^{\infty} \frac{\sin(k\,x)}{k}\right)\right)} \left(x + 2\sum_{k=1}^{\infty} \frac{\sin(k\,x)}{k}\right)^{-1\Big/\left(2\left(x+2\sum_{k=1}^{\infty} \frac{\sin(k\,x)}{k}\right)\right)}$$

for $(x \in \mathbb{R}$ and $x > 0)$

$$3.865 \Bigg/ \Bigg(\Bigg(3^{1/18-3/(2\pi)} (-1)\, 2^{-1/\pi} \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)}\right)} \Bigg) \sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9} \Bigg) = (-1.32175 + 2.28934\, i)\, 2^{-9/\left(8\sum_{k=1}^{\infty} \tan^{-1}\left(\frac{1}{F_{1+2k}}\right)\right)} \times 3^{-1/18+3/\left(8\sum_{k=1}^{\infty} \tan^{-1}\left(\frac{1}{F_{1+2k}}\right)\right)} \left(\sum_{k=1}^{\infty} \tan^{-1}\left(\frac{1}{F_{1+2k}}\right)\right)^{-1/\left(8\sum_{k=1}^{\infty} \tan^{-1}\left(\frac{1}{F_{1+2k}}\right)\right)}$$

$\mathbb{R}$ is the set of real numbers

F_n is the n^{th} Fibonacci number

$\tan^{-1}(x)$ is the inverse tangent function

Integral representations

$$3.865 \Bigg/ \Bigg(\Bigg(3^{1/18-3/(2\pi)} (-1)\, 2^{-1/\pi} \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)}\right)} \Bigg) \sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9} \Bigg) = (-1.24349 + 2.15379\, i)\, 3^{3/\left(4\int_0^{\infty} \frac{1}{1+t^2}\, dt\right)} \times 4^{-1/\left(\int_0^{\infty} \frac{1}{1+t^2}\, dt\right)} \left(\int_0^{\infty} \frac{1}{1+t^2}\, dt\right)^{-1/\left(4\int_0^{\infty} \frac{1}{1+t^2}\, dt\right)}$$

$$3.865 \Bigg/ \Bigg(\Bigg(3^{1/18-3/(2\pi)} (-1)\, 2^{-1/\pi} \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)}\right)} \Bigg) \Big/ \left(\sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9} \right) = (-1.24349 + 2.15379\, i)\, 3^{3/\left(4\int_0^\infty \frac{\sin(t)}{t}\, dt\right)} \times 4^{-1/\left(\int_0^\infty \frac{\sin(t)}{t}\, dt\right)} \left(\int_0^\infty \frac{\sin(t)}{t}\, dt \right)^{-1/\left(4\int_0^\infty \frac{\sin(t)}{t}\, dt\right)}$$

$$3.865 \Bigg/ \Bigg(\Bigg(3^{1/18-3/(2\pi)} (-1)\, 2^{-1/\pi} \sqrt[3]{6 - 3\sqrt[3]{3} + 3\left(\sqrt[3]{21 - 3\left(3\sqrt[3]{3} + 3^{2/3}\right)} - \sqrt[3]{3 + 3\left(3\sqrt[3]{3} + 3^{2/3}\right)}\right)} \Bigg) \Big/ \left(\sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9} \right) = (-1.24349 + 2.15379\, i)\, 3^{3/\left(4\int_0^\infty \frac{\sin^2(t)}{t^2}\, dt\right)} \times 4^{-1/\left(\int_0^\infty \frac{\sin^2(t)}{t^2}\, dt\right)} \left(\int_0^\infty \frac{\sin^2(t)}{t^2}\, dt \right)^{-1/\left(4\int_0^\infty \frac{\sin^2(t)}{t^2}\, dt\right)}$$

Furthermore, we obtain also:

(4 (1.61802)^(2 π))/(5 (3 + sqrt(5)))

Input interpretation

$$\frac{4 \times 1.61802^{2\pi}}{5\left(3 + \sqrt{5}\right)}$$

Result

3.14173239271420040205566693494119498004308431364244733426050882 69

…

3.1417323927142…. ≈ π (Ramanujan Recurring Number)

Series representations

$$\frac{4\times 1.61802^{2\pi}}{5\left(3+\sqrt{5}\right)}=\frac{4\,e^{0.962406\,\pi}}{5\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{4\times 1.61802^{2\pi}}{5\left(3+\sqrt{5}\right)}=\frac{4\,e^{0.962406\,\pi}}{5\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)}$$

$$\frac{4\times 1.61802^{2\pi}}{5\left(3+\sqrt{5}\right)}=\frac{8\,e^{0.962406\,\pi}\,\sqrt{\pi}}{5\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

And:

1/6((4(1.61802)^(2π))/(5(3+sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.61802^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64508040457161578992425110127211567776793674956253023586724520 17

…

1.645080404571…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.61802^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92481\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61802^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92481\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61802^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.92481\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

And again:

sqrt(1/(1/6((4(1.61802)^(2π))/(5(3+sqrt(5))))^2)*(4/3))

Input interpretation

$$\sqrt{\frac{1}{\frac{1}{6}\left(\frac{4\times 1.61802^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2}\times\frac{4}{3}}$$

Result

0.900276271558144988720405792748712262907185433870214753102536919 8

…

$0.90027\ldots \approx 0.9003163161571\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed form

$$\frac{2\sqrt{2}}{\pi} \approx 0.90031631$$

Series representations

$$\sqrt{\frac{4}{\frac{3}{6}\left(\frac{4\times 1.61802^{2}\,\pi}{5\left(3+\sqrt{5}\right)}\right)^{2}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-1+\frac{25}{2}\,e^{-1.92481\,\pi}\left(3+\sqrt{5}\right)^{2}\right)^k}{k!}$$

$$\sqrt{\frac{4}{\frac{3}{6}\left(\frac{4\times 1.61802^{2}\,\pi}{5\left(3+\sqrt{5}\right)}\right)^{2}}} = \sqrt{z_0}\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{25}{2}\,e^{-1.92481\,\pi}\left(3+\sqrt{5}\right)^{2}-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\sqrt{\frac{4}{\frac{3}{6}\left(\frac{4\times 1.61802^{2}\,\pi}{5\left(3+\sqrt{5}\right)}\right)^{2}}} = \exp\left(i\,\pi\left\lfloor \frac{\arg\left(-x+\frac{25}{2}\,e^{-1.92481\,\pi}\left(3+\sqrt{5}\right)^{2}\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty} \frac{(-1)^k\, x^{-k} \left(-\frac{1}{2}\right)_k \left(-x+\frac{25}{2}\,e^{-1.92481\,\pi}\left(3+\sqrt{5}\right)^{2}\right)^k}{k!}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

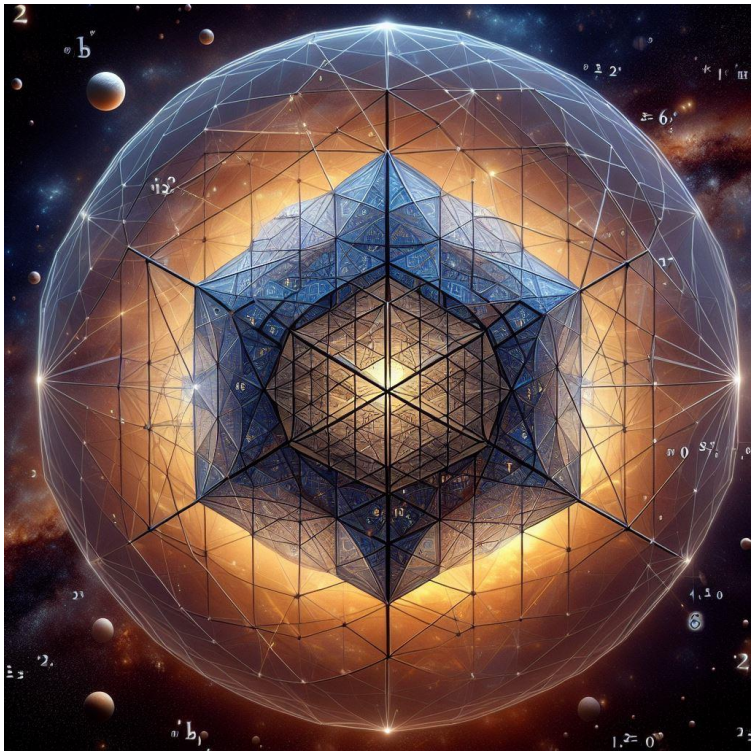


Fig. 2

A geometric representation of the Universe in a scenario of eternal inflation that emerges as a fluctuation from the quantum vacuum, which incorporates the solutions of Ramanujan's cubic equations, multiplying the solutions of these equations or the equations themselves, by the formula relating to the ratios (and/ or the inverses) between the volumes of the icosahedron, the octahedron and the tetrahedron and the volume of the sphere, where this formula includes the ratio $(2\sqrt{2})/\pi$, the so-called Del Gaudio-Nardelli Constant, whose result is very close to the values of the golden ratio and π.

Fig. 3

It was hypothesized that the solutions of Ramanujan's cubic equations which are related to the cubic equations solvable with the Cardano method and Padovan's recurrence relations, are boundary conditions to deepen the understanding of the formation of the bubbles of the universe with eternal inflation and an octahedron which could represent a mathematical or conceptual model of the primordial universe before the Big Bang. Finally, regarding cubic equations, they could represent the geometric characteristics of the quantum vacuum, while the spherical surface would represent the geometry of the universe bubble. We propose that the results obtained by multiplying the solution of the cubic equation by a formula relating the ratios (and/or inverses) between the volumes of the icosahedron, octahedron and tetrahedron and the volume of the sphere. including the ratio (2sqrt2)/Pi, the result of which provides the golden ratio and Pigreco, can be connected to the cosmological model described above.

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

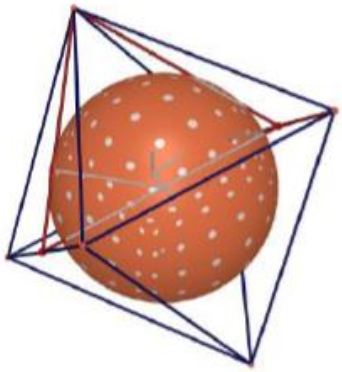


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2=\frac{2}{3}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2=\frac{2}{3}\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$
$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

Indeed:

From:

https://www.academia.edu/75787512/The_Theory_of_String_A_Candidate_for_a_Generalized_Unification_Model

The Einstein's field equation and the String Theory.

The Einstein's field equation which includes the cosmological constant is:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (8)$$

where $R_{\mu\nu}$ is the Ricci tensor, R its trace, λ the cosmological constant, $g_{\mu\nu}$ the metric tensor of the space geometry, G the Newton's gravitational constant and $T_{\mu\nu}$ the tensor representing the properties of energy, matter and momentum.

The left hand-side of (8) represents the gravitational field and, consequently, the warped space-time, while the right hand-side represents the matter, i.e. the sources of the gravitational field.

In string theory the gravity is related to the gravitons which are bosons, whereas the matter is related to fermions. It follows that the left and right hand of (8) may be respectively related to the action of bosonic and of superstrings.

The actions of bosonic string and superstring (also containing fermions) are connected by the Palumbo-Nardelli relation (Palumbo et al. 2005):

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\varphi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right]=$$

$$=\int_0^\infty \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|\widetilde{H}_3|^2-\frac{\kappa_{10}^2}{g_{10}^2}Tr_\nu\left(|F_2|^2\right)\right]$$

The sign minus in the above equation comes from the inversion of any relationship.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^{\,8}\right)^{-k}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^{\,8}\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4(-1)^k\, 1195^{-1-2k}\left(5^{1+2k} - 4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1 \sqrt{1-t^2}\, dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

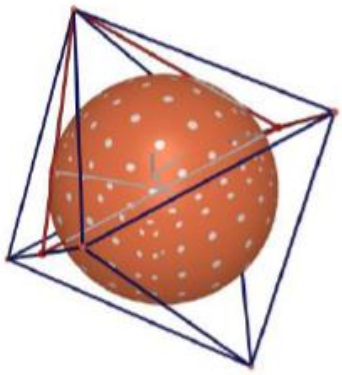
$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard to a sphere inscribed in an octahedron



we have the following formulas:

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\,2}{\pi} \quad (\text{for } l \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\,\pi)\rfloor)}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we have the following convex regular polyhedrons volumes:

$\frac{\sqrt{2}}{12}d^3$ Tetrahedron Volume

$\frac{\sqrt{2}}{3}d^3$ Octahedron Volume

$\frac{5}{12}(3+\sqrt{5})d^3$ Icosahedron Volume

Regarding the Sphere the volume is

$$V = \frac{4}{3}\pi r^3$$

With regard the sphere inscribed in a Tetrahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_T}{V_S} = \frac{\frac{\sqrt{2}}{12}d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(((√2 d^3)/12))*1/(4/3*π*(d/2)^3)

Input

$$\left(\frac{1}{12}\left(\sqrt{2}\ d^3\right)\right) \times \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.225079079039276517388799797751685145666143537488805156392859367 8…

0.225079079….

Property

$\frac{1}{\sqrt{2}\ \pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Octahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_O}{V_S} = \frac{\frac{\sqrt{2}}{3}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

((1/3*√2*d^3)/(4/3*π*(d/2)^3))

Input

$$\frac{\frac{1}{3}\sqrt{2}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\ 2}{\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Icosahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_I}{V_S} = \frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3)

Input

$$\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{5\left(3+\sqrt{5}\right)}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

4.166730504921372682725163141597223093105782245731637719697502271 9

…

4.166730504921….

Property

$\frac{5\left(3+\sqrt{5}\right)}{2\pi}$ is a transcendental number

Expanded form

$$\frac{15}{2\pi} + \frac{5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

Alternate forms

$$\frac{15+5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

$$\frac{\frac{15}{2}+\frac{5\sqrt{5}}{2}}{\pi} \quad (\text{for } d \neq 0)$$

Series representations

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\pi\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x\in\mathbb{R}\ \text{and}\ x<0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\ a^3\right)\left(\sqrt{2}\ d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}{3\times3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\ z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\right.\right.\right.$$

$$\left.\left.\left. z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\ z_0^{-k}}{k!}\right)\right)\Big/$$

$$\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\ z_0^{-k}}{k!}\right)^2\right)\wedge\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

Indeed, we obtain:

$$\frac{\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)\left(d^3\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2\right)}}{\sqrt[2\pi]{\frac{4}{3}\left(\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\right)\sqrt{2}\left(d^3 \times \frac{\sqrt{2}}{12}\right)d^3}}$$

= 1.6180085459001 07…

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4 \times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358487162422193250985871527893102518924557617007810061 3

…

3.1415926535…. = π (Ramanujan Recurring Number)

In conclusion, from the following further formula concerning the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron volumes and their ratios (and/or the inverses) with the sphere volume, raised the expression

$$(2\log(2)log^{2/3}(3^2))/(3e^{7/3})$$

$$\left(\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\frac{\frac{1}{4}\left(\left(15+7\sqrt{5}\right)d^3\right)}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\right)^{(2\log(2)\log^{2/3}(3^2))/(3e^{7/3})}$$

i.e.

$$\left(\frac{3}{2}\right)^{(4\log(2)\log^{2/3}(9))/(3e^{7/3})}\left(\frac{5\left(3+\sqrt{5}\right)\left(15+7\sqrt{5}\right)}{\pi}\right)^{(2\log(2)\log^{2/3}(9))/(3e^{7/3})}$$

we obtain:

1.6180381711519139857456817084743134673903021614989370712917722071

…

1.6180381711…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

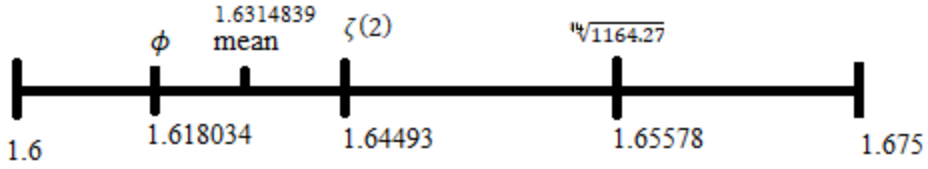
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

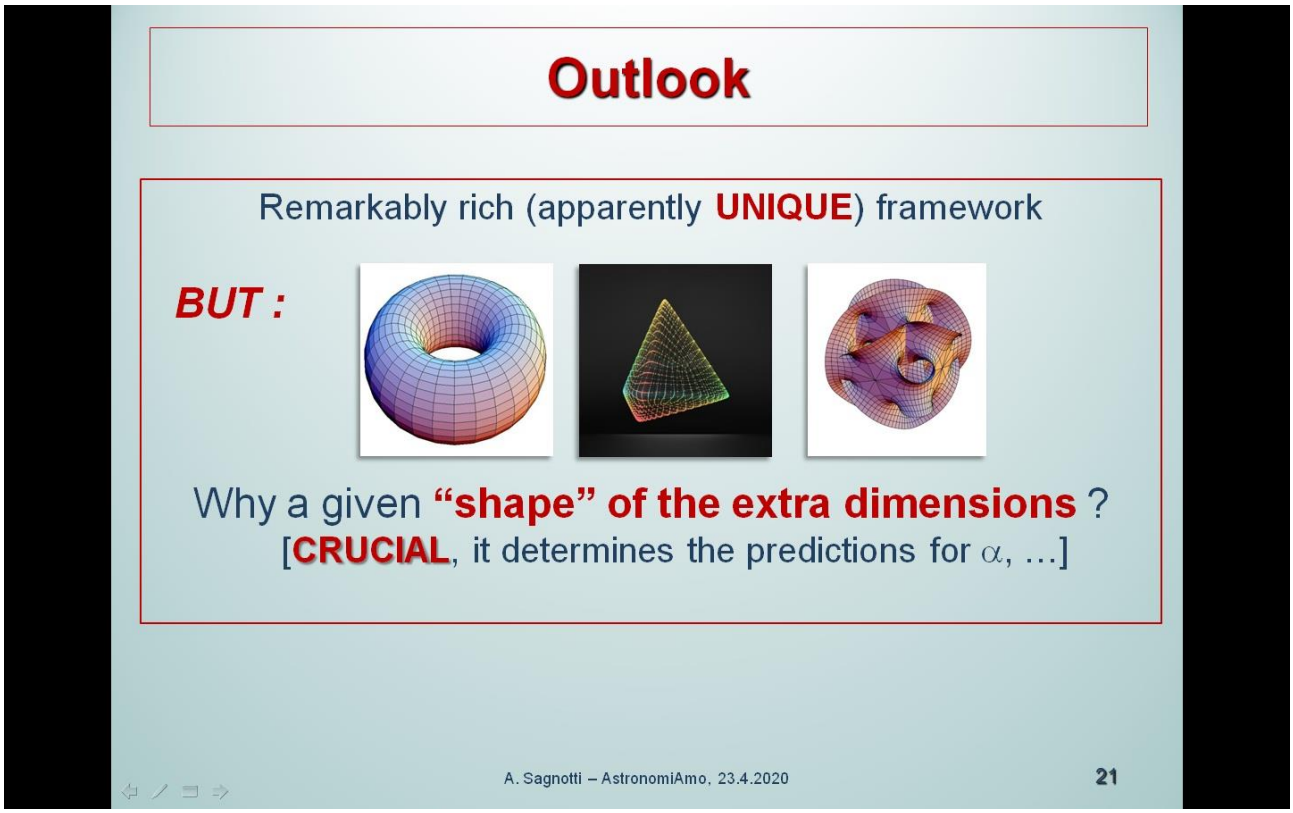
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix


Outlook
Remarkably rich (apparently UNIQUE) framework
BUT :
Why a given "shape" of the extra dimensions ?
[CRUCIAL, it determines the predictions for α, …]
A. Sagnotti – AstronomiAmo, 23.4.2020
21

From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

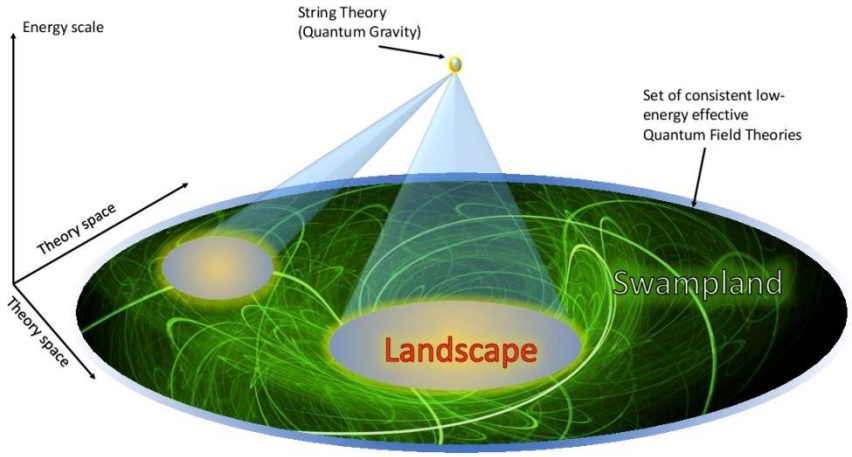


Fig. 1

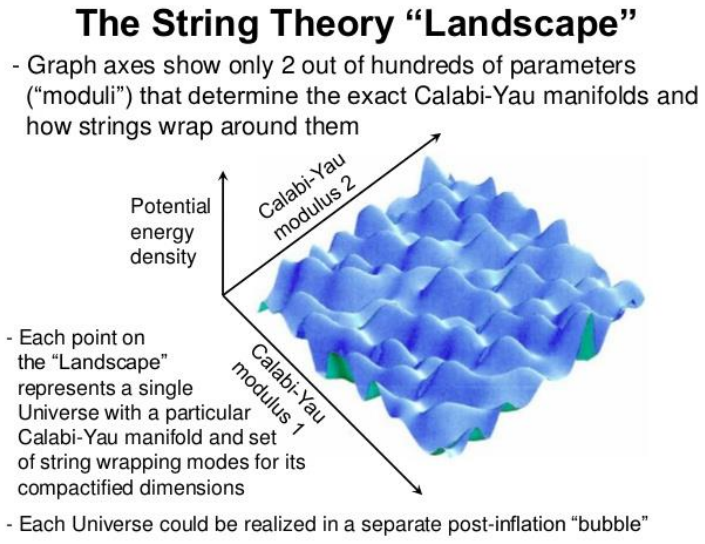

The String Theory "Landscape"
- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them
Potential energy density
Calabi-Yau modulus 2
Calabi-Yau modulus 1
- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

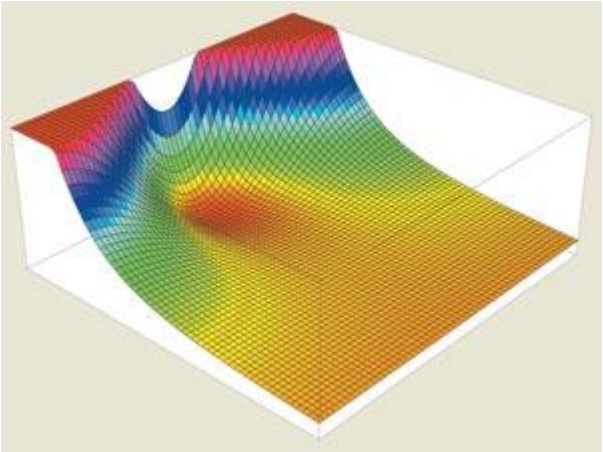


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

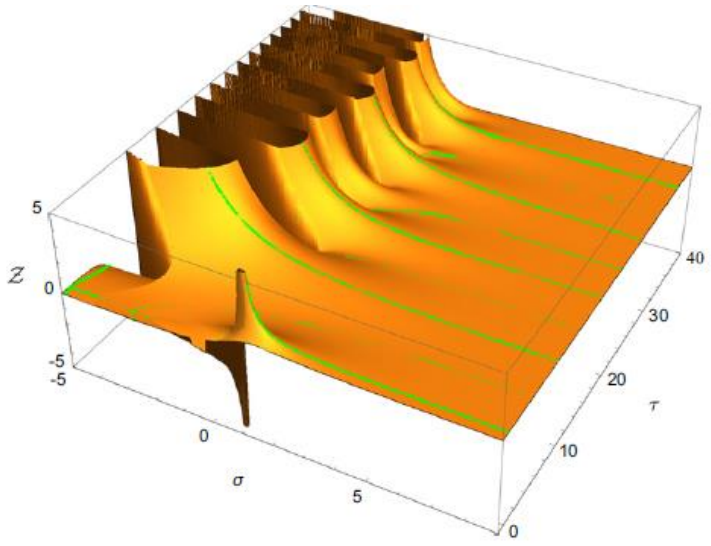


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

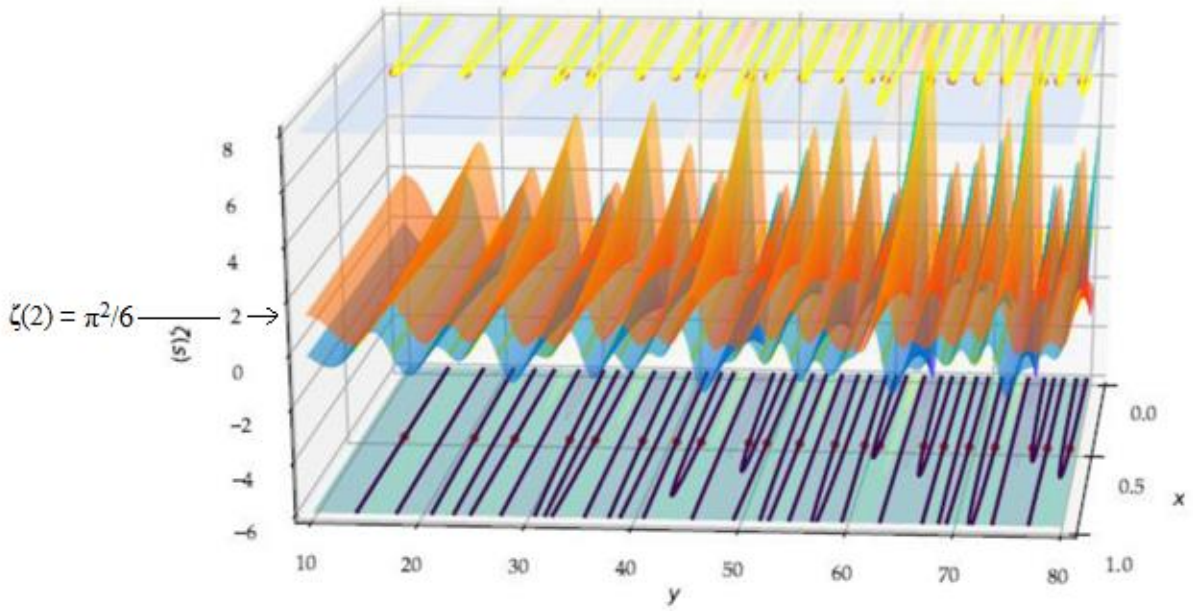


Figure 1. $C(x,y)$ and $S(x,y)$ surfaces of the Riemann $\zeta(x,y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

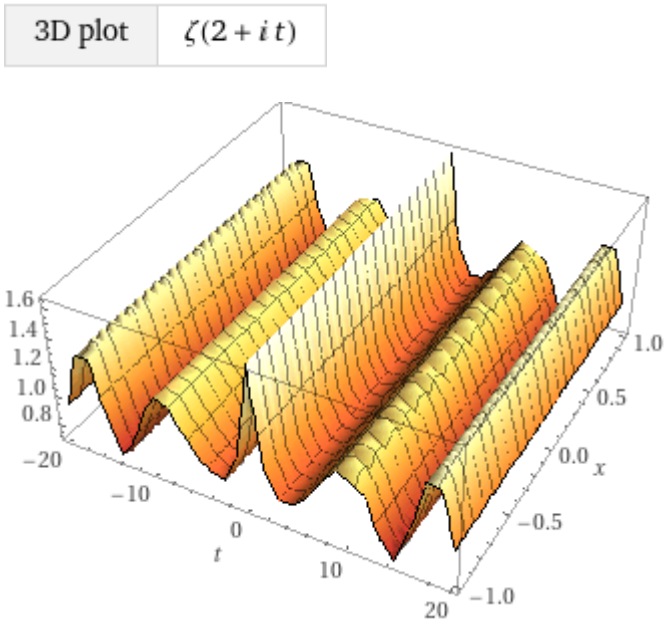


Fig. 6

Where ζ(2+it) :

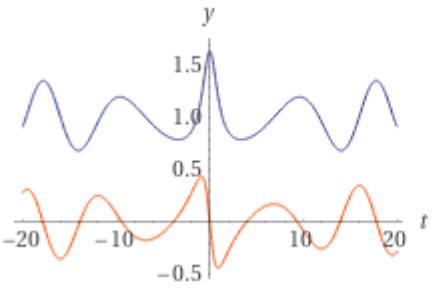
Input

$\zeta(2 + i\,t)$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

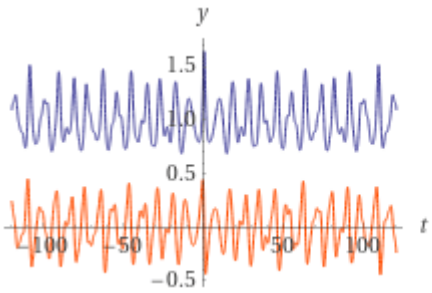
Plots



(t from −20 to 20)

real part

imaginary part



(t from −120 to 120)

real part

imaginary part

Roots

$t = 2\,i\,(n+1), \quad n \in \mathbb{Z}, \quad n \geq 1$

$t = -i\,(\rho_n - 2), \quad n \neq 0, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\,t\,\zeta'(2) - \frac{1}{2}\,t^2\,\zeta''(2) - \frac{1}{6}\,i\,\zeta^{(3)}(2)\,t^3 + \frac{1}{24}\,\zeta^{(4)}(2)\,t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2 + i\,t) = \zeta(2 + i\,t,\, 1)$$

$$\zeta(2 + i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2 + i\,t) = \frac{\zeta\left(2 + i\,t,\, \frac{1}{2}\right)}{-1 + 2^{2+i\,t}}$$

$\zeta(s,\, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2 + i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = \frac{\sum_{k=0}^{\infty} (1 + 2\,k)^{-2-i\,t}}{1 - 2^{-2-i\,t}} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \mathrm{Im}(t) < 1$$

$\mathrm{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^\infty \frac{\tau^{1+i\,t}}{-1+e^\tau}\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^\infty \tau^{2+i\,t}\,\mathrm{csch}^2(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^\infty e^{-\tau}\,\tau^{1+i\,t}\,\mathrm{csch}(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\mathrm{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+i\,t) = -i\,2^{2+i\,t}\,\pi^{1+i\,t}\,\Gamma(-1-i\,t)\sinh\left(\frac{\pi\,t}{2}\right)\zeta(-1-i\,t)$$

$$\zeta(2+i\,t) = \frac{\pi^{3/2+i\,t}\,\Gamma\left(-\frac{1}{2}-\frac{i\,t}{2}\right)\zeta(-1-i\,t)}{\Gamma\left(1+\frac{i\,t}{2}\right)}$$

$$\zeta(2+i\,t) = -\frac{i\sum_{k=0}^\infty \frac{\Gamma\left(k-\frac{i\,t}{2}\right)\sum_{j=0}^k (-1)^j\,(1+2\,j)\binom{k}{j}\zeta(2+2\,j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{i\,t}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372033095 8…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

1.64381856858…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\right/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$
for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^-}6$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right)=0.054323\,(0.0814845\,b+1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M}=-\frac{\frac{226\,802\,245}{18\,480\,874}+b}{M}$$

$$\frac{\partial M(b,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b}=-\frac{18\,480\,874\,M}{226\,802\,245+18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M}=\frac{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b}=\frac{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right\}=0 \text{ at } (b,M)=(-16,0)$$

Global minima:

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

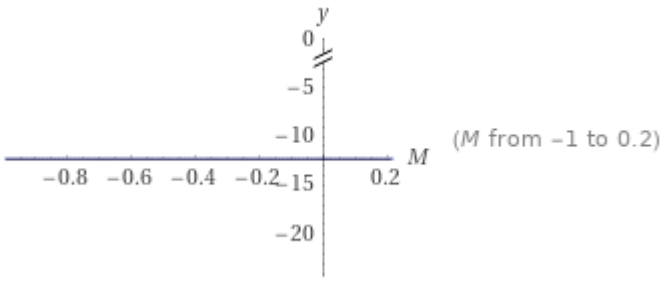
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

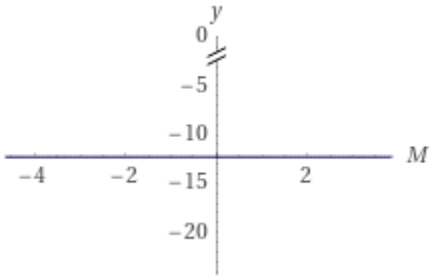
Result:

$$\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323\, M^2\right)}{M^2}\right) = \frac{3.55271 \times 10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

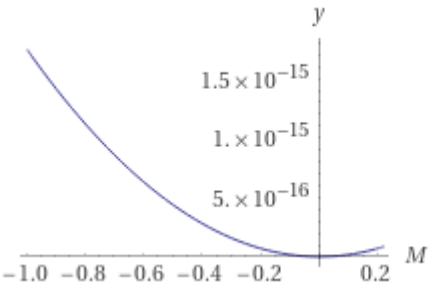
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

Result:

0

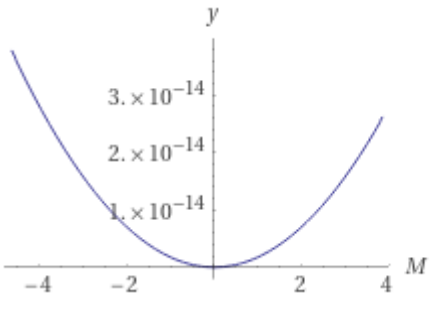
Plots: **(possible mathematical connection with an open string)**



(M from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(M from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O\left(M^{62194}\right)$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times10^{-15}\,M^2+O\left(\left(\frac{1}{M}\right)^{62\,194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}\,M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.38851344947464545348970783378088020833333333333333333333\ldots \times 10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 0.2^2 + 6.58545 \times 10^{-10}\sqrt{0.2^4}\right)}{0.2^2} + 1\right)^2 \times 0.2^2$$

Result:

$$7.02162151915943272558353253404940833333333333333333333333\ldots \times 10^{-17}$$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875\times10^{-14}$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots \times 10^{-8}$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([[((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left(\left(1/\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)\text{ ^ }(1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

$1.6181818182\ldots$

$1.6181818182\ldots$ result that is a very good approximation to the value of the golden ratio $1.618033988749...$

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

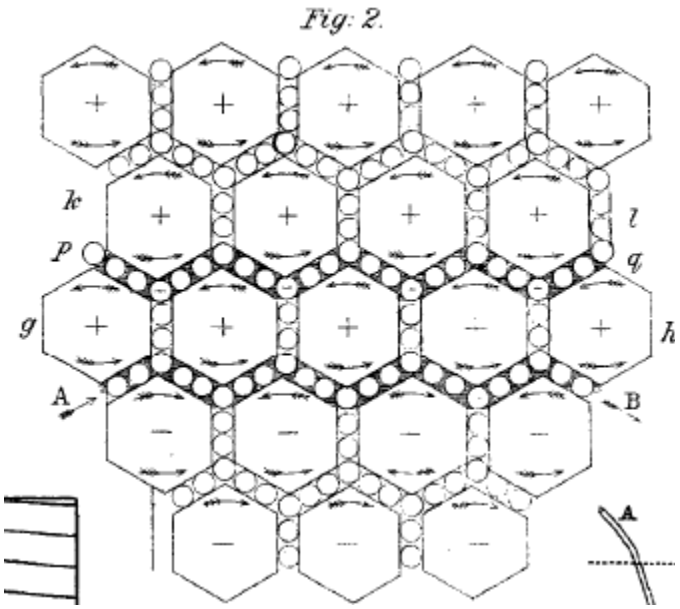
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

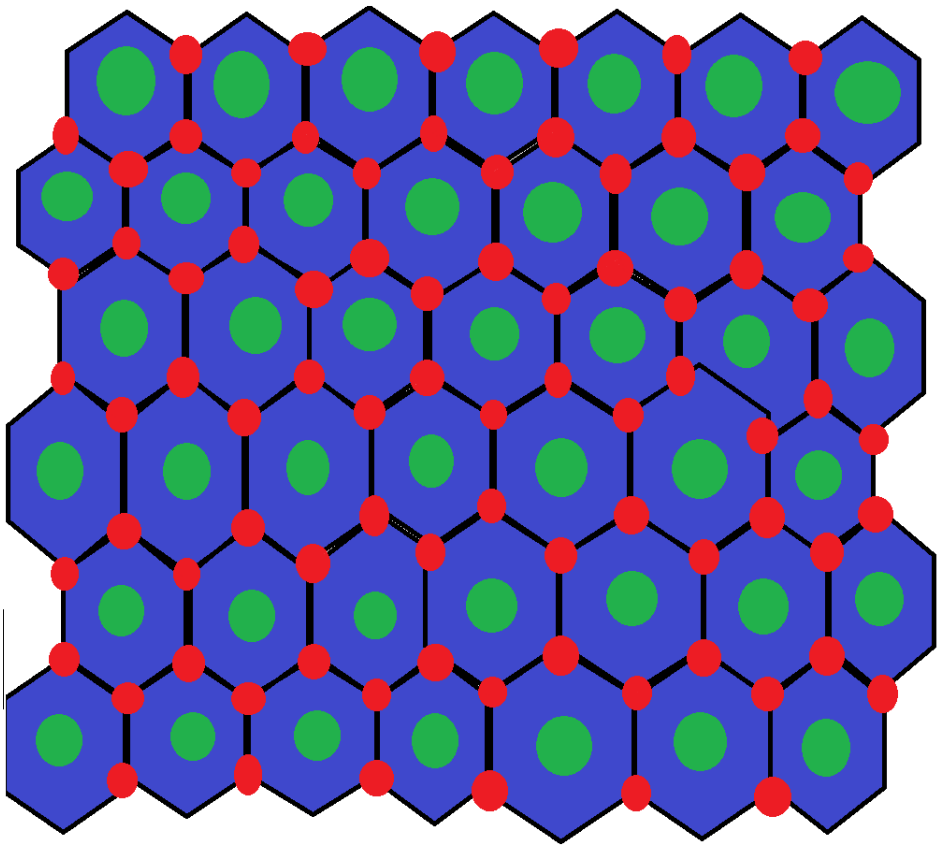
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Fig. 3

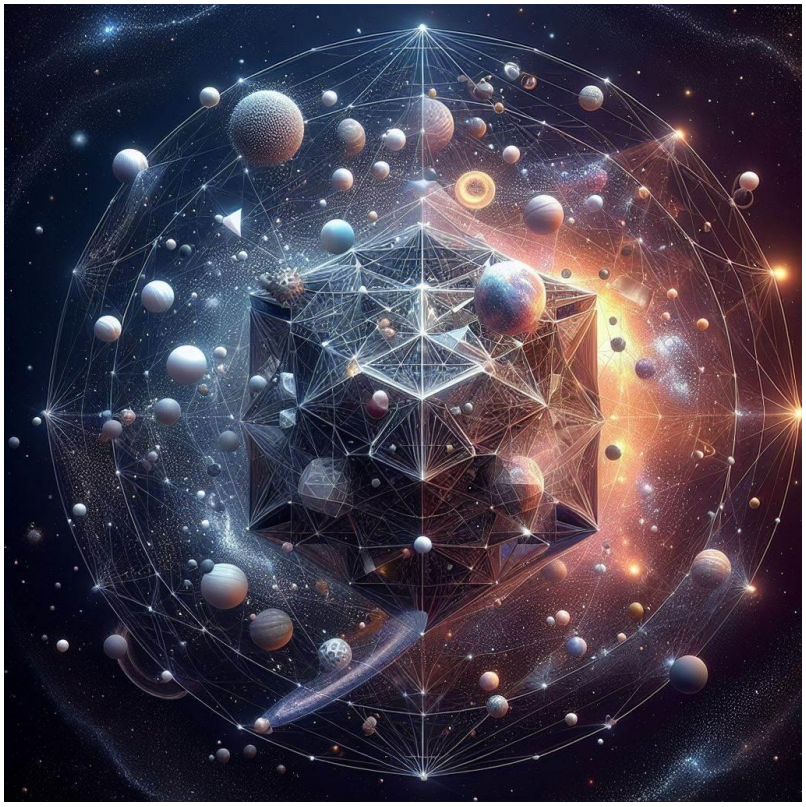


Image representing the solutions of the cubic equations in relation to the boundary conditions of the Plateau problem useful for understanding the formation of the bubbles of the eternally inflated universe and of an octahedron as a mathematical model of the primordial universe before the Big Bang, therefore highly symmetrical and with extremely low entropy. As for the cubic equations, they represent the geometric characteristics of the quantum vacuum, while the spherical surface would represent the geometry of the bubble of the universe.

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability. I would like to thank also my son **Francesco Nardelli**, student of Educational Sciences, majoring in History and Philosophy, for his useful suggestions regarding the intriguing image at the end of this work

References

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

Notes on efficient computation of Ramanujan cubic equations - *Peter J.-S. Shiue, Anthony G. Shannon, Shen C. Huang and Jorge E. Reyes* - Notes on Number Theory and Discrete Mathematics - Print ISSN 1310–5132, Online ISSN 2367–8275 2022, Volume 28, Number 2, Pages 350–375 - DOI: 10.7546/nntdm.2022.28.2.350-375

And also:

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *Emilian Dudas, Sergio Ferrara, Alex Kehagias and Augusto Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the analysis of various Cubic Equations that can be solved with Cardano's Formula, Quartic Equations and Quintic Equations. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory III

Michele Nardelli[1], Antonio Nardelli

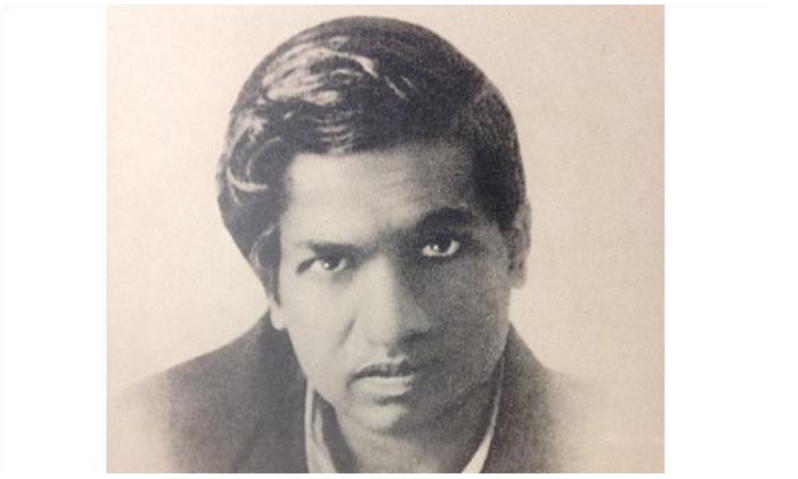
Abstract

In this paper (part III), we analyze various Cubic Equations that can be solved with Cardano's formula, Quartic Equations and Quintic Equations. We obtain, new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Introduction

Cardano's method for cubic equations involves cube roots and introduces symmetry concepts. Similarly, string theory and M-theory involve symmetries and transformation groups that play a fundamental role in describing particles and interactions. Solutions of complete cubic equations via Cardano's method involve complex cube roots. Similarly, equations and solutions in superstring theory can often involve advanced algebraic and mathematical complexities. Solutions of cubic equations can be understood geometrically through topology and algebraic geometry. In string theory, the geometric and topological configurations of strings and branes are also central. Duality is an important concept in string theory, where different mathematical descriptions can be equivalent. Similarly, in solving cubic equations, duality links exist through cube roots.

Thus, in string theory, duality refers to the fact that different mathematical formulations or physical descriptions of a theory can be equivalent. For example, two seemingly different theories can be dual to each other, meaning they are two different ways of describing the same physical reality. This concept of duality has been very useful for understanding profound aspects of string theory and field theories.

Similarly, in solving cubic equations, duality can emerge through cube roots. The cubic equation $ax^3+bx^2+cx+d=0$ can be solved using cube roots, as in the Cardano method I described above. However, cube roots involve the concept of complex numbers, and it is sometimes possible to express the solutions in ways that highlight a duality between the solutions.

For example, if we consider complex cube roots, we can see a duality between them, since there are connections via the cube roots of unity. This duality is similar to the concept of duality in string theory, where different descriptions can be connected by similar transformations.

In both cases, duality provides a deeper way of understanding the underlying mathematical or physical structure, allowing you to explore hidden connections between different concepts.

Cubic surfaces can be associated with geometric and topological concepts. In string theory, for example, some models require spaces with particular shapes, and the geometry of the spaces can influence the properties of strings and particles. Some mathematical and physical models involving surfaces might be described by cubic or

complex equations. The geometry of surfaces and the associated mathematical structure can influence the properties of universe bubbles in the multiverse. Cubic surfaces can represent certain topological spaces, and their properties can affect the structure of the associated string theory.

The connection between the surface of a D-brane and the roots of a cubic equation can be explored in specific contexts within string theory, especially when it comes to studying the geometric and topological properties of D-branes. D-branes are extended objects on whose surface open strings can terminate. The geometry of the D-brane surface can interfere with the physics of surrounding strings. In some cases, geometry can be described mathematically through objects known in algebraic surface theory, such as cubic equations.

D-branes can host open fields, which describe particles interacting with the D-brane. Field equations on a D-brane can involve surface geometry, and in some situations, these equations can be associated with higher degree algebraic equations, such as cubic equations.

The compactification of extra dimensions, which is an important part of string theory, can be associated with the surface geometry of a D-brane. Compactification can lead to a reduction in dimensions and a geometric description that can involve algebraic equations, including cubic equations.

The topological invariants associated with the surface of a D-brane, such as the Betti number or the Chern number, can be related to geometric characteristics that can be described via algebraic equations, including cubic equations.

In some situations of duality, the description of a D-brane in a context can be dualistic to a description based on algebraic objects such as curves or surfaces, which can be represented via algebraic equations, including cubic equations.

In this paper, we will hypothesize that the solutions of the cubic equations can be used as boundary conditions to further the understanding of the formation of universe bubbles in eternal inflation. Furthermore, an octahedron could serve as a mathematical or conceptual model of the universe in the pre-Big Bang phase. Cubic equations, in this context, could represent the geometric characteristics of the quantum vacuum, while the spherical surface could be used to describe the geometry of the universe bubble.

The results $(2\sqrt{2})/\pi$, the golden ratio φ and π obtained, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and universe bubbles in eternal inflation. It may reflect a fundamental relationship between the geometry of solids and the mathematical characteristics of solutions to cubic equations.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and universe bubbles.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of universe bubbles and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

Finally, it is also possible to hypothesize the solutions of Ramanujan's cubic equations which are related to the cubic equations solvable with the Cardano method and Padovan's recurrence relations, as boundary conditions to deepen the understanding of the formation of inflationary universe bubbles eternal and of an octahedron which could represent a mathematical or conceptual model of the primordial universe before the Big Bang. Furthermore, regarding the cubic equations, they can represent the geometric characteristics of the quantum vacuum, while the spherical surface represents the geometry of the universe bubble. The results above $(2\sqrt{2})/\pi$, the golden ratio φ and π which are obtained, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

These mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe. The cubic equations, in this context, could reflect the geometric and dynamic characteristics of the quantum vacuum, while the spherical surface could represent the geometry of the bubbles of the universe.

Regarding fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the primordial universe and universe bubbles.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

We will also give an example of how the result of multiplying the solutions of a quartic equation for the formula involving the ratios between the volumes of geometric solids and the Del Gaudio-Nardelli Constant approaches the values of the golden ratio and Pi, and as from the product of the solutions, the Recurring Numbers of Ramanujan are obtained. This discovery suggests that there are still many connections to be explored between mathematics and physics, and that these connections could lead to new insights into the nature of the universe and its fundamental laws.

From

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

Cubic equations and Cardano's Formulas

We analyze the following equation

$$x^3 + 26x = 12x^2 + 12.$$

i.e.

$$x^3 - 12x^2 + 26x - 12 = 0$$

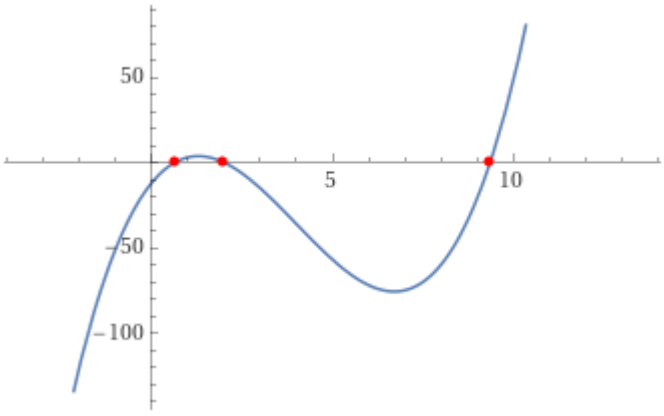
We obtain:

x^3-12x^2+26x-12=0

Input

$x^3 - 12\,x^2 + 26\,x - 12 = 0$

Root plot **(figure that can be related to an open string)**



Alternate forms

$(x-2)((x-10)x+6)=0$

$(x-2)\left(x^2-10x+6\right)=0$

$(x-4)^3-22(x-4)-36=0$

Solutions

$x=2$

$x=5-\sqrt{19}$

$x=5+\sqrt{19}$

Sum of roots

12

Product of roots

12

From

$$ax^3 + bx^2 + cx + d = 0$$

The Cardano's Formula is:

$$x = \sqrt[3]{-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a} + \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} + \sqrt[3]{-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a} - \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}$$

From

$$x^3 - 12x^2 + 26x - 12 = 0$$

We obtain:

(((12^3)/(27)+(-12*26)/(6)+(12)/(2)+sqrt(((12^3)/(27)+(-12*26)/(6)+(12)/(2))^2+((26)/(3)-(12^2)/(9))^3)))^1/3

Input

$$\sqrt[3]{\frac{12^3}{27} + \frac{1}{6}(-12\times 26) + \frac{12}{2} + \sqrt{\left(\frac{12^3}{27} + \frac{1}{6}(-12\times 26) + \frac{12}{2}\right)^2 + \left(\frac{26}{3} - \frac{12^2}{9}\right)^3}}$$

Result

$$\sqrt[3]{18 + \frac{10}{3}\, i \sqrt{\frac{19}{3}}}$$

Decimal approximation

2.679449471770336776118490991929807829568501962616222468445172 06…

+

0.392280335427352969442390963738083326232296490532039709021867661…

i

Alternate complex forms

$2.6794 + 0.3923\,i$

$$\sqrt{\frac{22}{3}}\left(\cos\left(\frac{1}{3}\tan^{-1}\left(\frac{5\sqrt{\frac{19}{3}}}{27}\right)\right)+i\sin\left(\frac{1}{3}\tan^{-1}\left(\frac{5\sqrt{\frac{19}{3}}}{27}\right)\right)\right)$$

$$\sqrt{\frac{22}{3}}\,e^{1/3\,i\tan^{-1}\left(\left(5\sqrt{\frac{19}{3}}\right)/27\right)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=\sqrt{\frac{22}{3}}\ \text{(radius)},\quad \theta=\frac{1}{3}\tan^{-1}\left(\frac{5\sqrt{\frac{19}{3}}}{27}\right)\ \text{(angle)}$$

Alternate forms

$$\sqrt[3]{\frac{2}{3}\,i\left(5\sqrt{\frac{19}{3}}+-27\,i\right)}$$

$$\frac{\sqrt[3]{2\left(81+5\,i\,\sqrt{57}\right)}}{3^{2/3}}$$

$$\frac{1}{2}\left(1+\sqrt{19}+i\sqrt{\frac{28}{3}-2\sqrt{19}}\right)$$

Minimal polynomial

$$9x^4-18x^3-30x^2-132x+484$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

All 3rd roots of 18+10/3 i sqrt(19/3)

$$\sqrt{\frac{22}{3}}\;e^{1/3\,i\tan^{-1}\left(\left(5\sqrt{\frac{19}{3}}\right)/27\right)}\approx 2.6794+0.3923\,i \quad \text{(principal root)}$$

$$\sqrt{\frac{22}{3}}\;e^{1/3\,i\left(2\pi+\tan^{-1}\left(\left(5\sqrt{\frac{19}{3}}\right)/27\right)\right)}\approx -1.6794+2.1243\,i$$

$$\sqrt{\frac{22}{3}}\;\exp\left(i\left(-2\pi+\frac{1}{3}\left(4\pi+\tan^{-1}\left(\frac{5\sqrt{\frac{19}{3}}}{27}\right)\right)\right)\right)\approx -1.0000-2.5166\,i$$

(((12^3)/(27)+(-12*26)/(6)+(12)/(2)-sqrt(((12^3)/(27)+(-12*26)/(6)+(12)/(2))^2+((26)/(3)-(12^2)/(9))^3)))^1/3

Input

$$\sqrt[3]{\frac{12^3}{27}+\frac{1}{6}(-12\times 26)+\frac{12}{2}-\sqrt{\left(\frac{12^3}{27}+\frac{1}{6}(-12\times 26)+\frac{12}{2}\right)^2+\left(\frac{26}{3}-\frac{12^2}{9}\right)^3}}$$

Result

$$\sqrt[3]{18-\frac{10}{3}\,i\,\sqrt{\frac{19}{3}}}$$

Decimal approximation

2.679449471770336776118490991929807829568501962616222468445172 06…

$-$

0.392280335427352969442390963738083326232296490532039709021867661…

i

Alternate complex forms

$2.6794 - 0.3923\,i$

$$\sqrt{\frac{22}{3}}\left(\cos\left(-\frac{1}{3}\tan^{-1}\left(\frac{5\sqrt{\frac{19}{3}}}{27}\right)\right)+i\sin\left(-\frac{1}{3}\tan^{-1}\left(\frac{5\sqrt{\frac{19}{3}}}{27}\right)\right)\right)$$

$$\sqrt{\frac{22}{3}}\;e^{-1/3\,i\tan^{-1}\left(\left(5\sqrt{\frac{19}{3}}\right)/27\right)}$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r = \sqrt{\frac{22}{3}} \text{ (radius)}, \quad \theta = -\frac{1}{3}\tan^{-1}\left(\frac{5\sqrt{\frac{19}{3}}}{27}\right) \text{ (angle)}$$

Alternate forms

$$\sqrt[3]{-\frac{2}{3}i\left(5\sqrt{\frac{19}{3}}+27i\right)}$$

$$\frac{\sqrt[3]{2\left(81-5i\sqrt{57}\right)}}{3^{2/3}}$$

$$\frac{1}{2}\left(1+\sqrt{19}-i\sqrt{\frac{28}{3}-2\sqrt{19}}\right)$$

Minimal polynomial

$$9x^4 - 18x^3 - 30x^2 - 132x + 484$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

All 3rd roots of 18-10/3 i sqrt(19/3)

$$\sqrt{\frac{22}{3}}\, e^{-1/3\, i \tan^{-1}\left(\left(5\sqrt{\frac{19}{3}}\right)/27\right)} \approx 2.6794 - 0.3923\, i$$

$$\sqrt{\frac{22}{3}}\; e^{1/3\,i\left(2\pi-\tan^{-1}\left(\left(5\sqrt{\frac{19}{3}}\right)/27\right)\right)} \approx -1.0000 + 2.5166\,i \quad \text{(principal root)}$$

$$\sqrt{\frac{22}{3}}\;\exp\left(i\left(-2\pi+\frac{1}{3}\left(4\pi-\tan^{-1}\left(\frac{5\sqrt{\frac{19}{3}}}{27}\right)\right)\right)\right) \approx -1.6794 - 2.1243\,i$$

We obtain:

$$\sqrt[3]{18+\frac{10}{3}\,i\sqrt{\frac{19}{3}}}$$

And

$$\sqrt[3]{18-\frac{10}{3}\,i\sqrt{\frac{19}{3}}}$$

(18 + 10/3 i sqrt(19/3))^(1/3) + (18 - 10/3 i sqrt(19/3))^(1/3) + 4

Input

$$\sqrt[3]{18+\frac{10}{3}\,i\sqrt{\frac{19}{3}}}+\sqrt[3]{18-\frac{10}{3}\,i\sqrt{\frac{19}{3}}}+4$$

i is the imaginary unit

Decimal approximation

9.358898943540673552236981983859615659137003925232444936890344138 1

…

9.35889894354….

Alternate forms

$$\frac{1}{3}\left(\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}+12\right)$$

$$5+\sqrt{19}$$

$$4+2\sqrt{\frac{22}{3}}\cos\left(\frac{1}{3}\tan^{-1}\left(\frac{5\sqrt{\frac{19}{3}}}{27}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Minimal polynomial

$$x^2-10\,x+6$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the alternate form

$$\frac{1}{3}\left(\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}+12\right)$$

we obtain:

((1/3 ((6 (81 - 5 i sqrt(57)))^(1/3) + (6 (81 + 5 i sqrt(57)))^(1/3) + 12))) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

Input

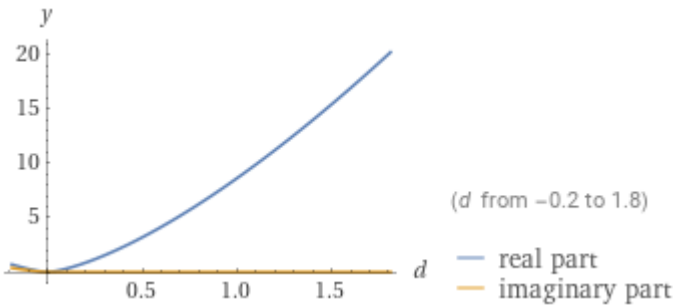
$$\left(\frac{1}{3}\left(\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}+12\right)\right)$$
$$\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3\right)^2}$$

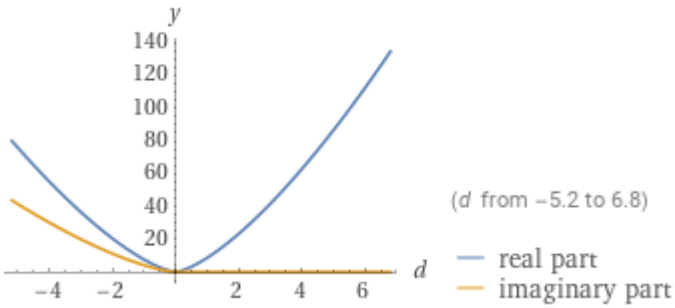
i is the imaginary unit

Exact result

$$3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(12+\sqrt[3]{6\left(81-5i\sqrt{57}\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate form

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(5+\sqrt{19}\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{d^9}$$

Expanded forms

$$2^{1/3-2/\pi} \times 3^{-2/3-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{81+5\,i\,\sqrt{57}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9} +$$
$$2^{1/3-2/\pi} \times 3^{-2/3-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{81-5\,i\,\sqrt{57}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9} +$$
$$2^{2-2/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$3^{-1-3/(2\pi)} \times 4^{-1/\pi}\ \sqrt[3]{486+30\,i\,\sqrt{57}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{5\sqrt{5}\ d^9+15\,d^9} +$$
$$3^{-1-3/(2\pi)} \times 4^{-1/\pi}\ \sqrt[3]{486-30\,i\,\sqrt{57}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{5\sqrt{5}\ d^9+15\,d^9} +$$
$$3^{-3/(2\pi)} \times 4^{1-1/\pi}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{5\sqrt{5}\ d^9+15\,d^9}$$

Alternate form assuming d>0

$$3^{-(9+4\pi)/(6\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(4 \times 3^{2/3} + \sqrt[3]{2\left(81-5\,i\,\sqrt{57}\right)} + \sqrt[3]{2\left(81+5\,i\,\sqrt{57}\right)}\right)$$
$$\sqrt[\pi]{\frac{\pi}{4}}\ d^{9/(2\pi)}$$

Root

$d = 0$ (assuming complex-valued fractional powers)

Series expansion at d=∞

$$3^{-1-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(12+\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)} + \sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}\right) \sqrt[\pi]{\frac{\pi}{4}}$$
$$d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{3}\left(\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}+12\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$

$$\frac{1}{d}2^{-(2+\pi)/\pi}\times 3^{1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(12+\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}\right)\sqrt[2\pi]{d^9}\;\pi^{1/\pi-1}$$

Indefinite integral

$$\int 3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(12+\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}\right)\sqrt[2\pi]{d^9}\;dd=$$

$$\sqrt[\pi]{\frac{\pi}{4}}\,\frac{1}{1+\frac{9}{2\pi}}3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(12+\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}\right)\sqrt[\pi]{\frac{\pi}{4}}\;d\sqrt[2\pi]{d^9}+\text{constant}$$

Series representations

$$\frac{1}{3}\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\left(\sqrt[3]{6\left(81-i\left(5\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}+12\right)=2^{-2/\pi}\times 3^{-2/3-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}\left(4\times 3^{2/3}+\sqrt[3]{2}\sqrt[3]{81-5i\sqrt{56}\sum_{k=0}^{\infty}56^{-k}\binom{\frac{1}{2}}{k}}+\sqrt[3]{2}\sqrt[3]{81+5i\sqrt{56}\sum_{k=0}^{\infty}56^{-k}\binom{\frac{1}{2}}{k}}\right)$$

$$\frac{1}{3}\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\left(\sqrt[3]{6\left(81-i\left(5\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}+12\right)=2^{-2/\pi}\times 3^{-2/3-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}\left(4\times 3^{2/3}+\sqrt[3]{2}\sqrt[3]{81-5i\sqrt{56}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{56}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\sqrt[3]{2}\sqrt[3]{81+5i\sqrt{56}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{56}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$\frac{1}{3}\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\left(\sqrt[3]{6\left(81-i\left(5\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}+12\right)=$$
$$2^{-2/\pi}\times 3^{-2/3-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}$$
$$\left(4\times 3^{2/3}+\sqrt[3]{2}\sqrt[3]{81-5i\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(57-z_0)^k z_0^{-k}}{k!}}+\sqrt[3]{2}\sqrt[3]{81+5i\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(57-z_0)^k z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

And:

((1/3 ((6 (81 - 5 i sqrt(57)))^(1/3) + (6 (81 + 5 i sqrt(57)))^(1/3) + 12))) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

Input

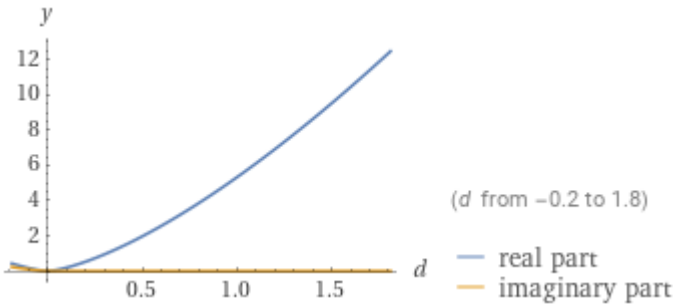
$$\left(\frac{1}{3}\left(\sqrt[3]{6\left(81-5i\sqrt{57}\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}+12\right)\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}d^3\right)}$$

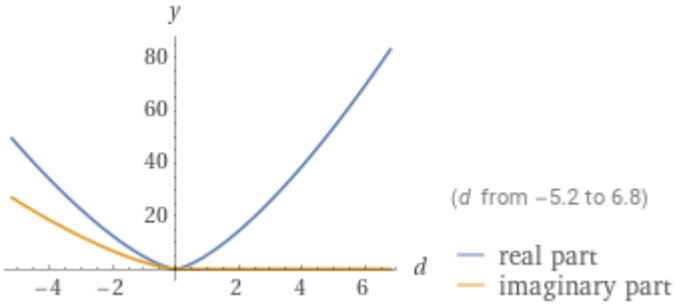
i is the imaginary unit

Exact result

$$2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(12+\sqrt[3]{6\left(81-5\,i\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\sqrt{57}\right)}\right)\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate forms

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\left(5+\sqrt{19}\right)\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(12+\sqrt[3]{6\,i\left(5\sqrt{57}+-81\,i\right)}+\sqrt[3]{-6\,i\left(5\sqrt{57}+81\,i\right)}\right)\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$\left(2^{2-1/\pi}\times 3^{-3/(2\pi)}+2^{1/3-1/\pi}\times 3^{-2/3-3/(2\pi)}\sqrt[3]{i\left(5\sqrt{57}+-81\,i\right)}+2^{1/3-1/\pi}\times 3^{-2/3-3/(2\pi)}\sqrt[3]{-i\left(5\sqrt{57}+81\,i\right)}\right)\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Expanded forms

$$2^{1/3-1/\pi}\times 3^{-2/3-3/(2\pi)}\sqrt[3]{81+5\,i\sqrt{57}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}+$$
$$2^{1/3-1/\pi}\times 3^{-2/3-3/(2\pi)}\sqrt[3]{81-5\,i\sqrt{57}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}+2^{2-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$2^{-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[3]{486+30\,i\sqrt{57}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}+$$
$$2^{-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[3]{486-30\,i\sqrt{57}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}+2^{2-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Alternate form assuming d>0

$$2^{-1/\pi}\times 3^{-(9+4\pi)/(6\pi)}\left(4\times 3^{2/3}+\sqrt[3]{2\left(81-5\,i\sqrt{57}\right)}+\sqrt[3]{2\left(81+5\,i\sqrt{57}\right)}\right)\sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Root

$d=0$ (assuming complex-valued fractional powers)

Series expansion at d=∞

$$2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(12+\sqrt[3]{6\left(81-5\,i\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\sqrt{57}\right)}\right)\sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{3}\left(\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}+12\right)\sqrt[2\pi]{\frac{\left(4\,\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\;d^3\right)\left(\sqrt{2}\;d^3\right)}{3\times3\times12}}\right)=$$

$$\frac{2^{-(1+\pi)/\pi}\times3^{1-3/(2\pi)}\left(12+\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}\right)\sqrt[2\pi]{d^9}\;\pi^{1/(2\pi)-1}}{d}$$

Indefinite integral

$$\int 2^{-1/\pi}\times3^{-1-3/(2\pi)}\left(12+\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}\right)\sqrt[2\pi]{d^9}\;\sqrt[2\pi]{\pi}\;dd=$$

$$\frac{1}{9+2\,\pi}2^{(\pi-1)/\pi}\times3^{-1-3/(2\pi)}\left(12+\sqrt[3]{6\left(81-5\,i\,\sqrt{57}\right)}+\sqrt[3]{6\left(81+5\,i\,\sqrt{57}\right)}\right)\pi^{1+1/(2\pi)}\;d\sqrt[2\pi]{d^9}\;+\text{constant}$$

Series representations

$$\frac{1}{3}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\left(\sqrt[3]{6\left(81-i\left(5\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}+12\right)=$$
$$2^{-3/(2\pi)}\times3^{-2/3-3/(2\pi)}\sqrt[2\pi]{d^9\pi\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2}$$
$$\left(4\times3^{2/3}+\sqrt[3]{2}\sqrt[3]{81-5i\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(57-z_0)^k z_0^{-k}}{k!}}+\right.$$
$$\left.\sqrt[3]{2}\sqrt[3]{81+5i\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(57-z_0)^k z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\frac{1}{3}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\left(\sqrt[3]{6\left(81-i\left(5\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}+12\right)=2^{-3/(2\pi)}\times3^{-2/3-3/(2\pi)}\sqrt[2\pi]{d^9\,\pi\exp^2\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}\left(4\times3^{2/3}+\sqrt[3]{2}\left(\sqrt[3]{81-5i\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(57-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(57-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\sqrt[3]{2}\sqrt[3]{81+5i\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(57-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(57-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)\right)$$

for $(x\in\mathbb{R}\text{ and }x<0)$

$$\frac{1}{3}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\left(\sqrt[3]{6\left(81-i\left(5\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}+12\right)=2^{-3/(2\pi)}\times3^{-2/3-3/(2\pi)}\sqrt[2\pi]{d^9\,\pi\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}\left(4\times3^{2/3}+\sqrt[3]{2}\left(81-5i\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(57-z_0)/(2\pi)\rfloor}z_0^{\,1/2+1/2\lfloor\arg(57-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(57-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)+\sqrt[3]{2}\left(81+5i\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(57-z_0)/(2\pi)\rfloor}z_0^{\,1/2+1/2\lfloor\arg(57-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(57-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Dividing the two exact results

$$3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(12+\sqrt[3]{6\left(81-5i\sqrt{57}\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)\sqrt[\pi]{\frac{\pi}{4}}$$

$$\sqrt[2\pi]{d^9}$$

and

$$2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(12+\sqrt[3]{6\left(81-5i\sqrt{57}\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

we obtain:

(3^(-1-3/(2π))(5(3+√(5)))^(1/(2π))(12+(6(81-5 i √(57)))^(1/3)+(6(81+5 i√(57)))^(1/3))(π/4)^(1/π))/(2^(-1/π)3^(-1-3/(2π))(12+(6(81-5 i√(57)))^(1/3)+(6(81+5 i√(57)))^(1/3)) π^(1/(2π)))

Input

$$\frac{3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(12+\sqrt[3]{6\left(81-5i\sqrt{57}\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)\sqrt[\pi]{\frac{\pi}{4}}}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(12+\sqrt[3]{6\left(81-5\left(i\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)\sqrt[2\pi]{\pi}}$$

i is the imaginary unit

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{3^{-1-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(12+\sqrt[3]{6\left(81-i\left(5\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)\sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(12+\sqrt[3]{6\left(81-5\left(i\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)\sqrt[2\pi]{\pi}}$$

$$=2^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-1-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(12+\sqrt[3]{6\left(81-i\left(5\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)\sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(12+\sqrt[3]{6\left(81-5\left(i\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)^{2\pi}\sqrt{\pi}}$$
$$= 2^{-1/\pi}\,\sqrt[2\pi]{5}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{3^{-1-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(12+\sqrt[3]{6\left(81-i\left(5\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)\sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(12+\sqrt[3]{6\left(81-5\left(i\sqrt{57}\right)\right)}+\sqrt[3]{6\left(81+5i\sqrt{57}\right)}\right)^{2\pi}\sqrt{\pi}}$$
$$= 2^{-1/\pi}\,\sqrt[2\pi]{5}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535884… ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298888445046920
…

1.644934066846… ≈ ζ(2) = π²/6 = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.618008545900002\pi}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900002\pi}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.92478440155986\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

And:

sqrt(1/(1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2)*(4/3))

Input interpretation

$$\sqrt{\frac{1}{\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\,\pi}}{5\left(3+\sqrt{5}\right)}\right)^2}\times\frac{4}{3}}$$

Result

0.900316316157480364283707666073791426354137001825739267605879203 3

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed form

$$\frac{2\sqrt{2}}{\pi} \approx 0.900316316157$$

Now, we analyze the following quintic polynomial

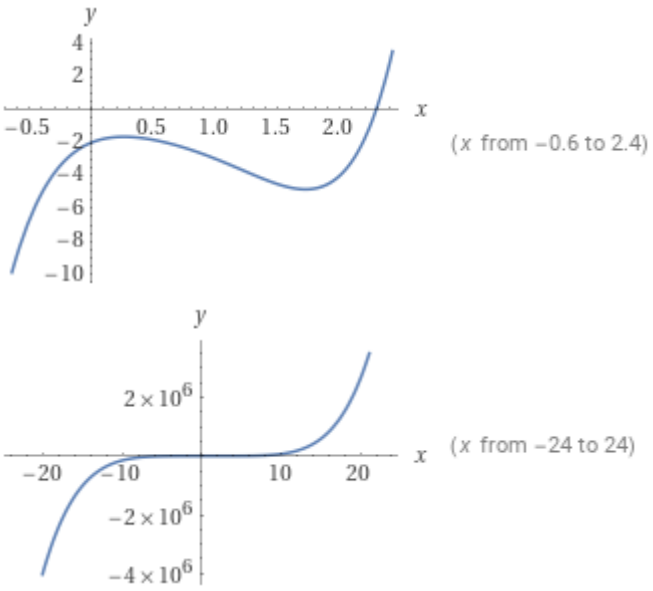
$x^5 - 4x^4 + 7x^3 - 8x^2 + 3x - 2$

x^5−4x^4+7x^3−8x^2+3x−2

Input

$x^5 - 4x^4 + 7x^3 - 8x^2 + 3x - 2$

Plots (figures that can be related to the open strings)



Alternate form

$x\,(x\,(x\,((x-4)\,x+7)-8)+3)-2$

Real root

$x =$ root of $x^5 - 4x^4 + 7x^3 - 8x^2 + 3x - 2$ near $x = 2.29209$

Complex roots

$x \approx 0.07713 - 0.58620\,i$

$x \approx 0.07713 + 0.58620\,i$

$x \approx 0.7768 - 1.3757\,i$

$x \approx 0.7768 + 1.3757\,i$

Polynomial discriminant

$\Delta = 117312$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Surjectivity

surjective onto $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Derivative

$$\frac{d}{dx}\left(x^5 - 4x^4 + 7x^3 - 8x^2 + 3x - 2\right) = 5x^4 - 16x^3 + 21x^2 - 16x + 3$$

Indefinite integral

$$\int (x^5 - 4x^4 + 7x^3 - 8x^2 + 3x - 2)\,dx =$$
$$\frac{x^6}{6} - \frac{4x^5}{5} + \frac{7x^4}{4} - \frac{8x^3}{3} + \frac{3x^2}{2} - 2x + \text{constant}$$

Local maximum

$$\max\{x^5 - 4x^4 + 7x^3 - 8x^2 + 3x - 2\} \approx -1.6549 \text{ at } x \approx 0.26015$$

Local minimum

$$\min\{x^5 - 4x^4 + 7x^3 - 8x^2 + 3x - 2\} \approx -4.8430 \text{ at } x \approx 1.7230$$

From the derivative result

$$\frac{d}{dx}(x^5 - 4x^4 + 7x^3 - 8x^2 + 3x - 2) = 5x^4 - 16x^3 + 21x^2 - 16x + 3$$

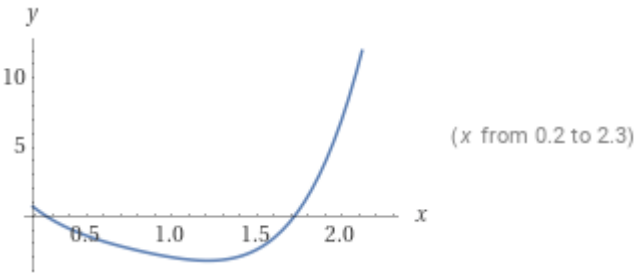
we obtain:

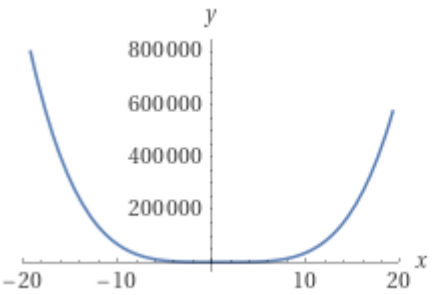
5 x^4 - 16 x^3 + 21 x^2 - 16 x + 3

Input

$$5x^4 - 16x^3 + 21x^2 - 16x + 3$$

Plots (figures that can be related to the open strings)





(x from −19.2 to 19.2)

Alternate form

$x\,(x\,(x\,(5\,x - 16) + 21) - 16) + 3$

Real roots

$$x = \frac{4}{5} + \frac{1}{10}\sqrt{-6 + \frac{5\sqrt[3]{459 + 2\sqrt{140907}}}{3^{2/3}} - \frac{245}{\sqrt[3]{3\left(459 + 2\sqrt{140907}\right)}}} -$$

$$\frac{1}{2}\sqrt{\left(-\frac{12}{25} - \frac{\sqrt[3]{459 + 2\sqrt{140907}}}{5 \times 3^{2/3}} + \frac{49}{5\sqrt[3]{3\left(459 + 2\sqrt{140907}\right)}} + \right.$$

$$\left.\frac{144}{25\sqrt{-6 + \frac{5\sqrt[3]{459+2\sqrt{140907}}}{3^{2/3}} - \frac{245}{\sqrt[3]{3\left(459+2\sqrt{140907}\right)}}}}\right)}$$

$$x = \frac{4}{5} + \frac{1}{10}\sqrt{-6 + \frac{5\sqrt[3]{459 + 2\sqrt{140907}}}{3^{2/3}} - \frac{245}{\sqrt[3]{3\left(459 + 2\sqrt{140907}\right)}}} + \frac{1}{2}\sqrt{\left(-\frac{12}{25} - \frac{\sqrt[3]{459 + 2\sqrt{140907}}}{5 \times 3^{2/3}} + \frac{49}{5\sqrt[3]{3\left(459 + 2\sqrt{140907}\right)}} + \frac{144}{25\sqrt{-6 + \frac{5\sqrt[3]{459+2\sqrt{140907}}}{3^{2/3}} - \frac{245}{\sqrt[3]{3\left(459+2\sqrt{140907}\right)}}}}\right)}$$

Complex roots

$$x = \frac{4}{5} - \frac{1}{10}\sqrt{-6 + \frac{5\sqrt[3]{459 + 2\sqrt{140907}}}{3^{2/3}} - \frac{245}{\sqrt[3]{3\left(459 + 2\sqrt{140907}\right)}}} - \frac{1}{2}i\sqrt{\left(\frac{12}{25} + \frac{\sqrt[3]{459 + 2\sqrt{140907}}}{5 \times 3^{2/3}} - \frac{49}{5\sqrt[3]{3\left(459 + 2\sqrt{140907}\right)}} + \frac{144}{25\sqrt{-6 + \frac{5\sqrt[3]{459+2\sqrt{140907}}}{3^{2/3}} - \frac{245}{\sqrt[3]{3\left(459+2\sqrt{140907}\right)}}}}\right)}$$

$$x = \frac{4}{5} - \frac{1}{10}\sqrt{-6 + \frac{5\sqrt[3]{459 + 2\sqrt{140907}}}{3^{2/3}} - \frac{245}{\sqrt[3]{3\left(459 + 2\sqrt{140907}\right)}}} + \frac{1}{2}i\sqrt{\left(\frac{12}{25} + \frac{\sqrt[3]{459 + 2\sqrt{140907}}}{5 \times 3^{2/3}} - \frac{49}{5\sqrt[3]{3\left(459 + 2\sqrt{140907}\right)}} + \frac{144}{25\sqrt{-6 + \frac{5\sqrt[3]{459+2\sqrt{140907}}}{3^{2/3}} - \frac{245}{\sqrt[3]{3\left(459+2\sqrt{140907}\right)}}}}\right)}$$

Polynomial discriminant

$\Delta = -751504$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\{y \in \mathbb{R} : y \geq -3.24096\}$

$\mathbb{R}$ is the set of real numbers

Derivative

$$\frac{d}{dx}\left(5x^4 - 16x^3 + 21x^2 - 16x + 3\right) = 20x^3 - 48x^2 + 42x - 16$$

Indefinite integral

$$\int \left(5x^4 - 16x^3 + 21x^2 - 16x + 3\right) dx = x^5 - 4x^4 + 7x^3 - 8x^2 + 3x + \text{constant}$$

Global minimum

$$\min\left\{5x^4 - 16x^3 + 21x^2 - 16x + 3\right\} = \frac{1}{500}\left(-1306 - \frac{5157}{\sqrt[3]{2000\sqrt{6} - 4127}} + 27\sqrt[3]{2000\sqrt{6} - 4127}\right)$$

$$\text{at } x = \frac{4}{5} - \frac{3^{2/3}}{5\sqrt[3]{2\left(12 + 5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12 + 5\sqrt{6}\right)}}{5 \times 2^{2/3}}$$

Definite integral

$$\int_{\text{root of } 3-16x+21x^2-16x^3+5x^4 \text{ near } x=0.260155}^{\text{root of } 3-16x+21x^2-16x^3+5x^4 \text{ near } x=1.72299} \left(3 - 16x + 21x^2 - 16x^3 + 5x^4\right) dx =$$

$$-\sqrt{\begin{array}{l}\text{root of } 931\,322\,574\,615\,478\,515\,625\,x^6 - 10\,269\,451\,904\,296\,875\,000\,000\,x^5 - \\ \quad 8\,945\,421\,673\,411\,250\,000\,000\,x^4 + 3\,935\,732\,509\,905\,949\,206\,994\,944\,x^3 - \\ \quad 14\,374\,766\,773\,633\,014\,107\,926\,528\,x^2 - \\ \quad 190\,932\,948\,083\,881\,012\,976\,418\,816\,x - \\ \quad 524\,437\,778\,113\,618\,606\,377\,926\,656 \text{ near } x = 10.1642\end{array}} \approx -3.18813$$

Again, from the derivative result

$$\frac{d}{dx}\left(5x^4 - 16x^3 + 21x^2 - 16x + 3\right) = 20x^3 - 48x^2 + 42x - 16$$

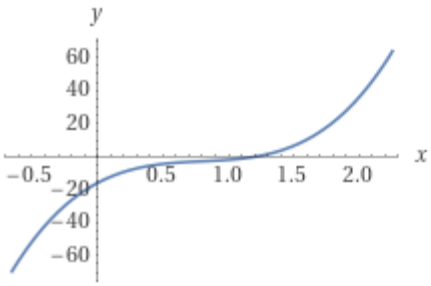
we obtain:

20 x^3 - 48 x^2 + 42 x – 16

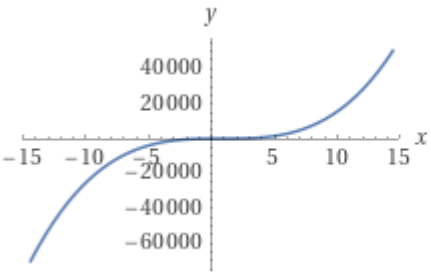
Input

$$20x^3 - 48x^2 + 42x - 16$$

Plots (figures that can be related to the open strings)



(x from −0.7 to 2.3)



(x from −14.4 to 14.4)

Alternate forms

$$2\left(10\,x^3 - 24\,x^2 + 21\,x - 8\right)$$

$$x\,(x\,(20\,x - 48) + 42) - 16$$

$$2\,x\,(2\,x\,(5\,x - 12) + 21) - 16$$

Real root

$$x = \frac{1}{5}\left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12 + 5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12 + 5\sqrt{6}\right)}}{2^{2/3}}\right)$$

Complex roots

$$x = \frac{4}{5} + \frac{3^{2/3}}{10\sqrt[3]{2\left(12+5\sqrt{6}\right)}} - \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{10\times 2^{2/3}} + i\left(-\frac{3^{5/6}\sqrt[3]{12+5\sqrt{6}}}{10\times 2^{2/3}} - \frac{3\sqrt[6]{3}}{10\sqrt[3]{2\left(12+5\sqrt{6}\right)}}\right)$$

$$x = \frac{4}{5} + \frac{3^{2/3}}{10\sqrt[3]{2\left(12+5\sqrt{6}\right)}} - \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{10\times 2^{2/3}} + i\left(\frac{3^{5/6}\sqrt[3]{12+5\sqrt{6}}}{10\times 2^{2/3}} + \frac{3\sqrt[6]{3}}{10\sqrt[3]{2\left(12+5\sqrt{6}\right)}}\right)$$

Polynomial discriminant

$\Delta = -93\,312$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Bijectivity

bijective from its domain to $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Derivative

$$\frac{d}{dx}\left(20x^3 - 48x^2 + 42x - 16\right) = 60x^2 - 96x + 42$$

Indefinite integral

$$\int \left(20x^3 - 48x^2 + 42x - 16\right) dx = 5x^4 - 16x^3 + 21x^2 - 16x + \text{constant}$$

Now, from the real root

$$x = \frac{1}{5}\left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12 + 5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12 + 5\sqrt{6}\right)}}{2^{2/3}}\right)$$

we obtain:

1+1/(Pi((1/5 (4 - 3^(2/3)/(2 (12 + 5 sqrt(6)))^(1/3) + (3 (12 + 5 sqrt(6)))^(1/3)/2^(2/3)))))^1/3

Input

$$1 + \frac{1}{\sqrt[3]{\pi\left(\frac{1}{5}\left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\right)}}$$

Exact result

$$1+\sqrt[3]{\frac{5}{\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\pi}}$$

Decimal approximation

1.640420987147318380659128532732348601656401792367466435736463983

…

1.64042098714…≈ $\zeta(2) = \pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$$1+\sqrt[3]{\frac{5}{\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\pi}}$$ is a transcendental number

Alternate forms

$$1+\sqrt[3]{\frac{10}{\left(8-\frac{6^{2/3}}{\sqrt[3]{12+5\sqrt{6}}}+\sqrt[3]{72+30\sqrt{6}}\right)\pi}}$$

$$1+\left(\frac{10\left(12+5\sqrt{6}\right)}{\left(96+40\sqrt{6}+5\times 6^{5/6}\sqrt[3]{12+5\sqrt{6}}+12\sqrt[3]{6\left(12+5\sqrt{6}\right)}-\left(6\left(12+5\sqrt{6}\right)\right)^{2/3}\right)\pi}\right)\text{^}(1/3)$$

$$2^{2/9}\left(\left(5\ \text{root of}\ x^6-24x^3-6\ \text{near}\ x=2.89438\right)\Big/\left(\pi\left(\sqrt[3]{3}\left(\text{root of}\ x^6-588x^3+36\ \text{near}\ x=8.37743\right)-\sqrt[3]{6}\right)+4\times 2^{2/3}\right.\right.$$
$$\left.\left.\text{root of}\ x^6-24x^3-6\ \text{near}\ x=2.89438\right)\right)\text{^}(1/3)+1$$

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the real root

$$x = \frac{1}{5}\left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)$$

we obtain:

((1/5 (4 - 3^(2/3)/(2 (12 + 5 sqrt(6)))^(1/3) + (3 (12 + 5 sqrt(6)))^(1/3)/2^(2/3)))
((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

Input

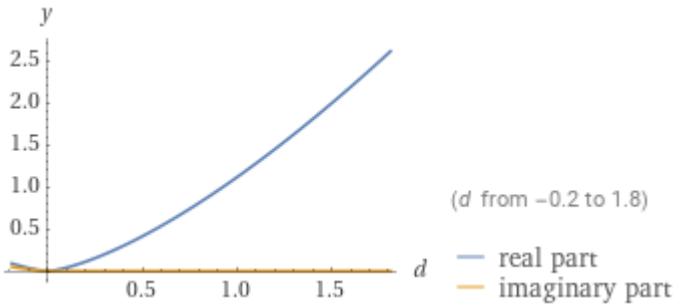
$$\left(\frac{1}{5}\left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\right)^{\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}}$$

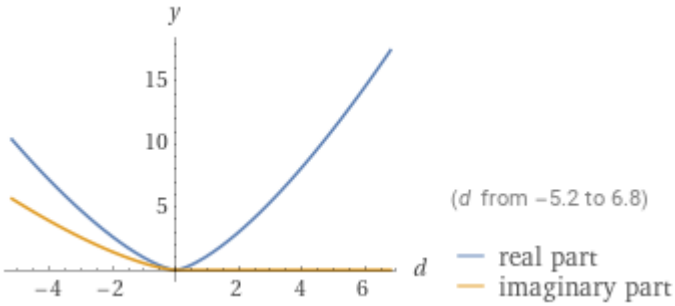
Exact result

$$3^{-3/(2\pi)} \times 4^{-1/\pi} \times 5^{1/(2\pi)-1}\ \sqrt[2\pi]{3+\sqrt{5}}\left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)$$

$$\sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Expanded form

$$2^{-2/3-2/\pi} \times 3^{1/3-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[3]{12+5\sqrt{6}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9} - \frac{2^{-1/3-2/\pi} \times 3^{2/3-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}}{\sqrt[3]{12+5\sqrt{6}}} + 2^{2-2/\pi} \times 3^{-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Alternate forms

$$-\frac{1}{\sqrt[3]{12+5\sqrt{6}}} 2^{-1-2/\pi} \times 3^{-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \left(6^{2/3} - 8\sqrt[3]{12+5\sqrt{6}} - \sqrt[3]{6}\left(12+5\sqrt{6}\right)^{2/3}\right) \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$2^{-2/3-2/\pi} \times 3^{-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}$$
$$\left(\sqrt[3]{12+5\sqrt{6}}\left(4 \times 2^{2/3}+\sqrt[3]{3\left(12+5\sqrt{6}\right)}\right)-\sqrt[3]{2}\ 3^{2/3}\right)$$
$$\sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9} \quad \text{root of } 6x^6+24x^3-1 \text{ near } x=0.345497$$

Series expansion at d=∞

$$2^{-(2+\pi)/\pi} \times 3^{-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}\left(8-\frac{6^{2/3}}{\sqrt[3]{12+5\sqrt{6}}}+\sqrt[3]{72+30\sqrt{6}}\right)\sqrt[\pi]{\pi}$$
$$d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{5}\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$
$$2^{-(2(1+\pi))/\pi} \times 3^{2-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}\left(8-\frac{6^{2/3}}{\sqrt[3]{12+5\sqrt{6}}}+\sqrt[3]{72+30\sqrt{6}}\right)$$
$$d^8\left(d^9\right)^{1/(2\pi)-1}\pi^{1/\pi-1}$$

Indefinite integral

$$\int 3^{-3/(2\pi)} \times 4^{-1/\pi} \times 5^{-1+1/(2\pi)} \sqrt[2\pi]{3+\sqrt{5}} \left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right) dd =$$

$$\sqrt[2\pi]{d^9}\ \sqrt[\pi]{\pi}$$

$$\frac{1}{9+2\pi} 3^{-3/(2\pi)} \times 4^{-1/\pi} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \left(8 - \frac{6^{2/3}}{\sqrt[3]{12+5\sqrt{6}}} + \sqrt[3]{72+30\sqrt{6}}\right)$$

$$\pi^{1+1/\pi}\, d\, \sqrt[2\pi]{d^9} + \text{constant}$$

And:

((1/5 (4 - 3^(2/3)/(2 (12 + 5 sqrt(6)))^(1/3) + (3 (12 + 5 sqrt(6)))^(1/3)/2^(2/3))) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

Input

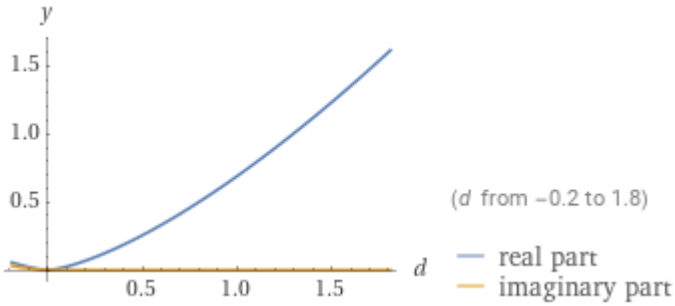
$$\left(\frac{1}{5}\left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\right)$$

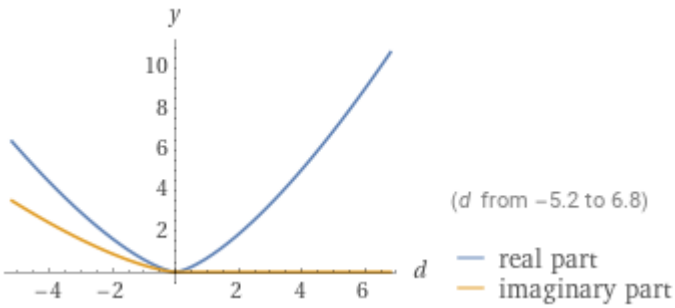
$$\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\, d^3\right)\left(\frac{\sqrt{2}}{12}\, d^3\right)}$$

Exact result

$$\frac{1}{5} \times 2^{-1/\pi} \times 3^{-3/(2\pi)} \left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate forms

$$2^{-(1+\pi)/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

root of $5x^3 - 24x^2 + 42x - 32$ near $x = 2.42373$

$$-\frac{2^{-1-1/\pi} \times 3^{-3/(2\pi)} \left(6^{2/3} - 8\sqrt[3]{12+5\sqrt{6}} - \sqrt[3]{6}\left(12+5\sqrt{6}\right)^{2/3}\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}{5\sqrt[3]{12+5\sqrt{6}}}$$

Expanded form

$$\frac{1}{5}\times 2^{-2/3-1/\pi}\times 3^{1/3-3/(2\pi)}\sqrt[3]{12+5\sqrt{6}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}\ -$$
$$\frac{2^{-1/3-1/\pi}\times 3^{2/3-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}{5\sqrt[3]{12+5\sqrt{6}}}+\frac{1}{5}\times 2^{2-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Alternate form assuming d>0

$$\frac{1}{5\sqrt[3]{12+5\sqrt{6}}}2^{-2/3-1/\pi}\times 3^{-3/(2\pi)}\left(-\sqrt[3]{2}\ 3^{2/3}+4\times 2^{2/3}\sqrt[3]{12+5\sqrt{6}}\ +\sqrt[3]{3}\left(12+5\sqrt{6}\right)^{2/3}\right)\sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Root

$d=0$

Series expansion at d=∞

$$\frac{1}{5}\times 2^{-(1+\pi)/\pi}\times 3^{-3/(2\pi)}\left(8-\frac{6^{2/3}}{\sqrt[3]{12+5\sqrt{6}}}+\sqrt[3]{72+30\sqrt{6}}\right)\sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{5}\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right)=$$

$$\frac{1}{5}\times2^{-2-1/\pi}\times3^{2-3/(2\pi)}\left(8-\frac{6^{2/3}}{\sqrt[3]{12+5\sqrt{6}}}+\sqrt[3]{72+30\sqrt{6}}\right)d^8\left(d^9\right)^{1/(2\pi)-1}\pi^{1/(2\pi)-1}$$

Indefinite integral

$$\int\frac{1}{5}\times2^{-1/\pi}\times3^{-3/(2\pi)}\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{d^9}\,\sqrt[2\pi]{\pi}\;dd=$$

$$\frac{2^{-1/\pi}\times3^{-3/(2\pi)}\left(8-\frac{6^{2/3}}{\sqrt[3]{12+5\sqrt{6}}}+\sqrt[3]{72+30\sqrt{6}}\right)\pi^{1+1/(2\pi)}\,d\,\sqrt[2\pi]{d^9}}{5\,(9+2\pi)}+\text{constant}$$

Global minimum

$$\min\left\{\frac{1}{5}\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right\}=0\ \text{at}\ d=0$$

Dividing the two exact results

$$3^{-3/(2\pi)} \times 4^{-1/\pi} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)$$

$$\sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

and

$$\frac{1}{5} \times 2^{-1/\pi} \times 3^{-3/(2\pi)} \left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

we obtain:

(3^(-3/(2π))4^(-1/π)5^(1/(2π)-1)(3+√(5))^(1/(2π))(4-3^(2/3)/(2(12+5√6))^1/3+(3(12+5√6))^(1/3)/2^(2/3))π^(1/π))/(1/5 2^(-1/π)3^(-3/(2π))(4-3^(2/3)/(2(12+5√6))^1/3+(3(12+5√6))^(1/3)/2^(2/3))π^(1/(2π)))

Input

$$\frac{3^{-3/(2\pi)} \times 4^{-1/\pi} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right) \sqrt[\pi]{\pi}}{\frac{1}{5} \times 2^{-1/\pi} \times 3^{-3/(2\pi)} \left(4 - \frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}} + \frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right) \sqrt[2\pi]{\pi}}$$

Exact result

$$2^{-1/\pi} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{3^{-3/(2\pi)}\left(4^{-1/\pi}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\sqrt[\pi]{\pi}\right)}{\frac{1}{5}\times 2^{-1/\pi}\times 3^{-3/(2\pi)}\left(\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\pi}\right)}=$$

$$2^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-3/(2\pi)}\left(4^{-1/\pi}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\sqrt[\pi]{\pi}\right)}{\frac{1}{5}\times 2^{-1/\pi}\times 3^{-3/(2\pi)}\left(\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\pi}\right)}=$$

$$2^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}$$

$$\frac{3^{-3/(2\pi)}\left(4^{-1/\pi}\times 5^{1/(2\pi)-1}\,\sqrt[2\pi]{3+\sqrt{5}}\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\sqrt[\pi]{\pi}\right)}{\frac{1}{5}\times 2^{-1/\pi}\times 3^{-3/(2\pi)}\left(\left(4-\frac{3^{2/3}}{\sqrt[3]{2\left(12+5\sqrt{6}\right)}}+\frac{\sqrt[3]{3\left(12+5\sqrt{6}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\pi}\right)}=$$

$$2^{-1/\pi}\,\sqrt[2\pi]{5}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the exact result

$$2^{-1/\pi}\,\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13…

3.1415926535884…≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298884450469 20

…

1.64493406684…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.618008545900 00^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900 00^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900 00^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{32\,e^{1.92478440155986\,\pi}\,\sqrt{\pi}^2}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Now, we have:

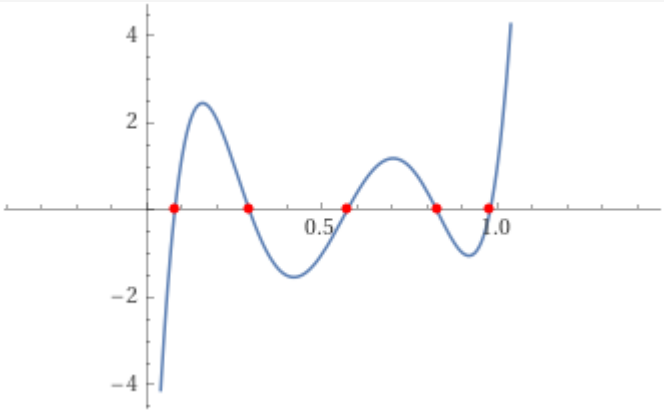
$1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11 = 0$

1024x^5-2816x^4+2816x^3-1232x^2+220x-11 = 0

Input

$1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11 = 0$

Root plot



Alternate forms

$4\,x\,(4\,x\,(16\,x\,(x\,(4\,x - 11) + 11) - 77) + 55) - 11 = 0$

$256\,x^5 - 704\,x^4 + 704\,x^3 - 308\,x^2 + 55\,x = \frac{11}{4}$

$4\,x\left(256\,x^4 + 704\,x^2 + 55\right) = 11\left(256\,x^4 + 112\,x^2 + 1\right)$

Solutions

$x \approx 0.0793732$

$x \approx 0.292292$

$x \approx 0.571157$

$x \approx 0.82743$

$x \approx 0.979746$

Sum of roots

$\frac{11}{4}$

Product of roots

$\frac{11}{1024}$

We have that:

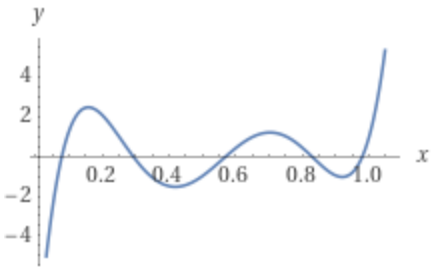
$1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11$

1024x^5-2816x^4+2816x^3-1232x^2+220x-11

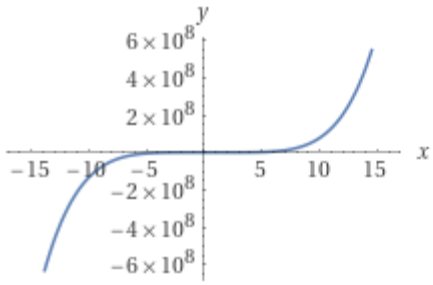
Input

$1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11$

Plots (figures that can be related to the open strings)



(x from 0.01 to 1.1)



(x from −16.5 to 16.5)

Alternate forms

$x\,(x\,(x\,(x\,(1024\,x - 2816) + 2816) - 1232) + 220) - 11$

$4\,x\,(4\,x\,(16\,x\,(x\,(4\,x - 11) + 11) - 77) + 55) - 11$

Roots

$x \approx 0.0793732$

$x \approx 0.292292$

$x \approx 0.571157$

$x \approx 0.82743$

$x \approx 0.979746$

Polynomial discriminant

$\Delta = 16097949742268416$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Surjectivity

surjective onto $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Derivative

$$\frac{d}{dx}\left(1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11\right) =$$
$$4\left(1280\,x^4 - 2816\,x^3 + 2112\,x^2 - 616\,x + 55\right)$$

Indefinite integral

$$\int \left(1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11\right) dx =$$
$$\frac{512\,x^6}{3} - \frac{2816\,x^5}{5} + 704\,x^4 - \frac{1232\,x^3}{3} + 110\,x^2 - 11\,x + \text{constant}$$

Local maxima

$$\max\left\{1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11\right\} \approx 2.4575 \text{ at } x \approx 0.15862$$

$$\max\left\{1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11\right\} \approx 1.1908 \text{ at } x \approx 0.70281$$

Local minima

$$\min\left\{1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11\right\} \approx -1.5356 \text{ at } x \approx 0.41927$$

$$\min\left\{1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11\right\} \approx -1.0426 \text{ at } x \approx 0.91930$$

Definite integral area below the axis between the smallest and largest real roots

$$\int_{\text{root of } -11+220\,x-1232\,x^2+2816\,x^3-2816\,x^4+1024\,x^5 \text{ near } x=0.0793732}^{\text{root of } -11+220\,x-1232\,x^2+2816\,x^3-2816\,x^4+1024\,x^5 \text{ near } x=0.979746} \left(-11 + 220\,x - 1232\,x^2 + 2816\,x^3 - 2816\,x^4 + 1024\,x^5\right)\theta\left(11 - 220\,x + 1232\,x^2 - dx = 2816\,x^3 + 2816\,x^4 - 1024\,x^5\right)$$

$$\text{root of } 24\,883\,200\,000\,x^5 - 4\,599\,936\,000\,x^3 - 210\,830\,400\,x^2 + 77\,304\,480\,x + 1\,771\,561 \text{ near } x = -0.376538 \approx -0.376538$$

$\theta(x)$ is the Heaviside step function

Definite integral area above the axis between the smallest and largest real roots

$$\int_{\text{root of } -11+220\,x-1232\,x^2+2816\,x^3-2816\,x^4+1024\,x^5 \text{ near } x=0.0793732}^{\text{root of } -11+220\,x-1232\,x^2+2816\,x^3-2816\,x^4+1024\,x^5 \text{ near } x=0.979746} \left(-11 + 220\,x - 1232\,x^2 + 2816\,x^3 - 2816\,x^4 + 1024\,x^5\right)\theta\left(-11 + 220\,x - 1232\,x^2 + dx = 2816\,x^3 - 2816\,x^4 + 1024\,x^5\right)$$

$$\text{root of } 24\,883\,200\,000\,x^5 - 4\,599\,936\,000\,x^3 - 1\,054\,152\,000\,x^2 - 77\,304\,480\,x - 1\,771\,561 \text{ near } x = 0.526354 \approx 0.526354$$

From the derivative result

$$\frac{d}{dx}\left(1024\,x^5 - 2816\,x^4 + 2816\,x^3 - 1232\,x^2 + 220\,x - 11\right) = 4\left(1280\,x^4 - 2816\,x^3 + 2112\,x^2 - 616\,x + 55\right)$$

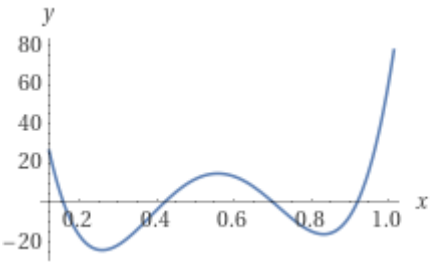
we obtain:

4 (55 - 616 x + 2112 x^2 - 2816 x^3 + 1280 x^4)

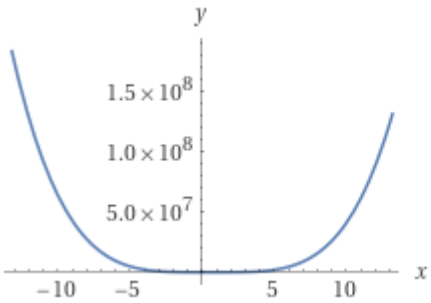
Input

$4\left(55 - 616\,x + 2112\,x^2 - 2816\,x^3 + 1280\,x^4\right)$

Plots **(figures that can be related to the open strings)**



(x from 0.1 to 1)



(x from −13.2 to 13.2)

Alternate forms

$5120\,x^4 - 11\,264\,x^3 + 8448\,x^2 - 2464\,x + 220$

$x\,(x\,(x\,(5120\,x - 11\,264) + 8448) - 2464) + 220$

$4\,(8\,x\,(8\,x\,(4\,x\,(5\,x-11)+33)-77)+55)$

Roots

$$x=\frac{11}{20}-\frac{1}{20}\sqrt{\frac{1}{2}\left(22+\frac{5\times 22^{2/3}}{\sqrt[3]{11+i\sqrt{55}}}+\frac{5\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{2^{2/3}}\right)}-$$

$$\frac{1}{2}\sqrt{\left(\frac{11}{50}-\frac{11^{2/3}}{20\sqrt[3]{2\left(11+i\sqrt{55}\right)}}-\frac{\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{40\times 2^{2/3}}+\right.}$$

$$\left.\frac{11}{100\sqrt{2\left(22+\frac{5\times 22^{2/3}}{\sqrt[3]{11+i\sqrt{55}}}+\frac{5\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{2^{2/3}}\right)}}\right)$$

$$x = \frac{11}{20} - \frac{1}{20}\sqrt{\frac{1}{2}\left(22 + \frac{5\times 22^{2/3}}{\sqrt[3]{11+i\sqrt{55}}} + \frac{5\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{2^{2/3}}\right)} +$$

$$\frac{1}{2}\sqrt{\left(\frac{11}{50} - \frac{11^{2/3}}{20\sqrt[3]{2\left(11+i\sqrt{55}\right)}} - \frac{\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{40\times 2^{2/3}} + \right.}$$

$$\left. \frac{11}{100\sqrt{2\left(22 + \frac{5\times 22^{2/3}}{\sqrt[3]{11+i\sqrt{55}}} + \frac{5\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{2^{2/3}}\right)}}\right)$$

$$x = \frac{11}{20} + \frac{1}{20}\sqrt{\frac{1}{2}\left(22 + \frac{5\times 22^{2/3}}{\sqrt[3]{11+i\sqrt{55}}} + \frac{5\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{2^{2/3}}\right)} -$$

$$\frac{1}{2}\sqrt{\left(\frac{11}{50} - \frac{11^{2/3}}{20\sqrt[3]{2\left(11+i\sqrt{55}\right)}} - \frac{\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{40\times 2^{2/3}} - \right.}$$

$$\left.\frac{11}{100\sqrt{2\left(22 + \frac{5\times 22^{2/3}}{\sqrt[3]{11+i\sqrt{55}}} + \frac{5\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{2^{2/3}}\right)}}\right)$$

$$x = \frac{11}{20} + \frac{1}{20}\sqrt{\frac{1}{2}\left(22 + \frac{5\times 22^{2/3}}{\sqrt[3]{11+i\sqrt{55}}} + \frac{5\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{2^{2/3}}\right)} +$$

$$\frac{1}{2}\sqrt{\left(\frac{11}{50} - \frac{11^{2/3}}{20\sqrt[3]{2\left(11+i\sqrt{55}\right)}} - \frac{\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{40\times 2^{2/3}} - \right.}$$

$$\left. \frac{11}{100\sqrt{2\left(22 + \frac{5\times 22^{2/3}}{\sqrt[3]{11+i\sqrt{55}}} + \frac{5\sqrt[3]{11\left(11+i\sqrt{55}\right)}}{2^{2/3}}\right)}}\right)$$

Polynomial discriminant

$\Delta = 197\,565\,746\,836\,930\,560$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\{y \in \mathbb{R} : y \geq -24.1371\}$

$\mathbb{R}$ is the set of real numbers

Derivative

$$\frac{d}{dx}\left(4\left(55-616\,x+2112\,x^2-2816\,x^3+1280\,x^4\right)\right)=$$
$$32\left(640\,x^3-1056\,x^2+528\,x-77\right)$$

Indefinite integral

$$\int 4\left(55-616\,x+2112\,x^2-2816\,x^3+1280\,x^4\right)dx=$$
$$4\left(256\,x^5-704\,x^4+704\,x^3-308\,x^2+55\,x\right)+\text{constant}$$

Local maximum

$$\max\left\{4\left(55-616\,x+2112\,x^2-2816\,x^3+1280\,x^4\right)\right\}\approx 14.843 \text{ at } x\approx 0.55834$$

Global minimum

$$\min\left\{4\left(55-616\,x+2112\,x^2-2816\,x^3+1280\,x^4\right)\right\}\approx -24.137 \text{ at } x\approx 0.25869$$

Again, from the derivative result

$$\frac{d}{dx}\left(4\left(55-616\,x+2112\,x^2-2816\,x^3+1280\,x^4\right)\right)=$$
$$32\left(640\,x^3-1056\,x^2+528\,x-77\right)$$

$$32\left(-77+528\,x-1056\,x^2+640\,x^3\right)$$

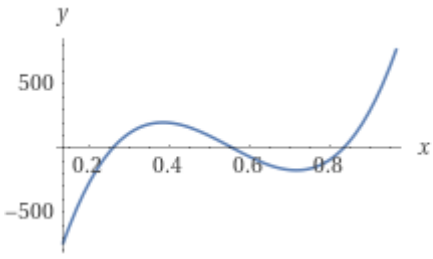
we obtain:

32 (-77 + 528 x - 1056 x^2 + 640 x^3)

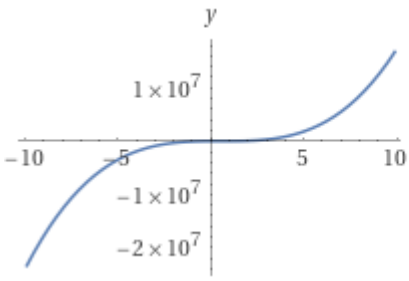
Input

$$32\left(-77+528\,x-1056\,x^2+640\,x^3\right)$$

Plots (figures that can be related to the open strings)



(x from 0.1 to 1)



(x from −9.9 to 9.9)

Alternate forms

$$20480\,x^3 - 33792\,x^2 + 16896\,x - 2464$$

$$x\,(x\,(20480\,x - 33792) + 16896) - 2464$$

$$32\left(16\,x\left(40\,x^2 - 66\,x + 33\right) - 77\right)$$

Roots

$$x = \frac{11}{20} - \frac{11^{2/3}\left(1 + i\sqrt{3}\right)}{20\sqrt[3]{2\left(-1 + 5\,i\sqrt{7}\right)}} - \frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{11\left(-1 + 5\,i\sqrt{7}\right)}}{40 \times 2^{2/3}}$$

$$x = \frac{11}{20} - \frac{11^{2/3}\left(1 - i\sqrt{3}\right)}{20\sqrt[3]{2\left(-1 + 5i\sqrt{7}\right)}} - \frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{11\left(-1 + 5i\sqrt{7}\right)}}{40 \times 2^{2/3}}$$

$$x = \frac{1}{20}\left(11 + \frac{22^{2/3}}{\sqrt[3]{-1 + 5i\sqrt{7}}} + \frac{\sqrt[3]{11\left(-1 + 5i\sqrt{7}\right)}}{2^{2/3}}\right)$$

Roots

$x \approx 0.25869$

$x \approx 0.55834$

$x \approx 0.83297$

Polynomial discriminant

$\Delta = 392\,886\,428\,368\,896$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Surjectivity

surjective onto $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Derivative

$$\frac{d}{dx}\left(32\left(-77+528\,x-1056\,x^2+640\,x^3\right)\right)=1536\left(40\,x^2-44\,x+11\right)$$

Indefinite integral

$$\int 32\left(-77+528\,x-1056\,x^2+640\,x^3\right)dx=$$
$$32\left(160\,x^4-352\,x^3+264\,x^2-77\,x\right)+\text{constant}$$

Local maximum

$$\max\left\{32\left(-77+528\,x-1056\,x^2+640\,x^3\right)\right\}=\frac{352}{25}\left(1+4\sqrt{11}\right)\text{ at }x=\frac{11}{20}-\frac{\sqrt{11}}{20}$$

Local minimum

$$\min\left\{32\left(-77+528\,x-1056\,x^2+640\,x^3\right)\right\}=-\frac{352}{25}\left(4\sqrt{11}-1\right)\text{ at }x=\frac{11}{20}+\frac{\sqrt{11}}{20}$$

Definite integrals

$$\int_{\text{root of } -77+528x-1056x^2+640x^3 \text{ near } x=0.258693}^{\text{root of } -77+528x-1056x^2+640x^3 \text{ near } x=0.55834} 32\left(-77+528x-1056x^2+640x^3\right) dx =$$

$$\frac{33}{125}\sqrt{\left(6\left(2024+\frac{93\,104}{\sqrt[3]{\frac{1}{11}\left(-3\,206\,193+3\,002\,125\,i\sqrt{7}\right)}}+11^{2/3}\right)\sqrt[3]{-3\,206\,193+3\,002\,125\,i\sqrt{7}}\right)} \approx 38.9798$$

$$\int_{\text{root of } -77+528x-1056x^2+640x^3 \text{ near } x=0.55834}^{\text{root of } -77+528x-1056x^2+640x^3 \text{ near } x=0.832967} 32\left(-77+528x-1056x^2+640x^3\right) dx =$$

$$\frac{33}{125}(-1)^{3/4}\sqrt{\left(3\left(4048\,i-\frac{93\,104\,i}{\sqrt[3]{\frac{1}{11}\left(-3\,206\,193+3\,002\,125\,i\sqrt{7}\right)}}-\frac{93\,104\sqrt{3}}{\sqrt[3]{\frac{1}{11}\left(-3\,206\,193+3\,002\,125\,i\sqrt{7}\right)}}-i\,11^{2/3}\sqrt[3]{-3\,206\,193+3\,002\,125\,i\sqrt{7}}+\sqrt{3}\;11^{2/3}\sqrt[3]{-3\,206\,193+3\,002\,125\,i\sqrt{7}}\right)\right)} \approx -30.8923$$

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the following root

$$x = \frac{1}{20}\left(11 + \frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}} + \frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)$$

we obtain:

((1/20 (11 + 22^(2/3)/(-1 + 5 i sqrt(7))^(1/3) + (11 (-1 + 5 i sqrt(7)))^(1/3)/2^(2/3))))
((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

Input

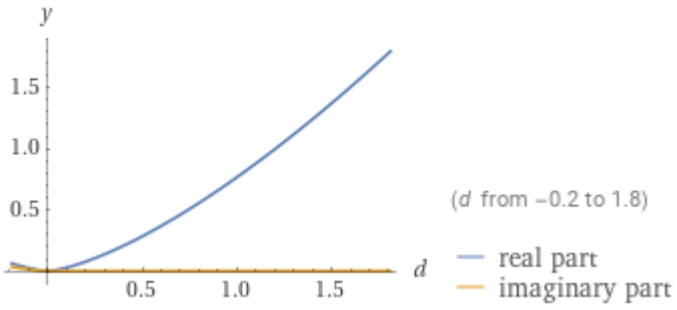
$$\left(\frac{1}{20}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\right)$$
$$\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3\right)^2}$$

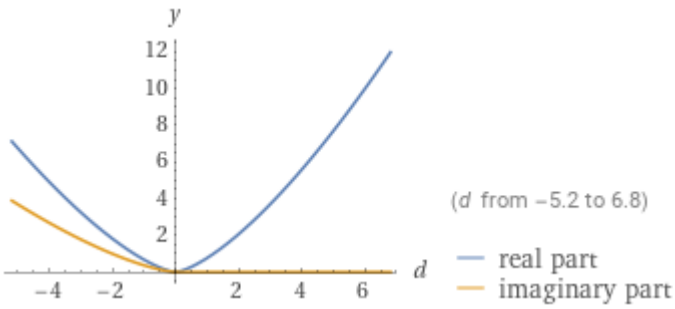
i is the imaginary unit

Exact result

$$3^{-3/(2\pi)}\times 4^{-1-1/\pi}\times 5^{1/(2\pi)-1}\,\sqrt[2\pi]{3+\sqrt{5}}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate form

$$\frac{1}{\sqrt[3]{i\left(5\sqrt{7}+i\right)}}2^{-8/3-2/\pi}\times 3^{-3/(2\pi)}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}$$
$$\left(2\sqrt[3]{2}\ 11^{2/3}+\sqrt[3]{i\left(5\sqrt{7}+i\right)}\left(11\times 2^{2/3}+\sqrt[3]{11\,i\left(5\sqrt{7}+i\right)}\right)\right)\sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Expanded form

$$2^{-8/3-2/\pi}\times 3^{-3/(2\pi)}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[3]{11\left(-1+5\,i\sqrt{7}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}+$$
$$\frac{2^{-4/3-2/\pi}\times 3^{-3/(2\pi)}\times 5^{1/(2\pi)-1}\times 11^{2/3}\sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}}{\sqrt[3]{-1+5\,i\sqrt{7}}}+$$
$$11\times 2^{-2-2/\pi}\times 3^{-3/(2\pi)}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Series expansion at d=∞

$$2^{-3-2/\pi}\times 3^{-3/(2\pi)}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\left(22+\frac{2\times 22^{2/3}}{\sqrt[3]{-1+5\,i\sqrt{7}}}+\right.$$
$$\left.\sqrt[3]{22\left(-1+5\,i\sqrt{7}\right)}\right)\sqrt[\pi]{\pi}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{20}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[2\,\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$

$$2^{-4-2/\pi}\times 3^{2-3/(2\,\pi)}\times 5^{1/(2\,\pi)-1}\,\sqrt[2\,\pi]{3+\sqrt{5}}\left(\frac{2\times 22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+22+\sqrt[3]{22\left(-1+5\,i\,\sqrt{7}\right)}\right)d^8\left(d^9\right)^{1/(2\,\pi)-1}\,\pi^{1/\pi-1}$$

Indefinite integral

$$\int 3^{-3/(2\,\pi)}\times 4^{-1-1/\pi}\times 5^{-1+1/(2\,\pi)}\,\sqrt[2\,\pi]{3+\sqrt{5}}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[2\,\pi]{d^9}\;\sqrt[\pi]{\pi}\,dd=$$

$$\frac{1}{9+2\,\pi}2^{-(2\,(1+\pi))/\pi}\times 3^{-3/(2\,\pi)}\times 5^{1/(2\,\pi)-1}\,\sqrt[2\,\pi]{3+\sqrt{5}}\left(22+\frac{2\times 22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\sqrt[3]{22\left(-1+5\,i\,\sqrt{7}\right)}\right)\pi^{1+1/\pi}\,d\,\sqrt[2\,\pi]{d^9}\;+\text{constant}$$

And:

((1/20 (11 + 22^(2/3)/(-1 + 5 i sqrt(7))^(1/3) + (11 (-1 + 5 i sqrt(7)))^(1/3)/2^(2/3))))
((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

Input

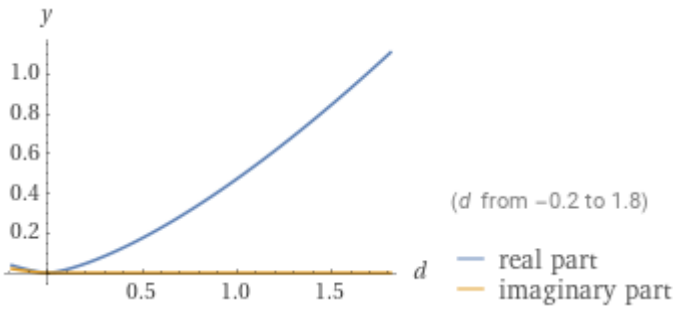
$$\left(\frac{1}{20}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\right)\sqrt[2\pi]{\left(\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\,\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

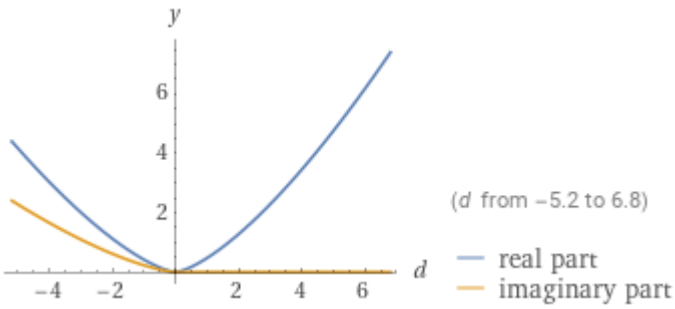
i is the imaginary unit

Exact result

$$\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

Plots **(figures that can be related to the open strings)**





Alternate forms

$$2^{-3-1/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

root of $5x^3 - 66x^2 + 264x - 308$ near $x = 6.66374$

$$\frac{1}{5\sqrt[3]{-1+5i\sqrt{7}}} 2^{-3-1/\pi} \times 3^{-3/(2\pi)} \left(2 \times 22^{2/3} + 22\sqrt[3]{-1+5i\sqrt{7}} + \sqrt[3]{22}\left(-1+5i\sqrt{7}\right)^{2/3}\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$\frac{1}{5\sqrt[3]{i\left(5\sqrt{7}+i\right)}} 2^{-8/3-1/\pi} \times 3^{-3/(2\pi)} \left(2\sqrt[3]{2}\ 11^{2/3} + \sqrt[3]{i\left(5\sqrt{7}+i\right)}\left(11 \times 2^{2/3} + \sqrt[3]{11\,i\left(5\sqrt{7}+i\right)}\right)\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Expanded form

$$\frac{1}{5} \times 2^{-8/3-1/\pi} \times 3^{-3/(2\pi)} \sqrt[3]{11\left(-1+5i\sqrt{7}\right)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9} + \frac{2^{-4/3-1/\pi} \times 3^{-3/(2\pi)} \times 11^{2/3}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}{5\sqrt[3]{-1+5i\sqrt{7}}} + \frac{11}{5} \times 2^{-2-1/\pi} \times 3^{-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Root

$d = 0$ (assuming complex-valued fractional powers)

Series expansion at d=∞

$$\frac{1}{5}\times 2^{-3-1/\pi}\times 3^{-3/(2\pi)}\left(22+\frac{2\times 22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\sqrt[3]{22\left(-1+5\,i\,\sqrt{7}\right)}\right)\sqrt[2\pi]{\pi}\;d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{20}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\frac{\left(4\,\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\;d^3\right)\left(\sqrt{2}\;d^3\right)}{3\times 3\times 12}}\right)=$$

$$\frac{1}{5}\times 2^{-4-1/\pi}\times 3^{2-3/(2\pi)}\left(\frac{2\times 22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+22+\sqrt[3]{22\left(-1+5\,i\,\sqrt{7}\right)}\right)d^8\left(d^9\right)^{1/(2\pi)-1}$$

$$\pi^{1/(2\pi)-1}$$

Indefinite integral

$$\int\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{d^9}\;\sqrt[2\pi]{\pi}$$

$$dd=$$

$$\frac{2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(22+\frac{2\times 22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\sqrt[3]{22\left(-1+5\,i\,\sqrt{7}\right)}\right)\pi^{1+1/(2\pi)}\,d\sqrt[2\pi]{d^9}}{5\,(9+2\,\pi)}+$$

constant

Dividing the two exact results

$$3^{-3/(2\pi)} \times 4^{-1-1/\pi} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \left(11 + \frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}} + \frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right) \sqrt[\pi]{\pi}\, \sqrt[2\pi]{d^9}$$

and

$$\frac{1}{5} \times 2^{-2-1/\pi} \times 3^{-3/(2\pi)} \left(11 + \frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}} + \frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right) \sqrt[2\pi]{\pi}\, \sqrt[2\pi]{d^9}$$

we obtain:

(3^(-3/(2π))4^(-1-1/π)5^(1/(2π)-1)(3+√(5))^(1/(2π))(11+22^(2/3)/(-1+5 i √(7))^(1/3)+(11(-1+5 i √(7)))^(1/3)/2^(2/3))π^(1/π))

Input

$$3^{-3/(2\pi)} \times 4^{-1-1/\pi} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \left(11 + \frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}} + \frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right) \sqrt[\pi]{\pi}$$

i is the imaginary unit

Exact result

$$3^{-3/(2\pi)} \times 4^{-1-1/\pi} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \left(11 + \frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}} + \frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right) \sqrt[\pi]{\pi}$$

Decimal approximation

0.767543654540148859982223535010778752113311934454003281 3427696023

…

0.76754365454….

Alternate complex forms

0.76754

0.767543654540148859982223535010778752113311934454003281

$(\cos(0) + i \sin(0))$

0.767543654540148859982223535010778752113311934454003281 e^0

Alternate forms

$$2^{-3-2/\pi} \times 3^{-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \sqrt[\pi]{\pi}$$

root of $x^3 - 66x^2 + 1320x - 7700$ near $x = 33.3187$

$$3^{-3/(2\pi)} \times 4^{-(1+\pi)/\pi} \times 5^{1/(2\pi)-1}\ \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}\left(11+2\sqrt{11}\ \sin\left(\frac{1}{6}\left(\pi+2\tan^{-1}(5\sqrt{7})\right)\right)\right)$$

$$\frac{1}{\sqrt[3]{i\left(5\sqrt{7}+i\right)}}2^{-8/3-2/\pi} \times 3^{-3/(2\pi)} \times 5^{1/(2\pi)-1}\ \sqrt[2\pi]{3+\sqrt{5}}\ \left(2\sqrt[3]{2}\ 11^{2/3}+\sqrt[3]{i\left(5\sqrt{7}+i\right)}\left(11\times 2^{2/3}+\sqrt[3]{11\,i\left(5\sqrt{7}+i\right)}\right)\right)\sqrt[\pi]{\pi}$$

$\tan^{-1}(x)$ is the inverse tangent function

Expanded form

$$11\times 2^{-2-2/\pi} \times 3^{-3/(2\pi)} \times 5^{1/(2\pi)-1}\ \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}+\frac{2^{-4/3-2/\pi} \times 3^{-3/(2\pi)} \times 5^{1/(2\pi)-1} \times 11^{2/3}\ \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}}{\sqrt[3]{-1+5i\sqrt{7}}}+2^{-8/3-2/\pi} \times 3^{-3/(2\pi)} \times 5^{1/(2\pi)-1}\ \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[3]{11\left(-1+5i\sqrt{7}\right)}\ \sqrt[\pi]{\pi}$$

Series representations

$$3^{-3/(2\pi)} \times 4^{-1-1/\pi} \times 5^{1/(2\pi)-1}\ \sqrt[2\pi]{3+\sqrt{5}}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[\pi]{\pi}=\left(2^{-8/3-2/\pi} \times 3^{-3/(2\pi)} \times 5^{-1+1/(2\pi)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}\left(2\sqrt[3]{2}\ 11^{2/3}+11\times 2^{2/3}\sqrt[3]{-1+5i\sqrt{6}\sum_{k=0}^{\infty}6^{-k}\binom{\frac{1}{2}}{k}}+\sqrt[3]{11}\left(-1+5i\sqrt{6}\sum_{k=0}^{\infty}6^{-k}\binom{\frac{1}{2}}{k}\right)^{2/3}\right)\right)\Big/\left(\sqrt[3]{-1+5i\sqrt{6}\sum_{k=0}^{\infty}6^{-k}\binom{\frac{1}{2}}{k}}\right)$$

$$3^{-3/(2\pi)} \times 4^{-1-1/\pi} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}$$

$$\left(11 + \frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}} + \frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right) \sqrt[\pi]{\pi} =$$

$$\left(2^{-8/3-2/\pi} \times 3^{-3/(2\pi)} \times 5^{-1+1/(2\pi)} \sqrt[\pi]{\pi} \sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right.$$

$$\left(2\sqrt[3]{2}\ 11^{2/3} + 11 \times 2^{2/3} \sqrt[3]{-1+5\,i\,\sqrt{6}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} +\right.$$

$$\left.\left.\sqrt[3]{11}\left(-1+5\,i\,\sqrt{6}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)^{2/3}\right)\right) \Big/$$

$$\left(\sqrt[3]{-1+5\,i\,\sqrt{6}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$3^{-3/(2\pi)} \times 4^{-1-1/\pi} \times 5^{1/(2\pi)-1}\ \sqrt[2\pi]{3+\sqrt{5}}$$
$$\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[\pi]{\pi}\ =$$
$$\left(2^{-8/3-2/\pi} \times 3^{-3/(2\pi)} \times 5^{-1+1/(2\pi)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (5-z_0)^k\ z_0^{-k}}{k!}}\right.$$
$$\left(2\sqrt[3]{2}\ 11^{2/3}+11\times 2^{2/3}\ \sqrt[3]{-1+5\,i\,\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (7-z_0)^k\ z_0^{-k}}{k!}}+\right.$$
$$\left.\left.\sqrt[3]{11}\left(-1+5\,i\,\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (7-z_0)^k\ z_0^{-k}}{k!}\right)^{2/3}\right)\right)\Big/$$
$$\left(\sqrt[3]{-1+5\,i\,\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (7-z_0)^k\ z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

0.76754*1/(1/5 2^(-2-1/π)3^(-3/(2π))(11+22^(2/3)/(-1+5 i √(7))^(1/3)+(11(-1+5 i √(7)))^(1/3)/2^(2/3))π^(1/(2π)))

Input

$$0.76754 \times \frac{1}{\frac{1}{5} \times 2^{-2-1/\pi} \times 3^{-3/(2\pi)}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\pi}}$$

i is the imaginary unit

Result

1.6180008420031922524578300073076725803562371077935878884…

1.618000842… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

1.618

Series representations

$$\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\pi}\right)\left(76.754\sqrt[\pi]{2}\;3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\left(-0.2+i\sqrt{6}\sum_{k=0}^{\infty}6^{-k}\binom{\frac{1}{2}}{k}\right)\right)\Big/\left(-11+55i\sqrt{6}\sum_{k=0}^{\infty}6^{-k}\binom{\frac{1}{2}}{k}-1.40102\sqrt[3]{-1+5i\sqrt{6}\sum_{k=0}^{\infty}6^{-k}\binom{\frac{1}{2}}{k}}+7.0051i\sqrt{6}\left(\sum_{k=0}^{\infty}6^{-k}\binom{\frac{1}{2}}{k}\right)\sqrt[3]{-1+5i\sqrt{6}\sum_{k=0}^{\infty}6^{-k}\binom{\frac{1}{2}}{k}}+7.85142\left(-1+5i\sqrt{6}\sum_{k=0}^{\infty}6^{-k}\binom{\frac{1}{2}}{k}\right)^{2/3}\right)}=$$

$$\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)^{2\pi}\sqrt{\pi}\right)}=$$

$$\left(76.754\,\sqrt[\pi]{2}\;3^{3/(2\pi)}\;\pi^{-1/(2\pi)}\left(-0.2+i\sqrt{6}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Bigg/$$

$$\left(-11+55\,i\sqrt{6}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-1.40102\sqrt[3]{-1+5\,i\sqrt{6}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\right.$$

$$7.0051\,i\sqrt{6}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sqrt[3]{-1+5\,i\sqrt{6}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\left.7.85142\left(-1+5\,i\sqrt{6}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{2/3}\right)$$

$$\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)\sqrt[2\pi]{\pi}\right)}=$$

$$\left(76.754\,\sqrt[\pi]{2}\;3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\left(-0.2+i\,\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k\,z_0^{-k}}{k!}\right)\right)\Bigg/$$

$$\left(-11+55\,i\,\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k\,z_0^{-k}}{k!}-\right.$$

$$1.40102\sqrt[3]{-1+5\,i\,\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k\,z_0^{-k}}{k!}}+$$

$$7.0051\,i\,\sqrt{z_0}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k\,z_0^{-k}}{k!}\right)$$

$$\sqrt[3]{-1+5\,i\,\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k\,z_0^{-k}}{k!}}+$$

$$\left.7.85142\left(-1+5\,i\,\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k\,z_0^{-k}}{k!}\right)^{2/3}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

we obtain also:

(4 (0.76754*1/(1/5 2^(-2-1/π)3^(-3/(2π))(11+22^(2/3)/(-1+5 i √(7))^(1/3)+(11(-1+5 i √(7)))^(1/3)/2^(2/3))π^(1/(2π))))^(2 π))/(5 (3 + sqrt(5)))

Input

$$\frac{4\left(0.76754\times\frac{1}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)^{2\pi\sqrt{\pi}}}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

i is the imaginary unit

Result

3.14149866958858719478436442326934532877749166834727916762…

3.14149866958…≈ π (Ramanujan Recurring Number)

Alternate form

3.1415

Series representations

$$\frac{4\left(\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)^{2\pi\sqrt{\pi}}\right)}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}=$$

$$4\times 3.8377^{2\pi}\left(\frac{2^{2+1/\pi}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}}{11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{6}\,\sum_{k=0}^{\infty}6^{-k}\left(\frac{1}{2}\right)_k}}+\frac{\sqrt[3]{11}\,\sqrt[3]{-1+5i\sqrt{6}\,\sum_{k=0}^{\infty}6^{-k}\left(\frac{1}{2}\right)_k}}{2^{2/3}}}\right)^{2\pi}$$

$$5\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\left(\frac{\frac{1}{2}}{k}\right)\right)$$

$$\frac{4\left(\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)2\pi\sqrt{\pi}\right)}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}=$$

$$\frac{4\times 3.8377^{2\pi}\left(\frac{2^{2+1/\pi}\times 3^{3/(2\pi)}\pi^{-1/(2\pi)}}{11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{6}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}+\frac{\sqrt[3]{11}\sqrt[3]{-1+5i\sqrt{6}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{2^{2/3}}}\right)^{2\pi}}{5\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$4\left(\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)2\pi\sqrt{\pi}\right)}\right)^{2\pi}\Bigg/\left(5\left(3+\sqrt{5}\right)\right)=$$

$$\left(4\times 3.8377^{2\pi}\left(\left(2^{2+1/\pi}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right)\Big/\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k\,z_0^{-k}}{k!}}}+\frac{\sqrt[3]{11}\,\sqrt[3]{-1+5\,i\,\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k\,z_0^{-k}}{k!}}}{2^{2/3}}\right)^{2\pi}\right)\Big/\left(5\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)\right)$$ for

(not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

1/6((4 (0.76754*1/(1/5 2^(-2-1/π)3^(-3/(2π))(11+22^(2/3)/(-1+5 i √(7))^(1/3)+(11(-1+5 i √(7)))^(1/3)/2^(2/3))π^(1/(2π))))^(2 π))/(5(3+sqrt(5))))^2

Input

$$\frac{1}{6}\left(\frac{4\left(0.76754\times\frac{1}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)2\pi\sqrt{\pi}}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

i is the imaginary unit

Result

1.64483564850446125714491808200341350734204979471112204 57…

1.6448356485…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate form

1.64484

Series representations

$$\frac{1}{6}\left(\frac{4\left(\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5\,i\,\sqrt{7}\right)}}{2^{2/3}}\right)2\pi\sqrt{\pi}\right)}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 =$$

$$8\times 3.8377^{4\pi}\left(\frac{2^{2+1/\pi}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}}{11+\frac{22^{2/3}}{\sqrt[3]{-1+5\,i\,\sqrt{6}\,\sum_{k=0}^{\infty}6^{-k}\left(\frac{1}{2}\right)_k}}+\frac{\sqrt[3]{11}\,\sqrt[3]{-1+5\,i\,\sqrt{6}\,\sum_{k=0}^{\infty}6^{-k}\left(\frac{1}{2}\right)_k}}{2^{2/3}}}\right)^{4\pi}$$

$$\frac{}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\left(\frac{\frac{1}{2}}{k}\right)\right)^2}$$

$$\frac{1}{6}\left(\frac{4\left(\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)2\pi\sqrt{\pi}\right)}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 =$$

$$8\times 3.8377^{4\pi}\left(\frac{2^{2+1/\pi}\times 3^{3/(2\pi)}\pi^{-1/(2\pi)}}{11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{6}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}+\frac{\sqrt[3]{11}\ \sqrt[3]{-1+5i\sqrt{6}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{2^{2/3}}}\right)^{4\pi}$$

$$\frac{}{75\left(3+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\left(4\left(\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)2\pi\sqrt{\pi}\right)}\right)^{2\pi}\right)^2}{5\left(3+\sqrt{5}\right)}\right)=$$

$$\left(8\times 3.8377^{4\pi}\left(\left(2^{2+1/\pi}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right)\Big/\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{z_0}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k\,z_0^{-k}}{k!}}}+\frac{\sqrt[3]{11}\;\sqrt[3]{-1+5i\sqrt{z_0}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(7-z_0)^k\,z_0^{-k}}{k!}}}{2^{2/3}}\right)^{4\pi}\right)\Big/\left(75\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)^2\right)\right)$$ for

(not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

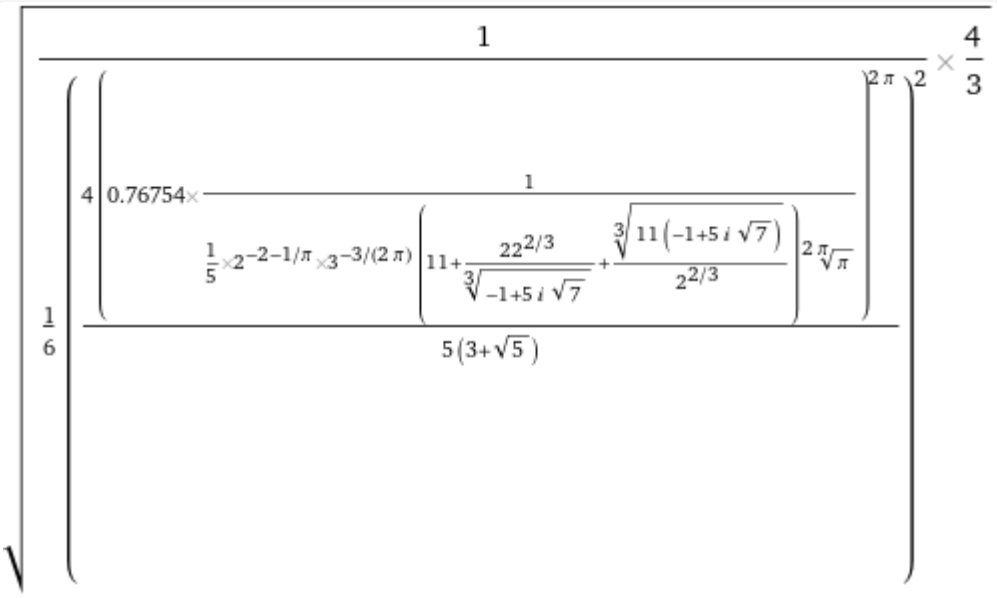
$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

And again:

sqrt(1/(1/6((4 (0.76754*1/(1/5 2^(-2-1/π)3^(-3/(2π))(11+22^(2/3)/(-1+5 i √(7))^(1/3)+(11(-1+5 i √(7)))^(1/3)/2^(2/3))π^(1/(2π))))^(2 π))/(5(3+sqrt(5))))^2)*(4/3))

Input



i is the imaginary unit

Result

0.900343…

Possible closed forms

$$\frac{2\sqrt{2}}{\pi} \approx 0.900316316$$

Alternate form

0.900343

0.900343…≈ 0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

All 2nd roots of 0.810618-1.35771×10^-16 i

$$0.900343\, e^{-8.37456\times10^{-17}\, i} \approx 0.90034 - 7.5400\times10^{-17}\, i \quad \text{(principal root)}$$

$$0.900343\, e^{3.14159\, i} \approx -0.90034 + 1.1026\times10^{-16}\, i$$

Series representations

$$\sqrt{\frac{4}{\frac{3}{6}\left(\frac{4\left(\frac{0.76754}{\frac{1}{5}\times2^{-2-1/\pi}\times3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)2\pi\sqrt{\pi}\right)}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2}} =$$

$$\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k\left(-\frac{1}{2}\right)_k$$

$$\left(-1+\frac{25}{2}\times3.8377^{-4\pi}\left(3+\sqrt{5}\right)^2\left(\frac{2^{2+1/\pi}\times3^{3/(2\pi)}\,\pi^{-1/(2\pi)}}{11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11}\,\sqrt[3]{-1+5i\sqrt{7}}}{2^{2/3}}}\right)^{-4\pi}\right)^k$$

$$\sqrt{\frac{4}{\frac{3}{6}\left(\frac{4\left(\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)2\pi\sqrt{\pi}\right)}\right)^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2}} =$$

$$\sqrt{z_0}\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k\left(-\frac{1}{2}\right)_k$$

$$\left(\frac{25}{2}\times 3.8377^{-4\pi}\left(3+\sqrt{5}\right)^2\left(\frac{2^{2+1/\pi}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}}{11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11}\,\sqrt[3]{-1+5i\sqrt{7}}}{2^{2/3}}}\right)^{-4\pi}-z_0\right)^k z_0^{-k} \quad \text{for } (\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\left(\frac{3}{6}\left(\frac{5\left(3+\sqrt{5}\right)}{4\left(\frac{0.76754}{\frac{1}{5}\times 2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(\left(11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11\left(-1+5i\sqrt{7}\right)}}{2^{2/3}}\right)2\pi\sqrt{\pi}\right)}\right)^{2\pi}}\right)^2}} =$$

$$\exp\left(\pi\mathcal{A}\left\lfloor\frac{1}{2\pi}\arg\left(-x+\frac{25}{2}\times 3.8377^{-4\pi}\left(3+\sqrt{5}\right)^2\left(\frac{2^{2+1/\pi}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}}{11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11}\,\sqrt[3]{-1+5i\sqrt{7}}}{2^{2/3}}}\right)^{-4\pi}\right)\right\rfloor\right)$$

$$\sqrt{x}\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+\frac{25}{2}\times 3.8377^{-4\pi}\left(3+\sqrt{5}\right)^2\left(\frac{2^{2+1/\pi}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}}{11+\frac{22^{2/3}}{\sqrt[3]{-1+5i\sqrt{7}}}+\frac{\sqrt[3]{11}\,\sqrt[3]{-1+5i\sqrt{7}}}{2^{2/3}}}\right)^{-4\pi}\right)^k \quad \text{for } (x\in\mathbb{R}$$

and $x<0$)

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Fig. 1

By multiplying the roots of a quintic equation, by the formula concerning the ratios (and/or the inverses) between the volumes of the icosahedron, the octahedron and the tetrahedron and the volume of the sphere, where this formula includes the ratio $(2\sqrt{2})/Pi$, the so-called Del Gaudio-Nardelli Constant, the result is very close to the values of the golden ratio and π. This is a possible geometric representation of the universe in its quantum phase with such data

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

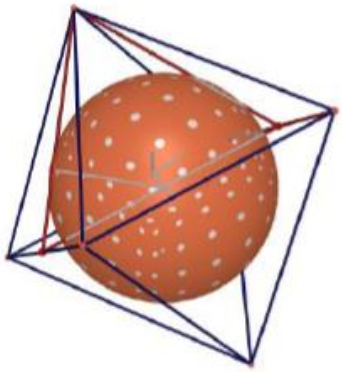


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\ z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\ x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^{0}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$
$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{\left(6\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

Indeed:

From:

https://www.academia.edu/75787512/The_Theory_of_String_A_Candidate_for_a_Generalized_Unification_Model

The Einstein's field equation and the String Theory.

The Einstein's field equation which includes the cosmological constant is:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (8)$$

where $R_{\mu\nu}$ is the Ricci tensor, R its trace, λ the cosmological constant, $g_{\mu\nu}$ the metric tensor of the space geometry, G the Newton's gravitational constant and $T_{\mu\nu}$ the tensor representing the properties of energy, matter and momentum.

The left hand-side of (8) represents the gravitational field and, consequently, the warped space-time, while the right hand-side represents the matter, i.e. the sources of the gravitational field.

In string theory, the gravity is related to the gravitons which are bosons, whereas the matter is related to fermions. It follows that the left and right hand of (8) may be respectively related to the action of bosonic and of superstrings.

The actions of bosonic string and superstring (also containing fermions) are connected by the Palumbo-Nardelli relation (Palumbo et al. 2005):

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\varphi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right]=$$

$$=\int_0^\infty \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|\widetilde{H}_3|^2-\frac{\kappa_{10}^2}{g_{10}^2}Tr_\nu\left(|F_2|^2\right)\right]$$

The sign minus in the above equation comes from the inversion of any relationship.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^{\,8}\right)^{-k}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^{\,8}\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^{0} \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

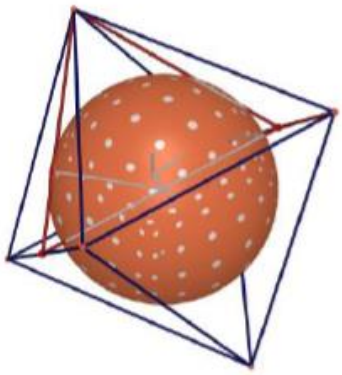
$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard to a sphere inscribed in an octahedron



we have the following formulas:

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\,2}{\pi} \quad (\text{for } l \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\,\pi)\rfloor)}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we have the following convex regular polyhedrons volumes:

$\frac{\sqrt{2}}{12}d^3$ Tetrahedron Volume

$\frac{\sqrt{2}}{3}d^3$ Octahedron Volume

$\frac{5}{12}(3+\sqrt{5})d^3$ Icosahedron Volume

Regarding the Sphere the volume is

$$V = \frac{4}{3}\pi r^3$$

With regard the sphere inscribed in a Tetrahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_T}{V_S} = \frac{\frac{\sqrt{2}}{12}d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(((√2 d^3)/12))*1/(4/3*π*(d/2)^3)

Input

$$\left(\frac{1}{12}\left(\sqrt{2}\ d^3\right)\right) \times \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.225079079039276517388799797751685145666143537488805156392859367 8

…

0.225079079….

Property

$\frac{1}{\sqrt{2}\ \pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Octahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_O}{V_S} = \frac{\frac{\sqrt{2}}{3}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

((1/3*√2*d^3)/(4/3*π*(d/2)^3))

Input

$$\frac{\frac{1}{3}\sqrt{2}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\ 2}{\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\, d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\,\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\,\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\,\pi)\rfloor)}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Icosahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_I}{V_S} = \frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3)

Input

$$\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{5\left(3+\sqrt{5}\right)}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

4.166730504921372682725163141597223093105782245731637719697502271 9

…

4.166730504921….

Property

$\frac{5\left(3+\sqrt{5}\right)}{2\pi}$ is a transcendental number

Expanded form

$$\frac{15}{2\pi}+\frac{5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

Alternate forms

$$\frac{15+5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

$$\frac{\frac{15}{2}+\frac{5\sqrt{5}}{2}}{\pi} \quad (\text{for } d \neq 0)$$

Series representations

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\pi\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x\in\mathbb{R}\text{ and }x<0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

Indeed, we obtain:

$$\frac{\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)\left(d^3\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2\right)}}{\sqrt[2\pi]{\frac{4}{3}\left(\pi\left(\frac{d}{2}\right)^3\times\frac{1}{3}\right)\sqrt{2}\left(d^3\times\frac{\sqrt{2}}{12}\right)d^3}}$$

= 1.618008545900107…

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535…. = π (Ramanujan Recurring Number)

In conclusion, from the following further formula concerning the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron volumes and their ratios (and/or the inverses) with the sphere volume, raised the expression

$$\left(2\log(2)log^{2/3}(3^2)\right)/\left(3e^{7/3}\right)$$

$$\left(\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\frac{\frac{1}{4}\left(\left(15+7\sqrt{5}\right)d^3\right)}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\right)^{(2\log(2)\log^{2/3}(3^2))/(3e^{7/3})}$$

i.e.

$$\left(\frac{3}{2}\right)^{(4\log(2)\log^{2/3}(9))/(3e^{7/3})}\left(\frac{5\left(3+\sqrt{5}\right)\left(15+7\sqrt{5}\right)}{\pi}\right)^{(2\log(2)\log^{2/3}(9))/(3e^{7/3})}$$

we obtain:

1.6180381711519139857456817084743134673903021614989370712917722071
…

1.6180381711…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

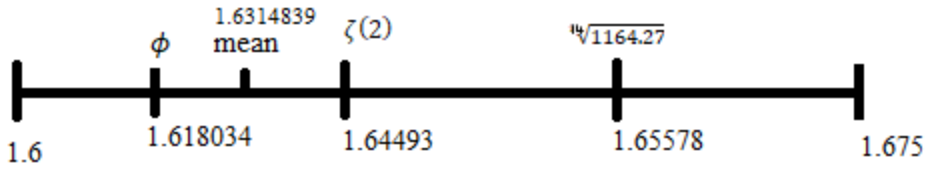
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

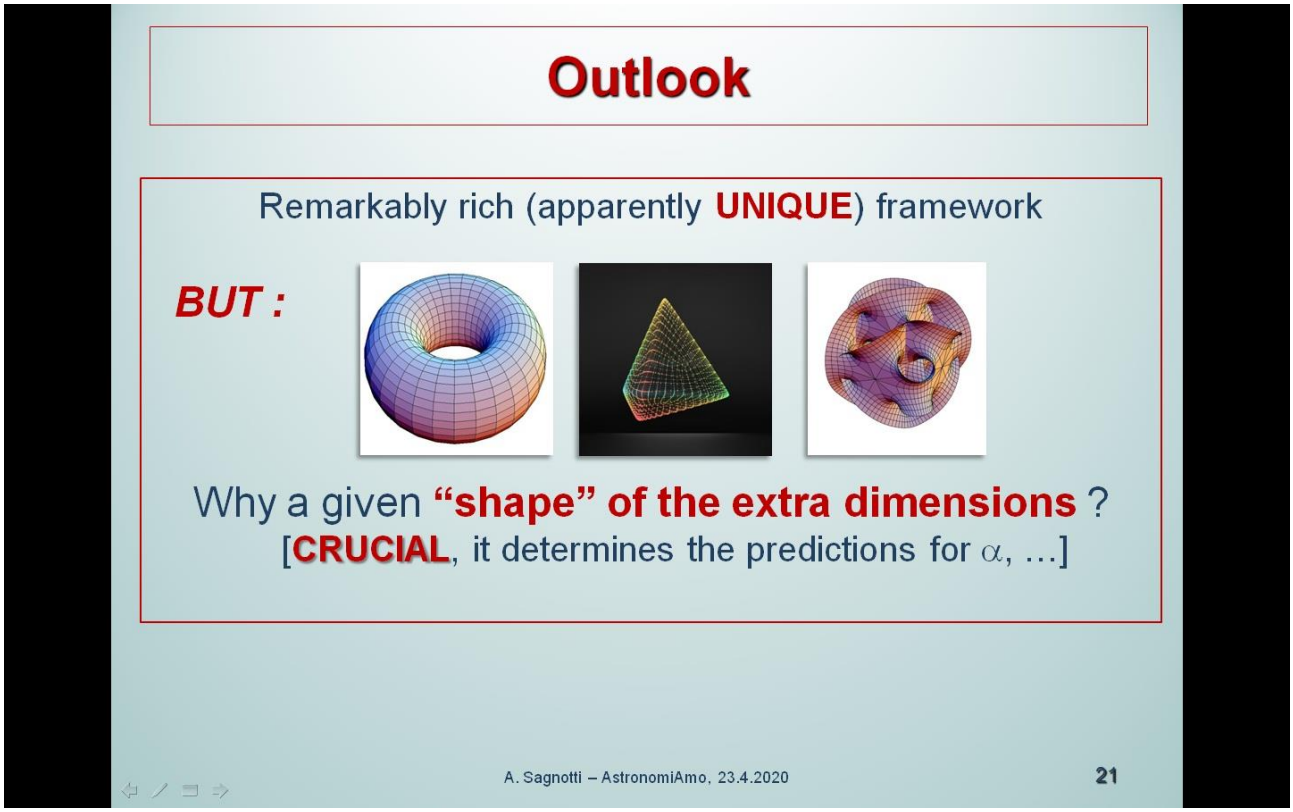
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix


Outlook
Remarkably rich (apparently UNIQUE) framework
BUT :
Why a given "shape" of the extra dimensions ?
[CRUCIAL, it determines the predictions for α, …]
A. Sagnotti – AstronomiAmo, 23.4.2020
21

From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

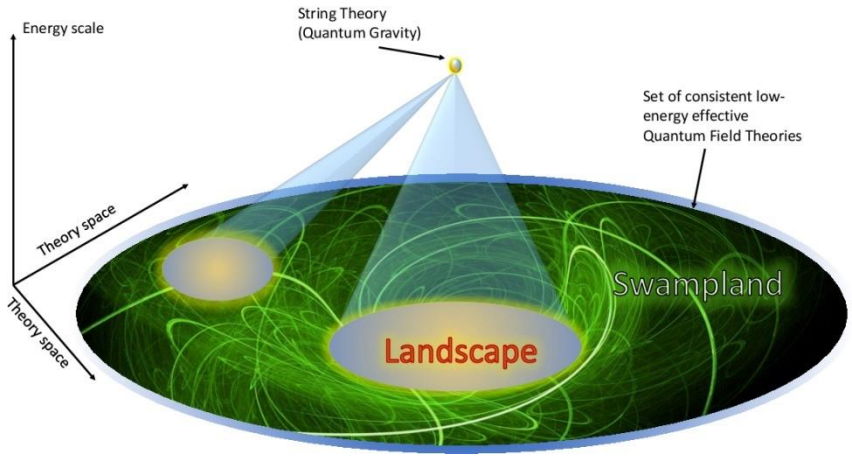


Fig. 1

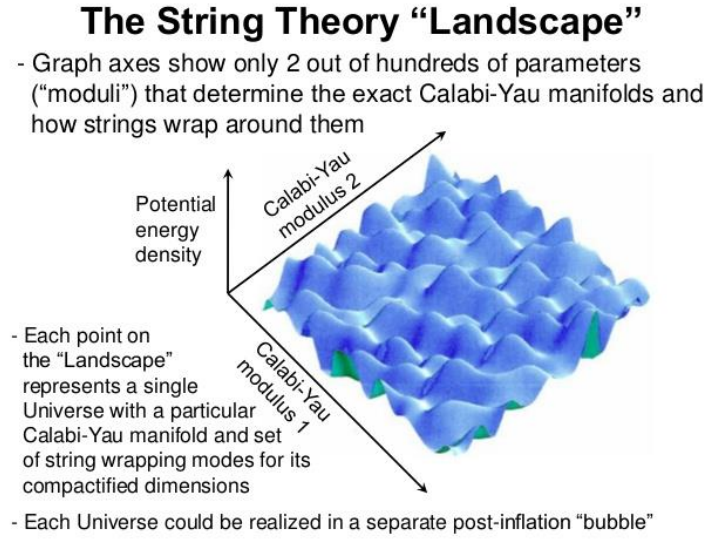

The String Theory "Landscape"
- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them
Potential energy density
Calabi-Yau modulus 2
Calabi-Yau modulus 1
- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

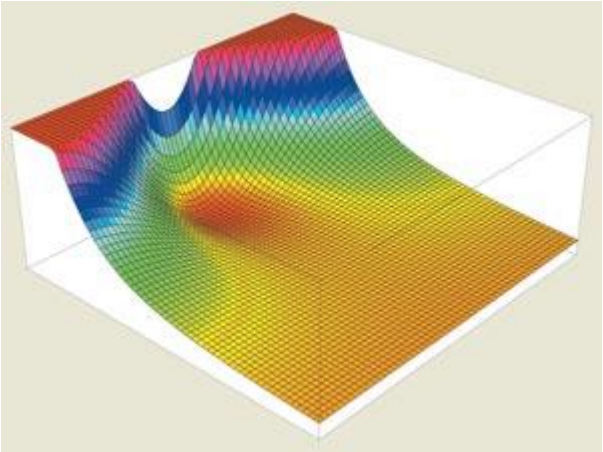


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

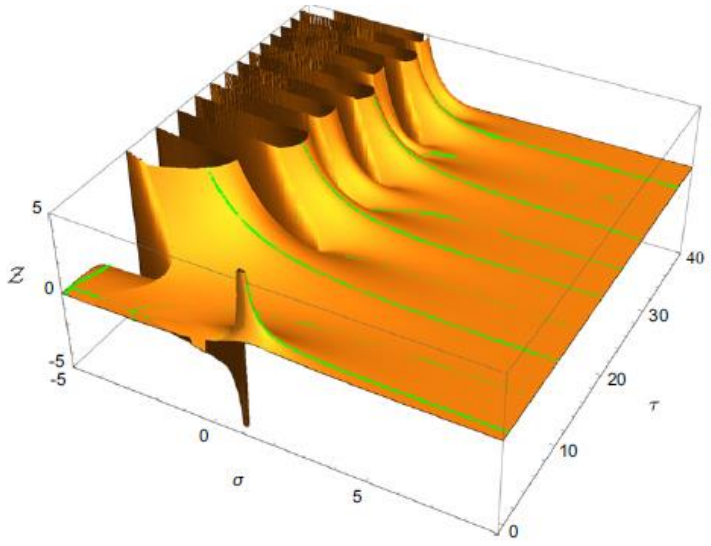


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

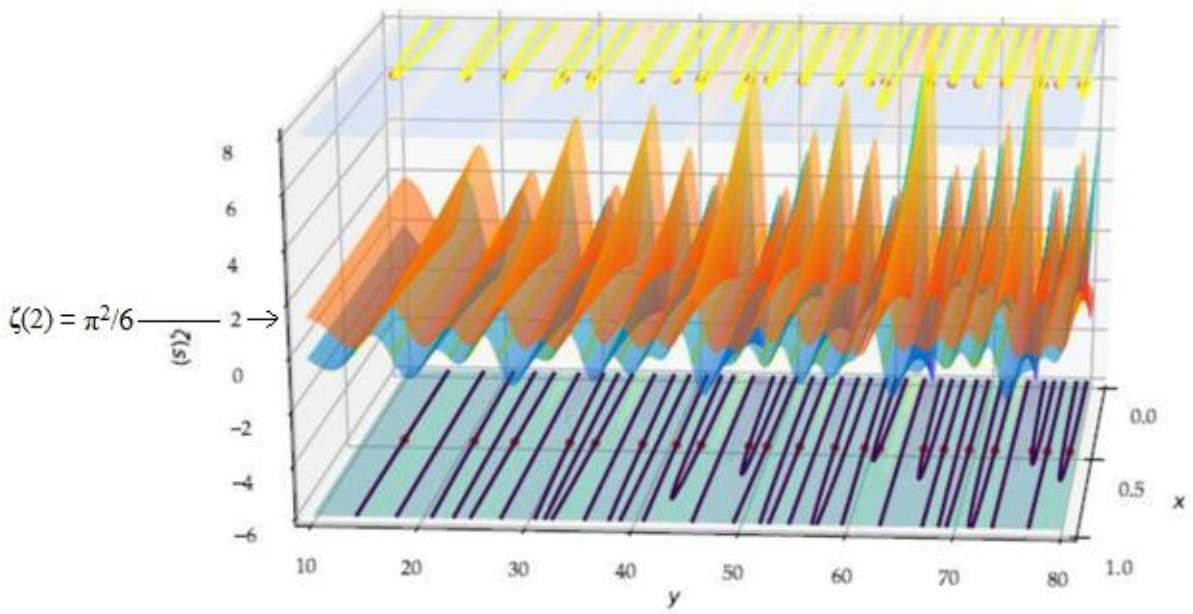


Figure 1. $C(x,y)$ and $S(x,y)$ surfaces of the Riemann $\zeta(x,y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

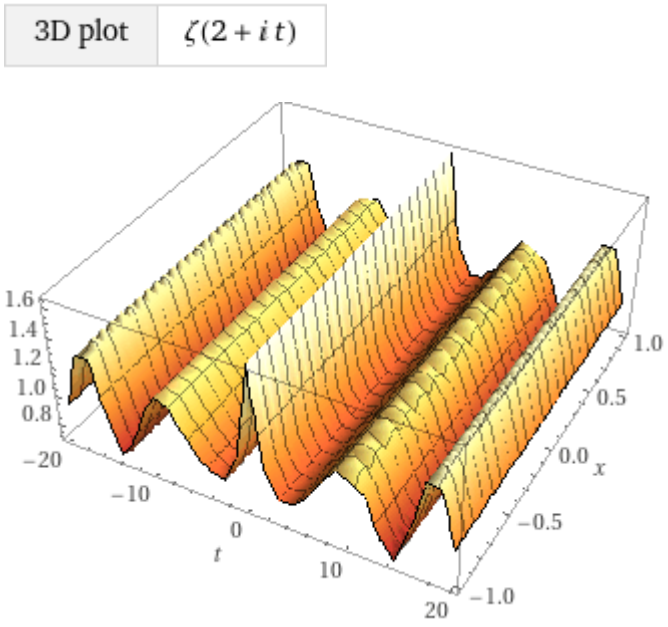


Fig. 6

Where ζ(2+it) :

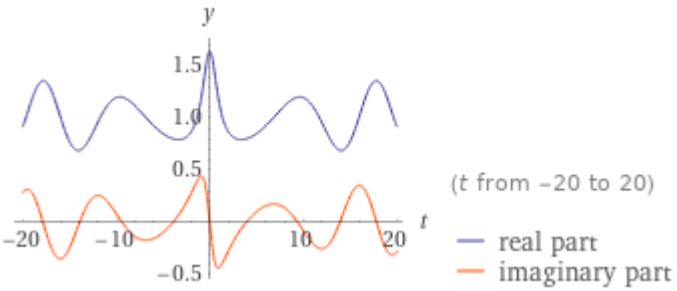
Input

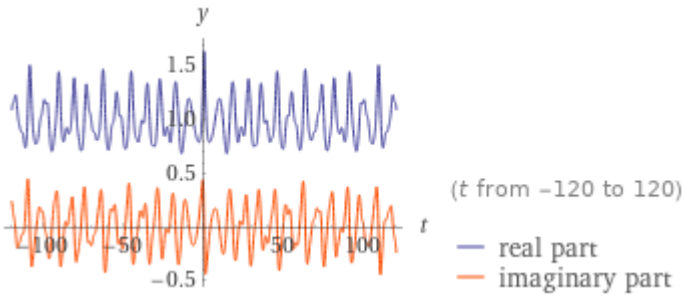
$\zeta(2 + i\,t)$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Plots





Roots

$t = 2\,i\,(n+1), \quad n \in \mathbb{Z}, \quad n \geq 1$

$t = -i\,(\rho_n - 2), \quad n \neq 0, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\,t\,\zeta'(2) - \frac{1}{2}\,t^2\,\zeta''(2) - \frac{1}{6}\,i\,\zeta^{(3)}(2)\,t^3 + \frac{1}{24}\,\zeta^{(4)}(2)\,t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2 + i\,t) = \zeta(2 + i\,t,\, 1)$$

$$\zeta(2 + i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2 + i\,t) = \frac{\zeta\left(2 + i\,t,\, \frac{1}{2}\right)}{-1 + 2^{2+i\,t}}$$

$\zeta(s,\, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2 + i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = \frac{\sum_{k=0}^{\infty} (1 + 2\,k)^{-2-i\,t}}{1 - 2^{-2-i\,t}} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \mathrm{Im}(t) < 1$$

$\mathrm{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^\infty \frac{\tau^{1+i\,t}}{-1+e^\tau}\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^\infty \tau^{2+i\,t}\,\mathrm{csch}^2(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^\infty e^{-\tau}\,\tau^{1+i\,t}\,\mathrm{csch}(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\mathrm{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+i\,t) = -i\,2^{2+i\,t}\,\pi^{1+i\,t}\,\Gamma(-1-i\,t)\sinh\left(\frac{\pi\,t}{2}\right)\zeta(-1-i\,t)$$

$$\zeta(2+i\,t) = \frac{\pi^{3/2+i\,t}\,\Gamma\left(-\frac{1}{2}-\frac{i\,t}{2}\right)\zeta(-1-i\,t)}{\Gamma\left(1+\frac{i\,t}{2}\right)}$$

$$\zeta(2+i\,t) = -\frac{i\sum_{k=0}^\infty \frac{\Gamma\left(k-\frac{i\,t}{2}\right)\sum_{j=0}^k (-1)^j\,(1+2\,j)\binom{k}{j}\zeta(2+2\,j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{i\,t}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372033095 8…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

1.64381856858…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^-}6$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right)=0.054323\,(0.0814845\,b+1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M}=-\frac{\frac{226\,802\,245}{18\,480\,874}+b}{M}$$

$$\frac{\partial M(b,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b}=-\frac{18\,480\,874\,M}{226\,802\,245+18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M}=\frac{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b}=\frac{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right\}=0 \text{ at } (b,M)=(-16,0)$$

Global minima:

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

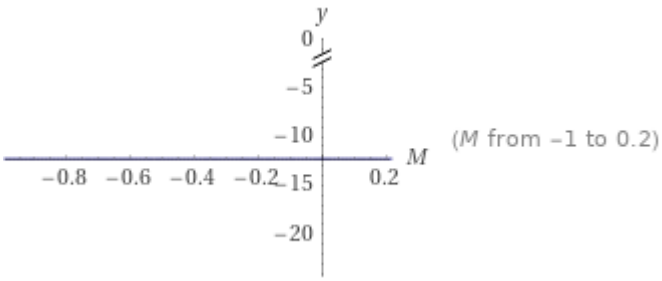
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

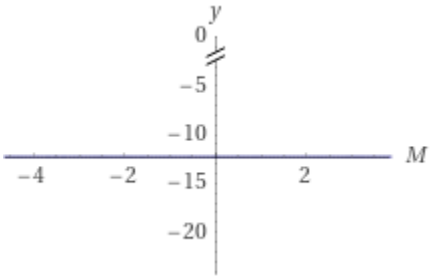
Result:

$$\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323\, M^2\right)}{M^2}\right) = \frac{3.55271 \times 10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

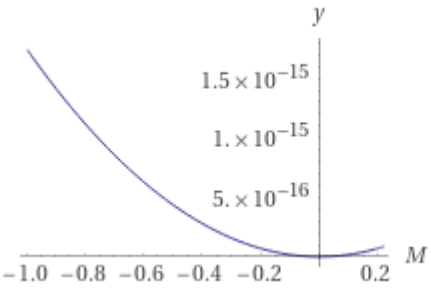
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

Result:

0

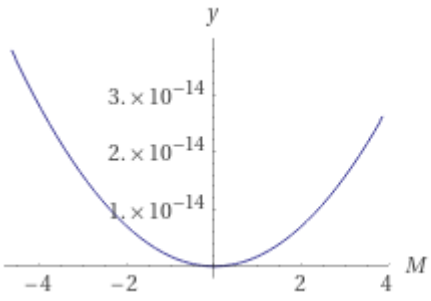
Plots: **(possible mathematical connection with an open string)**



(*M* from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$$O\left(M^{62194}\right)$$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times10^{-15}\,M^2+O\left(\left(\frac{1}{M}\right)^{62194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{1}{3}\,M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2(-0.5^2)$$

Result:

$$-4.388513449474645453489707833780880208333333333333333333\ldots\times10^{-16}$$

-4.38851344947*10^{-16}

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^2+6.58545\times10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times0.2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots \times 10^{-17}$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875 \times 10^{-14}$

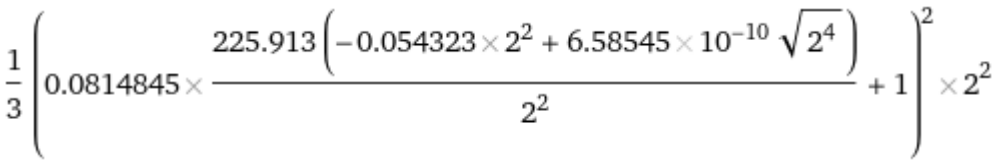
1.57986484181*10^-14

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:



Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots \times 10^{-15}$

7.021621519*10^-15

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times10^{-15}+1.57986484181\times10^{-14}+7.021621519\times10^{-17}-4.38851344947\times10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots\times10^{-8}$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_P^E=\mathbf{E}_P l_P^2=\phi_P l_P=\sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left((1/(7.021621519\times10^{-15}+1.57986484181\times10^{-14}+7.021621519\times10^{-17}-4.38851344947\times10^{-16}))\text{ ^ }(1/7)-\frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

$1.6181818182\ldots$

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

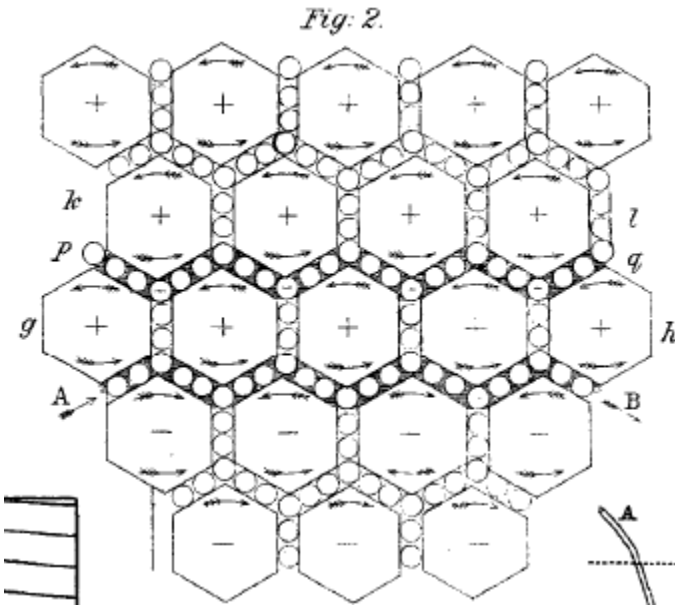
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

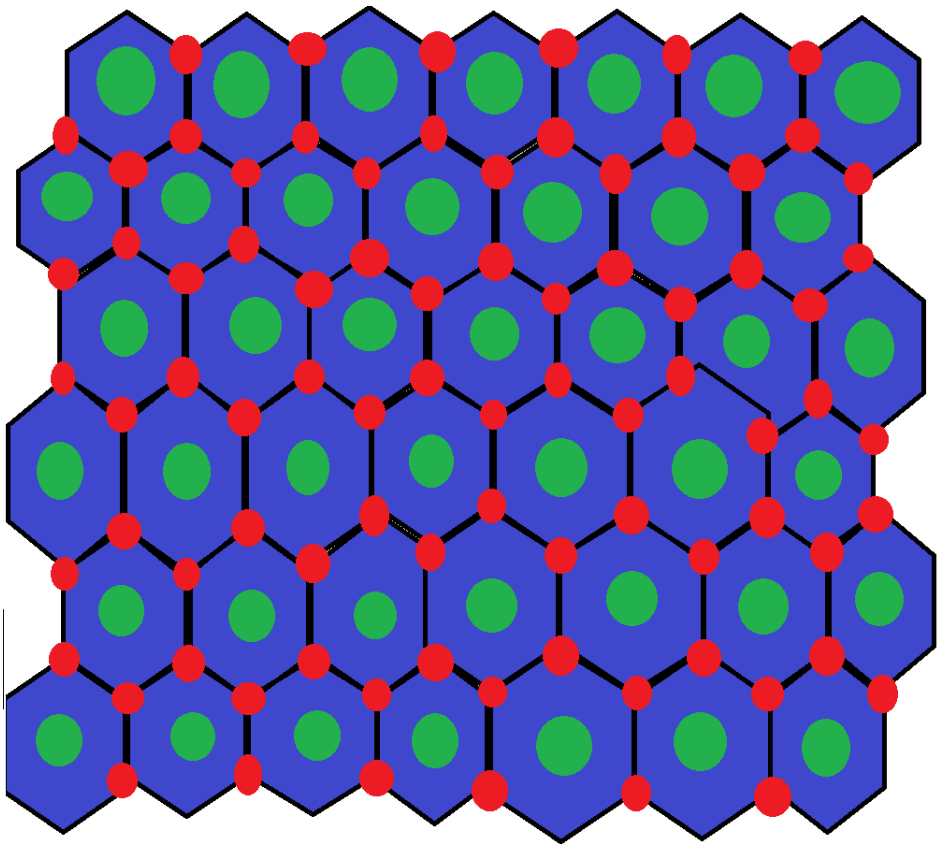
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Fig. 3

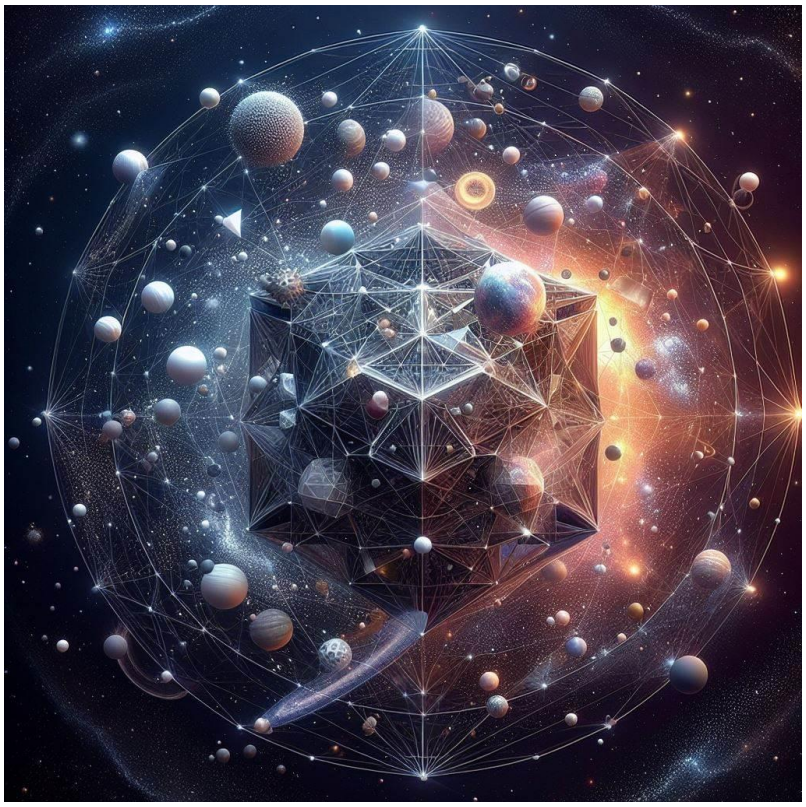


Image representing the solutions of the cubic equations in relation to the boundary conditions of the Plateau problem useful for understanding the formation of the bubbles of the eternally inflated universe and of an octahedron as a mathematical model of the primordial universe before the Big Bang, therefore highly symmetrical and with extremely low entropy. As for the cubic equations, they represent the geometric characteristics of the quantum vacuum, while the spherical surface would represent the geometry of the bubble of the universe.

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability. I would like to thank also my son **Francesco Nardelli**, student of Educational Sciences, majoring in History and Philosophy, for his useful suggestions regarding the intriguing image at the end of this work

References

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

See also:

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *Emilian Dudas, Sergio Ferrara, Alex Kehagias and Augusto Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the analysis of various Cubic Equations that can be solved with Cardano's Formula and of Superconformal Gravity and the Topology of Diffeomorphism Groups. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory IV

Michele Nardelli[1], Antonio Nardelli

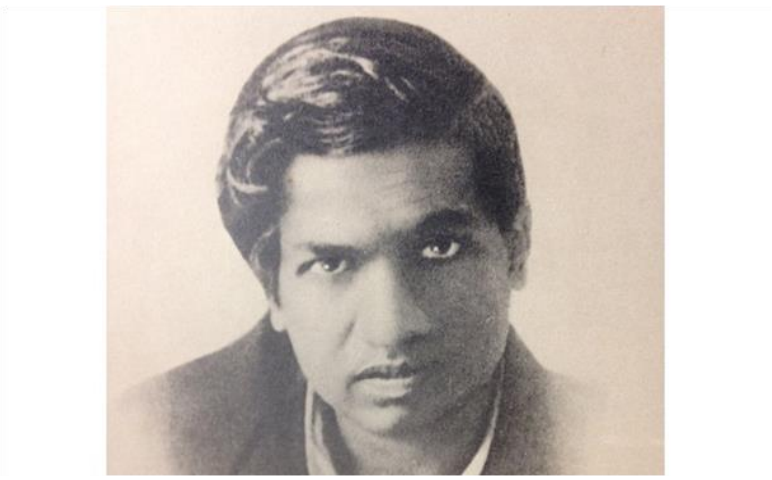
Abstract

In this paper (part IV), we analyze various Cubic Equations that can be solved with Cardano's formula and some equations of Superconformal Gravity and the Topology of Diffeomorphism Groups. We obtain new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Introduction

Cardano's method for cubic equations involves cube roots and introduces symmetry concepts. Similarly, string theory and M-theory involve symmetries and transformation groups that play a fundamental role in describing particles and interactions. Solutions of complete cubic equations via Cardano's method involve complex cube roots. Similarly, equations and solutions in superstring theory can often involve advanced algebraic and mathematical complexities. Solutions of cubic equations can be understood geometrically through topology and algebraic geometry. In string theory, the geometric and topological configurations of strings and branes are also central. Duality is an important concept in string theory, where different mathematical descriptions can be equivalent. Similarly, in solving cubic equations, duality links exist through cube roots.

Thus, in string theory, duality refers to the fact that different mathematical formulations or physical descriptions of a theory can be equivalent. For example, two seemingly different theories can be dual to each other, meaning they are two different ways of describing the same physical reality. This concept of duality has been very useful for understanding profound aspects of string theory and field theories.

Similarly, in solving cubic equations, duality can emerge through cube roots. The cubic equation $ax^3+bx^2+cx+d=0$ can be solved using cube roots, as in the Cardano method I described above. However, cube roots involve the concept of complex numbers, and it is sometimes possible to express the solutions in ways that highlight a duality between the solutions.

For example, if we consider complex cube roots, we can see a duality between them, since there are connections via the cube roots of unity. This duality is similar to the concept of duality in string theory, where different descriptions can be connected by similar transformations.

In both cases, duality provides a deeper way of understanding the underlying mathematical or physical structure, allowing you to explore hidden connections between different concepts.

Cubic surfaces can be associated with geometric and topological concepts. In string theory, for example, some models require spaces with particular shapes, and the geometry of the spaces can influence the properties of strings and particles. Some

mathematical and physical models involving surfaces might be described by cubic or complex equations. The geometry of surfaces and the associated mathematical structure can influence the properties of universe bubbles in the multiverse. Cubic surfaces can represent certain topological spaces, and their properties can affect the structure of the associated string theory.

The connection between the surface of a D-brane and the roots of a cubic equation can be explored in specific contexts within string theory, especially when it comes to studying the geometric and topological properties of D-branes. D-branes are extended objects on whose surface open strings can terminate. The geometry of the D-brane surface can interfere with the physics of surrounding strings. In some cases, geometry can be described mathematically through objects known in algebraic surface theory, such as cubic equations.

D-branes can host open fields, which describe particles interacting with the D-brane. Field equations on a D-brane can involve surface geometry, and in some situations, these equations can be associated with higher degree algebraic equations, such as cubic equations.

The compactification of extra dimensions, which is an important part of string theory, can be associated with the surface geometry of a D-brane. Compactification can lead to a reduction in dimensions and a geometric description that can involve algebraic equations, including cubic equations.

The topological invariants associated with the surface of a D-brane, such as the Betti number or the Chern number, can be related to geometric characteristics that can be described via algebraic equations, including cubic equations.

In some situations of duality, the description of a D-brane in a context can be dualistic to a description based on algebraic objects such as curves or surfaces, which can be represented via algebraic equations, including cubic equations.

In this paper, we will hypothesize that the solutions of the cubic equations can be used as boundary conditions to further the understanding of the formation of universe bubbles in eternal inflation. Furthermore, an octahedron could serve as a mathematical or conceptual model of the universe in the pre-Big Bang phase. Cubic equations, in this context, could represent the geometric characteristics of the quantum vacuum, while the spherical surface could be used to describe the geometry of the universe bubble.

The results $(2\sqrt{2})/\pi$, the golden ratio φ and π obtained, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and universe bubbles in eternal inflation. It may reflect a fundamental relationship between the geometry of solids and the mathematical characteristics of solutions to cubic equations.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and universe bubbles.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of universe bubbles and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

Finally, it is also possible to hypothesize the solutions of Ramanujan's cubic equations which are related to the cubic equations solvable with the Cardano method and Padovan's recurrence relations, as boundary conditions to deepen the understanding of the formation of inflationary universe bubbles eternal and of an octahedron which could represent a mathematical or conceptual model of the primordial universe before the Big Bang. Furthermore, regarding the cubic equations, they can represent the geometric characteristics of the quantum vacuum, while the spherical surface represents the geometry of the universe bubble. The results above $(2\sqrt{2})/\pi$, the golden ratio φ and π which are obtained, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

These mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe. The cubic equations, in this context, could reflect the geometric and dynamic characteristics of the quantum vacuum, while the spherical surface could represent the geometry of the bubbles of the universe.

Regarding fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the primordial universe and universe bubbles.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

We will also give an example of how the result of multiplying the solutions of a quartic equation and formula and of Superconformal Gravity and the Topology of Diffeomorphism Groups for the formula involving the ratios between the volumes of geometric solids and the Del Gaudio-Nardelli Constant approaches the values of the golden ratio and Pi, and as from the product of the solutions, the Recurring Numbers of Ramanujan are obtained.

This discovery suggests that there are still many connections to be explored between mathematics and physics, and that these connections could lead to new insights into the nature of the universe and its fundamental laws.

From

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

Cubic equations and Cardano's Formulas

Now, we analyze the following equation:

$x^3 + 20x = 6x^2 + 33.$

i.e.

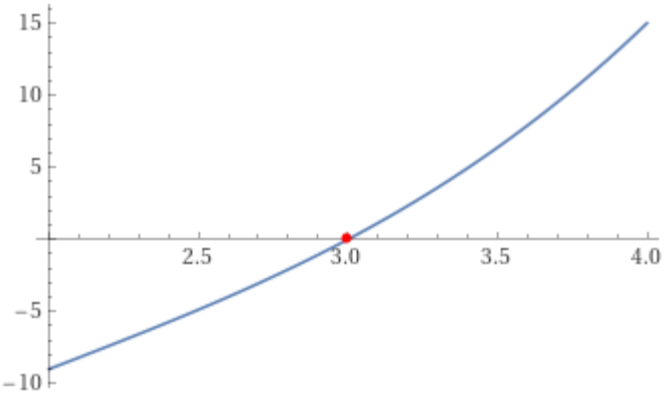
$$x^3 - 6x^2 + 20x - 33 = 0$$

x^3-6x^2+20x-33 = 0

Input

$x^3 - 6x^2 + 20x - 33 = 0$

Root plot **(figure that can be related to an open string)**



Alternate forms

$(x-3)\,((x-3)\,x+11)=0$

$(x-3)\left(x^2-3\,x+11\right)=0$

$(x-2)^3+8\,(x-2)-9=0$

Real solution

$x=3$

Complex solutions

$$x=\frac{3}{2}-\frac{i\sqrt{35}}{2}$$

$$x=\frac{3}{2}+\frac{i\sqrt{35}}{2}$$

Sum of roots

6

Product of roots

33

From

$$ax^3 + bx^2 + cx + d = 0$$

The Cardano's Formula is:

$$x = \sqrt[3]{-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a} + \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} + \sqrt[3]{-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a} - \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}$$

$$x^3 - 6x^2 + 20x - 33 = 0$$

From which, we consider:

(((216)/(27)-(120)/(6)+(33/2)+√(((216)/(27)-((120)/(6))+(33/2))^2+((20/3-36/9)^3))))^1/3+(((216)/(27)-((120)/(6))+(33/2)-√(((216)/(27)-((120)/(6))+(33/2))^2+((20/3-36/9)^3))))^1/3

Input

$$\sqrt[3]{\frac{216}{27} - \frac{120}{6} + \frac{33}{2} + \sqrt{\left(\frac{216}{27} - \frac{120}{6} + \frac{33}{2}\right)^2 + \left(\frac{20}{3} - \frac{36}{9}\right)^3}} + \sqrt[3]{\frac{216}{27} - \frac{120}{6} + \frac{33}{2} - \sqrt{\left(\frac{216}{27} - \frac{120}{6} + \frac{33}{2}\right)^2 + \left(\frac{20}{3} - \frac{36}{9}\right)^3}}$$

Exact result

$$\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}$$

Decimal approximation

2.811737691489899595805259670130262997683758165863708232385494624…

+

1.046007243882684687259970487513936170368173994245989923705978172…

i

Alternate complex forms

$$\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)+\frac{1}{2}\left(\sqrt{3}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)i$$

$$\frac{1}{\sqrt[3]{2}\,3^{2/3}}\sqrt{8\times 2^{2/3}\sqrt[3]{3}+\left(11\sqrt{105}-81\right)^{2/3}+\left(81+11\sqrt{105}\right)^{2/3}}$$

$$\left(\cos\left(\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}}{2\left(\frac{1}{2}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}}{2\left(\frac{1}{2}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)}\right)\right)\right)$$

$$\frac{1}{\sqrt[3]{2}\,3^{2/3}}\sqrt{8\times 2^{2/3}\sqrt[3]{3}+\left(11\sqrt{105}-81\right)^{2/3}+\left(81+11\sqrt{105}\right)^{2/3}}$$

$$\exp\left(i\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}}{2\left(\frac{1}{2}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)}\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 3$ (radius), $\theta \approx 0.35615$ (angle)

3

Alternate forms

$$\frac{1}{4}\left(1+\sqrt{105}+i\sqrt{2\left(19-\sqrt{105}\right)}\right)$$

---

root of $x^4-x^3-8x^2-9x+81$ near $x=2.81174+1.04601\,i$

$$\frac{\sqrt[3]{27-11\sqrt{\frac{35}{3}}}+\sqrt[3]{27+11\sqrt{\frac{35}{3}}}}{\sqrt[3]{6}}$$

Minimal polynomial

$x^4-x^3-8x^2-9x+81$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

And:

(((216)/(27)-(120)/(6)+(33/2)+√(((216)/(27)-((120)/(6))+(33/2))^2+((20/3-36/9)^3))))^1/3 (((216)/(27)-((120)/(6))+(33/2)-√(((216)/(27)-((120)/(6))+(33/2))^2+((20/3-36/9)^3))))^1/3

Input

$$\sqrt[3]{\frac{216}{27}-\frac{120}{6}+\frac{33}{2}+\sqrt{\left(\frac{216}{27}-\frac{120}{6}+\frac{33}{2}\right)^2+\left(\frac{20}{3}-\frac{36}{9}\right)^3}}\;\sqrt[3]{\frac{216}{27}-\frac{120}{6}+\frac{33}{2}-\sqrt{\left(\frac{216}{27}-\frac{120}{6}+\frac{33}{2}\right)^2+\left(\frac{20}{3}-\frac{36}{9}\right)^3}}$$

Exact result

$$\sqrt[3]{-\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}$$

Decimal approximation

1.33333333333333333333333333333333333333333333333333333333333333333333333333…

+

2.30940107675850305803659512200782982259040700508050750407440930 5…

i

Alternate complex forms

$$\frac{1}{2}\sqrt[3]{\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}+\frac{1}{2}\left(\sqrt{3}\sqrt[3]{\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}\right)i$$

$$\frac{8}{3}\left(\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)\right)$$

$$\frac{8}{3}e^{(i\pi)/3}$$

i is the imaginary unit

Polar coordinates

$r = \frac{8}{3}$ (radius), $\theta = \frac{\pi}{3}$ (angle)

Alternate forms

$$\frac{8\sqrt[3]{-1}}{3}$$

$$\frac{1}{3}\left(4 + 4\,i\,\sqrt{3}\right)$$

$$\frac{1}{2}\sqrt[3]{\left(\frac{11\sqrt{\frac{35}{3}}}{6} - \frac{9}{2}\right)\left(\frac{9}{2} + \frac{11\sqrt{\frac{35}{3}}}{6}\right)} + \frac{1}{2}\,i\,\sqrt{3}\,\sqrt[3]{\left(\frac{11\sqrt{\frac{35}{3}}}{6} - \frac{9}{2}\right)\left(\frac{9}{2} + \frac{11\sqrt{\frac{35}{3}}}{6}\right)}$$

Minimal polynomial

$9x^2 - 24x + 64$

And again:

(((-((11 sqrt(35/3))/6 - 9/2) (9/2 + (11 sqrt(35/3))/6))^(1/3)))(((9/2 - (11 sqrt(35/3))/6)^(1/3) + (9/2 + (11 sqrt(35/3))/6)^(1/3)))

Input

$$\sqrt[3]{-\left(\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right) - \frac{9}{2}\right)\left(\frac{9}{2} + \frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)\right)}$$
$$\left(\sqrt[3]{\frac{9}{2} - \frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)} + \sqrt[3]{\frac{9}{2} + \frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}\right)$$

Result

$$\sqrt[3]{-\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)$$

Decimal approximation

1.333333333333333333333333333333333333333333333333333333333333333333333333…

+

7.888106377466154723423104388748822731220668307725787097172959558…

i

Alternate complex forms

$$\left(-\frac{3}{4}\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)^{2/3}\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\frac{1}{2}\sqrt[3]{\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}\right.$$
$$\left.\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\right)+$$
$$\left(\frac{1}{4}\sqrt{3}\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)^{2/3}\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\right.$$
$$\frac{1}{2}\sqrt{3}\sqrt[3]{\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}$$
$$\left.\left.\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\right)i\right.$$

$$\frac{4}{3}\left(\frac{2}{3}\right)^{2/3}\sqrt{8\times 2^{2/3}\sqrt[3]{3}+\left(11\sqrt{105}-81\right)^{2/3}+\left(81+11\sqrt{105}\right)^{2/3}}$$
$$\left(\cos\left(\tan^{-1}\left(\left(\frac{1}{4}\sqrt{3}\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)^{2/3}\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\right.\right.\right.\right.$$
$$\frac{1}{2}\sqrt{3}\sqrt[3]{\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}$$
$$\left.\left.\left.\left.\left(\frac{1}{2}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\right)\right/$$
$$\left(\frac{1}{2}\sqrt[3]{\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}\right.$$
$$\left(\frac{1}{2}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)-$$
$$\left.\left.\left.\frac{3}{4}\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)^{2/3}\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\right)\right)+$$
$$i\sin\left(\tan^{-1}\left(\left(\frac{1}{4}\sqrt{3}\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)^{2/3}\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\right.\right.\right.$$
$$\frac{1}{2}\sqrt{3}\sqrt[3]{\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}$$
$$\left.\left.\left.\left(\frac{1}{2}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\right)\right/$$
$$\left(\frac{1}{2}\sqrt[3]{\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}\right.$$
$$\left(\frac{1}{2}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)-$$
$$\left.\left.\left.\left.\frac{3}{4}\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)^{2/3}\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\right)\right)\right)$$

$$\frac{4}{3}\left(\frac{2}{3}\right)^{2/3}\sqrt{8\times 2^{2/3}\sqrt[3]{3}+\left(11\sqrt{105}-81\right)^{2/3}+\left(81+11\sqrt{105}\right)^{2/3}}$$

$$\exp\left(i\tan^{-1}\left(\left(\frac{1}{4}\sqrt{3}\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)^{2/3}\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\right.\right.\right.$$

$$\frac{1}{2}\sqrt{3}\sqrt[3]{\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}$$

$$\left.\left(\frac{1}{2}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\right)\Big/$$

$$\left(\frac{1}{2}\sqrt[3]{\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}\left(\frac{1}{2}\sqrt[3]{\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}}+\right.\right.$$

$$\left.\left.\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)-\frac{3}{4}\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)^{2/3}\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\Bigg)\Bigg)\Bigg)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 8.0000$ (radius), $\theta \approx 1.4033$ (angle)

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Alternate forms

$$\frac{4}{9}\left((-2)^{2/3}\sqrt[3]{3\left(11\sqrt{105}-81\right)}+2^{2/3}\sqrt[3]{-3\left(81+11\sqrt{105}\right)}\right)$$

$$\frac{4}{3}\left(1+i\sqrt{35}\right)$$

$$\frac{1}{3}\left(4+4\,i\sqrt{35}\right)$$

Expanded form

$$\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)^{2/3}+\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}\sqrt[3]{-\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}$$

Minimal polynomial

$$3x^2-8x+192$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$2\pi\sqrt{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$2\pi\sqrt{\frac{5}{12}(3+\sqrt{5})d^3\times\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$2\pi\sqrt{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\times\frac{1}{3}\sqrt{2}d^3\times\frac{\sqrt{2}}{12}d^3}$$

From the previous exact result

$$\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}$$

we obtain:

(((9/2 - (11 sqrt(35/3))/6)^(1/3) + (9/2 + (11 sqrt(35/3))/6)^(1/3))) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

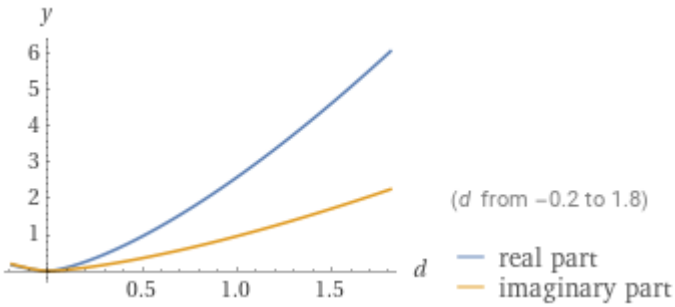
Input

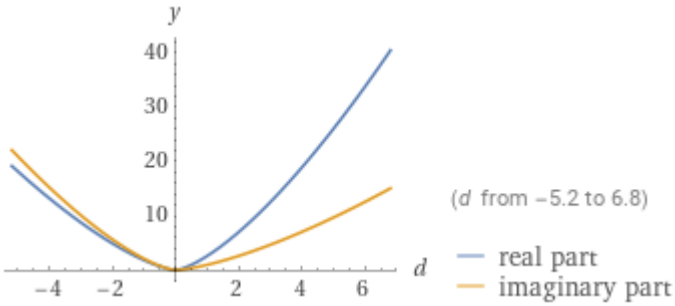
$$\left(\sqrt[3]{\frac{9}{2}-\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}+\sqrt[3]{\frac{9}{2}+\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}\right)\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Exact result

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\sqrt[\pi]{\frac{\pi}{4}}\,\sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate forms

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{13+4\,i\sqrt{35}}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}$$

$$3^{-3/(2\pi)}\times 4^{-1-1/\pi}\sqrt[2\pi]{15+5\sqrt{5}}\left(1+\sqrt{105}+i\sqrt{2\left(19-\sqrt{105}\right)}\right)\sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$2^{-1/3-2/\pi}\times 3^{-2/3-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(\sqrt[3]{81-11\sqrt{105}}+\sqrt[3]{81+11\sqrt{105}}\right)\sqrt[\pi]{\pi}$$
$$\sqrt[2\pi]{d^9}$$

Expanded form

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}+$$
$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}$$

Root

$d=0$ (assuming complex-valued fractional powers)

Series expansion at d=∞

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\sqrt[\pi]{\frac{\pi}{4}}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$

$$\frac{1}{d}2^{-4/3-2/\pi}\times 3^{4/3-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(\sqrt[3]{81-11\sqrt{105}}+\sqrt[3]{81+11\sqrt{105}}\right)$$
$$\sqrt[2\pi]{d^9}\ \pi^{1/\pi-1}$$

Indefinite integral

$$\int 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}}\ dd=$$

$$\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\sqrt[\pi]{\frac{\pi}{4}}\ d^{\sqrt[2\pi]{d^9}}}{1+\frac{9}{2\pi}}+\text{constant}$$

And

(((9/2 - (11 sqrt(35/3))/6)^(1/3) + (9/2 + (11 sqrt(35/3))/6)^(1/3))) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

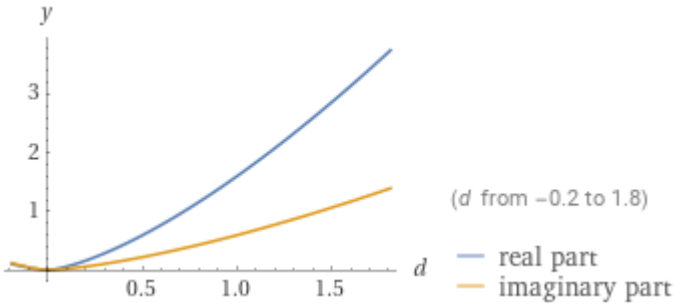
Input

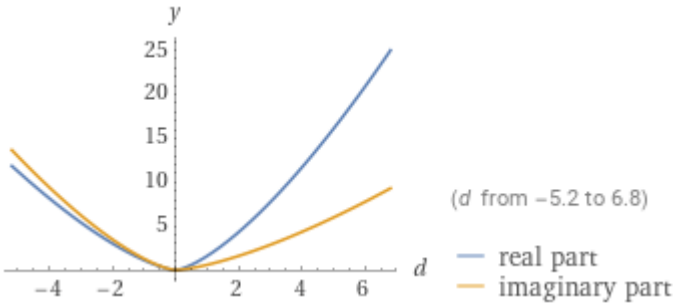
$$\left(\sqrt[3]{\frac{9}{2}-\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}+\sqrt[3]{\frac{9}{2}+\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}d^3\right)}$$

Exact result

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate forms

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[3]{13+4\,i\sqrt{35}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$2^{-2-1/\pi}\times 3^{-3/(2\pi)}\left(1+\sqrt{105}+i\sqrt{2\left(19-\sqrt{105}\right)}\right)\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$2^{-1/3-1/\pi} \times 3^{-2/3-3/(2\pi)} \left(\sqrt[3]{81-11\sqrt{105}}+\sqrt[3]{81+11\sqrt{105}}\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Expanded form

$$2^{-1/\pi} \times 3^{-3/(2\pi)} \sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9} + 2^{-1/\pi} \times 3^{-3/(2\pi)} \sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Alternate form assuming d>0

$$2^{-(3+\pi)/(3\pi)} \times 3^{-(9+4\pi)/(6\pi)} \left(\sqrt[3]{81-11\sqrt{105}}+\sqrt[3]{81+11\sqrt{105}}\right) \sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Root

$d = 0$ (assuming complex-valued fractional powers)

Series expansion at d=∞

$$2^{-1/\pi} \times 3^{-3/(2\pi)} \left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right) \sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right)=$$

$$\frac{2^{-4/3-1/\pi}\times3^{4/3-3/(2\pi)}\left(\sqrt[3]{81-11\sqrt{105}}+\sqrt[3]{81+11\sqrt{105}}\right)\sqrt[2\pi]{d^9}\;\pi^{1/(2\pi)-1}}{d}$$

Indefinite integral

$$\int 2^{-1/\pi}\times3^{-3/(2\pi)}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)\sqrt[2\pi]{d^9}\;\sqrt[2\pi]{\pi}\;dd=$$

$$\frac{2^{2/3-1/\pi}\times3^{-(9+4\pi)/(6\pi)}\left(\sqrt[3]{81-11\sqrt{105}}+\sqrt[3]{81+11\sqrt{105}}\right)\pi^{1+1/(2\pi)}\,d\,\sqrt[2\pi]{d^9}}{9+2\pi}+$$

constant

Dividing the two previous alternate forms

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[3]{13+4\,i\sqrt{35}}\;\sqrt[\pi]{\frac{\pi}{4}}\;\sqrt[2\pi]{d^9}$$

and

$$2^{-1/\pi}\times3^{-3/(2\pi)}\sqrt[3]{13+4\,i\sqrt{35}}\;\sqrt[2\pi]{\pi}\;\sqrt[2\pi]{d^9}$$

we obtain:

(3^(-3/(2 π)) (5 (3 + sqrt(5)))^(1/(2 π)) (13 + 4 i sqrt(35))^(1/3) (d^9)^(1/(2 π)) (π/4)^(1/π))/(2^(-1/π) 3^(-3/(2 π)) (13 + 4 i sqrt(35))^(1/3) π^(1/(2 π)) (d^9)^(1/(2 π)))

Input

$$\frac{3^{-3/(2\pi)}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{13+4\,i\,\sqrt{35}}\ \sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}}}{2^{-1/\pi}\times 3^{-3/(2\pi)}\ \sqrt[3]{13+4\,i\,\sqrt{35}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}$$

i is the imaginary unit

Exact result

$$2^{-1/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\ \sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{13+4\,i\,\sqrt{35}}\ \sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-3/(2\pi)}\ \sqrt[3]{13+4\,i\,\sqrt{35}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}} =$$

$$2^{-1/\pi}\ \sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{13+4\,i\,\sqrt{35}}\ \sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-3/(2\pi)}\ \sqrt[3]{13+4\,i\,\sqrt{35}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}} =$$

$$2^{-1/\pi}\ \sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{13+4\,i\,\sqrt{35}}\ \sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-3/(2\pi)}\ \sqrt[3]{13+4\,i\,\sqrt{35}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}} =$$

$$2^{-1/\pi}\ \sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the above alternate form

$$2^{-1/\pi}\ \sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13

…

3.1415926535884…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298888445046920

…

1.644934066846…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.618008545900002\pi}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900002\pi}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900002\pi}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.92478440155986\pi}\sqrt{\pi}^2}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Furthermore, from the two previous results

$$\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}$$

and

$$\sqrt[3]{-\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)}$$

we obtain also:

(((9/2 - (11 sqrt(35/3))/6)^(1/3) + (9/2 + (11 sqrt(35/3))/6)^(1/3))((-((11 sqrt(35/3))/6 - 9/2) (9/2 + (11 sqrt(35/3))/6))^(1/3)))^4

Input

$$\left(\left(\sqrt[3]{\frac{9}{2}-\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}+\sqrt[3]{\frac{9}{2}+\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}\right)\sqrt[3]{-\left(\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)\right)}\right)^4$$

Result

$$-\sqrt[3]{-1}\left(\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\right)^{4/3}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)^4$$

Decimal approximation

3211.06172839506172839506172839506

Alternate complex forms

$$\left(\frac{1}{2}\sqrt{3}\left(\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\right)^{4/3}\right.$$
$$\left(-\frac{3}{2}\sqrt{3}\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)+\right.$$
$$\left.2\sqrt{3}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)^3\right)-$$
$$\frac{1}{2}\left(\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\right)^{4/3}\left(\frac{9}{16}\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)^{4/3}-\right.$$
$$\frac{9}{2}\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)^{2/3}\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)^2+$$
$$\left.\left.\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)^4\right)\right)+$$
$$\left(-\frac{1}{2}\left(\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\right)^{4/3}\left(-\frac{3}{2}\sqrt{3}\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\right.\right.$$
$$\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)+2\sqrt{3}$$
$$\left.\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)^3\right)-$$
$$\frac{1}{2}\sqrt{3}\left(\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\right)^{4/3}\left(\frac{9}{16}\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)^{4/3}-\right.$$
$$\frac{9}{2}\left(-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)^{2/3}\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)^2+$$
$$\left.\left.\left(\frac{1}{2}\sqrt[3]{-\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)^4\right)\right)i$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 4096$ (radius), $\theta \approx -0.66979$ (angle)

4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{256\left((-2)^{2/3}\sqrt[3]{3\left(11\sqrt{105}-81\right)}+2^{2/3}\sqrt[3]{-3\left(81+11\sqrt{105}\right)}\right)^4}{6561}$$

$$\frac{2048}{81}\left(127-17\,i\sqrt{35}\right)$$

$$\frac{1}{81}\left(260096-34816\,i\sqrt{35}\right)$$

Expanded form

$$\frac{524288}{243} - \frac{256}{3}\left(\frac{9}{2} - \frac{11\sqrt{\frac{35}{3}}}{6}\right)^{2/3}\sqrt[3]{\frac{9}{2} + \frac{11\sqrt{\frac{35}{3}}}{6}} + \frac{2816\sqrt{35}\left(\frac{9}{2} - \frac{11\sqrt{\frac{35}{3}}}{6}\right)^{2/3}\sqrt[3]{\frac{9}{2} + \frac{11\sqrt{\frac{35}{3}}}{6}}}{81\sqrt{3}} - \frac{1280}{3}\sqrt[3]{\frac{9}{2} - \frac{11\sqrt{\frac{35}{3}}}{6}}\left(\frac{9}{2} + \frac{11\sqrt{\frac{35}{3}}}{6}\right)^{2/3} + \frac{2816\sqrt{35}\sqrt[3]{\frac{9}{2} - \frac{11\sqrt{\frac{35}{3}}}{6}}\left(\frac{9}{2} + \frac{11\sqrt{\frac{35}{3}}}{6}\right)^{2/3}}{27\sqrt{3}} - \frac{1024}{3}\sqrt[3]{\frac{9}{2} - \frac{11\sqrt{\frac{35}{3}}}{6}}\sqrt[3]{-\left(\frac{11\sqrt{\frac{35}{3}}}{6} - \frac{9}{2}\right)\left(\frac{9}{2} + \frac{11\sqrt{\frac{35}{3}}}{6}\right)} - \frac{11264\sqrt{35}\sqrt[3]{\frac{9}{2} - \frac{11\sqrt{\frac{35}{3}}}{6}}\sqrt[3]{-\left(\frac{11\sqrt{\frac{35}{3}}}{6} - \frac{9}{2}\right)\left(\frac{9}{2} + \frac{11\sqrt{\frac{35}{3}}}{6}\right)}}{81\sqrt{3}}$$

Minimal polynomial

$81x^2 - 520192x + 1358954496$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

27*sqrt((((9/2 - (11 sqrt(35/3))/6)^(1/3) + (9/2 + (11 sqrt(35/3))/6)^(1/3))((-((11 sqrt(35/3))/6 - 9/2) (9/2 + (11 sqrt(35/3))/6))^(1/3)))^4)+1

Input

$$27 \sqrt{\left(\left(\sqrt[3]{\frac{9}{2}-\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}+\sqrt[3]{\frac{9}{2}+\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}\right)\sqrt[3]{-\left(\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)\right)}\right)^4}+1$$

Exact result

$$1+27\sqrt{\left(\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\right)^{2/3}\left(-\sqrt[3]{-1}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)^4\right)}$$

Decimal approximation

1633.000000000000000000000000000000000000000000000000000000000000…

$-$

567.943659177563140086463515989915236647888118156256670996453088 2…

i

Alternate complex forms

$1633 - 567.9\,i$

$1728.9\,(\cos(-0.33471) + i\sin(-0.33471))$

$1728.9\, e^{-0.33471\, i}$

Polar coordinates

$r = 1728.93$ (radius), $\theta = -0.334682$ (angle)

1728.93

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string

boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$1633 - 96\,i\sqrt{35}$$

$$1 + 32\sqrt[3]{6}\sqrt{-\sqrt[3]{-1}\left(\sqrt[3]{27 - 11\sqrt{\frac{35}{3}}} + \sqrt[3]{27 + 11\sqrt{\frac{35}{3}}}\right)^4}$$

$$1 + 192\sqrt{-\sqrt[3]{-1}\left(\sqrt[3]{\frac{9}{2} - \frac{11\sqrt{\frac{35}{3}}}{6}} + \sqrt[3]{\frac{9}{2} + \frac{11\sqrt{\frac{35}{3}}}{6}}\right)^4}$$

Minimal polynomial

$$x^2 - 3266\,x + 2989249$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

(27*sqrt((((9/2 - (11 sqrt(35/3))/6)^(1/3) + (9/2 + (11 sqrt(35/3))/6)^(1/3))((-((11 sqrt(35/3))/6 - 9/2) (9/2 + (11 sqrt(35/3))/6))^(1/3)))^4)+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\left(27\sqrt{\left(\left(\sqrt[3]{\frac{9}{2}-\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}+\sqrt[3]{\frac{9}{2}+\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}\right)\sqrt[3]{-\left(\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)\right)}\right)^4}+1\right)\text{^}(1/15)+{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Exact result

$${C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\left(1+27\left(\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\right)^{2/3}\sqrt{-\sqrt[3]{-1}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)^4}\right)\text{^}(1/15)$$

Decimal approximation

1.644525292523239917059960364310256515415321875844038571830107 8… −

0.036676538882067267251381732980979493225059427509335988438493086…

i

Alternate complex forms

$$1.6445 - 0.0367\,i$$

$$1.6449\,(\cos(-0.022299) + i\sin(-0.022299))$$

$$1.6449\,e^{-0.022299\,i}$$

Polar coordinates

$r = 1.64491$ (radius), $\theta = -0.0223131$ (angle)

$1.64491 \approx \zeta(2) = \pi^2/6 = 1.644934$ (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \sqrt[15]{1633 - 96\,i\,\sqrt{35}}$$

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \sqrt[15]{1 + 32\sqrt[3]{6}\,\sqrt{-\sqrt[3]{-1}\left(\sqrt[3]{27 - 11\sqrt{\frac{35}{3}}} + \sqrt[3]{27 + 11\sqrt{\frac{35}{3}}}\right)^4}}$$

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \sqrt[15]{1 + 192\,\sqrt{-\sqrt[3]{-1}\left(\sqrt[3]{\frac{9}{2} - \frac{11\sqrt{\frac{35}{3}}}{6}} + \sqrt[3]{\frac{9}{2} + \frac{11\sqrt{\frac{35}{3}}}{6}}\right)^4}}$$

sqrt(1/((27*sqrt((((9/2 - (11 sqrt(35/3))/6)^(1/3) + (9/2 + (11 sqrt(35/3))/6)^(1/3))((-((11 sqrt(35/3))/6 - 9/2) (9/2 + (11 sqrt(35/3))/6))^(1/3)))^4)+1)^1/15+(MRB const)^(1-1/(4π)+π))*(4/3))

Input

$$\sqrt{\left(1\Big/\left(\left(27\sqrt{\left(\left(\sqrt[3]{\frac{9}{2}-\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}+\sqrt[3]{\frac{9}{2}+\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)}\right)\sqrt[3]{-\left(\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{1}{6}\left(11\sqrt{\frac{35}{3}}\right)\right)}\right)^4}+1\right)\text{^}\,(1/15)+{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}\right)\times\frac{4}{3}\right)}$$

C_{MRB} is the MRB constant

Exact result

$$2\Bigg/\Bigg(\sqrt{\Bigg(\Bigg(3\Bigg({C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\Bigg(1+27\Bigg(\left(\frac{11\sqrt{\frac{35}{3}}}{6}-\frac{9}{2}\right)\left(\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}\right)\Bigg)^{2/3}\sqrt{-\sqrt[3]{-1}\left(\sqrt[3]{\frac{9}{2}-\frac{11\sqrt{\frac{35}{3}}}{6}}+\sqrt[3]{\frac{9}{2}+\frac{11\sqrt{\frac{35}{3}}}{6}}\right)^4}\Bigg)\text{^}\,(1/15)\Bigg)\Bigg)\Bigg)\Bigg)$$

Decimal approximation

0.900260316022120595294410111065991186153536605181820848787557 47…

+

0.010037646702614743807634848721877189990768396394157282156108616…

i

Alternate complex forms

$0.90026 + 0.01004\, i$

$0.90032\,(\cos(0.011149) + i\sin(0.011149))$

$0.90032\, e^{0.011149\, i}$

Polar coordinates

$r = 0.900316$ (radius), $\theta = 0.0111519$ (angle)

$0.900316 \approx 0.9003163161571\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed form

$\frac{2\sqrt{2}}{\pi} \approx 0.900316\,31615$

Now, we analyze the following equation:

$$x^3 = 6x^2 + 72x + 729.$$

i.e.

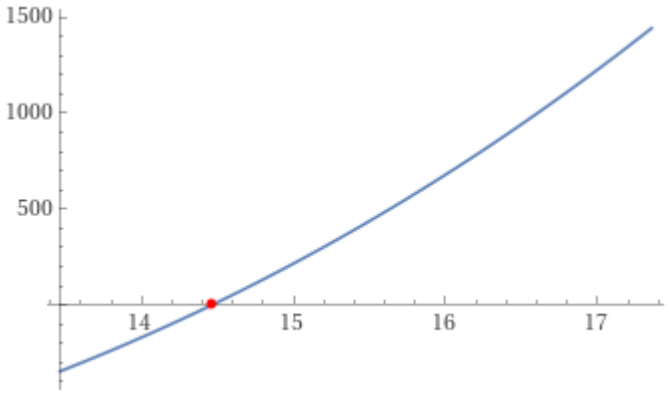
$$x^3 - 6x^2 - 72x - 729 = 0$$

x^3-6x^2-72x-729=0

Input

$$x^3 - 6x^2 - 72x - 729 = 0$$

Root plot (figure that can be related to an open string)



Alternate forms

$$(x - 12)\,x\,(x + 6) - 729 = 0$$

$$x^3 = 6x^2 + 72x + 729$$

$$x^3 - 6x^2 - 72x = 729$$

Real solution

$$x = 2 + \frac{1}{3}\sqrt[3]{\frac{24003}{2} - \frac{1701\sqrt{177}}{2}} + \sqrt[3]{\frac{7}{2}\left(127 + 9\sqrt{177}\right)}$$

Sum of roots

6

Complex solutions

$$x = 2 - \frac{1}{6}\sqrt[3]{\frac{24003}{2} - \frac{1701\sqrt{177}}{2}} - \frac{1}{2}\sqrt[3]{\frac{7}{2}\left(127 + 9\sqrt{177}\right)} + i\left(\frac{\sqrt[3]{\frac{24003}{2} - \frac{1701\sqrt{177}}{2}}}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\sqrt[3]{\frac{7}{2}\left(127 + 9\sqrt{177}\right)}\right)$$

$$x = 2 - \frac{1}{6}\sqrt[3]{\frac{24003}{2} - \frac{1701\sqrt{177}}{2}} - \frac{1}{2}\sqrt[3]{\frac{7}{2}\left(127 + 9\sqrt{177}\right)} + i\left(\frac{1}{2}\sqrt{3}\sqrt[3]{\frac{7}{2}\left(127 + 9\sqrt{177}\right)} - \frac{\sqrt[3]{\frac{24003}{2} - \frac{1701\sqrt{177}}{2}}}{2\sqrt{3}}\right)$$

Product of roots

729

From

$$ax^3 + bx^2 + cx + d = 0$$

The Cardano's Formula is:

$$x = \sqrt[3]{-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a} + \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} + \sqrt[3]{-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a} - \sqrt{\left(-\frac{b^3}{27a^3} + \frac{bc}{6a^2} - \frac{d}{2a}\right)^2 + \left(\frac{c}{3a} - \frac{b^2}{9a^2}\right)^3}} - \frac{b}{3a}$$

$$x^3 - 6x^2 - 72x - 729 = 0$$

(216/27+432/6+729/2+sqrt((216/27+432/6+729/2)^2+(-72/3-36/9)^3))^1/3+(216/27+432/6+729/2-sqrt((216/27+432/6+729/2)^2+(-72/3-36/9)^3))^1/3+2

Input

$$\sqrt[3]{\frac{216}{27}+\frac{432}{6}+\frac{729}{2}+\sqrt{\left(\frac{216}{27}+\frac{432}{6}+\frac{729}{2}\right)^2+\left(-\frac{72}{3}-\frac{36}{9}\right)^3}}+\sqrt[3]{\frac{216}{27}+\frac{432}{6}+\frac{729}{2}-\sqrt{\left(\frac{216}{27}+\frac{432}{6}+\frac{729}{2}\right)^2+\left(-\frac{72}{3}-\frac{36}{9}\right)^3}}+2$$

Exact result

$$2+\sqrt[3]{\frac{889}{2}-\frac{63\sqrt{177}}{2}}+\sqrt[3]{\frac{889}{2}+\frac{63\sqrt{177}}{2}}$$

Decimal approximation

14.463155668846454608585374208532490060088159854256520855737801726…

14.4631556688464546….

Alternate forms

root of $x^3 - 6x^2 - 72x - 729$ near $x = 14.4632$

$$2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}$$

$$2+\frac{1}{3}\sqrt[3]{\frac{24003}{2}-\frac{1701\sqrt{177}}{2}}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}$$

Minimal polynomial

$x^3-6x^2-72x-729$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

From the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume,

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)d^3\times\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the alternate form:

$$2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}$$

we obtain:

(2 + (7/2 (127 - 9 sqrt(177)))^(1/3) + (7/2 (127 + 9 sqrt(177)))^(1/3)) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

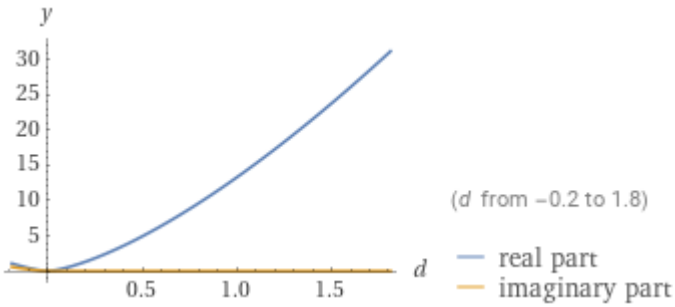
Input

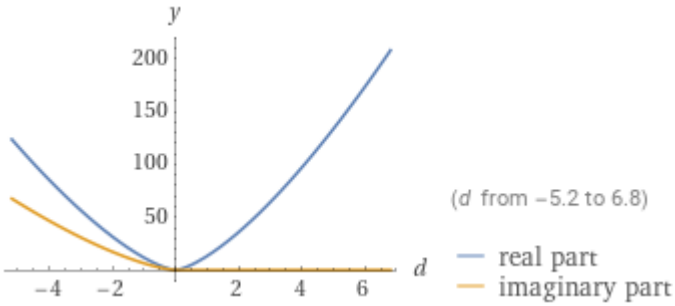
$$\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)$$
$$\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Exact result

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\sqrt[\pi]{\frac{\pi}{4}}$$
$$\sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate forms

$$2^{-(2+\pi)/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

root of $x^3 - 12x^2 - 288x - 5832$ near $x = 28.9263$

$$2^{-1-2/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(4 + 2^{2/3} \sqrt[3]{7\left(127 - 9\sqrt{177}\right)} + 2^{2/3} \sqrt[3]{7\left(127 + 9\sqrt{177}\right)}\right) \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Expanded forms

$$2^{-1/3-2/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \sqrt[3]{7\left(127 + 9\sqrt{177}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9} +$$
$$2^{-1/3-2/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \sqrt[3]{7\left(127 - 9\sqrt{177}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9} +$$
$$2^{1-2/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$3^{-3/(2\pi)} \times 4^{-1/\pi} \sqrt[3]{\frac{889}{2} + \frac{63\sqrt{177}}{2}} \sqrt[\pi]{\pi} \sqrt[2\pi]{5\sqrt{5}\, d^9 + 15 d^9} +$$
$$3^{-3/(2\pi)} \times 4^{-1/\pi} \sqrt[3]{\frac{889}{2} - \frac{63\sqrt{177}}{2}} \sqrt[\pi]{\pi} \sqrt[2\pi]{5\sqrt{5}\, d^9 + 15 d^9} +$$
$$2^{1-2/\pi} \times 3^{-3/(2\pi)} \sqrt[\pi]{\pi} \sqrt[2\pi]{5\sqrt{5}\, d^9 + 15 d^9}$$

Alternate form assuming d>0

$$2^{-(6+\pi)/(3\pi)} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(2\sqrt[3]{2} + \sqrt[3]{889 - 63\sqrt{177}} + \sqrt[3]{889 + 63\sqrt{177}}\right) \sqrt[\pi]{\pi}\, d^{9/(2\pi)}$$

Root

$d = 0$

Series expansion at d=∞

$$3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(2 + \sqrt[3]{\frac{7}{2}\left(127 - 9\sqrt{177}\right)} + \sqrt[3]{\frac{7}{2}\left(127 + 9\sqrt{177}\right)}\right) \sqrt[\pi]{\frac{\pi}{4}}\, d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\left(2 + \sqrt[3]{\frac{7}{2}\left(127 - 9\sqrt{177}\right)} + \sqrt[3]{\frac{7}{2}\left(127 + 9\sqrt{177}\right)}\right) \sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right) d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right) =$$
$$2^{-(2+\pi)/\pi} \times 3^{2-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(2 + \sqrt[3]{\frac{7}{2}\left(127 - 9\sqrt{177}\right)} + \sqrt[3]{\frac{7}{2}\left(127 + 9\sqrt{177}\right)}\right) d^8 \left(d^9\right)^{1/(2\pi)-1} \pi^{1/\pi - 1}$$

Indefinite integral

$$\int 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\sqrt[2\pi]{d^9}\;dd=$$

$$\sqrt[\pi]{\frac{\pi}{4}}$$

$$\frac{1}{1+\frac{9}{2\pi}}3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)$$

$$\sqrt[\pi]{\frac{\pi}{4}}\;d\sqrt[2\pi]{d^9}\;+\text{constant}$$

Global minimum

$$\min\left\{\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right\}=0\;\text{at}\;d=0$$

Series representations

$$\left(2+\sqrt[3]{\frac{1}{2}\left(127-9\sqrt{177}\right)7}+\sqrt[3]{\frac{1}{2}\left(127+9\sqrt{177}\right)7}\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=2^{-1/3-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}$$

$$\sqrt[2\pi]{d^9\,\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\left(\frac{\frac{1}{2}}{k}\right)\right)}\left(2\sqrt[3]{2}+\sqrt[3]{7}\sqrt[3]{127-9\sqrt{176}\sum_{k=0}^{\infty}176^{-k}\left(\frac{\frac{1}{2}}{k}\right)}+\sqrt[3]{7}\sqrt[3]{127+9\sqrt{176}\sum_{k=0}^{\infty}176^{-k}\left(\frac{\frac{1}{2}}{k}\right)}\right)$$

$$\left(2+\sqrt[3]{\frac{1}{2}\left(127-9\sqrt{177}\right)7}+\sqrt[3]{\frac{1}{2}\left(127+9\sqrt{177}\right)7}\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=$$

$$2^{-1/3-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}\left(2\sqrt[3]{2}+\sqrt[3]{7}\sqrt[3]{127-9\sqrt{176}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{176}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\sqrt[3]{7}\sqrt[3]{127+9\sqrt{176}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{176}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$\left(2+\sqrt[3]{\frac{1}{2}\left(127-9\sqrt{177}\right)7}+\sqrt[3]{\frac{1}{2}\left(127+9\sqrt{177}\right)7}\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)d^3\right)}=$$

$$2^{-1/3-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}\left(2\sqrt[3]{2}+\sqrt[3]{7}\sqrt[3]{127-9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(177-z_0)^k z_0^{-k}}{k!}}+\sqrt[3]{7}\sqrt[3]{127+9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(177-z_0)^k z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

And:

(2 + (7/2 (127 - 9 sqrt(177)))^(1/3) + (7/2 (127 + 9 sqrt(177)))^(1/3)) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

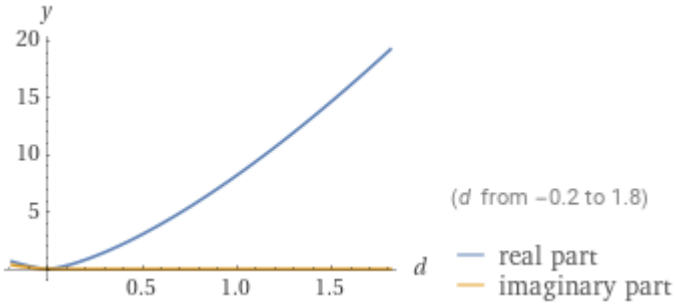
Input

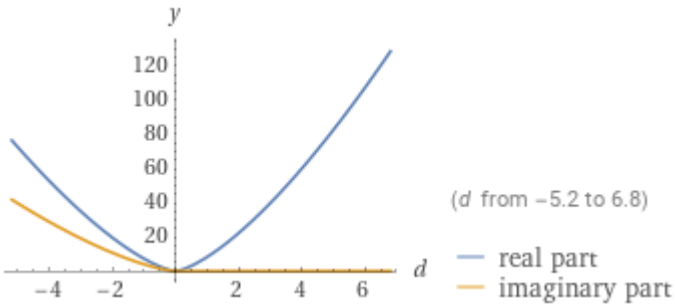
$$\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

Exact result

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Plots **(figures that can be related to the open strings)**





Alternate forms

$$2^{-(1+\pi)/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

root of $x^3 - 12x^2 - 288x - 5832$ near $x = 28.9263$

$$2^{-1-1/\pi} \times 3^{-3/(2\pi)} \left(4 + 2^{2/3} \sqrt[3]{7\left(127 - 9\sqrt{177}\right)} + 2^{2/3} \sqrt[3]{7\left(127 + 9\sqrt{177}\right)}\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Expanded form

$$2^{-1/3-1/\pi} \times 3^{-3/(2\pi)} \sqrt[3]{7\left(127 + 9\sqrt{177}\right)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9} +$$
$$2^{-1/3-1/\pi} \times 3^{-3/(2\pi)} \sqrt[3]{7\left(127 - 9\sqrt{177}\right)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9} + 2^{1-1/\pi} \times 3^{-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Alternate form assuming d>0

$$2^{-(3+\pi)/(3\pi)} \times 3^{-3/(2\pi)} \left(2\sqrt[3]{2} + \sqrt[3]{889 - 63\sqrt{177}} + \sqrt[3]{889 + 63\sqrt{177}}\right) \sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Root

$d = 0$

Series expansion at d=∞

$$2^{-1/\pi} \times 3^{-3/(2\pi)} \left(2 + \sqrt[3]{\frac{7}{2}\left(127 - 9\sqrt{177}\right)} + \sqrt[3]{\frac{7}{2}\left(127 + 9\sqrt{177}\right)}\right) \sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right)=$$

$$2^{-(1+\pi)/\pi}\times3^{2-3/(2\pi)}\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)d^8\left(d^9\right)^{1/(2\pi)-1}\pi^{1/(2\pi)-1}$$

Indefinite integral

$$\int 2^{-1/\pi}\times3^{-3/(2\pi)}\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\sqrt[2\pi]{d^9}\,\sqrt[2\pi]{\pi}\;dd=$$

$$\frac{1}{9+2\pi}2^{(\pi-1)/\pi}\times3^{-3/(2\pi)}\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\pi^{1+1/(2\pi)}\,d\,\sqrt[2\pi]{d^9}+\text{constant}$$

Global minimum

$$\min\left\{\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right\}=0\ \text{at}\ d=0$$

Series representations

$$\left(2+\sqrt[3]{\frac{1}{2}\left(127-9\sqrt{177}\right)7}+\sqrt[3]{\frac{1}{2}\left(127+9\sqrt{177}\right)7}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=$$
$$2^{-1/3-3/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{d^9\,\pi\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}$$
$$\left(2\sqrt[3]{2}+\sqrt[3]{7}\sqrt[3]{127-9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(177-z_0)^k\,z_0^{-k}}{k!}}+\sqrt[3]{7}\sqrt[3]{127+9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(177-z_0)^k\,z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\left(2+\sqrt[3]{\frac{1}{2}\left(127-9\sqrt{177}\right)7}+\sqrt[3]{\frac{1}{2}\left(127+9\sqrt{177}\right)7}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=2^{-1/3-3/(2\pi)}\times 3^{-3/(2\pi)}$$
$$\sqrt[2\pi]{d^9\,\pi\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$
$$\left(2\sqrt[3]{2}+\sqrt[3]{7}\sqrt[3]{127-9\exp\left(i\,\pi\left\lfloor\frac{\arg(177-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(177-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+\sqrt[3]{7}\sqrt[3]{127+9\exp\left(i\,\pi\left\lfloor\frac{\arg(177-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(177-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\left(2+\sqrt[3]{\frac{1}{2}\left(127-9\sqrt{177}\right)7}+\sqrt[3]{\frac{1}{2}\left(127+9\sqrt{177}\right)7}\right)$$

$$\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)\right)4\left(\pi\left(\frac{d}{2}\right)^3\right)}{(3\times 12)\,3}}=2^{-1/3-3/(2\pi)}\times 3^{-3/(2\pi)}$$

$$\sqrt[2\pi]{d^9\,\pi\left(\frac{1}{z_0}\right)^{\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1+\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}$$

$$\left(2\sqrt[3]{2}+\sqrt[3]{7}\left(127-9\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(177-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(177-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(177-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)+\sqrt[3]{7}\left(127+9\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(177-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(177-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(177-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Dividing the two exact results

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{d^9}$$

and

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\left(2+\sqrt[3]{\frac{7}{2}\left(127-9\sqrt{177}\right)}+\sqrt[3]{\frac{7}{2}\left(127+9\sqrt{177}\right)}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

we obtain:

((3^(-3/(2 π)) (5 (3 + sqrt(5)))^(1/(2 π)) (π/4)^(1/π) (d^9)^(1/(2 π))))/((2^(-1/π) 3^(-3/(2 π)) π^(1/(2 π)) (d^9)^(1/(2 π))))

Input

$$\frac{3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\,\sqrt[\pi]{\frac{\pi}{4}}\,\sqrt[2\pi]{d^9}}{2^{-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}\right)}{2^{-1/\pi}\times 3^{-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}} = 2^{-1/\pi}\ \sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}\right)}{2^{-1/\pi}\times 3^{-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}} =$$
$$2^{-1/\pi}\ \sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{3^{-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}\right)}{2^{-1/\pi}\times 3^{-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}} =$$
$$2^{-1/\pi}\ \sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535884…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298888445046920
…

1.644934066846…. ≈ ζ(2) = π²/6 = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\left(\frac{\frac{1}{2}}{k}\right)\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.92478440155986\,\pi}\,\sqrt{\pi}^2}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From

SUPERCONFORMAL GRAVITY AND THE TOPOLOGY OF DIFFEOMORPHISM GROUPS - *Jay Cushing, Gregory W. Moore, Martin Roček, Vivek Saxena* - arXiv:2311.08394v2 [hep-th] 17 Nov 2023

We analyze the following equation:

$$\Lambda_{\mu\nu} = -2\left(\operatorname{Im}\tau\right)F^{-}_{\rho(\mu}\chi_{\nu)}{}^{\rho} - \frac{i}{\sqrt{2}}g_{\mu\nu}\left(\operatorname{Im}\tau\right)\psi_\rho\boldsymbol{\nabla}^\rho\overline{a} + \frac{i}{\sqrt{2}}\left(\operatorname{Im}\tau\right)\psi_{(\mu}\boldsymbol{\nabla}_{\nu)}\overline{a}$$
$$+\frac{i}{8\sqrt{2}}\mathcal{F}'''\left(\psi_\mu\psi_\rho\right)^{-}\chi^\rho{}_\nu + \frac{i}{8\sqrt{2}}\mathcal{F}'''\left(\psi_\nu\psi_\rho\right)^{-}\chi^\rho{}_\mu + \frac{1}{12\sqrt{2}}g_{\mu\nu}\overline{\mathcal{F}}'''\chi_\kappa{}^\rho\chi^{\kappa\sigma}\chi_{\rho\sigma}\ .$$

-2*F-i/√2*g*ψ*a+i/√2*ψ*a+i/(8√2)*F*(ψ*ψ)*χ+
i/(8√2)*F*(ψ*ψ)*χ+1/(12√2)*g*F*χ*χ*χ

Input

$$-2F - \frac{i}{\sqrt{2}} g \psi a + \frac{i}{\sqrt{2}} \psi a + \frac{i}{8\sqrt{2}} F(\psi\psi)\chi + \frac{i}{8\sqrt{2}} F(\psi\psi)\chi + \frac{1}{12\sqrt{2}} g F \chi\chi\chi$$

i is the imaginary unit

Result

$$-\frac{i a g \psi}{\sqrt{2}} + \frac{i a \psi}{\sqrt{2}} + \frac{F g \chi^3}{12\sqrt{2}} + \frac{i F \chi \psi^2}{4\sqrt{2}} - 2F$$

Alternate forms

$$\frac{1}{24}\left(-12 i \sqrt{2}\, a g \psi + i\, 12\sqrt{2}\, a \psi + \sqrt{2}\, F g \chi^3 + i\, 3\sqrt{2}\, F \chi \psi^2 - 48 F\right)$$

$$\frac{1}{24} F\left(-48 + \sqrt{2}\, \chi\left(g \chi^2 + 3 i \psi^2\right)\right) - \frac{i a (g-1) \psi}{\sqrt{2}}$$

$$\frac{1}{24} F\left(\sqrt{2}\, g \chi^3 + 3 i \sqrt{2}\, \chi \psi^2 - 48\right) - \frac{i a (g-1) \psi}{\sqrt{2}}$$

Alternate form assuming a, F, g, χ, and ψ are real

$$\frac{1}{24} F\left(\sqrt{2}\, g \chi^3 - 48\right) + \frac{i \psi (F \chi \psi - 4 a (g-1))}{4\sqrt{2}}$$

Roots

From the result (2nd degree equation type), we obtain:

$$F\,\chi \neq 0, \quad \psi = \frac{-i\sqrt{3}\,\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\sqrt{2}\,F^2\,\chi}+6\,a\,g-6\,a}{3\,F\,\chi}$$

$$F\,\chi \neq 0, \quad \psi = \frac{i\sqrt{3}\,\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\sqrt{2}\,F^2\,\chi}+6\,a\,g-6\,a}{3\,F\,\chi}$$

$$F=0, \quad a\neq 0, \quad g=1$$

$$F=0, \quad a\,(g-1)\neq 0, \quad \psi=0$$

$$F\neq 0, \quad \chi=0, \quad a\,(g-1)\neq 0, \quad \psi=\frac{2\,i\sqrt{2}\,F}{a\,(g-1)}$$

Root

$$a=0, \quad F=0$$

Polynomial discriminant

$$\Delta_\chi = \frac{1}{768\sqrt{2}}\left(36\sqrt{2}\,a^2\,F^2\,g^4\,\psi^2-72\sqrt{2}\,a^2\,F^2\,g^3\,\psi^2+36\sqrt{2}\,a^2\,F^2\,g^2\,\psi^2-288\,i\,a\,F^3\,g^3\,\psi+288\,i\,a\,F^3\,g^2\,\psi-288\sqrt{2}\,F^4\,g^2+i\sqrt{2}\,F^4\,g\,\psi^6\right)$$

Roots for the variable ψ

$$\psi = -\frac{2i\sqrt{2}\left(-\frac{\sqrt{-6a^2g^2+12a^2g-6a^2-\frac{1}{2}iF^2g\chi^4+12i\sqrt{2}F^2\chi}}{2\sqrt{3}}+\frac{iag}{\sqrt{2}}-\frac{ia}{\sqrt{2}}\right)}{F\chi}$$

$$\psi = -\frac{2i\sqrt{2}\left(\frac{\sqrt{-6a^2g^2+12a^2g-6a^2-\frac{1}{2}iF^2g\chi^4+12i\sqrt{2}F^2\chi}}{2\sqrt{3}}+\frac{iag}{\sqrt{2}}-\frac{ia}{\sqrt{2}}\right)}{F\chi}$$

Derivative

$$\frac{\partial}{\partial a}\left(-2F-\frac{igψa}{\sqrt{2}}+\frac{iψa}{\sqrt{2}}+\frac{iF(ψψ)\chi}{8\sqrt{2}}+\frac{iF(ψψ)\chi}{8\sqrt{2}}+\frac{gF\chi\chi\chi}{12\sqrt{2}}\right)=-\frac{i(g-1)ψ}{\sqrt{2}}$$

Indefinite integral

$$\int\left(-2F+\frac{Fg\chi^3}{12\sqrt{2}}+\frac{iaψ}{\sqrt{2}}-\frac{iagψ}{\sqrt{2}}+\frac{iF\chi ψ^2}{4\sqrt{2}}\right)da=$$
$$-\frac{ia^2gψ}{2\sqrt{2}}+\frac{ia^2ψ}{2\sqrt{2}}+\frac{aFg\chi^3}{12\sqrt{2}}+\frac{iaF\chi ψ^2}{4\sqrt{2}}-2aF+\text{constant}$$

Definite integral over a hypercube of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L}\left(-2F+\frac{Fg\chi^3}{12\sqrt{2}}+\frac{iaψ}{\sqrt{2}}-\frac{iagψ}{\sqrt{2}}+\frac{iF\chi ψ^2}{4\sqrt{2}}\right)dψ\,d\chi\,dg\,dF\,da=$$
$$0$$

From the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3\times\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\times\frac{1}{3}\sqrt{2}d^3\times\frac{\sqrt{2}}{12}d^3}$$

Thence, from this root

$$F\,\chi\neq 0,$$

$$\psi=\frac{i\sqrt{3}\,\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\sqrt{2}\,F^2\,\chi}+6\,a\,g-6\,a}{3\,F\,\chi}$$

we obtain:

(((i sqrt(3) sqrt(-12 a^2 g^2 + 24 a^2 g - 12 a^2 - i F^2 g χ^4 + 24 i sqrt(2) F^2 χ) + 6 a g - 6 a)/(3 F χ))) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

Input

$$\frac{i\sqrt{3}\ \sqrt{-12a^2g^2+24a^2g-12a^2-iF^2g\chi^4+24i\sqrt{2}\ F^2\chi}+6ag-6a}{3F\chi\ \sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}}$$

i is the imaginary unit

Exact result

$$\frac{1}{F\chi}3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}\left(i\sqrt{3}\ \sqrt{-12a^2g^2+24a^2g-12a^2-iF^2g\chi^4+24i\sqrt{2}\ F^2\chi}+6ag-6a\right)$$

Expanded form

$$\frac{1}{F\chi}i\,3^{-1/2-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}\ \sqrt{-12a^2g^2+24a^2g-12a^2-iF^2g\chi^4+24i\sqrt{2}\ F^2\chi}+\frac{2^{1-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ a\ \sqrt[2\pi]{d^9}\ g}{F\chi}-\frac{2^{1-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ a\ \sqrt[2\pi]{d^9}}{F\chi}$$

Alternate form assuming a, d, F, g, and χ are positive

$$\frac{1}{F\chi}3^{-(3+\pi)/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ d^{9/(2\pi)}\left(2\sqrt{3}\ a(g-1)+i\sqrt{-12a^2(g-1)^2-iF^2\chi\left(g\chi^3-24\sqrt{2}\right)}\right)$$

Series expansion at a=0

$$\frac{i\,3^{-(3+\pi)/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\frac{\pi}{4}}\;\sqrt[2\pi]{d^9}\;\sqrt{-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}}{F\,\chi}+$$
$$\frac{2^{(\pi-2)/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\pi}\;a\;\sqrt[2\pi]{d^9}\;(g-1)}{F\,\chi}-$$
$$\frac{i\,2^{(\pi-2)/\pi}\times 3^{(\pi-3)/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\pi}\;a^2\;\sqrt[2\pi]{d^9}\;(g-1)^2}{F\,\chi\sqrt{-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}}-$$
$$\frac{i\,2^{(\pi-2)/\pi}\times 3^{(3(\pi-1))/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\pi}\;a^4\;\sqrt[2\pi]{d^9}\;(g-1)^4}{F\,\chi\left(-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)\right)^{3/2}}+O\left(a^5\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial a}\left(\frac{1}{3\,F\,\chi}\left(i\sqrt{3}\;\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\sqrt{2}\;F^2\,\chi}+6\,a\,g-6\,a\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$
$$\frac{1}{F\,\chi}3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\frac{\pi}{4}}\;\sqrt[2\pi]{d^9}$$
$$\left(-\frac{12\,i\,a\,(g-1)^2}{\sqrt{-4\,a^2\,(g-1)^2-\frac{1}{3}\,i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}}+6\,g-6\right)$$

Indefinite integral

$$\int \frac{1}{F\chi} 3^{-1-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[2\pi]{d^9}\ \sqrt[\pi]{\frac{\pi}{4}} \left(-6a+6ag+i\sqrt{3}\ da = \sqrt{-12a^2+24a^2g-12a^2g^2+24i\sqrt{2}\ F^2\chi - iF^2g\chi^4}\right)$$

$$\frac{1}{F\chi} 3^{-1-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}$$
$$\left(\frac{1}{2} i\sqrt{3}\ a\sqrt{-12a^2g^2+24a^2g-12a^2-iF^2g\chi^4+24i\sqrt{2}\ F^2\chi}\ + \frac{1}{4(g-1)} iF^2\chi\left(g\chi^3-24\sqrt{2}\right)\right.$$
$$\log\left(2\sqrt{-12a^2g^2+24a^2g-12a^2-iF^2g\chi^4+24i\sqrt{2}\ F^2\chi}\ - 4i\sqrt{3}\ a(g-1)\right) + 3a^2(g-1)\Big) + \text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

And:

(((i sqrt(3) sqrt(-12 a^2 g^2 + 24 a^2 g - 12 a^2 - i F^2 g χ^4 + 24 i sqrt(2) F^2 χ) + 6 a g - 6 a)/(3 F χ))) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

Input

$$\frac{i\sqrt{3}\ \sqrt{-12a^2g^2+24a^2g-12a^2-iF^2g\chi^4+24i\sqrt{2}\ F^2\chi}\ +6ag-6a}{3F\chi}$$
$$\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\ d^3\right)\left(\frac{\sqrt{2}}{12}\ d^3\right)}$$

i is the imaginary unit

Exact result

$$\frac{1}{F\chi} 2^{-1/\pi} \times 3^{-1-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}\ \left(i\sqrt{3}\right.$$
$$\left.\sqrt{-12a^2g^2+24a^2g-12a^2-iF^2g\chi^4+24i\sqrt{2}\ F^2\chi}\ +6ag-6a\right)$$

Alternate form

$$\sqrt[2\pi]{d^9}\left(\frac{2^{1-1/\pi}\,a\left(3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\;g-3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\right)}{F\,\chi}+\frac{i\,2^{-1/\pi}\times 3^{-1/2-3/(2\pi)}\sqrt[2\pi]{\pi}\;\sqrt{-12\,a^2\left(g^2-2\,g+1\right)-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}}{F\,\chi}\right)$$

Expanded form

$$\frac{1}{F\,\chi}i\,2^{-1/\pi}\times 3^{-1/2-3/(2\pi)}\sqrt[2\pi]{\pi}\;\sqrt[2\pi]{d^9}\;\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\sqrt{2}\;F^2\,\chi}+\frac{2^{1-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\;a\;\sqrt[2\pi]{d^9}\;g}{F\,\chi}-\frac{2^{1-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\;a\;\sqrt[2\pi]{d^9}}{F\,\chi}$$

Alternate forms assuming a, d, F, g, and χ are positive

$$\frac{1}{F\,\chi}2^{-1/\pi}\times 3^{-(3+\pi)/(2\pi)}\sqrt[2\pi]{\pi}\;d^{9/(2\pi)}\left(2\sqrt{3}\;a\,(g-1)+i\sqrt{-12\,a^2\,(g-1)^2-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}\right)$$

$$\frac{1}{F\,\chi}2^{-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{\pi}\;d^{9/(2\pi)}\left(6\,a\,(g-1)+i\sqrt{-36\,a^2\,(g-1)^2-3\,i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}\right)$$

Series expansion at a=0

$$\frac{i\,2^{-1/\pi}\times 3^{-(3+\pi)/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\,\sqrt{-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}}{F\,\chi}+$$
$$\frac{2^{(\pi-1)/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;a\,\sqrt[2\pi]{d^9}\;(g-1)}{F\,\chi}-$$
$$\frac{i\,2^{(\pi-1)/\pi}\times 3^{(\pi-3)/(2\pi)}\,\sqrt[2\pi]{\pi}\;a^2\,\sqrt[2\pi]{d^9}\;(g-1)^2}{F\,\chi\,\sqrt{-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}}-$$
$$\frac{i\,2^{(\pi-1)/\pi}\times 3^{(3(\pi-1))/(2\pi)}\,\sqrt[2\pi]{\pi}\;a^4\,\sqrt[2\pi]{d^9}\;(g-1)^4}{F\,\chi\left(-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)\right)^{3/2}}+O\left(a^5\right)$$

(Taylor series)

Series expansion at a=∞

$$\frac{2^{(\pi-1)/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;a\,\sqrt[2\pi]{d^9}\;\left(a\,(g-1)+i\sqrt{-a^2\,(g-1)^2}\right)}{a\,F\,\chi}+$$
$$\frac{2^{-2-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;a\,\sqrt[2\pi]{d^9}\;F\left(g\,\chi^3-24\sqrt{2}\right)}{a\,\sqrt{-a^2\,(g-1)^2}}+$$
$$\frac{i\,2^{-6-1/\pi}\times 3^{-2-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;a^3\,\sqrt[2\pi]{d^9}\;F^3\,\chi\left(g\,\chi^3-24\sqrt{2}\right)^2}{a^3\left(-a^2\,(g-1)^2\right)^{3/2}}+$$
$$\frac{2^{-9-1/\pi}\times 3^{-3-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;\sqrt[2\pi]{d^9}\;F^5\,\chi^2\,\sqrt{-a^2\,(g-1)^2}\;\left(g\,\chi^3-24\sqrt{2}\right)^3}{a\,a^5\,(g-1)^6}+O\left(\left(\frac{1}{a}\right)^9\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial a}\left(\frac{1}{3\,F\,\chi}\left(i\sqrt{3}\,\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\sqrt{2}\,F^2\,\chi}\,+\right.\right.$$
$$\left.\left.6\,a\,g-6\,a\right)\sqrt[2\pi]{\frac{\left(4\,\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right)=$$
$$\frac{2^{-1/\pi}\times3^{-1-3/(2\pi)}\,\sqrt[2\pi]{d^9}\,\sqrt[2\pi]{\pi}\left(-\frac{12\,i\,a\,(g-1)^2}{\sqrt{-4\,a^2\,(g-1)^2-\frac{1}{3}\,i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}}+6\,g-6\right)}{F\,\chi}$$

Indefinite integral

$$\int\frac{2^{-1/\pi}\times3^{-1-3/(2\pi)}\,\sqrt[2\pi]{d^9}\,\sqrt[2\pi]{\pi}\left(-6\,a+6\,a\,g+i\sqrt{3}\,\sqrt{-12\,a^2+24\,a^2\,g-12\,a^2\,g^2+24\,i\sqrt{2}\,F^2\,\chi-i\,F^2\,g\,\chi^4}\right)}{F\,\chi}\,da=\frac{1}{F\,\chi}2^{-1/\pi}\times3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$
$$\left(\frac{1}{2}\,i\sqrt{3}\,a\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\sqrt{2}\,F^2\,\chi}\,+\right.$$
$$\frac{1}{4\,(g-1)}\,i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)$$
$$\log\left(2\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\sqrt{2}\,F^2\,\chi}\,-\right.$$
$$\left.\left.4\,i\sqrt{3}\,a\,(g-1)\right)+3\,a^2\,(g-1)\right)+\text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

Dividing the two exact results

$$\frac{1}{F\,\chi}3^{-1-3/(2\pi)}\,\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\,\sqrt[\pi]{\frac{\pi}{4}}\,\sqrt[2\pi]{d^9}\left(i\sqrt{3}\right.$$
$$\left.\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\sqrt{2}\,F^2\,\chi}\,+6\,a\,g-6\,a\right)$$

and

$$\frac{1}{F\,\chi}2^{-1/\pi}\times3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\left(i\sqrt{3}\right.$$
$$\left.\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\sqrt{2}\,F^2\,\chi}\,+6\,a\,g-6\,a\right)$$

we obtain:

(((3^(-1 - 3/(2 π)) (5 (3 + sqrt(5)))^(1/(2 π)) (π/4)^(1/π)))/(((2^(-1/π) 3^(-1 - 3/(2 π)) π^(1/(2 π))))))

Input

$$\frac{3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[\pi]{\frac{\pi}{4}}}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{\pi}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{3^{-1-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{\pi}}=2^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-1-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\ \sqrt[2\pi]{\pi}} = 2^{-1/\pi}\ \sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{3^{-1-3/(2\pi)}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\right)}{2^{-1/\pi}\times 3^{-1-3/(2\pi)}\ \sqrt[2\pi]{\pi}} =$$

$$2^{-1/\pi}\ \sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\ 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From this other root for variable ψ

$$\psi = -\frac{2i\sqrt{2}\left(\frac{\sqrt{-6a^2g^2+12a^2g-6a^2-\frac{1}{2}iF^2g\chi^4+12i\sqrt{2}F^2\chi}}{2\sqrt{3}}+\frac{iag}{\sqrt{2}}-\frac{ia}{\sqrt{2}}\right)}{F\chi}$$

we obtain:

(-(2 i sqrt(2) (-(i a)/sqrt(2) + (i a g)/sqrt(2) + sqrt(-6 a^2 + 12 a^2 g - 6 a^2 g^2 + 12 i sqrt(2) F^2 χ - 1/2 i F^2 g χ^4)/(2 sqrt(3))))/(F χ)) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

Input

$$-\frac{2i\sqrt{2}\left(-\frac{ia}{\sqrt{2}}+\frac{iag}{\sqrt{2}}+\frac{\sqrt{-6a^2+12a^2g-6a^2g^2+12i\sqrt{2}F^2\chi-\frac{1}{2}iF^2g\chi^4}}{2\sqrt{3}}\right)}{F\chi\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}}$$

i is the imaginary unit

Exact result

$$-\frac{1}{F\chi}i\,2^{3/2-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\left(\frac{\sqrt{-6a^2g^2+12a^2g-6a^2-\frac{1}{2}iF^2g\chi^4+12i\sqrt{2}F^2\chi}}{2\sqrt{3}}+\frac{iag}{\sqrt{2}}-\frac{ia}{\sqrt{2}}\right)$$

Alternate form

$$\frac{1}{F\chi}3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}\left(6a(g-1)-i\sqrt{-36a^2(g-1)^2-3iF^2\chi\left(g\chi^3-24\sqrt{2}\right)}\right)$$

Expanded form

$$-\frac{1}{F\chi}i\,2^{1/2-2/\pi}\times 3^{-1/2-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\pi}\;\sqrt[2\pi]{d^9}$$
$$\sqrt{-6a^2g^2+12a^2g-6a^2-\frac{1}{2}iF^2g\chi^4+12i\sqrt{2}\,F^2\chi}\;+$$
$$\frac{2^{1-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\pi}\;a\sqrt[2\pi]{d^9}\;g}{F\chi}-$$
$$\frac{2^{1-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\pi}\;a\sqrt[2\pi]{d^9}}{F\chi}$$

Alternate form assuming a, d, F, g, and χ are positive

$$\frac{1}{F\chi}3^{-(3+\pi)/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\frac{\pi}{4}}\;d^{9/(2\pi)}\left(2\sqrt{3}\,a(g-1)-i\right.$$
$$\left.\sqrt{-12a^2(g-1)^2-iF^2\chi\left(g\chi^3-24\sqrt{2}\right)}\right)$$

Derivative

$$\frac{\partial}{\partial a}\left(-\frac{1}{F\chi}\left(2i\sqrt{2}\left(-\frac{ia}{\sqrt{2}}+\frac{iag}{\sqrt{2}}+\frac{\sqrt{-6a^2+12a^2g-6a^2g^2+12i\sqrt{2}\,F^2\chi-\frac{1}{2}iF^2g\chi^4}}{2\sqrt{3}}\right)\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$

$$\left(2^{(\pi-2)/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[2\pi]{d^9}\,(g-1)\sqrt[\pi]{\pi}\left(\sqrt{-12a^2(g-1)^2-iF^2\chi\left(g\chi^3-24\sqrt{2}\right)}+2i\sqrt{3}\,a(g-1)\right)\right)\Big/\left(F\chi\sqrt{-12a^2(g-1)^2-iF^2\chi\left(g\chi^3-24\sqrt{2}\right)}\right)$$

Indefinite integral

$$\int -\frac{1}{F\chi}i\,2^{3/2-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[2\pi]{d^9}\sqrt[\pi]{\pi}\left(-\frac{ia}{\sqrt{2}}+\frac{iag}{\sqrt{2}}+\frac{\sqrt{-6a^2+12a^2g-6a^2g^2+12i\sqrt{2}\,F^2\chi-\frac{1}{2}iF^2g\chi^4}}{2\sqrt{3}}\right)da=$$

$$\frac{1}{F\chi}3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{d^9}\left(-\frac{1}{2}i\sqrt{3}\,a\sqrt{-12a^2g^2+24a^2g-12a^2-iF^2g\chi^4+24i\sqrt{2}\,F^2\chi}-\frac{1}{4(g-1)}iF^2\chi\left(g\chi^3-24\sqrt{2}\right)\log\left(2\sqrt{-12a^2g^2+24a^2g-12a^2-iF^2g\chi^4+24i\sqrt{2}\,F^2\chi}-4i\sqrt{3}\,a(g-1)\right)+3a^2(g-1)\right)+\text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

And:

(-(2 i sqrt(2) (-(i a)/sqrt(2) + (i a g)/sqrt(2) + sqrt(-6 a^2 + 12 a^2 g - 6 a^2 g^2 + 12 i sqrt(2) F^2 χ - 1/2 i F^2 g χ^4)/(2 sqrt(3))))/(F χ)) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

Input

$$-\frac{2\,i\,\sqrt{2}\left(-\frac{i\,a}{\sqrt{2}}+\frac{i\,a\,g}{\sqrt{2}}+\frac{\sqrt{-6\,a^2+12\,a^2\,g-6\,a^2\,g^2+12\,i\,\sqrt{2}\,F^2\,\chi-\frac{1}{2}\,i\,F^2\,g\,\chi^4}}{2\sqrt{3}}\right)}{F\,\chi}$$
$$\sqrt[2\pi]{\left(\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\,\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

i is the imaginary unit

Exact result

$$-\frac{1}{F\,\chi}\,i\,2^{3/2-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$
$$\left(\frac{\sqrt{-6\,a^2\,g^2+12\,a^2\,g-6\,a^2-\frac{1}{2}\,i\,F^2\,g\,\chi^4+12\,i\,\sqrt{2}\,F^2\,\chi}}{2\sqrt{3}}+\frac{i\,a\,g}{\sqrt{2}}-\frac{i\,a}{\sqrt{2}}\right)$$

Alternate forms

$$\frac{1}{F\,\chi}\,2^{-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\left(6\,a\,(g-1)-i\right.$$
$$\left.\sqrt{-36\,a^2\,(g-1)^2-3\,i\,F^2\,\chi\left(g\,\chi^3-24\,\sqrt{2}\right)}\right)$$

$$-\frac{1}{F\,\chi}\,i\,2^{-1/\pi}\times 3^{-1/2-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$
$$\left(\sqrt{-12\,a^2\left(g^2-2\,g+1\right)-i\,F^2\,\chi\left(g\,\chi^3-24\,\sqrt{2}\right)}+2\,i\,\sqrt{3}\,a\,(g-1)\right)$$

Expanded form

$$-\frac{1}{F\,\chi}i\,2^{1/2-1/\pi}\times 3^{-1/2-3/(2\pi)}\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\sqrt{-6a^2g^2+12a^2g-6a^2-\frac{1}{2}i\,F^2\,g\,\chi^4+12i\sqrt{2}\,F^2\,\chi}+\frac{2^{1-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\,a\,\sqrt[2\pi]{d^9}\,g}{F\,\chi}-\frac{2^{1-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\,a\,\sqrt[2\pi]{d^9}}{F\,\chi}$$

Alternate form assuming a, d, F, g, and χ are positive

$$\frac{1}{F\,\chi}2^{-1/\pi}\times 3^{-(3+\pi)/(2\pi)}\sqrt[2\pi]{\pi}\,d^{9/(2\pi)}\left(2\sqrt{3}\,a\,(g-1)-i\sqrt{-12a^2(g-1)^2-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}\right)$$

Series expansion at a=0

$$-\frac{i\,2^{-1/\pi}\times 3^{-(3+\pi)/(2\pi)}\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\,\sqrt{-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}}{F\,\chi}+\frac{2^{(\pi-1)/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\,a\,\sqrt[2\pi]{d^9}\,(g-1)}{F\,\chi}+\frac{i\,2^{(\pi-1)/\pi}\times 3^{(\pi-3)/(2\pi)}\sqrt[2\pi]{\pi}\,a^2\,\sqrt[2\pi]{d^9}\,(g-1)^2}{F\,\chi\sqrt{-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}}+\frac{i\,2^{(\pi-1)/\pi}\times 3^{(3(\pi-1))/(2\pi)}\sqrt[2\pi]{\pi}\,a^4\,\sqrt[2\pi]{d^9}\,(g-1)^4}{F\,\chi\left(-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)\right)^{3/2}}+O\left(a^5\right)$$

(Taylor series)

Series expansion at a=∞

$$\frac{2^{(\pi-1)/\pi}\times 3^{-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ a\ \sqrt[2\pi]{d^9}\ \left(a\,(g-1)-i\sqrt{-a^2\,(g-1)^2}\right)}{a\,F\,\chi}+$$
$$\frac{2^{-2-1/\pi}\times 3^{-1-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ a\ \sqrt[2\pi]{d^9}\ F\left(24\sqrt{2}-g\,\chi^3\right)}{a\sqrt{-a^2\,(g-1)^2}}-$$
$$\frac{i\,2^{-6-1/\pi}\times 3^{-2-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ a^3\ \sqrt[2\pi]{d^9}\ F^3\,\chi\left(g\,\chi^3-24\sqrt{2}\right)^2}{a^3\left(-a^2\,(g-1)^2\right)^{3/2}}+$$
$$\frac{2^{-9-1/\pi}\times 3^{-3-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ a\ \sqrt[2\pi]{d^9}\ F^5\,\chi^2\left(g\,\chi^3-24\sqrt{2}\right)^3}{a^5\,(g-1)^4\sqrt{-a^2\,(g-1)^2}}+O\left(\left(\frac{1}{a}\right)^9\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial a}\left(-\frac{1}{F\,\chi}\left(2\,i\sqrt{2}\left(-\frac{i\,a}{\sqrt{2}}+\frac{i\,a\,g}{\sqrt{2}}+\frac{\sqrt{-6\,a^2+12\,a^2\,g-6\,a^2\,g^2+12\,i\sqrt{2}\ F^2\,\chi-\frac{1}{2}\,i\,F^2\,g\,\chi^4}}{2\sqrt{3}}\right)\right)\right.$$
$$\left.\sqrt[2\pi]{\frac{\left(4\,\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)}{3\times 3\times 12}}\right)=\left(2^{(\pi-1)/\pi}\times 3^{-3/(2\pi)}\ \sqrt[2\pi]{d^9}\ (g-1)\right.$$
$$\left.\sqrt[2\pi]{\pi}\left(\sqrt{-12\,a^2\,(g-1)^2-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}+2\,i\sqrt{3}\ a\,(g-1)\right)\right)\Big/$$
$$\left(F\,\chi\sqrt{-12\,a^2\,(g-1)^2-i\,F^2\,\chi\left(g\,\chi^3-24\sqrt{2}\right)}\right)$$

Indefinite integral

$$\int -\frac{i\,2^{3/2-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{d^9}\,\sqrt[2\pi]{\pi}\left(-\frac{i\,a}{\sqrt{2}}+\frac{i\,a\,g}{\sqrt{2}}+\frac{\sqrt{-6\,a^2+12\,a^2\,g-6\,a^2\,g^2+12\,i\,\sqrt{2}\,F^2\,\chi-\frac{1}{2}\,i\,F^2\,g\,\chi^4}}{2\,\sqrt{3}}\right)}{F\,\chi}$$

$$da = \frac{1}{F\,\chi}2^{-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$
$$\left(-\frac{1}{2}\,i\,\sqrt{3}\;a\,\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\,\sqrt{2}\;F^2\,\chi}\;-\right.$$
$$\frac{1}{4\,(g-1)}\,i\,F^2\,\chi\left(g\,\chi^3-24\,\sqrt{2}\right)$$
$$\log\left(2\,\sqrt{-12\,a^2\,g^2+24\,a^2\,g-12\,a^2-i\,F^2\,g\,\chi^4+24\,i\,\sqrt{2}\;F^2\,\chi}\;-\right.$$
$$\left.\left.4\,i\,\sqrt{3}\;a\,(g-1)\right)+3\,a^2\,(g-1)\right)+\text{constant}$$

(assuming a complex-valued logarithm)

$\log(x)$ is the natural logarithm

Dividing the two exact results

$$-\frac{1}{F\,\chi}\,i\,2^{3/2-2/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\pi}\,\sqrt[2\pi]{d^9}$$
$$\left(\frac{\sqrt{-6\,a^2\,g^2+12\,a^2\,g-6\,a^2-\frac{1}{2}\,i\,F^2\,g\,\chi^4+12\,i\,\sqrt{2}\;F^2\,\chi}}{2\,\sqrt{3}}+\frac{i\,a\,g}{\sqrt{2}}-\frac{i\,a}{\sqrt{2}}\right)$$

and

$$-\frac{1}{F\,\chi}\,i\,2^{3/2-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$
$$\left(\frac{\sqrt{-6\,a^2\,g^2+12\,a^2\,g-6\,a^2-\frac{1}{2}\,i\,F^2\,g\,\chi^4+12\,i\,\sqrt{2}\;F^2\,\chi}}{2\,\sqrt{3}}+\frac{i\,a\,g}{\sqrt{2}}-\frac{i\,a}{\sqrt{2}}\right)$$

we obtain:

((-(i 2^(3/2 - 2/π) 3^(-3/(2 π)) (5 (3 + sqrt(5)))^(1/(2 π)) π^(1/π)))/((-(i 2^(3/2 - 1/π) 3^(-3/(2 π)) π^(1/(2 π))))

Input

$$\frac{-\left(i\times 2^{3/2-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\right)}{-\left(i\times 2^{3/2-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\right)}$$

i is the imaginary unit

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{-\left(i\,2^{3/2-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\right)}{-\left(i\,2^{3/2-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\right)}=$$

$$2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{-\left(i\,2^{3/2-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\right)}{-\left(i\,2^{3/2-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\right)}=$$

$$2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{-\left(i\,2^{3/2-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\right)}{-\left(i\,2^{3/2-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\right)}=$$

$$2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the both roots analyzed, from the exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13

…

3.1415926535884…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298884450469 20

…

1.64493406684…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.618008545900 00^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\, e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900 00^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\, e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900 00^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\, e^{1.92478440155986\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

We analyze the following equation:

$$\int^{\infty} y^{\frac{1}{2}b-1} e^{-2\pi y\lambda_+^2} dy = \frac{\Gamma(b/2)}{\left(\sqrt{2\pi}|\lambda_+|\right)^b} \left(1+\mathcal{O}(|\lambda_+|)\right) \ .$$

We consider:

(Gamma(b/2))/((sqrt(2π)*λ)^b) * (1+λ)

Input

$$\frac{\Gamma\left(\frac{b}{2}\right)}{\left(\sqrt{2\,\pi}\ \lambda\right)^b}\ (1+\lambda)$$

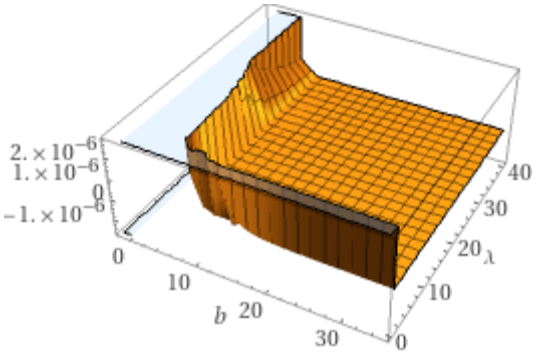
$\Gamma(x)$ is the gamma function

Exact result

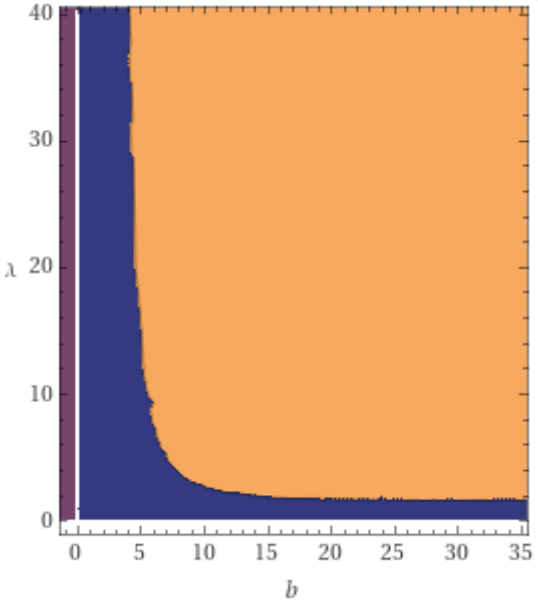
$$(2\,\pi)^{-b/2}\ (\lambda+1)\ \lambda^{-b}\ \Gamma\left(\frac{b}{2}\right)$$

3D plot **(figure that can be related to a D-brane/Instanton)**

The key observation from the above plots and is that at $b = 1$, which is taken as the energy density of the universe at the Big Bang, with $\lambda = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.



Contour plot



Alternate form

$$\frac{2^{1-b/2}\,\pi^{-b/2}\,(\lambda+1)\,\frac{b}{2}!\,\lambda^{-b}}{b}$$

$n!$ is the factorial function

Expanded form

$$(2\pi)^{-b/2}\,\lambda^{1-b}\,\Gamma\left(\frac{b}{2}\right)+(2\pi)^{-b/2}\,\lambda^{-b}\,\Gamma\left(\frac{b}{2}\right)$$

Roots

$\mathrm{Re}(b) > 0\,, \quad \lambda = -1$

$\mathrm{Re}(b) < 0\,, \quad \lambda = 0$

$\mathrm{Re}(z)$ is the real part of z

Series expansion at b=0

$$\frac{2(\lambda+1)}{b} - (\lambda+1)(2\log(\lambda)+\gamma+\log(2\pi)) + \frac{1}{24}b(\lambda+1)$$
$$\left(24\log^2(\lambda)+24(\gamma+\log(2\pi))\log(\lambda)+\pi^2+6\gamma^2+6\log^2(2\pi)+12\gamma\log(2\pi)\right) -$$
$$\frac{1}{48}b^2\left((\lambda+1)\left(16\log^3(\lambda)+24(\gamma+\log(2\pi))\log^2(\lambda)+\right.\right.$$
$$2\left(6\gamma^2+\pi^2+6\log^2(2\pi)+12\gamma\log(2\pi)\right)\log(\lambda)+2\gamma^3+2\log^3(2\pi)+$$
$$\left.\left.\gamma\left(\pi^2+6\log^2(2\pi)\right)+\pi^2\log(2\pi)+6\gamma^2\log(2\pi)-2\psi^{(2)}(1)\right)\right)+$$
$$\frac{1}{3840}b^3(\lambda+1)\left(320\log^4(\lambda)+640(\gamma+\log(2\pi))\log^3(\lambda)+\right.$$
$$80\left(6\gamma^2+\pi^2+6\log^2(2\pi)+12\gamma\log(2\pi)\right)\log^2(\lambda)+$$
$$80\log(\lambda)\left(2\gamma^3+2\log^3(2\pi)+\gamma\left(\pi^2+6\log^2(2\pi)\right)+6\gamma^2\log(2\pi)+\right.$$
$$\left.\pi^2\log(2\pi)-2\psi^{(2)}(1)\right)+3\pi^4+20\gamma^4+20\log^4(2\pi)+$$
$$20\gamma^2\left(\pi^2+6\log^2(2\pi)\right)+20\pi^2\log^2(2\pi)+80\gamma^3\log(2\pi)+$$
$$\left.40\gamma\left(2\log^3(2\pi)+\pi^2\log(2\pi)-2\psi^{(2)}(1)\right)-80\log(2\pi)\psi^{(2)}(1)\right)+O\left(b^4\right)$$

(Laurent series)

$\log(x)$ is the natural logarithm

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

γ is the Euler-Mascheroni constant

Series expansion at b=∞

$$(2\pi)^{-b/2}\lambda^{-b}e^{1/2\,b\,(\log(b/2)-1)}$$
$$\left(2\sqrt{\pi}(\lambda+1)\sqrt{\frac{1}{b}}+\frac{1}{3}\sqrt{\pi}(\lambda+1)\left(\frac{1}{b}\right)^{3/2}+\frac{1}{36}\sqrt{\pi}(\lambda+1)\left(\frac{1}{b}\right)^{5/2}-\right.$$
$$\left.\frac{139\left(\sqrt{\pi}(\lambda+1)\right)\left(\frac{1}{b}\right)^{7/2}}{3240}-\frac{571\left(\sqrt{\pi}(\lambda+1)\right)\left(\frac{1}{b}\right)^{9/2}}{77760}+O\left(\left(\frac{1}{b}\right)^{11/2}\right)\right)$$

Derivative

$$\frac{\partial}{\partial b}\left(\frac{\Gamma\left(\frac{b}{2}\right)(1+\lambda)}{\left(\sqrt{2\pi}\,\lambda\right)^b}\right)=$$
$$-2^{-b-1}\pi^{-b}(1+\lambda)\lambda^{-b}\Gamma\left(\frac{b}{2}\right)\left(2^{b/2+1}\pi^{b/2}\log(\lambda)+(2\pi)^{b/2}\log(2\pi)-(2\pi)^{b/2}\psi^{(0)}\left(\frac{b}{2}\right)\right)$$

From the

$$(2\pi)^{-b/2}\,\lambda^{-b}\,e^{1/2\,b\,(\log(b/2)-1)}\left(2\sqrt{\pi}\,(\lambda+1)\sqrt{\frac{1}{b}}+\frac{1}{3}\sqrt{\pi}\,(\lambda+1)\left(\frac{1}{b}\right)^{3/2}+\frac{1}{36}\sqrt{\pi}\,(\lambda+1)\left(\frac{1}{b}\right)^{5/2}-\frac{139\left(\sqrt{\pi}\,(\lambda+1)\right)\left(\frac{1}{b}\right)^{7/2}}{3240}-\frac{571\left(\sqrt{\pi}\,(\lambda+1)\right)\left(\frac{1}{b}\right)^{9/2}}{77\,760}+O\left(\left(\frac{1}{b}\right)^{11/2}\right)\right)$$

we obtain:

(2π)^(-b/2) λ^(-b) e^(1/2 b(log(b/2)-1))(2√(π)(λ+1)√(1/b)+1/3√(π)(λ+1)(1/b)^(3/2)+1/36√(π)(λ+1)(1/b)^(5/2)-(139(√(π)(λ+1)) (1/b)^(7/2))/3240-(571(√(π)(λ+1))(1/b)^(9/2))/77760+((1/b)^(11/2)))

Input

$$(2\pi)^{-b/2}\,\lambda^{-b}\left(e^{1/2\,b\,(\log(b/2)-1)}\left(2\sqrt{\pi}\,(\lambda+1)\sqrt{\frac{1}{b}}+\frac{1}{3}\sqrt{\pi}\left((\lambda+1)\left(\frac{1}{b}\right)^{3/2}\right)+\frac{1}{36}\sqrt{\pi}\,(\lambda+1)\left(\frac{1}{b}\right)^{5/2}-\frac{139\left(\sqrt{\pi}\,(\lambda+1)\right)\left(\frac{1}{b}\right)^{7/2}}{3240}-\frac{571\left(\sqrt{\pi}\,(\lambda+1)\right)\left(\frac{1}{b}\right)^{9/2}}{77\,760}+\left(\frac{1}{b}\right)^{11/2}\right)\right)$$

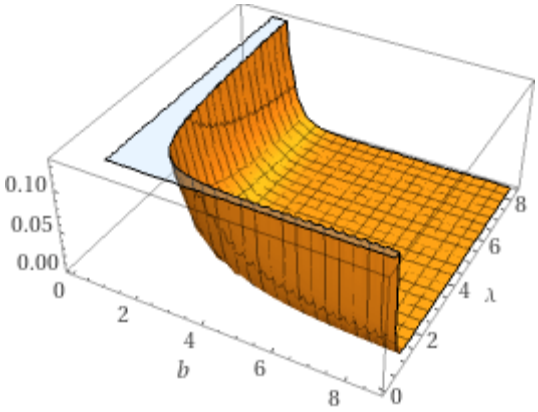
$\log(x)$ is the natural logarithm

Exact result

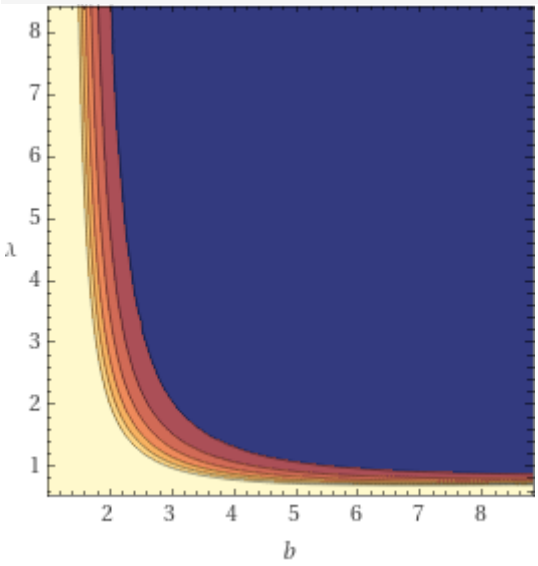
$$(2\pi)^{-b/2}\,\lambda^{-b}\left(-\frac{571\sqrt{\pi}\left(\frac{1}{b}\right)^{9/2}(\lambda+1)}{77\,760}-\frac{139\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2}(\lambda+1)}{3240}+\frac{1}{36}\sqrt{\pi}\left(\frac{1}{b}\right)^{5/2}(\lambda+1)+\frac{1}{3}\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}(\lambda+1)+2\sqrt{\pi}\sqrt{\frac{1}{b}}\,(\lambda+1)+\left(\frac{1}{b}\right)^{11/2}\right)e^{1/2\,b\,(\log(b/2)-1)}$$

3D plot **(figure that can be related to a D-brane/Instanton)**

The key observation from the above plots and is that at $b = 1$, which is taken as the energy density of the universe at the Big Bang, with $\lambda = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.



Contour plot



From the exact result

$$(2\pi)^{-b/2}\,\lambda^{-b}\left(-\frac{571\sqrt{\pi}\left(\frac{1}{b}\right)^{9/2}(\lambda+1)}{77760}-\frac{139\sqrt{\pi}\left(\frac{1}{b}\right)^{7/2}(\lambda+1)}{3240}+\frac{1}{36}\sqrt{\pi}\left(\frac{1}{b}\right)^{5/2}(\lambda+1)+\frac{1}{3}\sqrt{\pi}\left(\frac{1}{b}\right)^{3/2}(\lambda+1)+2\sqrt{\pi}\sqrt{\frac{1}{b}}(\lambda+1)+\left(\frac{1}{b}\right)^{11/2}\right)e^{1/2\,b\,(\log(b/2)-1)}$$

for b = 4 :

(2π)^(-4/2) λ^(-4)(-(571√(π)(1/4)^(9/2)(λ+1))/77760-(139sqrt(π)(1/4)^(7/2)(λ+1))/3240+1/36 √(π) (1/4)^(5/2) (λ+1)+1/3√(π)(1/4)^(3/2)(λ+1)+2 sqrt(π) sqrt(1/4)(λ+1)+(1/4)^(11/2)) e^(1/2*4(log(4/2)-1))

Input

$$(2\pi)^{-4/2}\times\frac{1}{\lambda(n)^4}\left(-\frac{571\sqrt{\pi}\left(\frac{1}{4}\right)^{9/2}(\lambda+1)}{77760}-\frac{139\sqrt{\pi}\left(\frac{1}{4}\right)^{7/2}(\lambda+1)}{3240}+\frac{1}{36}\sqrt{\pi}\left(\frac{1}{4}\right)^{5/2}(\lambda+1)+\frac{1}{3}\sqrt{\pi}\left(\frac{1}{4}\right)^{3/2}(\lambda+1)+2\sqrt{\pi}\left(\sqrt{\frac{1}{4}}(\lambda+1)\right)+\left(\frac{1}{4}\right)^{11/2}\right)e^{1/2\times4\,(\log(4/2)-1)}$$

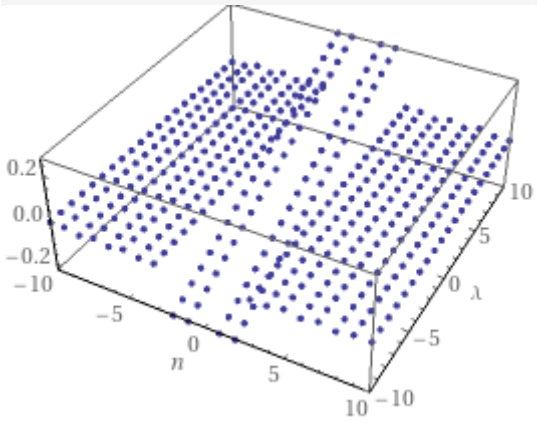
$\lambda(n)$ is the Carmichael function

$\log(x)$ is the natural logarithm

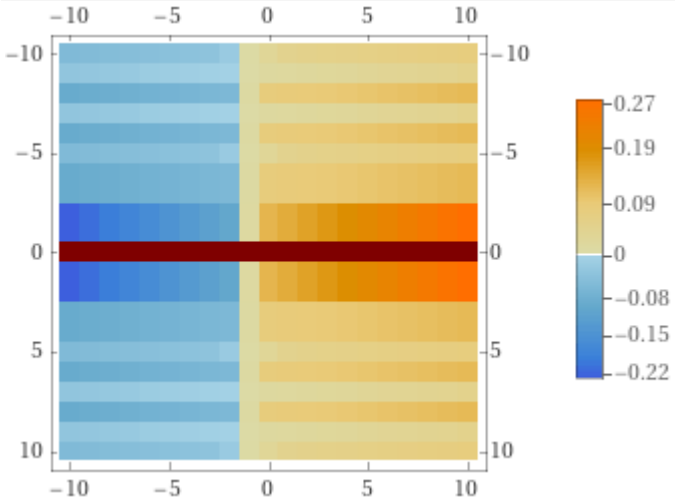
Exact result

$$\frac{\left(\frac{8\,298\,529\sqrt{\pi}\,(\lambda+1)}{7962\,624}+\frac{1}{2048}\right)e^{2(\log(2)-1)}}{4\,\pi^2\,\lambda(n)^4}$$

Plot



Matrix plot



Alternate forms

$$\frac{8\,298\,529\,\sqrt{\pi}\,(\lambda+1)+3888}{7\,962\,624\,e^2\,\pi^2\,\lambda(n)^4}$$

$$\frac{8\,298\,529\,\sqrt{\pi}\,\lambda+8\,298\,529\,\sqrt{\pi}\,+3888}{7\,962\,624\,e^2\,\pi^2\,\lambda(n)^4}$$

$$\frac{8\,298\,529\,\pi^2\,\lambda + \left(3888 + 8\,298\,529\,\sqrt{\pi}\right)\pi^{3/2}}{7\,962\,624\,e^2\,\pi^{7/2}\,\lambda(n)^4}$$

Expanded form

$$\frac{8\,298\,529\,\lambda}{7\,962\,624\,e^2\,\pi^{3/2}\,\lambda(n)^4} + \frac{8\,298\,529}{7\,962\,624\,e^2\,\pi^{3/2}\,\lambda(n)^4} + \frac{1}{2048\,e^2\,\pi^2\,\lambda(n)^4}$$

Alternate form assuming n and λ are positive

$$\frac{8\,298\,529\,(\lambda+1)\,e^{2(\log(2)-1)}}{31\,850\,496\,\pi^{3/2}\,\lambda(n)^4} + \frac{e^{2(\log(2)-1)}}{8192\,\pi^2\,\lambda(n)^4}$$

From the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

Thence, from the exact result

$$\frac{\left(\frac{8\,298\,529\,\sqrt{\pi}\ (\lambda+1)}{7962\,624}+\frac{1}{2048}\right)e^{2(\log(2)-1)}}{4\,\pi^2\,\lambda(n)^4}$$

we obtain:

((((8298529 sqrt(π) (λ + 1))/7962624 + 1/2048) e^(2 (log(2) - 1)))/(4 π^2 λ(n)^4))
((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

Input

$$\frac{\left(\frac{8\,298\,529\,\sqrt{\pi}\ (\lambda+1)}{7962\,624}+\frac{1}{2048}\right)e^{2(\log(2)-1)}}{4\,\pi^2\,\lambda(n)^4}\ \sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3\right)^2}$$

$\log(x)$ is the natural logarithm

$\lambda(n)$ gives the Liouville function

Exact result

$$\frac{3^{-3/(2\pi)}\times 4^{-1-1/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \pi^{1/\pi-2}\ \sqrt[2\pi]{d^9}\ \left(\frac{8\,298\,529\,\sqrt{\pi}\ (\lambda+1)}{7\,962\,624}+\frac{1}{2048}\right)e^{2(\log(2)-1)}}{\lambda(n)^4}$$

And:

((((8298529 sqrt(π) (λ + 1))/7962624 + 1/2048) e^(2 (log(2) - 1)))/(4 π^2 λ(n)^4)) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

Input

$$\frac{\left(\frac{8298529\sqrt{\pi}\ (\lambda+1)}{7962624}+\frac{1}{2048}\right)e^{2(\log(2)-1)}}{4\pi^2\lambda(n)^4}\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\ d^3\right)\left(\frac{\sqrt{2}}{12}\ d^3\right)}$$

$\log(x)$ is the natural logarithm

$\lambda(n)$ gives the Liouville function

Exact result

$$\frac{2^{-2-1/\pi}\times 3^{-3/(2\pi)}\ \pi^{1/(2\pi)-2}\ \sqrt[2\pi]{d^9}\ \left(\frac{8298529\sqrt{\pi}\ (\lambda+1)}{7962624}+\frac{1}{2048}\right)e^{2(\log(2)-1)}}{\lambda(n)^4}$$

Dividing the two exact results

$$\frac{3^{-3/(2\pi)}\times 4^{-1-1/\pi}\ \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \pi^{1/\pi-2}\ \sqrt[2\pi]{d^9}\ \left(\frac{8298529\sqrt{\pi}\ (\lambda+1)}{7962624}+\frac{1}{2048}\right)e^{2(\log(2)-1)}}{\lambda(n)^4}$$

and

$$\frac{2^{-2-1/\pi}\times 3^{-3/(2\pi)}\ \pi^{1/(2\pi)-2}\ \sqrt[2\pi]{d^9}\ \left(\frac{8298529\sqrt{\pi}\ (\lambda+1)}{7962624}+\frac{1}{2048}\right)e^{2(\log(2)-1)}}{\lambda(n)^4}$$

we obtain:

(((3^(-3/(2 π)) 4^(-1 - 1/π) (5 (3 + sqrt(5)))^(1/(2 π)) π^(1/π - 2))))/(((2^(-2 - 1/π) 3^(-3/(2 π)) π^(1/(2 π) - 2))))

Input

$$\frac{3^{-3/(2\pi)} \times 4^{-1-1/\pi} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \pi^{1/\pi-2}}{2^{-2-1/\pi} \times 3^{-3/(2\pi)}\ \pi^{1/(2\pi)-2}}$$

Exact result

$$2^{-1/\pi} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi} \sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{3^{-3/(2\pi)}\left(4^{-1-1/\pi} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \pi^{1/\pi-2}\right)}{2^{-2-1/\pi} \times 3^{-3/(2\pi)}\ \pi^{1/(2\pi)-2}} = 2^{-1/\pi} \sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{3^{-3/(2\pi)}\left(4^{-1-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\pi^{1/\pi-2}\right)}{2^{-2-1/\pi}\times 3^{-3/(2\pi)}\;\pi^{1/(2\pi)-2}} = 2^{-1/\pi}\sqrt[2\pi]{5}\;\sqrt[2\pi]{\pi}\;\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{3^{-3/(2\pi)}\left(4^{-1-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\pi^{1/\pi-2}\right)}{2^{-2-1/\pi}\times 3^{-3/(2\pi)}\;\pi^{1/(2\pi)-2}} = 2^{-1/\pi}\sqrt[2\pi]{5}\;\sqrt[2\pi]{\pi}\;\sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13

…

3.1415926535884…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298888445046920

…

1.644934066846…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.618008545900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.92478440155986\,\pi}\,\sqrt{\pi}^2}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

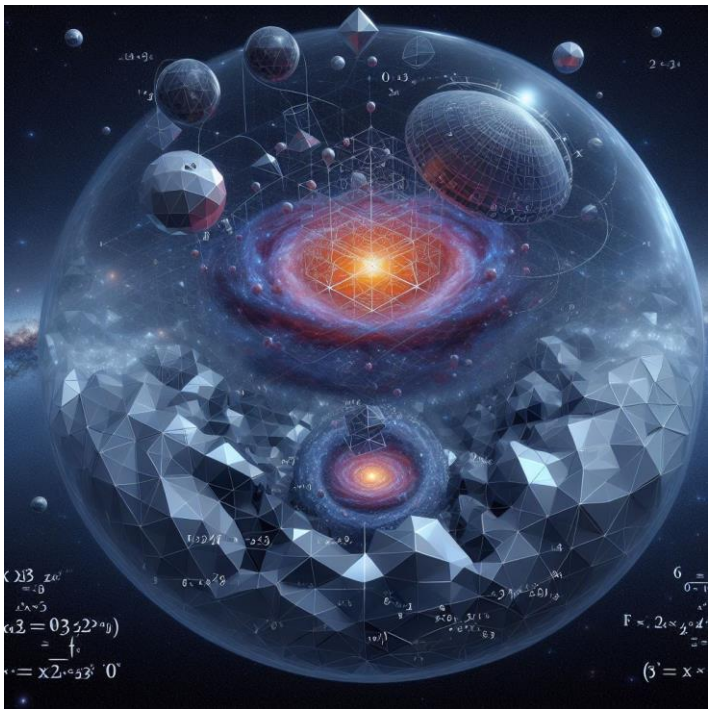


Fig. 1

A geometric representation of the universe emerging from the quantum vacuum in a cosmological model of eternal inflation, taking into account the solutions of the cubic equation $x^3-6x^2+20x-33 = 0$, the equation related to the Coulomb branch and superconformal gravity, the multiplication of two of its solutions by the formula concerning the ratios (and/or inverses) between the volumes of the icosahedron, octahedron and tetrahedron and the volume of the sphere, where in this formula is included the ratio $(2\sqrt{2})/\pi$, the so-called Del Gaudio-Nardelli Constant, the result is very close to the values of the golden ratio and π.

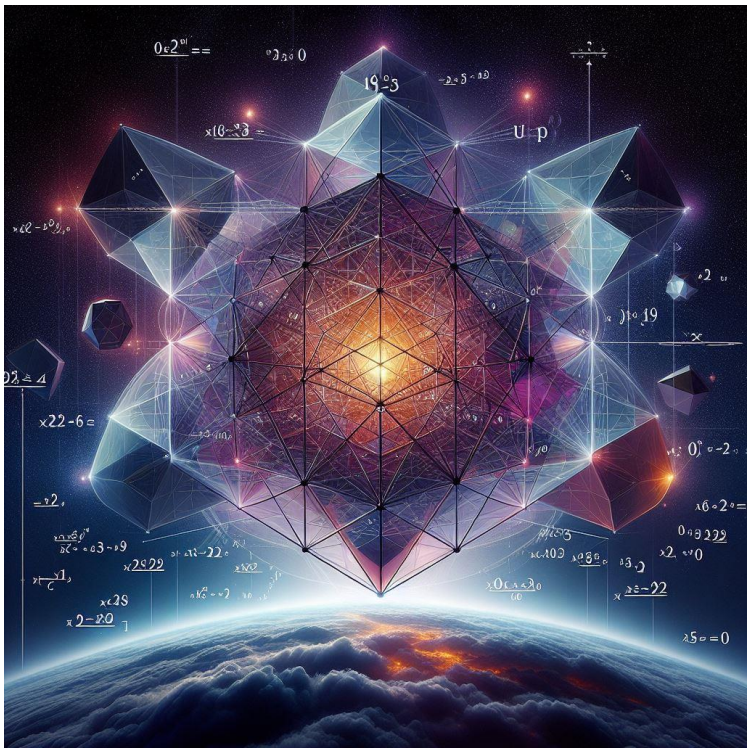


Fig.2

A geometric representation of the universe emerging from the quantum vacuum in a cosmological model of eternal inflation, taking into account the solutions of the cubic equations $x^3-6x^2+20x-33 = 0$ and $x^3-6x^2-72x-729=0$, the equation related to the Coulomb branch and superconformal gravity, the multiplication of two of its solutions by the formula concerning the ratios (and/or inverses) between the volumes of the icosahedron, octahedron and tetrahedron and the volume of the sphere, where in this formula is included the ratio $(2\sqrt{2})/\pi$, the so-called Del Gaudio-Nardelli Constant, the result is very close to the values of the golden ratio and π.

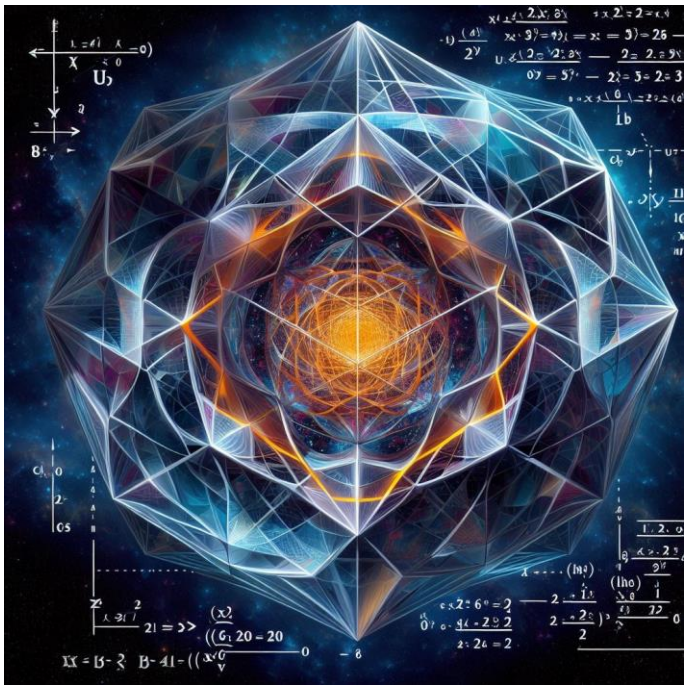


Fig. 3

A geometric representation of the universe emerging from the quantum vacuum in a cosmological model of eternal inflation, taking into account the solutions of the cubic equations x^3-6x^2+20x-33 = 0 and x^3-6x^2-72x-729=0, the equation related to the Coulomb branch and superconformal gravity, a general formula for the main singularity, the multiplication of the solutions by the formula concerning the ratios (and/or inverses) between the volumes of the icosahedron, octahedron and tetrahedron and the volume of the sphere, where in this formula is included the ratio (2sqrt2)/Pi, the so-called Del Gaudio-Nardelli Constant, the result is very close to the values of the golden ratio and Pi.

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

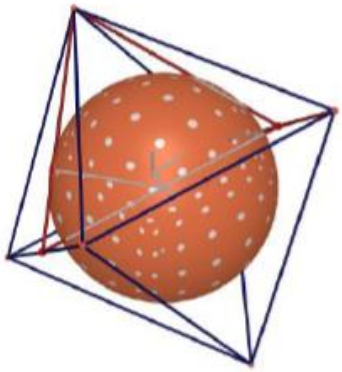


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

Indeed:

From:

https://www.academia.edu/75787512/The_Theory_of_String_A_Candidate_for_a_Generalized_Unification_Model

The Einstein's field equation and the String Theory.

The Einstein's field equation which includes the cosmological constant is:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (8)$$

where $R_{\mu\nu}$ is the Ricci tensor, R its trace, λ the cosmological constant, $g_{\mu\nu}$ the metric tensor of the space geometry, G the Newton's gravitational constant and $T_{\mu\nu}$ the tensor representing the properties of energy, matter and momentum.

The left hand-side of (8) represents the gravitational field and, consequently, the warped space-time, while the right hand-side represents the matter, i.e. the sources of the gravitational field.

In string theory the gravity is related to the gravitons which are bosons, whereas the matter is related to fermions. It follows that the left and right hand of (8) may be respectively related to the action of bosonic and of superstrings.

The actions of bosonic string and superstring (also containing fermions) are connected by the Palumbo-Nardelli relation (Palumbo et al. 2005):

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\varphi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right]=$$

$$=\int_0^\infty \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|\widetilde{H}_3|^2-\frac{\kappa_{10}^2}{g_{10}^2}Tr_\nu\left(|F_2|^2\right)\right]$$

The sign minus in the above equation comes from the inversion of any relationship.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^{\,8}\right)^{-k}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^{\,8}\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k z_0^{-k}}{k!}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^{0} \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

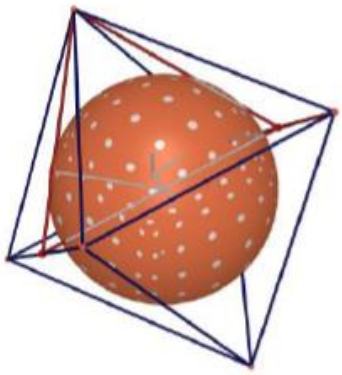
$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard to a sphere inscribed in an octahedron



we have the following formulas:

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\,2}{\pi} \quad (\text{for } l \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\,\pi)\rfloor)}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we have the following convex regular polyhedrons volumes:

$\frac{\sqrt{2}}{12}d^3$ Tetrahedron Volume

$\frac{\sqrt{2}}{3}d^3$ Octahedron Volume

$\frac{5}{12}(3+\sqrt{5})d^3$ Icosahedron Volume

Regarding the Sphere the volume is

$$V = \frac{4}{3}\pi r^3$$

With regard the sphere inscribed in a Tetrahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_T}{V_S} = \frac{\frac{\sqrt{2}}{12}d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(((√2 d^3)/12))*1/(4/3*π*(d/2)^3)

Input

$$\left(\frac{1}{12}\left(\sqrt{2}\ d^3\right)\right) \times \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.225079079039276517388799797751685145666143537488805156392859367 8

…

0.225079079….

Property

$\frac{1}{\sqrt{2}\ \pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Octahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_O}{V_S} = \frac{\frac{\sqrt{2}}{3}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

((1/3*√2*d^3)/(4/3*π*(d/2)^3))

Input

$$\frac{\frac{1}{3}\sqrt{2}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\ 2}{\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Icosahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_I}{V_S} = \frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3)

Input

$$\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{5\left(3+\sqrt{5}\right)}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

4.166730504921372682725163141597223093105782245731637719697502271 9

…

4.166730504921….

Property

$\frac{5\left(3+\sqrt{5}\right)}{2\pi}$ is a transcendental number

Expanded form

$$\frac{15}{2\pi} + \frac{5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

Alternate forms

$$\frac{15+5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

$$\frac{\frac{15}{2}+\frac{5\sqrt{5}}{2}}{\pi} \quad (\text{for } d \neq 0)$$

Series representations

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\pi\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\ a^3\right)\left(\sqrt{2}\ d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}{3\times3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\ z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\right.\right.\right.$$

$$\left.\left.\left. z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\ z_0^{-k}}{k!}\right)\right)\Big/$$

$$\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\ z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

Indeed, we obtain:

$$\frac{\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)\left(d^3\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2\right)}}{\sqrt[2\pi]{\frac{4}{3}\left(\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\right)\sqrt{2}\left(d^3 \times \frac{\sqrt{2}}{12}\right)d^3}}$$

= 1.6180085459001 07…

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4 \times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358487162422193250985871527893102518924557617007810061 3

…

3.1415926535…. = π (Ramanujan Recurring Number)

In conclusion, from the following further formula concerning the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron volumes and their ratios (and/or the inverses) with the sphere volume, raised the expression

$$(2\log(2)log^{2/3}(3^2))/(3e^{7/3})$$

$$\left(\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\frac{\frac{1}{4}\left(\left(15+7\sqrt{5}\right)d^3\right)}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\right)^{(2\log(2)\log^{2/3}(3^2))/(3e^{7/3})}$$

i.e.

$$\left(\frac{3}{2}\right)^{(4\log(2)\log^{2/3}(9))/(3e^{7/3})}\left(\frac{5\left(3+\sqrt{5}\right)\left(15+7\sqrt{5}\right)}{\pi}\right)^{(2\log(2)\log^{2/3}(9))/(3e^{7/3})}$$

we obtain:

1.6180381711519139857456817084743134673903021614989370712917722071

…

1.6180381711…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

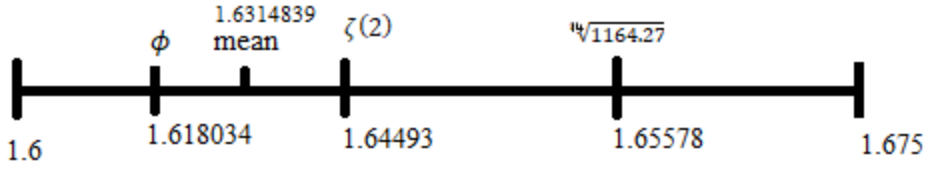
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

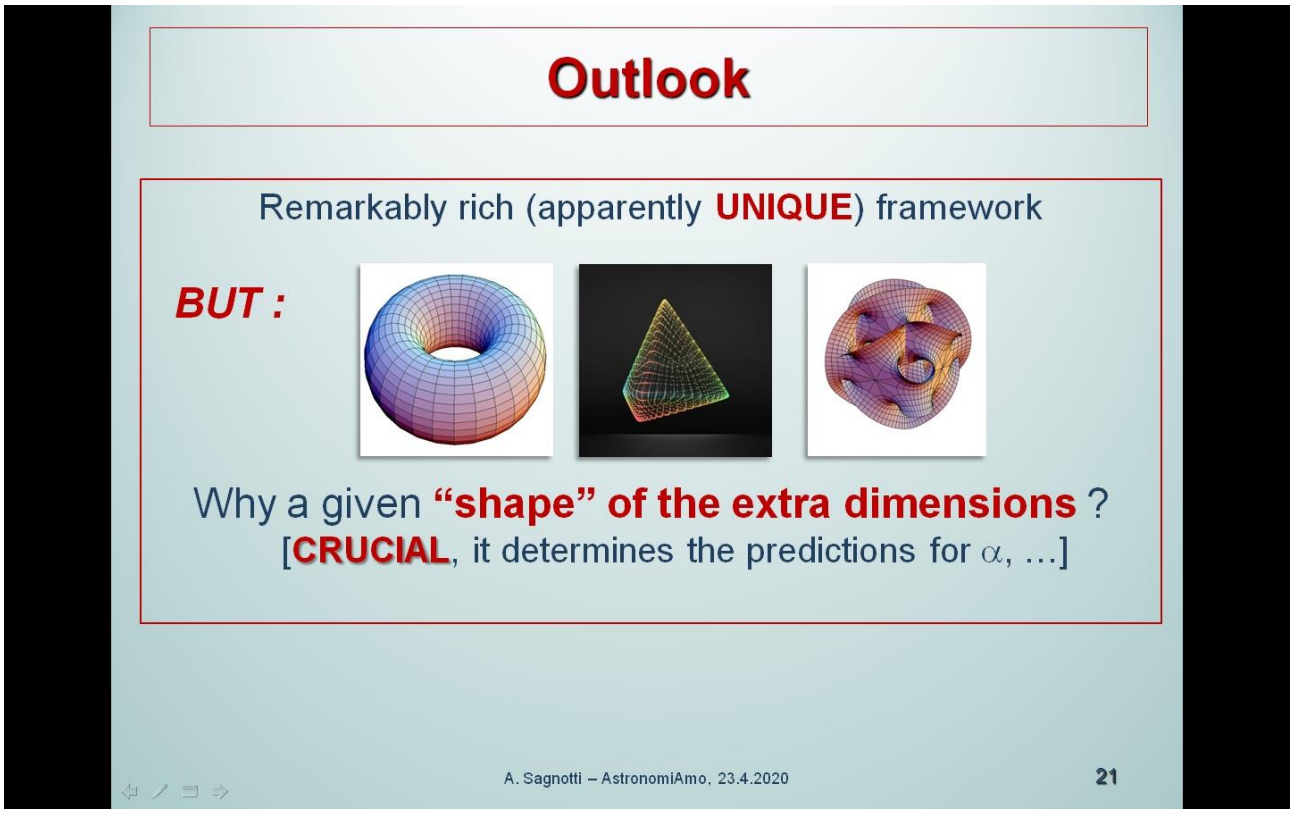
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix


Outlook
Remarkably rich (apparently UNIQUE) framework
BUT :
Why a given "shape" of the extra dimensions ?
[CRUCIAL, it determines the predictions for α, …]
A. Sagnotti – AstronomiAmo, 23.4.2020
21

From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

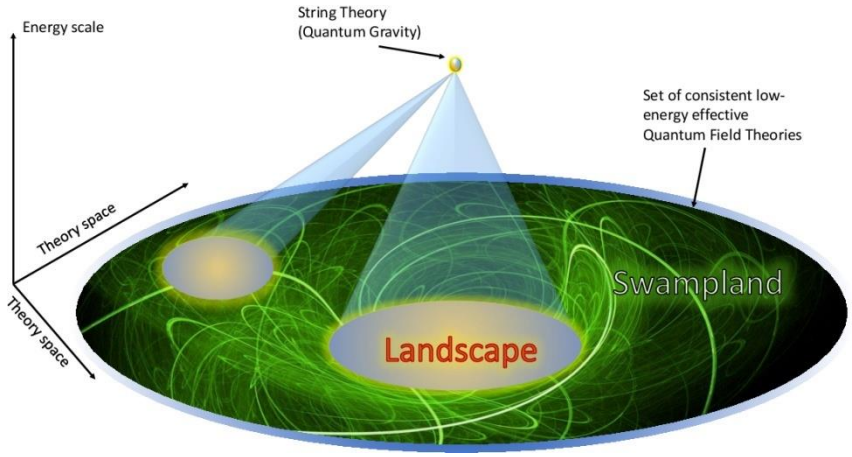


Fig. 1

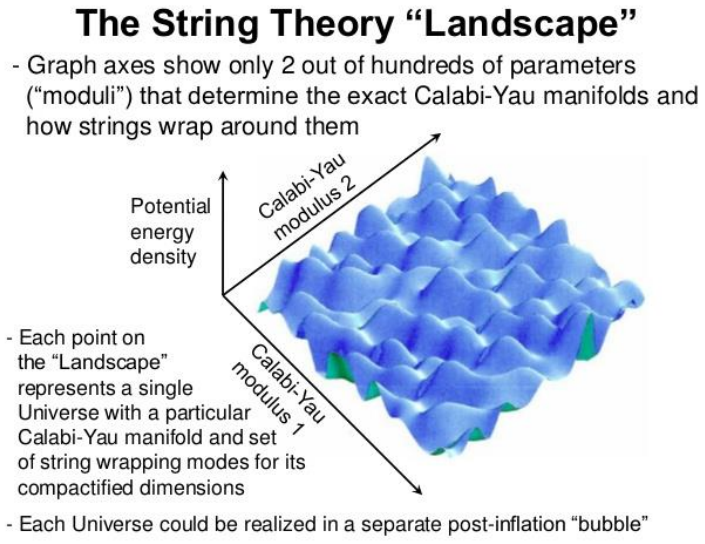

The String Theory "Landscape"
- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them
Potential energy density
Calabi-Yau modulus 2
Calabi-Yau modulus 1
- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

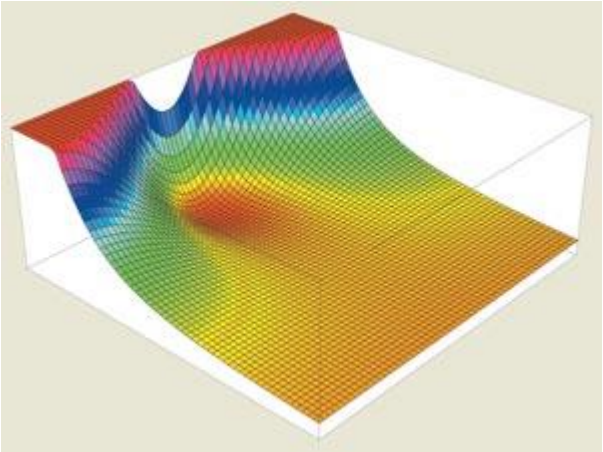


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

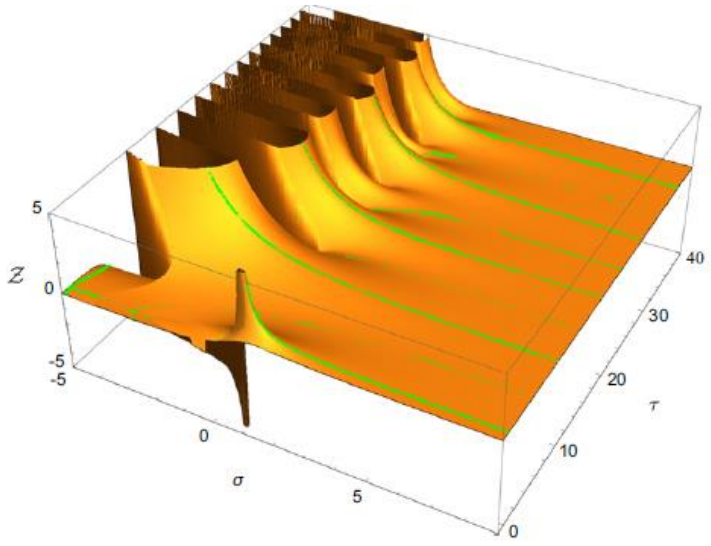


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

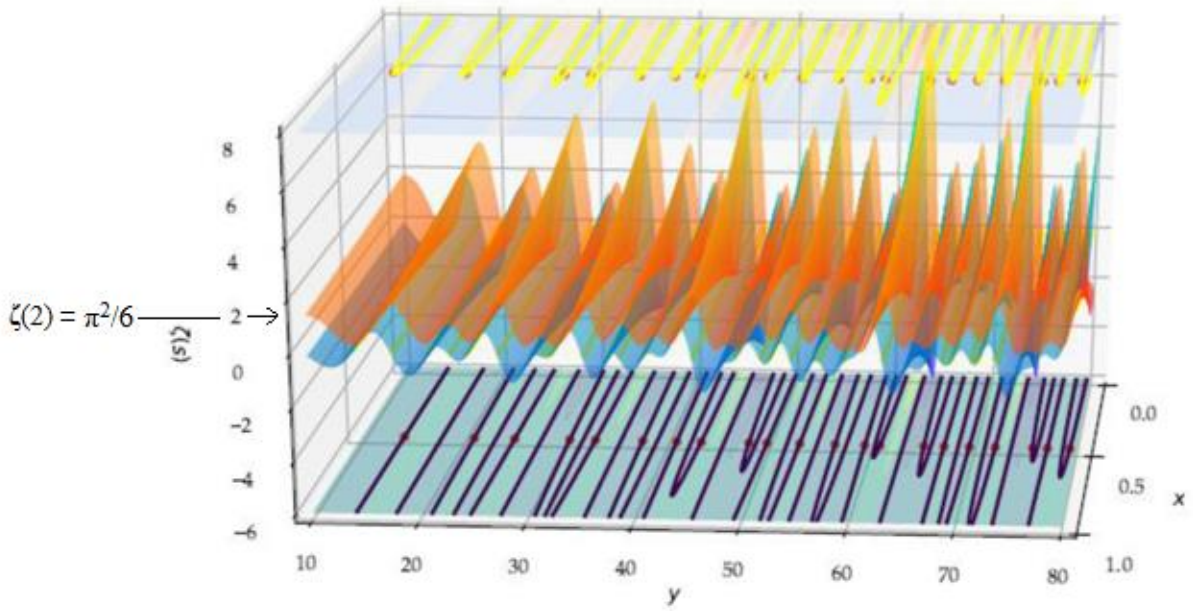


Figure 1. $C(x,y)$ and $S(x,y)$ surfaces of the Riemann $\zeta(x,y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

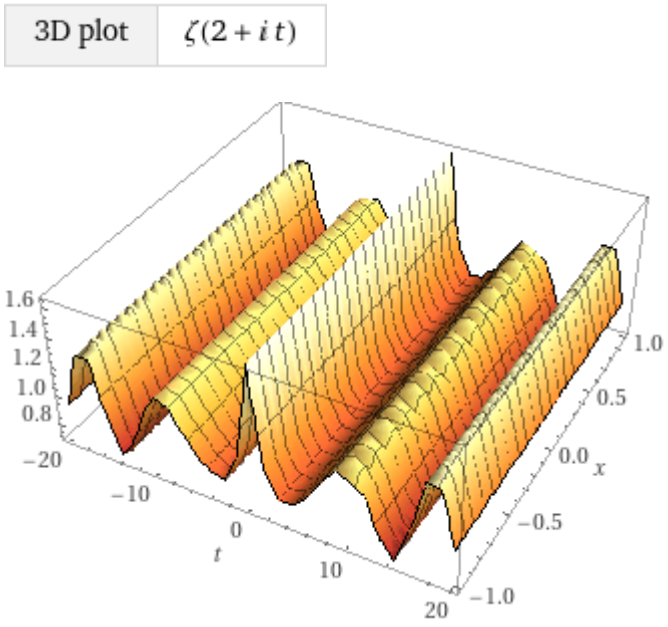


Fig. 6

Where ζ(2+it) :

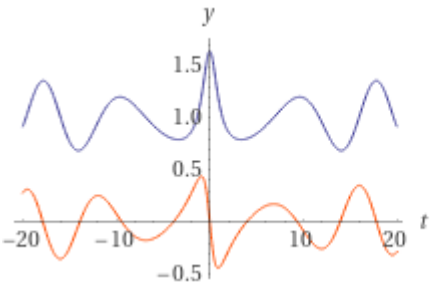
Input

$\zeta(2 + i\,t)$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

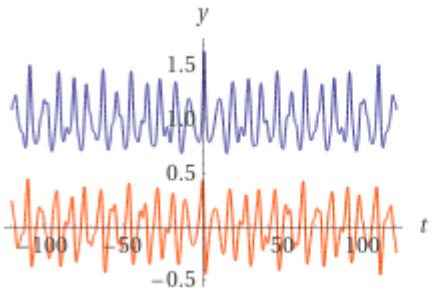
Plots



(t from −20 to 20)

real part

imaginary part



(t from −120 to 120)

real part

imaginary part

Roots

$t = 2\,i\,(n+1), \quad n \in \mathbb{Z}, \quad n \geq 1$

$t = -i\,(\rho_n - 2), \quad n \neq 0, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\,t\,\zeta'(2) - \frac{1}{2}\,t^2\,\zeta''(2) - \frac{1}{6}\,i\,\zeta^{(3)}(2)\,t^3 + \frac{1}{24}\,\zeta^{(4)}(2)\,t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2 + i\,t) = \zeta(2 + i\,t,\,1)$$

$$\zeta(2 + i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2 + i\,t) = \frac{\zeta\left(2 + i\,t,\,\frac{1}{2}\right)}{-1 + 2^{2+i\,t}}$$

$\zeta(s,\,a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2 + i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = \frac{\sum_{k=0}^{\infty} (1 + 2\,k)^{-2-i\,t}}{1 - 2^{-2-i\,t}} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \mathrm{Im}(t) < 1$$

$\mathrm{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^\infty \frac{\tau^{1+i\,t}}{-1+e^\tau}\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^\infty \tau^{2+i\,t}\,\mathrm{csch}^2(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^\infty e^{-\tau}\,\tau^{1+i\,t}\,\mathrm{csch}(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\mathrm{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+i\,t) = -i\,2^{2+i\,t}\,\pi^{1+i\,t}\,\Gamma(-1-i\,t)\sinh\left(\frac{\pi\,t}{2}\right)\zeta(-1-i\,t)$$

$$\zeta(2+i\,t) = \frac{\pi^{3/2+i\,t}\,\Gamma\left(-\frac{1}{2}-\frac{i\,t}{2}\right)\zeta(-1-i\,t)}{\Gamma\left(1+\frac{i\,t}{2}\right)}$$

$$\zeta(2+i\,t) = -\frac{i\sum_{k=0}^\infty \frac{\Gamma\left(k-\frac{i\,t}{2}\right)\sum_{j=0}^k (-1)^j\,(1+2\,j)\binom{k}{j}\zeta(2+2\,j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{i\,t}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372033095 8…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi =$$
$$\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

1.64381856858…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^-}6$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right)=0.054323\,(0.0814845\,b+1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M}=-\frac{\frac{226\,802\,245}{18\,480\,874}+b}{M}$$

$$\frac{\partial M(b,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b}=-\frac{18\,480\,874\,M}{226\,802\,245+18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M}=\frac{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b}=\frac{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right\}=0 \text{ at } (b,M)=(-16,0)$$

Global minima:

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

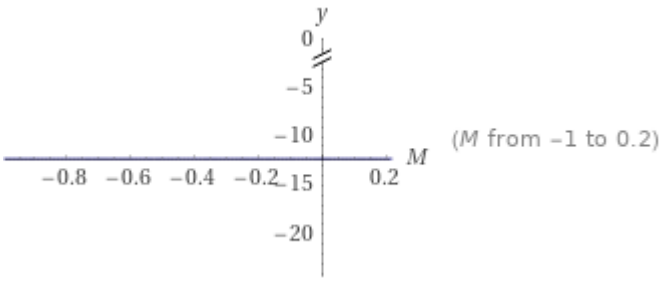
Input interpretation:

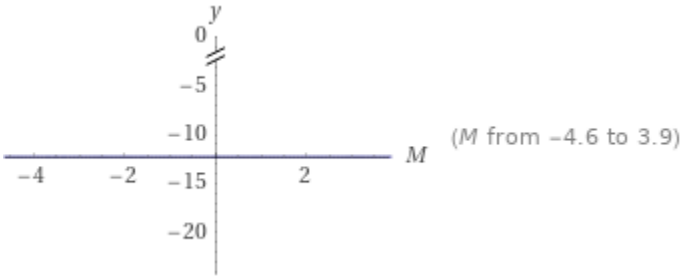
$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

Result:

$$\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:





Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323\, M^2\right)}{M^2}\right) = \frac{3.55271 \times 10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM = \frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2} = -12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM = 0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

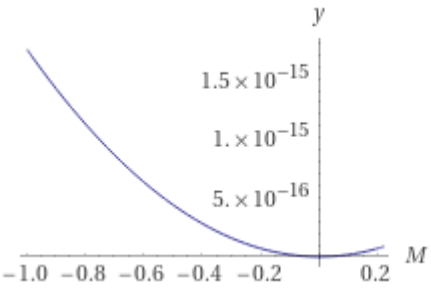
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

Result:

0

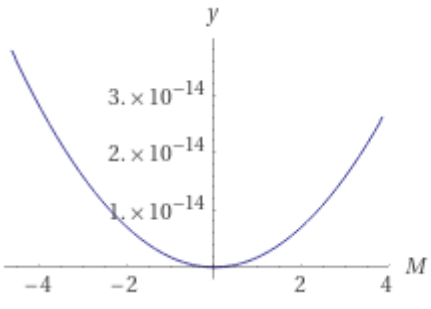
Plots: **(possible mathematical connection with an open string)**



(*M* from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O(M^{62194})$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times10^{-15}\,M^2+O\left(\left(\frac{1}{M}\right)^{62\,194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}\,M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.38851344947464545348970783378088020833333333333333333333\ldots \times 10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 0.2^2 + 6.58545 \times 10^{-10}\sqrt{0.2^4}\right)}{0.2^2} + 1\right)^2 \times 0.2^2$$

Result:

$$7.02162151915943272558353253404940833333333333333333333333\ldots \times 10^{-17}$$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875\times10^{-14}$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots \times 10^{-8}$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([[((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left(\left(1/\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)\text{ ^ }(1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

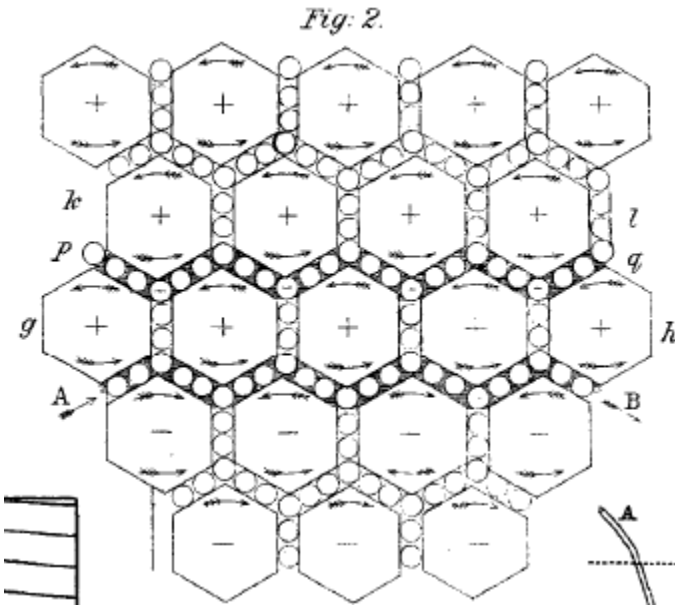
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

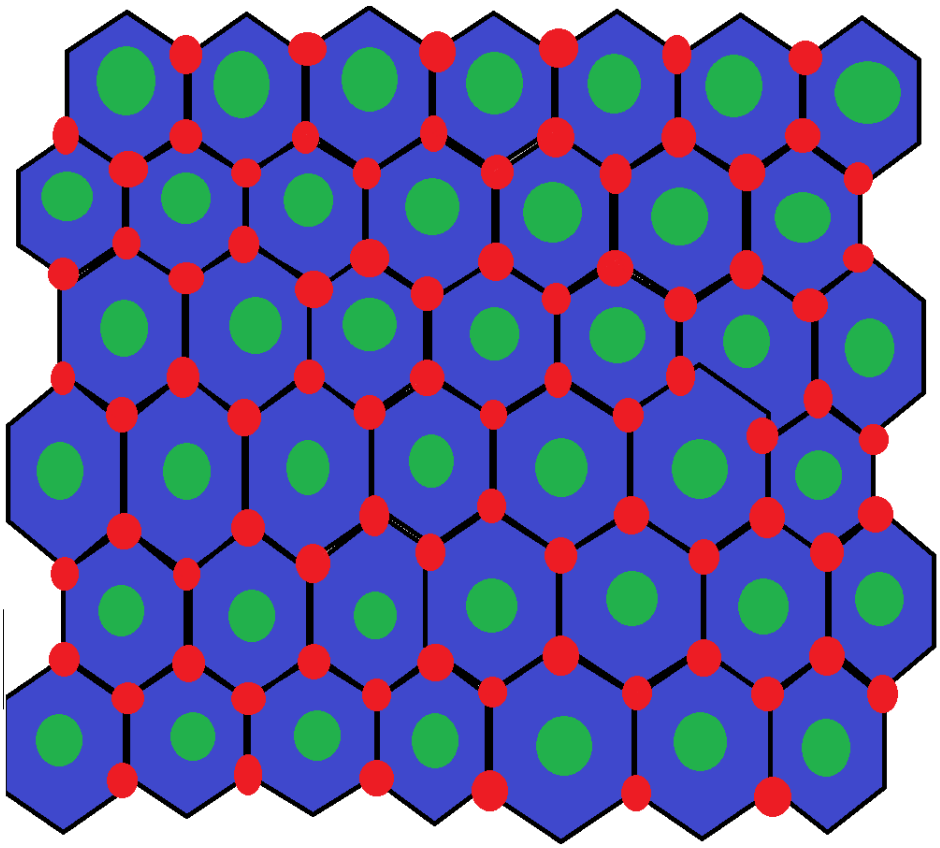
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Fig. 3

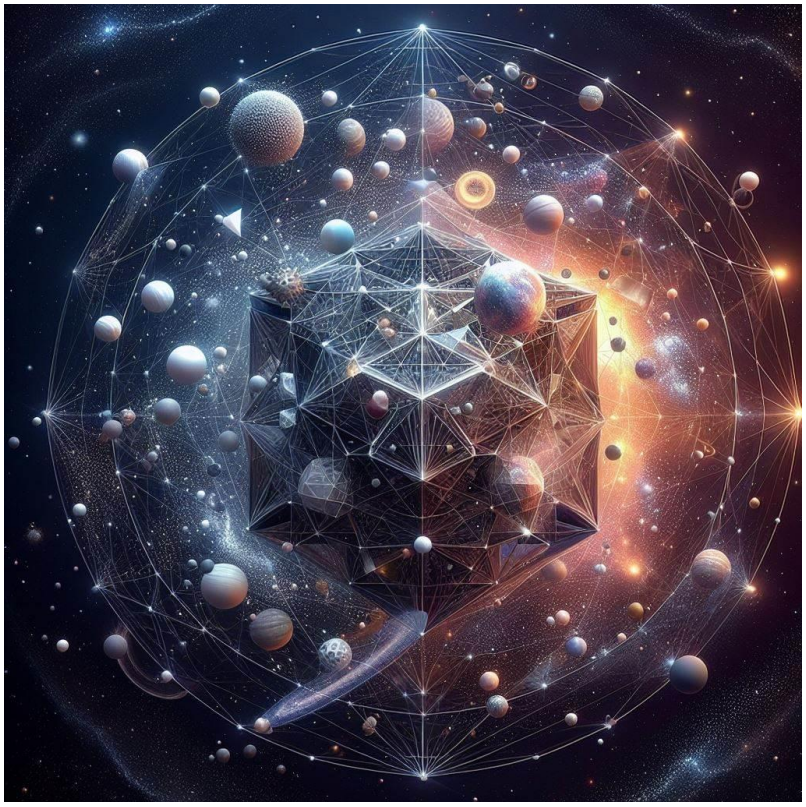


Image representing the solutions of the cubic equations in relation to the boundary conditions of the Plateau problem useful for understanding the formation of the bubbles of the eternally inflated universe and of an octahedron as a mathematical model of the primordial universe before the Big Bang, therefore highly symmetrical and with extremely low entropy. As for the cubic equations, they represent the geometric characteristics of the quantum vacuum, while the spherical surface would represent the geometry of the bubble of the universe.

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability. I would like to thank also my son **Francesco Nardelli**, student of Educational Sciences, majoring in History and Philosophy, for his useful suggestions regarding the intriguing image at the end of this work

References

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

See also:

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

SUPERCONFORMAL GRAVITY AND THE TOPOLOGY OF DIFFEOMORPHISM GROUPS - *Jay Cushing, Gregory W. Moore, Martin Roček, Vivek Saxena* - arXiv:2311.08394v2 [hep-th] 17 Nov 2023

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *Emilian Dudas, Sergio Ferrara, Alex Kehagias and Augusto Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the analysis of various Cubic equations and Cardano's Formulas. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory V

Michele Nardelli[1], Antonio Nardelli

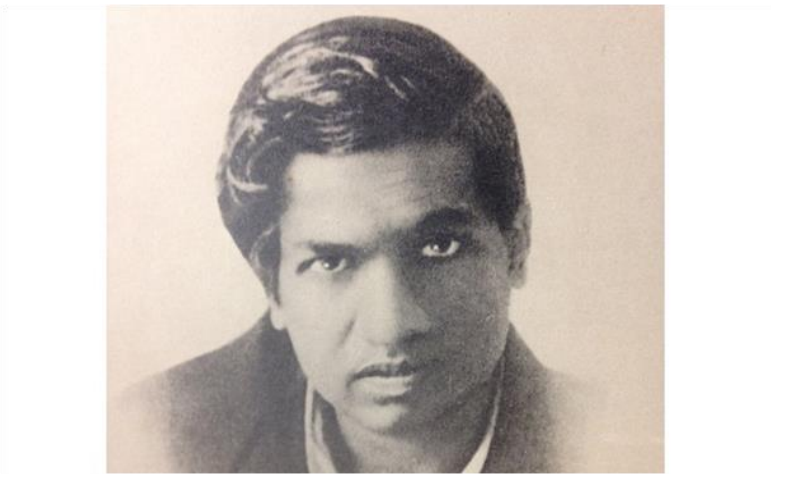
Abstract

In this paper (part V), we analyze various Cubic equations and Cardano's Formulas. We obtain new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Introduction

Cardano's method for cubic equations involves cube roots and introduces symmetry concepts. Similarly, string theory and M-theory involve symmetries and transformation groups that play a fundamental role in describing particles and interactions. Solutions of complete cubic equations via Cardano's method involve complex cube roots. Similarly, equations and solutions in superstring theory can often involve advanced algebraic and mathematical complexities. Solutions of cubic equations can be understood geometrically through topology and algebraic geometry. In string theory, the geometric and topological configurations of strings and branes are also central. Duality is an important concept in string theory, where different mathematical descriptions can be equivalent. Similarly, in solving cubic equations, duality links exist through cube roots.

Thus, in string theory, duality refers to the fact that different mathematical formulations or physical descriptions of a theory can be equivalent. For example, two seemingly different theories can be dual to each other, meaning they are two different ways of describing the same physical reality. This concept of duality has been very useful for understanding profound aspects of string theory and field theories.

Similarly, in solving cubic equations, duality can emerge through cube roots. The cubic equation $ax^3+bx^2+cx+d=0$ can be solved using cube roots, as in the Cardano method I described above. However, cube roots involve the concept of complex numbers, and it is sometimes possible to express the solutions in ways that highlight a duality between the solutions.

For example, if we consider complex cube roots, we can see a duality between them, since there are connections via the cube roots of unity. This duality is similar to the concept of duality in string theory, where different descriptions can be connected by similar transformations.

In both cases, duality provides a deeper way of understanding the underlying mathematical or physical structure, allowing you to explore hidden connections between different concepts.

Cubic surfaces can be associated with geometric and topological concepts. In string theory, for example, some models require spaces with particular shapes, and the geometry of the spaces can influence the properties of strings and particles. Some mathematical and physical models involving surfaces might be described by cubic or

complex equations. The geometry of surfaces and the associated mathematical structure can influence the properties of universe bubbles in the multiverse. Cubic surfaces can represent certain topological spaces, and their properties can affect the structure of the associated string theory.

The connection between the surface of a D-brane and the roots of a cubic equation can be explored in specific contexts within string theory, especially when it comes to studying the geometric and topological properties of D-branes. D-branes are extended objects on whose surface open strings can terminate. The geometry of the D-brane surface can interfere with the physics of surrounding strings. In some cases, geometry can be described mathematically through objects known in algebraic surface theory, such as cubic equations.

D-branes can host open fields, which describe particles interacting with the D-brane. Field equations on a D-brane can involve surface geometry, and in some situations, these equations can be associated with higher degree algebraic equations, such as cubic equations.

The compactification of extra dimensions, which is an important part of string theory, can be associated with the surface geometry of a D-brane. Compactification can lead to a reduction in dimensions and a geometric description that can involve algebraic equations, including cubic equations.

The topological invariants associated with the surface of a D-brane, such as the Betti number or the Chern number, can be related to geometric characteristics that can be described via algebraic equations, including cubic equations.

In some situations of duality, the description of a D-brane in a context can be dualistic to a description based on algebraic objects such as curves or surfaces, which can be represented via algebraic equations, including cubic equations.

In this paper, we will hypothesize that the solutions of the cubic equations can be used as boundary conditions to further the understanding of the formation of universe bubbles in eternal inflation. Furthermore, an octahedron could serve as a mathematical or conceptual model of the universe in the pre-Big Bang phase. Cubic equations, in this context, could represent the geometric characteristics of the quantum vacuum, while the spherical surface could be used to describe the geometry of the universe bubble.

The results $(2\sqrt{2})/\pi$, the golden ratio φ and π obtained, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and universe bubbles in eternal inflation. It may reflect a fundamental relationship between the geometry of solids and the mathematical characteristics of solutions to cubic equations.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and universe bubbles.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of universe bubbles and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

Finally, it is also possible to hypothesize the solutions of the cubic equations solvable with the Cardano method and Padovan's recurrence relations, as boundary conditions to deepen the understanding of the formation of inflationary universe bubbles eternal and of an octahedron which could represent a mathematical or conceptual model of the primordial universe before the Big Bang. Furthermore, regarding the cubic equations, they can represent the geometric characteristics of the quantum vacuum, while the spherical surface represents the geometry of the universe bubble. The results above $(2\sqrt{2})/\pi$, the golden ratio φ and π which are obtained, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

These mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe. The cubic equations, in this context, could reflect the geometric and dynamic characteristics of the quantum vacuum, while the spherical surface could represent the geometry of the bubbles of the universe.

Regarding fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the primordial universe and universe bubbles.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

From

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

Cubic equations and Cardano's Formulas

We analyze the following formula:

$$\sqrt[3]{84\tfrac{9}{14} + \sqrt{7013\tfrac{1}{3}}} + \sqrt[3]{84\tfrac{9}{14} - \sqrt{7013\tfrac{1}{3}}}$$

(84*9/14+sqrt(7013*1/3))^(1/3)+(84*9/14-sqrt(7013*1/3))^(1/3)

Input

$$\sqrt[3]{84 \times \frac{9}{14} + \sqrt{7013 \times \frac{1}{3}}} + \sqrt[3]{84 \times \frac{9}{14} - \sqrt{7013 \times \frac{1}{3}}}$$

Result

$$\sqrt[3]{54 - \sqrt{\frac{7013}{3}}} + \sqrt[3]{54 + \sqrt{\frac{7013}{3}}}$$

Decimal approximation

6.458796161592982515706989786682472083017206577252833406785890435 1

…

6.458791615929….

Alternate forms

$$\frac{1}{3}\times 3^{2/3}\left(\sqrt[3]{162-\sqrt{21039}}+\sqrt[3]{162+\sqrt{21039}}\right)$$

---

root of $x^9-324\,x^6+19377\,x^3-1259712$ near $x=6.4588$

$$\frac{\sqrt[3]{162-\sqrt{21039}}+\sqrt[3]{162+\sqrt{21039}}}{\sqrt[3]{3}}$$

Minimal polynomial

$x^9-324\,x^6+19377\,x^3-1259712$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

From

$$\sqrt[3]{165\tfrac{8}{27}+\sqrt{27{,}161\tfrac{13}{27}}}+\sqrt[3]{165\tfrac{8}{27}-\sqrt{27{,}161\tfrac{13}{27}}}-2\tfrac{1}{3}.$$

(165*8/27+sqrt(27161*3/14))^1/3+(165*8/27-sqrt(27161*3/14))^1/3 – 2/3

Input

$$\sqrt[3]{165\times\frac{8}{27}+\sqrt{27161\times\frac{3}{14}}}+\sqrt[3]{165\times\frac{8}{27}-\sqrt{27161\times\frac{3}{14}}}-\frac{2}{3}$$

Result

$$-\frac{2}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}$$

Decimal approximation

5.843119295162315196501074218640446588870513218292767218093352505…

+

2.610889113030967045476170307559989203233160825467758590799343312…

i

Alternate complex forms

$$\left(-\frac{2}{3}+\frac{1}{2}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\frac{1}{2}\left(\sqrt{3}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)i$$

$$\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)}\left(\cos\left(\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}}{2\left(-\frac{2}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}}{2\left(-\frac{2}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)}\right)\right)\right)$$

$$\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)}\exp\left(i\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}}{2\left(-\frac{2}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)}\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)}\quad\text{(radius)},$$

$$\theta=\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}}{2\left(-\frac{2}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)}\right)\quad\text{(angle)}$$

Alternate forms

$$\frac{1}{42}\left(14^{2/3}\sqrt[3]{-3\left(9\sqrt{1\,140\,762}-6160\right)}+14^{2/3}\sqrt[3]{3\left(6160+9\sqrt{1\,140\,762}\right)}-28\right)$$

$$-\frac{2}{3}+\frac{\sqrt[3]{\frac{1}{14}\left(6160-9\sqrt{1\,140\,762}\right)}}{3^{2/3}}+\frac{\sqrt[3]{\frac{1}{14}\left(6160+9\sqrt{1\,140\,762}\right)}}{3^{2/3}}$$

$$\frac{-2\times 3^{2/3}+3\sqrt[3]{440-9\sqrt{\frac{81483}{14}}}+3\sqrt[3]{440+9\sqrt{\frac{81483}{14}}}}{3\times 3^{2/3}}$$

Minimal polynomial

$$275562\,x^9+1653372\,x^8+4408992\,x^7-73973088\,x^6-316467648\,x^5-534304512\,x^4+32947029531\,x^3+66609077814\,x^2+44501561892\,x-247700098040$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

Performing the sum of the two previous results:

$$\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}$$

and

$$-\frac{2}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}$$

we obtain:

(((54 - sqrt(7013/3))^(1/3) + (54 + sqrt(7013/3))^(1/3)))+((-2/3 + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3)))

Input

$$\left(\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}\right)+\left(-\frac{2}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)$$

Result

$$-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}$$

Decimal approximation

12.301915456755297712208064005322918671887719795545600624879242 94…

+

2.610889113030967045476170307559989203233160825467758590799343312…

i

Alternate complex forms

$$\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\frac{1}{2}\left(\sqrt{3}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)i$$

$$\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.}$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)$$

$$\left(\cos\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\right.\right.\right.$$

$$\left.\left.\left.\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)+$$

$$i\sin\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\right.\right.$$

$$\left.\left.\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)$$

$$\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)}$$

$$\exp\left(i\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)}$$

(radius),

$$\theta=\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)$$

(angle)

Alternate forms

$$\frac{1}{42}\left(14\times 3^{2/3}\sqrt[3]{162-\sqrt{21\,039}}+14\times 3^{2/3}\sqrt[3]{162+\sqrt{21\,039}}+14^{2/3}\sqrt[3]{-3\left(9\sqrt{1\,140\,762}-6160\right)}+14^{2/3}\sqrt[3]{3\left(6160+9\sqrt{1\,140\,762}\right)}-28\right)$$

$$-\frac{2}{3}+\sqrt[3]{\frac{1}{3}\left(162-\sqrt{21039}\right)}+\sqrt[3]{\frac{1}{3}\left(162+\sqrt{21039}\right)}+\frac{\sqrt[3]{\frac{1}{14}\left(6160-9\sqrt{1140762}\right)}}{3^{2/3}}+\frac{\sqrt[3]{\frac{1}{14}\left(6160+9\sqrt{1140762}\right)}}{3^{2/3}}$$

$$-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81483}{14}}-\frac{440}{9}}+\frac{1}{2}i\sqrt{3}$$
$$\sqrt[3]{\sqrt{\frac{81483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}$$

From the result

$$-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}$$

multiplying by 1/4, we obtain:

1/4(-2/3 + (54 - sqrt(7013/3))^(1/3) + (54 + sqrt(7013/3))^(1/3) + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3))

Input

$$\frac{1}{4}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)$$

Decimal approximation

3.075478864188824428052016001330729667971929948886400156219810 73…

+

0.652722278257741761369042576889997300808290206366939647699835828…

i

Alternate complex forms

$$\frac{1}{4}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\frac{1}{8}\left(\sqrt{3}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)i$$

$$\frac{1}{4}\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.}$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)$$

$$\left(\cos\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\right.\right.\right.$$

$$\left.\left.\left.\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)+$$

$$i\sin\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\right.\right.$$

$$\left.\left.\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)$$

$$\frac{1}{4}\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)}$$

$$\exp\left(i\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=\frac{1}{4}\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)}\ \text{(radius)},$$

$$\theta=\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\ \text{(angle)}$$

$r\approx 3.1440$ (radius), $\theta\approx 0.20913$ (angle)

3.1440 ≈ π (Ramanujan Recurring Number)

From which:

1/6(1/4(-2/3 + (54 - sqrt(7013/3))^(1/3) + (54 + sqrt(7013/3))^(1/3) + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3)))^2

Input

$$\frac{1}{6}\left(\frac{1}{4}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)^2$$

Exact result

$$\frac{1}{96}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2$$

Decimal approximation

1.505420645256367451282980653251201004475914319834691222868969 93…

+

0.669144523655620480776483025565985036408341629469527217644279233…

i

Alternate complex forms

$$\frac{1}{96}\left(-\frac{3}{4}\left(-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}\right)^{2/3}+\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)+\frac{1}{32\sqrt{3}}\left(\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)i$$

$$\left(\frac{1}{216}+\frac{\sqrt[3]{\frac{1735}{3}}}{48}+\frac{\sqrt[3]{\frac{3\,889\,723}{42}}}{288}-\frac{1}{72}\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\frac{1}{96}\left(54-\sqrt{\frac{7013}{3}}\right)^{2/3}-\right.$$

$$\frac{1}{72}\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{96}\left(54+\sqrt{\frac{7013}{3}}\right)^{2/3}-\frac{1}{144}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+$$

$$\frac{1}{96}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\frac{1}{96}\sqrt[3]{\left(54-\sqrt{\frac{7013}{3}}\right)\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)}+$$

$$\frac{1}{96}\sqrt[3]{\left(54+\sqrt{\frac{7013}{3}}\right)\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)}-\frac{1}{72}\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+$$

$$\frac{1}{96}\left(\frac{440}{9}+\sqrt{\frac{81\,483}{14}}\right)^{2/3}+\frac{1}{48}\sqrt[3]{\left(54-\sqrt{\frac{7013}{3}}\right)\left(\frac{440}{9}+\sqrt{\frac{81\,483}{14}}\right)}+$$

$$\left.\frac{1}{48}\sqrt[3]{\left(54+\sqrt{\frac{7013}{3}}\right)\left(\frac{440}{9}+\sqrt{\frac{81\,483}{14}}\right)}\right)\left(\cos\left(\tan^{-1}\left(\right.\right.\right.$$

$$\left(\sqrt{3}\,\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\Big/$$

$$\left(\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2-\right.$$

$$\left.\left.\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}\right)\right)+i\sin\left(\tan^{-1}\left(\left(\left(\sqrt{3}\,\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right.\right.\right.\right.$$

$$\left.\left.\left.\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.$$

$$\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\Big/$$

$$\left(\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2-\right.$$

$$\left.\left.\left.\left.\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}\right)\right)\right)\right)$$

$$\left(\frac{1}{216}+\frac{\sqrt[3]{\frac{1735}{3}}}{48}+\frac{\sqrt[3]{\frac{3\,889\,723}{42}}}{288}-\frac{1}{72}\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\frac{1}{96}\left(54-\sqrt{\frac{7013}{3}}\right)^{2/3}-\right.$$

$$\frac{1}{72}\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{96}\left(54+\sqrt{\frac{7013}{3}}\right)^{2/3}-\frac{1}{144}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+$$

$$\frac{1}{96}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\frac{1}{96}\sqrt[3]{\left(54-\sqrt{\frac{7013}{3}}\right)\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)}+$$

$$\frac{1}{96}\sqrt[3]{\left(54+\sqrt{\frac{7013}{3}}\right)\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)}-\frac{1}{72}\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+$$

$$\frac{1}{96}\left(\frac{440}{9}+\sqrt{\frac{81\,483}{14}}\right)^{2/3}+\frac{1}{48}\sqrt[3]{\left(54-\sqrt{\frac{7013}{3}}\right)\left(\frac{440}{9}+\sqrt{\frac{81\,483}{14}}\right)}+$$

$$\left.\frac{1}{48}\sqrt[3]{\left(54+\sqrt{\frac{7013}{3}}\right)\left(\frac{440}{9}+\sqrt{\frac{81\,483}{14}}\right)}\right)\exp\left(i\tan^{-1}\left(\right.\right.$$

$$\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\Big/$$

$$\left(\left(\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\right.\right.\right.$$

$$\left.\left.\left.\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2-\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r = \frac{1}{216} + \frac{\sqrt[3]{\frac{1735}{3}}}{48} + \frac{\sqrt[3]{\frac{3\,889\,723}{42}}}{288} - \frac{1}{72}\sqrt[3]{54 - \sqrt{\frac{7013}{3}}} + \frac{1}{96}\left(54 - \sqrt{\frac{7013}{3}}\right)^{2/3} - \frac{1}{72}\sqrt[3]{54 + \sqrt{\frac{7013}{3}}} + \frac{1}{96}\left(54 + \sqrt{\frac{7013}{3}}\right)^{2/3} - \frac{1}{144}\sqrt[3]{\sqrt{\frac{81\,483}{14}} - \frac{440}{9}} + \frac{1}{96}\left(\sqrt{\frac{81\,483}{14}} - \frac{440}{9}\right)^{2/3} + \frac{1}{96}\sqrt[3]{\left(54 - \sqrt{\frac{7013}{3}}\right)\left(\sqrt{\frac{81\,483}{14}} - \frac{440}{9}\right)} + \frac{1}{96}\sqrt[3]{\left(54 + \sqrt{\frac{7013}{3}}\right)\left(\sqrt{\frac{81\,483}{14}} - \frac{440}{9}\right)} - \frac{1}{72}\sqrt[3]{\frac{440}{9} + \sqrt{\frac{81\,483}{14}}} + \frac{1}{96}\left(\frac{440}{9} + \sqrt{\frac{81\,483}{14}}\right)^{2/3} + \frac{1}{48}\sqrt[3]{\left(54 - \sqrt{\frac{7013}{3}}\right)\left(\frac{440}{9} + \sqrt{\frac{81\,483}{14}}\right)} + \frac{1}{48}\sqrt[3]{\left(54 + \sqrt{\frac{7013}{3}}\right)\left(\frac{440}{9} + \sqrt{\frac{81\,483}{14}}\right)} \text{ (radius)}, \quad \theta = \tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}} - \frac{440}{9}}\left(-\frac{2}{3} + \sqrt[3]{54 - \sqrt{\frac{7013}{3}}} + \sqrt[3]{54 + \sqrt{\frac{7013}{3}}} + \frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}} - \frac{440}{9}} + \sqrt[3]{\frac{440}{9} + \sqrt{\frac{81\,483}{14}}}\right)\right) \Big/ \left(\left(-\frac{2}{3} + \sqrt[3]{54 - \sqrt{\frac{7013}{3}}} + \sqrt[3]{54 + \sqrt{\frac{7013}{3}}} + \frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}} - \frac{440}{9}} + \sqrt[3]{\frac{440}{9} + \sqrt{\frac{81\,483}{14}}}\right)^2 - \frac{3}{4}\left(\sqrt{\frac{81\,483}{14}} - \frac{440}{9}\right)^{2/3}\right)\right) \text{ (angle)}$$

$r \approx 1.6474$ (radius), $\theta \approx 0.41826$ (angle)

1.6474 ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the previous formula

$$\left(\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}\right)+$$
$$\left(-\frac{2}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)$$

we obtain:

((((54 - sqrt(7013/3))^(1/3) + (54 + sqrt(7013/3))^(1/3)))+((-2/3 + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3)))) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

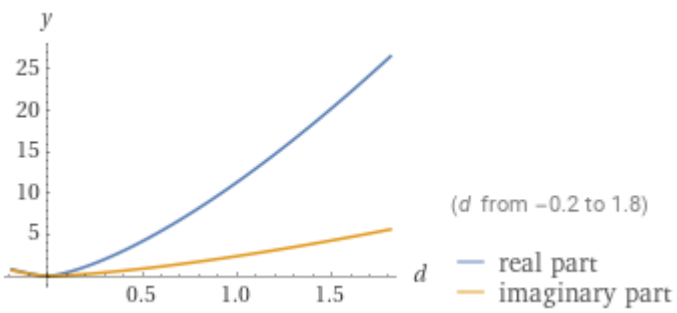
Input

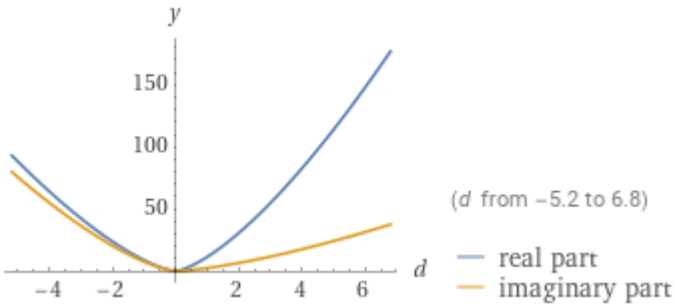
$$\left(\left(\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}\right)+\left(-\frac{2}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Exact result

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{d^9}$$

Plots **(figures that can be related to the open strings)**





And:

((((54 - sqrt(7013/3))^(1/3) + (54 + sqrt(7013/3))^(1/3)))+((-2/3 + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3)))) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

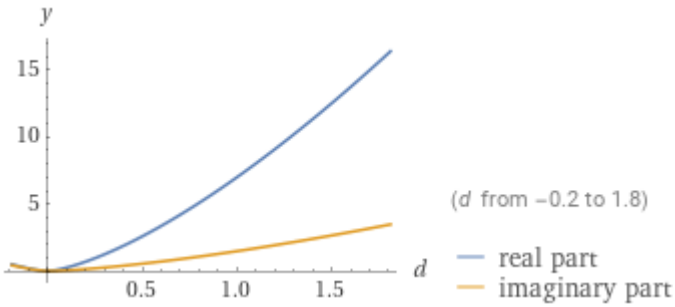
Input

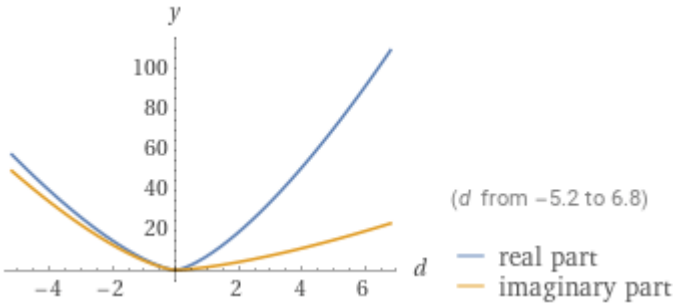
$$\left(\left(\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}\right)+\left(-\frac{2}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

Exact result

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

Plots **(figures that can be related to the open strings)**





Dividing the two exact results

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{d^9}$$

and

$$2^{-1/\pi}\times 3^{-3/(2\pi)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\sqrt[2\pi]{\pi}\sqrt[2\pi]{d^9}$$

we obtain, for d = 2 :

3^(-3/(2 π)) (5 (3 + sqrt(5)))^(1/(2 π)) (-2/3 + (54 - sqrt(7013/3))^(1/3) + (54 + sqrt(7013/3))^(1/3) + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3)) (π/4)^(1/π) (2^9)^(1/(2 π))

Input

$$3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right) \sqrt[\pi]{\frac{\pi}{4}} \sqrt[2\pi]{2^9}$$

Exact result

$$2^{5/(2\pi)} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right) \sqrt[\pi]{\pi}$$

Decimal approximation

30.594381174782310015291163811947262482767608586844966845756231 81…

\+

6.493178806987764252376405774990943062634967357870447124120456272…

i

Alternate complex forms

$$2^{5/(2\pi)} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.$$
$$\left.\frac{1}{2}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\sqrt[\pi]{\pi}+$$
$$\left(2^{-1+5/(2\pi)} \times 3^{1/2-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\ \sqrt[\pi]{\pi}\right) i$$

$$2^{5/(2\pi)} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$

$$\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.}$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)$$

$$\sqrt[\pi]{\pi}\left(\cos\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\right.\right.\right.$$

$$\left.\left.\left.\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)+$$

$$i\sin\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\right.\right.$$

$$\left.\left.\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)$$

$$2^{5/(2\pi)} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.}$$
$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)$$
$$\sqrt[\pi]{\pi}\,\exp\left(i\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\right.\right.$$
$$\left(2\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.$$
$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 31.276$ (radius), $\theta \approx 0.20913$ (angle)

31.276

Alternate forms

$$2^{5/(2\pi)} \times 3^{-5/3-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\left(-2\times 3^{2/3}+3\times 3^{2/3}\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+3\times 3^{2/3}\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.$$
$$\left.3\sqrt[3]{440-9\sqrt{\frac{81\,483}{14}}}+3\sqrt[3]{440+9\sqrt{\frac{81\,483}{14}}}\right)\sqrt[\pi]{\pi}$$

$$\frac{1}{7}\times 2^{5/(2\pi)-2}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\left(-56+28\times 3^{2/3}\sqrt[3]{162-\sqrt{21039}}+28\times 3^{2/3}\sqrt[3]{162+\sqrt{21039}}+\right.$$
$$\left.14^{2/3}\sqrt[3]{3\left(9\sqrt{1140762}-6160\right)}+2\times 14^{2/3}\sqrt[3]{3\left(6160+9\sqrt{1140762}\right)}\right)$$
$$\sqrt[\pi]{\pi}+i\,2^{5/(2\pi)-1}\times 3^{(\pi-3)/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[3]{\sqrt{\frac{81483}{14}}-\frac{440}{9}}\sqrt[\pi]{\pi}$$

$$\frac{1}{7}\times 2^{5/(2\pi)-1}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\left(-28+14\times 3^{2/3}\sqrt[3]{162-\sqrt{21039}}+14\times 3^{2/3}\sqrt[3]{162+\sqrt{21039}}+\right.$$
$$\left.14^{2/3}\sqrt[3]{18480-27\sqrt{1140762}}+14^{2/3}\sqrt[3]{3\left(6160+9\sqrt{1140762}\right)}\right)\sqrt[\pi]{\pi}$$

Expanded form

$$-2^{1+5/(2\pi)}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[\pi]{\pi}+$$
$$2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[3]{54-\sqrt{\frac{7013}{3}}}\sqrt[\pi]{\pi}+$$
$$2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[3]{54+\sqrt{\frac{7013}{3}}}\sqrt[\pi]{\pi}+$$
$$2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}\sqrt[\pi]{\pi}+$$
$$2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\sqrt[\pi]{\pi}$$

Series representations

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{2^9}=$$

$$2^{5/(2\pi)}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\left(-2+3\sqrt[3]{54-\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\left(\frac{7010}{3}\right)^{-k}\binom{\frac{1}{2}}{k}}+3\sqrt[3]{54+\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\left(\frac{7010}{3}\right)^{-k}\binom{\frac{1}{2}}{k}}+\sqrt[3]{3}\sqrt[3]{440-9\sqrt{\frac{81469}{14}}\sum_{k=0}^{\infty}\left(\frac{81469}{14}\right)^{-k}\binom{\frac{1}{2}}{k}}+\sqrt[3]{3}\sqrt[3]{440+9\sqrt{\frac{81469}{14}}\sum_{k=0}^{\infty}\left(\frac{81469}{14}\right)^{-k}\binom{\frac{1}{2}}{k}}\right)$$

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{2^9}=$$

$$2^{5/(2\pi)}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\left(-2+3\sqrt[3]{54-\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{7010}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+3\sqrt[3]{54+\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{7010}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\sqrt[3]{3}\sqrt[3]{440-9\sqrt{\frac{81469}{14}}\sum_{k=0}^{\infty}\frac{\left(-\frac{14}{81469}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\sqrt[3]{3}\sqrt[3]{440+9\sqrt{\frac{81469}{14}}\sum_{k=0}^{\infty}\frac{\left(-\frac{14}{81469}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\sqrt[\pi]{\frac{\pi}{4}}\sqrt[2\pi]{2^9}=$$

$$2^{5/(2\pi)}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[\pi]{\pi}\sqrt[2\pi]{3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}$$

$$\left(-2+3\sqrt[3]{54-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{7013}{3}-z_0\right)^k z_0^{-k}}{k!}}+3\sqrt[3]{54+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{7013}{3}-z_0\right)^k z_0^{-k}}{k!}}+\sqrt[3]{3}\sqrt[3]{440-9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{81\,483}{14}-z_0\right)^k z_0^{-k}}{k!}}+\sqrt[3]{3}\sqrt[3]{440+9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{81\,483}{14}-z_0\right)^k z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

31.276/((2^(-1/π) 3^(-3/(2 π)) (-2/3 + (54 - sqrt(7013/3))^(1/3) + (54 + sqrt(7013/3))^(1/3) + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3)) π^(1/(2 π)) (2^9)^(1/(2 π))))

Input interpretation

$$31.276 \Big/ \left(2^{-1/\pi} \times 3^{-3/(2\pi)} \left(-\frac{2}{3} + \sqrt[3]{54 - \sqrt{\frac{7013}{3}}} + \sqrt[3]{54 + \sqrt{\frac{7013}{3}}} + \sqrt[3]{\frac{440}{9} - \sqrt{\frac{81\,483}{14}}} + \sqrt[3]{\frac{440}{9} + \sqrt{\frac{81\,483}{14}}}\right) \sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9}\right)$$

Result

1.58276… −
0.335917… i

Alternate complex forms

$1.58276 - 0.335917\,i$

$1.61802\,(\cos(-0.209131) + i\sin(-0.209131))$

$1.61802\,e^{-0.209131\,i}$

i is the imaginary unit

Polar coordinates

$r = 1.61802$ (radius), $\theta = -0.209131$ (angle)

1.61802 result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Series representations

$$31.276 \Bigg/ \left(2^{-1/\pi} \times 3^{-3/(2\pi)} \left(-\frac{2}{3} + \sqrt[3]{54 - \sqrt{\frac{7013}{3}}} + \sqrt[3]{54 + \sqrt{\frac{7013}{3}}} + \sqrt[3]{\frac{440}{9} - \sqrt{\frac{81483}{14}}} + \sqrt[3]{\frac{440}{9} + \sqrt{\frac{81483}{14}}}\right) \sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9}\right) =$$

$$\left(31.276 \times 2^{-7/(2\pi)} \times 3^{3/(2\pi)}\, \pi^{-1/(2\pi)}\right) \Bigg/ \left(-0.666667 + \sqrt[3]{54 - \sqrt{\frac{7010}{3}} \sum_{k=0}^{\infty} \left(\frac{7010}{3}\right)^{-k} \binom{\frac{1}{2}}{k}} + \sqrt[3]{54 + \sqrt{\frac{7010}{3}} \sum_{k=0}^{\infty} \left(\frac{7010}{3}\right)^{-k} \binom{\frac{1}{2}}{k}} + 0.48075 \sqrt[3]{440 - 9\sqrt{\frac{81469}{14}} \sum_{k=0}^{\infty} \left(\frac{81469}{14}\right)^{-k} \binom{\frac{1}{2}}{k}} + 0.48075 \sqrt[3]{440 + 9\sqrt{\frac{81469}{14}} \sum_{k=0}^{\infty} \left(\frac{81469}{14}\right)^{-k} \binom{\frac{1}{2}}{k}}\right)$$

$$31.276 \Big/ \left(2^{-1/\pi} \times 3^{-3/(2\pi)} \left(-\frac{2}{3} + \sqrt[3]{54 - \sqrt{\frac{7013}{3}}} + \sqrt[3]{54 + \sqrt{\frac{7013}{3}}} + \sqrt[3]{\frac{440}{9} - \sqrt{\frac{81483}{14}}} + \sqrt[3]{\frac{440}{9} + \sqrt{\frac{81483}{14}}} \right) \sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9} \right) =$$

$$\left(31.276 \times 2^{-7/(2\pi)} \times 3^{3/(2\pi)}\, \pi^{-1/(2\pi)}\right) \Big/ \left(-0.666667 + \sqrt[3]{54 - \sqrt{\frac{7010}{3}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{7010}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + \sqrt[3]{54 + \sqrt{\frac{7010}{3}} \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{7010}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + 0.48075 \sqrt[3]{440 - 9\sqrt{\frac{81469}{14}} \sum_{k=0}^{\infty} \frac{\left(-\frac{14}{81469}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + 0.48075 \sqrt[3]{440 + 9\sqrt{\frac{81469}{14}} \sum_{k=0}^{\infty} \frac{\left(-\frac{14}{81469}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} \right)$$

$$31.276 \Big/ \left(2^{-1/\pi} \times 3^{-3/(2\pi)} \left(-\frac{2}{3} + \sqrt[3]{54 - \sqrt{\frac{7013}{3}}} + \sqrt[3]{54 + \sqrt{\frac{7013}{3}}} + \sqrt[3]{\frac{440}{9} - \sqrt{\frac{81\,483}{14}}} + \sqrt[3]{\frac{440}{9} + \sqrt{\frac{81\,483}{14}}} \right) \sqrt[2\pi]{\pi}\, \sqrt[2\pi]{2^9} \right) =$$

$$\left(31.276 \times 2^{-7/(2\pi)} \times 3^{3/(2\pi)}\, \pi^{-1/(2\pi)}\right) \Big/ \left(-0.666667 + \sqrt[3]{54 - \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{7013}{3} - z_0\right)^k z_0^{-k}}{k!}} + \sqrt[3]{54 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{7013}{3} - z_0\right)^k z_0^{-k}}{k!}} + 0.48075 \sqrt[3]{440 - 9\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{81\,483}{14} - z_0\right)^k z_0^{-k}}{k!}} + 0.48075 \sqrt[3]{440 + 9\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{81\,483}{14} - z_0\right)^k z_0^{-k}}{k!}} \right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

Furthermore, we obtain also:

(4(31.276/((2^(-1/π)3^(-3/(2π))(-2/3+(54-√(7013/3))^(1/3)+(54+√(7013/3))^(1/3)+(440/9-√(81483/14))^(1/3)+(440/9+√(81483/14))^(1/3))π^(1/(2π))(2^9)^(1/(2π)))))^(2π))/(5(3+√(5)))

Input interpretation

$$\frac{1}{5\left(3+\sqrt{5}\right)}\,4\left(31.276\Big/\left(2^{-1/\pi}\times 3^{-3/(2\pi)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{2^9}\right)\right)^{2\pi}$$

Result

0.797908… −
3.03869… i

Alternate complex forms

$0.797908 - 3.03869\,i$

$3.1417\,(\cos(-1.31401) + i\sin(-1.31401))$

$3.1417\,e^{-1.31401\,i}$

i is the imaginary unit

Polar coordinates

$r = 3.1417$ (radius), $\theta = -1.31401$ (angle)

3.1417 ≈ π (Ramanujan Recurring Number)

Series representations

$$\frac{1}{5\left(3+\sqrt{5}\right)}$$

$$4\left(31.276\Big/\left(2^{-1/\pi}\times 3^{-3/(2\pi)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.$$

$$\left.\left.\left.\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{2^9}\right)\right)^{2\pi}=$$

$$\left(4\times 31.276^{2\pi}\left(\left(2^{-7/(2\pi)}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right)\Big/\right.\right.$$

$$\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\left(\frac{7010}{3}\right)^{-k}\binom{\frac{1}{2}}{k}}+\right.$$

$$\sqrt[3]{54+\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\left(\frac{7010}{3}\right)^{-k}\binom{\frac{1}{2}}{k}}+$$

$$\frac{\sqrt[3]{440-9\sqrt{\frac{81\,469}{14}}\sum_{k=0}^{\infty}\left(\frac{81\,469}{14}\right)^{-k}\binom{\frac{1}{2}}{k}}}{3^{2/3}}+$$

$$\left.\left.\left.\frac{\sqrt[3]{440+9\sqrt{\frac{81\,469}{14}}\sum_{k=0}^{\infty}\left(\frac{81\,469}{14}\right)^{-k}\binom{\frac{1}{2}}{k}}}{3^{2/3}}\right)\right)^{2\pi}\right)\Big/\left(5\right.$$

$$\left.\left.\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)$$

$$\frac{1}{5\left(3+\sqrt{5}\right)}$$

$$4\left(31.276\Big/\left(2^{-1/\pi}\times 3^{-3/(2\pi)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.$$

$$\left.\left.\left.\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\sqrt[2\pi]{\pi}\sqrt[2\pi]{2^9}\right)\right)^{2\pi}=$$

$$\left(4\times 31.276^{2\pi}\left(\left(2^{-7/(2\pi)}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right)\Big/\right.\right.$$

$$\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{7010}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\right.$$

$$\sqrt[3]{54+\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{7010}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+$$

$$\frac{\sqrt[3]{440-9\sqrt{\frac{81\,469}{14}}\sum_{k=0}^{\infty}\frac{\left(-\frac{14}{81\,469}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{3^{2/3}}+$$

$$\left.\left.\left.\frac{\sqrt[3]{440+9\sqrt{\frac{81\,469}{14}}\sum_{k=0}^{\infty}\frac{\left(-\frac{14}{81\,469}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{3^{2/3}}\right)\right)^{2\pi}\right)\Big/$$

$$\left(5\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$\frac{1}{5\left(3+\sqrt{5}\right)}$$

$$4\left(31.276\Big/\left(2^{-1/\pi}\times 3^{-3/(2\pi)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.$$

$$\left.\left.\left.\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^{2\pi\sqrt{\pi}}\right.\right.$$

$$\left.\left.{}^{2\pi}\sqrt{2^9}\right)\right)^{2\pi}=\left(4\times 31.276^{2\pi}\left(\left(2^{-7/(2\pi)}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right)\Big/\right.\right.$$

$$\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{7013}{3}-z_0\right)^k z_0^{-k}}{k!}}+\right.$$

$$\sqrt[3]{54+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{7013}{3}-z_0\right)^k z_0^{-k}}{k!}}+$$

$$\frac{\sqrt[3]{440-9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{81\,483}{14}-z_0\right)^k z_0^{-k}}{k!}}}{3^{2/3}}+$$

$$\left.\left.\left.\frac{\sqrt[3]{440+9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{81\,483}{14}-z_0\right)^k z_0^{-k}}{k!}}}{3^{2/3}}\right)^{2\pi}\right)\Big/\right.$$

$$\left.\left(5\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\right.$$ for

$$\text{(not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

1/6((4(31.276/((2^(-1/π)3^(-3/(2π))(-2/3+(54-√(7013/3))^(1/3)+(54+√(7013/3))^(1/3)+(440/9-√(81483/14))^(1/3)+(440/9+√(81483/14))^(1/3))π^(1/(2π))(2^9)^(1/(2π)))))^(2π))/(5(3+√(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{1}{5\left(3+\sqrt{5}\right)}\right.$$
$$4\left(31.276\Big/\left(2^{-1/\pi}\times 3^{-3/(2\pi)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.$$
$$\left.\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\right.$$
$$\left.\left.\left.\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\sqrt[2\pi]{\pi}\sqrt[2\pi]{2^9}\right)\right)^{2\pi}\Bigg)^2$$

Result

$-1.43283\ldots - 0.808198\ldots\, i$

Alternate complex forms

$-1.43283 - 0.808198\, i$

$1.64505\,(\cos(-2.62802) + i\sin(-2.62802))$

$1.64505\, e^{-2.62802\, i}$

Polar coordinates

$r = 1.64505$ (radius), $\theta = -2.62802$ (angle)

1.64505 ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{1}{5\left(3+\sqrt{5}\right)}\right.$$
$$4\left(31.276\Big/\left(2^{-1/\pi}\times 3^{-3/(2\pi)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.$$
$$\left.\left.\left.\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right.\right.$$
$$\left.\left.\left.{}^{2\pi}\sqrt{\pi}\;{}^{2\pi}\sqrt{2^9}\right)\right)^{2\pi}\right)^2=\left(8\times 31.276^{4\pi}\right.$$
$$\left(\left(2^{-7/(2\pi)}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right)\Big/\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\left(\frac{7010}{3}\right)^{-k}\binom{\frac{1}{2}}{k}}+\right.\right.$$
$$\sqrt[3]{54+\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\left(\frac{7010}{3}\right)^{-k}\binom{\frac{1}{2}}{k}}+$$
$$\frac{\sqrt[3]{440-9\sqrt{\frac{81\,469}{14}}\sum_{k=0}^{\infty}\left(\frac{81\,469}{14}\right)^{-k}\binom{\frac{1}{2}}{k}}}{3^{2/3}}+$$
$$\left.\left.\frac{\sqrt[3]{440+9\sqrt{\frac{81\,469}{14}}\sum_{k=0}^{\infty}\left(\frac{81\,469}{14}\right)^{-k}\binom{\frac{1}{2}}{k}}}{3^{2/3}}\right)^{4\pi}\right)\Big/$$
$$\left.\left(75\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2\right)\right)$$

$$\frac{1}{6}\left(\frac{1}{5\left(3+\sqrt{5}\right)}\right.$$
$$4\left(31.276\Big/\left(2^{-1/\pi}\times 3^{-3/(2\pi)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\right.\right.\right.$$
$$\left.\left.\left.\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right.\right.$$
$$\left.\left.\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{2^9}\right)\right)^{2\pi}\Bigg)^2=\left(8\times 31.276^{4\pi}\right.$$
$$\left(\left(2^{-7/(2\pi)}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right)\Big/\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{7010}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\right.\right.$$
$$\sqrt[3]{54+\sqrt{\frac{7010}{3}}\sum_{k=0}^{\infty}\frac{\left(-\frac{3}{7010}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+$$
$$\frac{\sqrt[3]{440-9\sqrt{\frac{81\,469}{14}}\sum_{k=0}^{\infty}\frac{\left(-\frac{14}{81\,469}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{3^{2/3}}+$$
$$\left.\left.\frac{\sqrt[3]{440+9\sqrt{\frac{81\,469}{14}}\sum_{k=0}^{\infty}\frac{\left(-\frac{14}{81\,469}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}{3^{2/3}}\right)^{4\pi}\right)\Big/$$
$$\left(75\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2\right)$$

$$\frac{1}{6}\left(\frac{1}{5\left(3+\sqrt{5}\right)}4\left(31.276\Big/\left(2^{-1/\pi}\times 3^{-3/(2\pi)}\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{\frac{7013}{3}}}+\sqrt[3]{54+\sqrt{\frac{7013}{3}}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{2^9}\right)\right)^{2\pi}\right)^2=\left(8\times 31.276^{4\pi}\left(\left(2^{-7/(2\pi)}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right)\Big/\left(-\frac{2}{3}+\sqrt[3]{54-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{7013}{3}-z_0\right)^k z_0^{-k}}{k!}}+\sqrt[3]{54+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{7013}{3}-z_0\right)^k z_0^{-k}}{k!}}+\frac{\sqrt[3]{440-9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{81483}{14}-z_0\right)^k z_0^{-k}}{k!}}}{3^{2/3}}+\frac{\sqrt[3]{440+9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{81483}{14}-z_0\right)^k z_0^{-k}}{k!}}}{3^{2/3}}\right)^{4\pi}\right)\Big/\left(75\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^2\right)$$

for (not
$(z_0 \in \mathbb{R}$ and
$-\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

ℝ is the set of real numbers

Now, we analyze the following equation:

$47\,x^3 - 29\,x^2 + 18\,x - 4 = 0$

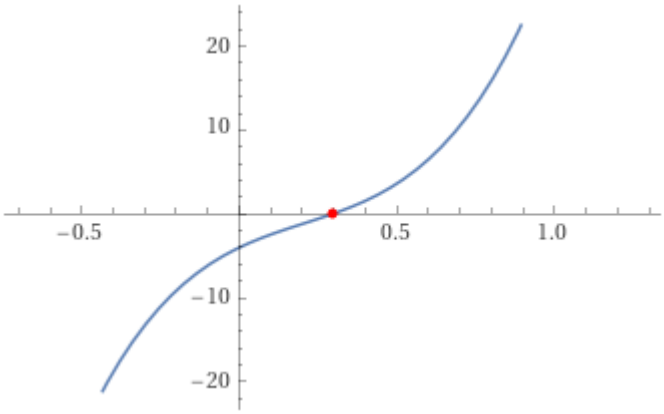
We note that 4, 18, 29 and 47 are Lucas numbers. We obtain:

47x^3-29x^2+18x-4=0

Input

$47\,x^3 - 29\,x^2 + 18\,x - 4 = 0$

Root plot **(figure that can be related to an open string)**



Alternate forms

$x\,(x\,(47\,x - 29) + 18) - 4 = 0$

$47\,x^3 - 29\,x^2 + 18\,x = 4$

$$47x^3 + 18x = 29x^2 + 4$$

Real solution

$$x = \frac{1}{141}\left(29 - \frac{1697}{\sqrt[3]{33272 + 987\sqrt{6153}}} + \sqrt[3]{33272 + 987\sqrt{6153}}\right)$$

Complex solutions

$$x = \frac{29}{141} + \frac{1697}{282\sqrt[3]{33272 + 987\sqrt{6153}}} - \frac{1}{282}\sqrt[3]{33272 + 987\sqrt{6153}} + i$$
$$\left(-\frac{1697}{94\sqrt{3}\sqrt[3]{33272 + 987\sqrt{6153}}} - \frac{\sqrt[3]{33272 + 987\sqrt{6153}}}{94\sqrt{3}}\right)$$

$$x = \frac{29}{141} + \frac{1697}{282\sqrt[3]{33272 + 987\sqrt{6153}}} - \frac{1}{282}\sqrt[3]{33272 + 987\sqrt{6153}} + i$$
$$\left(\frac{1697}{94\sqrt{3}\sqrt[3]{33272 + 987\sqrt{6153}}} + \frac{\sqrt[3]{33272 + 987\sqrt{6153}}}{94\sqrt{3}}\right)$$

Real solution

$x \approx 0.29554$

Complex solutions

$x \approx 0.16074 - 0.51199\,i$

$x \approx 0.16074 + 0.51199\,i$

Sum of roots

$$\frac{29}{47}$$

0.61702127659574 result that is a very good approximation to the value of the golden ratio conjugate 0.618033988749…

Product of roots

$\frac{4}{47}$

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the real solution

$$x = \frac{1}{141}\left(29 - \frac{1697}{\sqrt[3]{33\,272 + 987\sqrt{6153}}} + \sqrt[3]{33\,272 + 987\sqrt{6153}}\right)$$

we obtain:

((1/141 (29 - 1697/(33272 + 987 sqrt(6153))^(1/3) + (33272 + 987 sqrt(6153))^(1/3)))) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

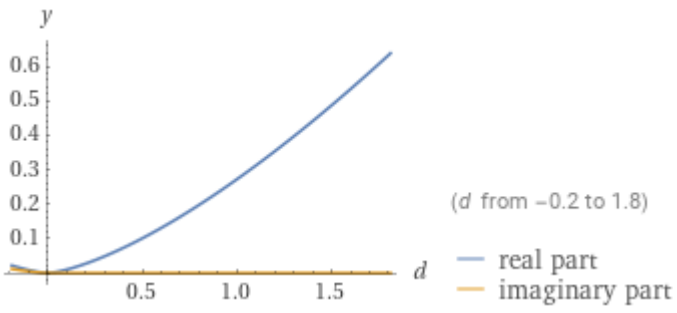
Input

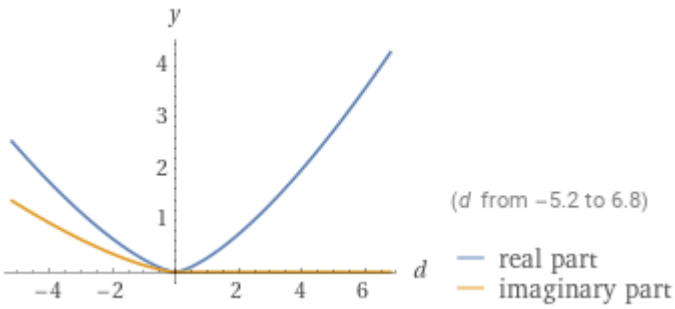
$$\left(\frac{1}{141}\left(29 - \frac{1697}{\sqrt[3]{33\,272 + 987\sqrt{6153}}} + \sqrt[3]{33\,272 + 987\sqrt{6153}}\right)\right)$$
$$\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Exact result

$$\frac{1}{47} \times 3^{-1-3/(2\pi)} \times 4^{-1/\pi} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(29 - \frac{1697}{\sqrt[3]{33\,272 + 987\sqrt{6153}}} + \sqrt[3]{33\,272 + 987\sqrt{6153}}\right) \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Plots **(figures that can be related to the open strings)**





Alternate forms

$$3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}$$

root of $47x^3-87x^2+162x-108$ near $x=0.886623$

$$\frac{1}{47}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(\sqrt[3]{33272+987\sqrt{6153}}\left(29+\sqrt[3]{33272+987\sqrt{6153}}\right)-1697\right)\sqrt[\pi]{\frac{\pi}{4}}\ \sqrt[2\pi]{d^9}$$

root of $4887035873x^6+66544x^3-1$ near $x=0.020827$

Expanded form

$$\frac{1}{47}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{33272+987\sqrt{6153}}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}\ -\ \frac{1697\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}}{47\sqrt[3]{33272+987\sqrt{6153}}}+\ \frac{29}{47}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Root

$d=0$

Series expansion at d=∞

$$\frac{1}{47}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[\pi]{\pi}\;d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{141}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=$$

$$\frac{1}{47}\times 2^{-(2+\pi)/\pi}\times 3^{1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)d^8\left(d^9\right)^{1/(2\pi)-1}\pi^{1/\pi-1}$$

Indefinite integral

$$\int\frac{1}{47}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{d^9}\;\sqrt[\pi]{\pi}\;dd=$$

$$\frac{1}{47\left(1+\frac{9}{2\pi}\right)}3^{-1-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[\pi]{\frac{\pi}{4}}\;d^{\sqrt[2\pi]{d^9}}+\text{constant}$$

Global minimum

$$\min\left\{\frac{1}{141}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right\}=0 \text{ at } d=0$$

Series representations

$$\frac{1}{141}\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)=$$
$$\left(3^{-1-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}\left(-1697+29\sqrt[3]{33\,272+987\sqrt{6152}}\sum_{k=0}^{\infty}6152^{-k}\binom{\frac{1}{2}}{k}+\left(33\,272+987\sqrt{6152}\sum_{k=0}^{\infty}6152^{-k}\binom{\frac{1}{2}}{k}\right)^{2/3}\right)\right)\Big/$$
$$\left(47\sqrt[3]{33\,272+987\sqrt{6152}}\sum_{k=0}^{\infty}6152^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{1}{141}\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)=$$
$$\left(3^{-1-3/(2\pi)}\times 4^{-1/\pi\,\sqrt[2\pi]{5}}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}\left(-1697+29\sqrt[3]{33\,272+987\sqrt{6152}}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6152}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\left(33\,272+987\sqrt{6152}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6152}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{2/3}\right)\right)\Big/$$
$$\left(47\sqrt[3]{33\,272+987\sqrt{6152}}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6152}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{1}{141}\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)=$$
$$\left(3^{-1-3/(2\pi)}\times 4^{-1/\pi\,\sqrt[2\pi]{5}}\sqrt[2\pi]{d^9\pi^2\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}\left(-1697+29\sqrt[3]{33\,272+987\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6153-z_0)^k z_0^{-k}}{k!}}+\left(33\,272+987\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6153-z_0)^k z_0^{-k}}{k!}\right)^{2/3}\right)\right)\Big/$$
$$\left(47\sqrt[3]{33\,272+987\sqrt{z_0}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6153-z_0)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

And:

((1/141 (29 - 1697/(33272 + 987 sqrt(6153))^(1/3) + (33272 + 987 sqrt(6153))^(1/3)))) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

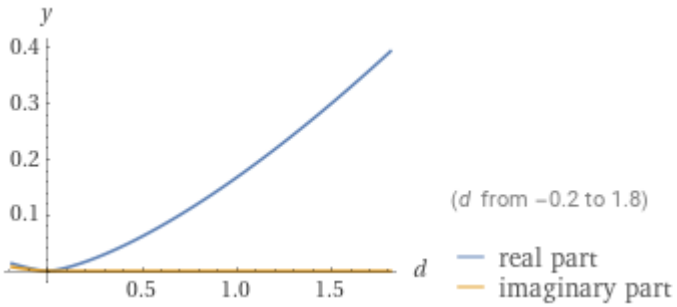
Input

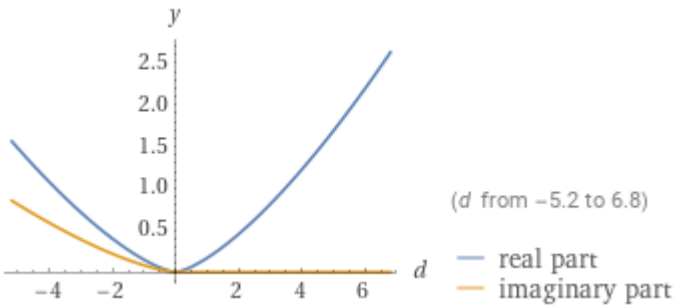
$$\left(\frac{1}{141}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\right)\Big/\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

Exact result

$$\frac{1}{47}\times 2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\Big/\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate forms

$$2^{-1/\pi} \times 3^{-1-3/(2\pi)} \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

root of $47x^3 - 87x^2 + 162x - 108$ near $x = 0.886623$

$$\left(\frac{29}{47} \times 2^{-1/\pi} \times 3^{-1-3/(2\pi)} - \frac{1697 \times 2^{-1/\pi} \times 3^{-1-3/(2\pi)}}{47\sqrt[3]{33\,272 + 987\sqrt{6153}}} + \frac{1}{47} \times 2^{-1/\pi} \times 3^{-1-3/(2\pi)} \sqrt[3]{33\,272 + 987\sqrt{6153}}\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Expanded form

$$\frac{1}{47} \times 2^{-1/\pi} \times 3^{-1-3/(2\pi)} \sqrt[3]{33\,272 + 987\sqrt{6153}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9} - \frac{1697 \times 2^{-1/\pi} \times 3^{-1-3/(2\pi)} \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}}{47\sqrt[3]{33\,272 + 987\sqrt{6153}}} + \frac{29}{47} \times 2^{-1/\pi} \times 3^{-1-3/(2\pi)} \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Root

$d = 0$

Series expansion at d=∞

$$\frac{1}{47} \times 2^{-1/\pi} \times 3^{-1-3/(2\pi)} \left(29 - \frac{1697}{\sqrt[3]{33\,272 + 987\sqrt{6153}}} + \sqrt[3]{33\,272 + 987\sqrt{6153}}\right) \sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{141}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right)=$$

$$\frac{1}{47}\times2^{-(1+\pi)/\pi}\times3^{1-3/(2\pi)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)d^8\left(d^9\right)^{1/(2\pi)-1}\pi^{1/(2\pi)-1}$$

Indefinite integral

$$\int\frac{1}{47}\times2^{-1/\pi}\times3^{-1-3/(2\pi)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{d^9}\,\sqrt[2\pi]{\pi}\,dd=\frac{1}{47\,(9+2\pi)}2^{(\pi-1)/\pi}\times3^{-1-3/(2\pi)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\pi^{1+1/(2\pi)}\,d\,\sqrt[2\pi]{d^9}+\text{constant}$$

Global minimum

$$\min\left\{\frac{1}{141}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right\}=0\ \text{at}\ d=0$$

Series representations

$$\frac{1}{141}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)=$$
$$\left(2^{-3/(2\pi)}\times3^{-1-3/(2\pi)}\sqrt[2\pi]{d^9\,\pi\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}\right.$$
$$\left(-1697+29\sqrt[3]{33\,272+987\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6153-z_0)^k\,z_0^{-k}}{k!}}+\right.$$
$$\left.\left.\left(33\,272+987\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6153-z_0)^k\,z_0^{-k}}{k!}\right)^{2/3}\right)\right)\Big/$$
$$\left(47\sqrt[3]{33\,272+987\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6153-z_0)^k\,z_0^{-k}}{k!}}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{141}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times 3\times 12}}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)=$$

$$\left(2^{-3/(2\pi)}\times 3^{-1-3/(2\pi)}\right.$$

$$\sqrt[2\pi]{d^9\,\pi\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\left(-1697+29\left(33\,272+987\exp\left(i\pi\left\lfloor\frac{\arg(6153-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(6153-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\wedge(1/3)+$$

$$\left(33\,272+987\exp\left(i\pi\left\lfloor\frac{\arg(6153-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(6153-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{2/3}\right)\right)\Big/$$

$$\left(47\left(33\,272+987\exp\left(i\pi\left\lfloor\frac{\arg(6153-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(6153-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\wedge(1/3)\right)\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$\frac{1}{141}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\ d^3\right)\left(\sqrt{2}\ d^3\right)}{3\times 3\times 12}}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)=$$

$$\left(2^{-3/(2\pi)}\times 3^{-1-3/(2\pi)}\right.$$

$$\sqrt[2\pi]{d^9\,\pi\left(\frac{1}{z_0}\right)^{\lfloor \arg(2-z_0)/(2\pi)\rfloor}\ z_0^{1+\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\ z_0^{-k}}{k!}\right)^2}$$

$$\left(-1697+29\left(33\,272+987\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(6153-z_0)/(2\pi)\rfloor}\ z_0^{1/2+1/2\lfloor \arg(6153-z_0)/(2\pi)\rfloor}\right.\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6153-z_0)^k\ z_0^{-k}}{k!}\right)\text{^}(1/3)+$$

$$\left(33\,272+987\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(6153-z_0)/(2\pi)\rfloor}\ z_0^{1/2+1/2\lfloor \arg(6153-z_0)/(2\pi)\rfloor}\right.$$

$$\left.\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6153-z_0)^k\ z_0^{-k}}{k!}\right)^{2/3}\right)\right/$$

$$\left(47\left(33\,272+987\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(6153-z_0)/(2\pi)\rfloor}\ z_0^{1/2+1/2\lfloor \arg(6153-z_0)/(2\pi)\rfloor}\right.\right.$$

$$\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(6153-z_0)^k\ z_0^{-k}}{k!}\right)\text{^}(1/3)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Dividing the two exact results

$$\frac{1}{47}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[\pi]{\pi}\sqrt[2\pi]{d^9}$$

and

$$\frac{1}{47}\times 2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{\pi}\sqrt[2\pi]{d^9}$$

we obtain:

(1/47 3^(-1-3/(2π))4^(-1/π)(5(3+√(5)))^(1/(2π))(29-1697/(33272+987√(6153))^1/3+(33272+987√(6153))^1/3)π^(1/π))/(1/47 2^(-1/π)3^(-1-3/(2π))(29-1697/(33272+987√6153)^1/3+(33272+987√6153)^1/3)π^(1/(2π)))

Input

$$\left(\frac{1}{47}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[\pi]{\pi}\right)\Big/\left(\frac{1}{47}\times 2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{\pi}\right)$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\left(3^{-1-3/(2\pi)}\left(4^{-1/\pi}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[\pi]{\pi}\right)\right)\right)\Big/$$

$$\left(\frac{1}{47}\left(2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{\pi}\right)\right)47\right)=$$

$$2^{-1/\pi}\sqrt[2\pi]{5}\,\sqrt[2\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\left(3^{-1-3/(2\pi)}\left(4^{-1/\pi}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\right.\right.\right.$$
$$\left.\left.\left.\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[\pi]{\pi}\right)\right)\right)\Big/$$
$$\left(\frac{1}{47}\left(2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{\pi}\right)\right)47\right)=$$
$$2^{-1/\pi}\sqrt[2\pi]{5}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\left(3^{-1-3/(2\pi)}\left(4^{-1/\pi}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\right.\right.\right.$$
$$\left.\left.\left.\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[\pi]{\pi}\right)\right)\right)\Big/$$
$$\left(\frac{1}{47}\left(2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(\left(29-\frac{1697}{\sqrt[3]{33\,272+987\sqrt{6153}}}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\sqrt[3]{33\,272+987\sqrt{6153}}\right)\sqrt[2\pi]{\pi}\right)\right)47\right)=$$
$$2^{-1/\pi}\sqrt[2\pi]{5}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

We analyze the following equation:

$55\,x^3 - 34\,x^2 + 21\,x - 5 = 0$

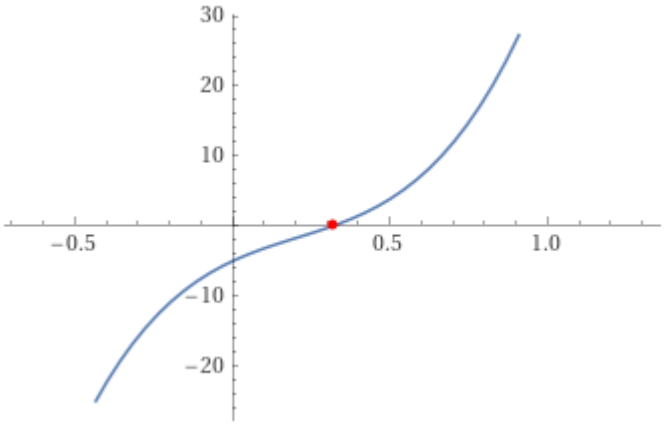
We note that 5, 21, 34 and 55 are Fibonacci numbers. We obtain:

55x^3-34x^2+21x-5 = 0

Input

$55\,x^3 - 34\,x^2 + 21\,x - 5 = 0$

Root plot **(figure that can be related to an open string)**



Alternate forms

$x\,(x\,(55\,x - 34) + 21) - 5 = 0$

$55\,x^3 - 34\,x^2 + 21\,x = 5$

$55\,x^3 + 21\,x = 34\,x^2 + 5$

Real solution

$$x = \frac{1}{165}\left(34 - 2309\sqrt[3]{\frac{2}{133\,553 + 165\sqrt{2\,463\,837}}} + \sqrt[3]{\frac{1}{2}\left(133\,553 + 165\sqrt{2\,463\,837}\right)}\right.$$

Complex solutions

$$x = \frac{34}{165} - \frac{1}{330}\sqrt[3]{\frac{1}{2}\left(133\,553 + 165\sqrt{2\,463\,837}\right)} + \frac{2309}{165 \times 2^{2/3}\sqrt[3]{133\,553 + 165\sqrt{2\,463\,837}}} + i\left(-\frac{\sqrt[3]{\frac{1}{2}\left(133\,553 + 165\sqrt{2\,463\,837}\right)}}{110\sqrt{3}} - \frac{2309}{55 \times 2^{2/3}\sqrt{3}\sqrt[3]{133\,553 + 165\sqrt{2\,463\,837}}}\right)$$

$$x = \frac{34}{165} - \frac{1}{330}\sqrt[3]{\frac{1}{2}\left(133\,553 + 165\sqrt{2\,463\,837}\right)} + \frac{2309}{165 \times 2^{2/3}\sqrt[3]{133\,553 + 165\sqrt{2\,463\,837}}} + i\left(\frac{\sqrt[3]{\frac{1}{2}\left(133\,553 + 165\sqrt{2\,463\,837}\right)}}{110\sqrt{3}} + \frac{2309}{55 \times 2^{2/3}\sqrt{3}\sqrt[3]{133\,553 + 165\sqrt{2\,463\,837}}}\right)$$

Real solution

$x \approx 0.31747$

Complex solutions

$x \approx 0.15035 - 0.51356\,i$

$x \approx 0.15035 + 0.51356\,i$

Sum of roots

$$\frac{34}{55}$$

0.618181818…. result that is a very good approximation to the value of the golden ratio conjugate 0.618033988749…

Product of roots

$$\frac{1}{11}$$

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the real solution

$$x = \frac{1}{165}\left(34 - 2309\sqrt[3]{\frac{2}{133\,553 + 165\sqrt{2\,463\,837}}} + \sqrt[3]{\frac{1}{2}\left(133\,553 + 165\sqrt{2\,463\,837}\right)}\right)$$

we obtain also:

(1/165 (34 - 2309 (2/(133553 + 165 sqrt(2463837)))^(1/3) + (1/2 (133553 + 165 sqrt(2463837)))^(1/3))) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

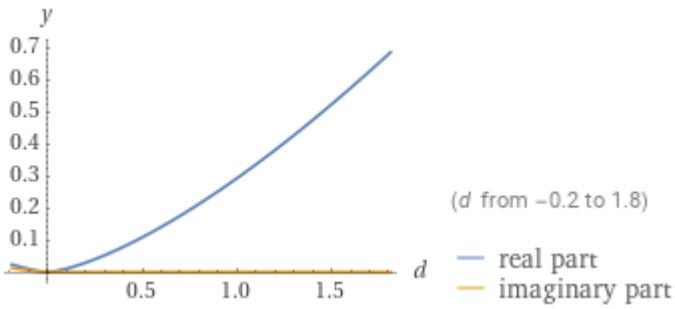
Input

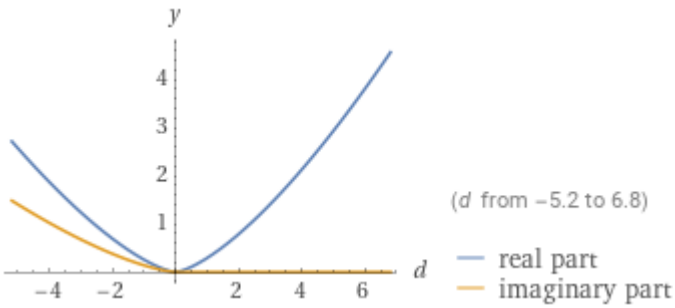
$$\left(\frac{1}{165}\left(34 - 2309\sqrt[3]{\frac{2}{133\,553 + 165\sqrt{2\,463\,837}}} + \sqrt[3]{\frac{1}{2}\left(133\,553 + 165\sqrt{2\,463\,837}\right)}\right)\right)\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

Exact result

$$\frac{1}{11}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[\pi]{\pi}\sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate forms

$$\left(\sqrt[2\pi]{3+\sqrt{5}}\sqrt[\pi]{\pi}\sqrt[2\pi]{d^9}\left(\text{root of } 11x^3-204x^2+3780x-27\,000 \text{ near } x=9.52422\right)\right)\Big/\left(30\left(\sinh\left(\frac{\log\left(\frac{432}{5}\right)}{2\pi}\right)+\cosh\left(\frac{\log\left(\frac{432}{5}\right)}{2\pi}\right)\right)\right)$$

$$-\frac{1}{11}\times 2^{-1/3-2/\pi}\times 3^{-1-3/(2\pi)}\times 5^{1/(2\pi)-1}\,\sqrt[2\pi]{3+\sqrt{5}}$$
$$\left(-34\,\sqrt[3]{2}+\frac{2309\times 2^{2/3}}{\sqrt[3]{133\,553+165\,\sqrt{2\,463\,837}}}-\sqrt[3]{133\,553+165\,\sqrt{2\,463\,837}}\right)$$
$$\sqrt[\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

$\log(x)$ is the natural logarithm

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Expanded form

$$\frac{1}{11}\times 2^{-1/3-2/\pi}\times 3^{-1-3/(2\pi)}\times 5^{1/(2\pi)-1}\,\sqrt[2\pi]{3+\sqrt{5}}\;\sqrt[3]{133\,553+165\,\sqrt{2\,463\,837}}\;\sqrt[\pi]{\pi}$$
$$\sqrt[2\pi]{d^9}-\frac{2309\times 2^{1/3-2/\pi}\times 3^{-1-3/(2\pi)}\times 5^{1/(2\pi)-1}\,\sqrt[2\pi]{3+\sqrt{5}}\;\sqrt[\pi]{\pi}\,\sqrt[2\pi]{d^9}}{11\,\sqrt[3]{133\,553+165\,\sqrt{2\,463\,837}}}+$$
$$\frac{17}{11}\times 2^{1-2/\pi}\times 3^{-1-3/(2\pi)}\times 5^{1/(2\pi)-1}\,\sqrt[2\pi]{3+\sqrt{5}}\;\sqrt[\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

Root

$d=0$

Series expansion at d=∞

$$\frac{1}{11}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\times 5^{1/(2\pi)-1}\,\sqrt[2\pi]{3+\sqrt{5}}\left(34-2309\right.$$
$$\left.\sqrt[3]{\frac{2}{133\,553+165\,\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\,\sqrt{2\,463\,837}\right)}\right)\sqrt[\pi]{\pi}\;d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{165}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right)=\frac{1}{11\,d}$$
$$2^{-(2(1+\pi))/\pi}\times 3^{1-3/(2\pi)}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\left(68-4618\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+2^{2/3}\sqrt[3]{133\,553+165\sqrt{2\,463\,837}}\right)\sqrt[2\pi]{d^9}\;\pi^{1/\pi-1}$$

Indefinite integral

$$\int\frac{1}{11}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\times 5^{-1+1/(2\pi)}\sqrt[2\pi]{3+\sqrt{5}}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[2\pi]{d^9}\;\sqrt[\pi]{\pi}\,dd=\frac{1}{11\left(1+\frac{9}{2\pi}\right)}3^{-1-3/(2\pi)}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[\pi]{\frac{\pi}{4}}\;d\sqrt[2\pi]{d^9}+\text{constant}$$

Global minimum

$$\min\left\{\frac{1}{165}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[2\pi]{\frac{1}{12}\left(5\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}\right\}=0 \text{ at } d=0$$

And:

(1/165 (34 - 2309 (2/(133553 + 165 sqrt(2463837)))^(1/3) + (1/2 (133553 + 165 sqrt(2463837)))^(1/3))) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

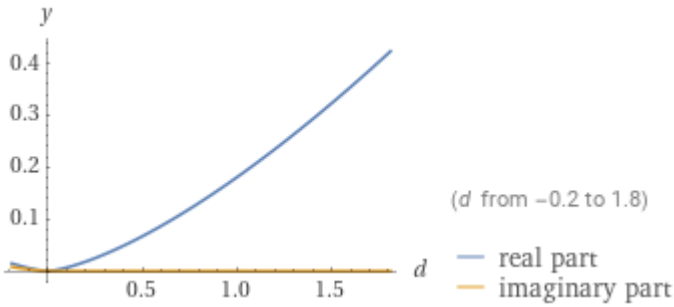
Input

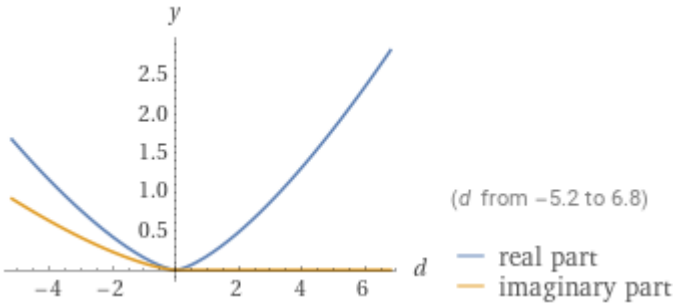
$$\left(\frac{1}{165}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

Exact result

$$\frac{1}{55}\times 2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

Plots (figures that can be related to the open strings)





Alternate forms

$$2^{-(1+\pi)/\pi} \times 3^{-1-3/(2\pi)} \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

root of $55\,x^3 - 204\,x^2 + 756\,x - 1080$ near $x = 1.90484$

$$-\frac{1}{55} \times 2^{-1/3-1/\pi} \times 3^{-1-3/(2\pi)} \left(-34\sqrt[3]{2} + \frac{2309 \times 2^{2/3}}{\sqrt[3]{133\,553 + 165\sqrt{2\,463\,837}}} - \sqrt[3]{133\,553 + 165\sqrt{2\,463\,837}}\right) \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Expanded form

$$\frac{1}{55}\times 2^{-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[3]{\frac{133\,553}{2}+\frac{165\sqrt{2\,463\,837}}{2}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}\ -$$
$$\frac{2309}{55}\times 2^{-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[3]{\frac{165\sqrt{2\,463\,837}}{24\,620\,779\,258}-\frac{133\,553}{24\,620\,779\,258}}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}\ +$$
$$\frac{17}{55}\times 2^{1-1/\pi}\times 3^{-1-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

Alternate forms assuming d>0

$$\left(2^{-(3+\pi)/(3\pi)}\times 3^{-1-3/(2\pi)}\left(-2309\times 2^{2/3}+\left(133\,553+165\sqrt{2\,463\,837}\right)^{2/3}+34\right.\right.$$
$$\left.\left.\sqrt[3]{267\,106+330\sqrt{2\,463\,837}}\right)\sqrt[2\pi]{\pi}\ d^{9/(2\pi)}\right)\Big/\left(55\sqrt[3]{133\,553+165\sqrt{2\,463\,837}}\right)$$

$$\frac{1}{55}\times 2^{-1/3-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[3]{133\,553+165\sqrt{2\,463\,837}}\ \sqrt[2\pi]{\pi}\ d^{9/(2\pi)}\ -$$
$$\frac{2309\times 2^{1/3-1/\pi}\times 3^{-1-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ d^{9/(2\pi)}}{55\sqrt[3]{133\,553+165\sqrt{2\,463\,837}}}+\frac{17}{55}\times 2^{1-1/\pi}\times 3^{-1-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Root

$d=0$

Series expansion at d=∞

$$\frac{1}{55}\times 2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}\ +\right.$$
$$\left.\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[2\pi]{\pi}\ d^{9/(2\pi)}$$

Derivative

$$\frac{d}{dd}\left(\frac{1}{165}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right)=$$

$$\frac{1}{55\,d}2^{-2-1/\pi}\times3^{1-3/(2\pi)}\left(68-4618\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+2^{2/3}\sqrt[3]{133\,553+165\sqrt{2\,463\,837}}\right)\sqrt[2\pi]{d^9}\;\pi^{1/(2\pi)-1}$$

Indefinite integral

$$\int\frac{1}{55}\times2^{-1/\pi}\times3^{-1-3/(2\pi)}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[2\pi]{d^9}\;\sqrt[2\pi]{\pi}\,dd=$$

$$\frac{1}{55\,(9+2\pi)}2^{-1/\pi}\times3^{-1-3/(2\pi)}\left(68-4618\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+2^{2/3}\sqrt[3]{133\,553+165\sqrt{2\,463\,837}}\right)\pi^{1+1/(2\pi)}\,d\,\sqrt[2\pi]{d^9}+\text{constant}$$

Global minimum

$$\min\left\{\frac{1}{165}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times3\times12}}\right\}=0\ \text{at}\ d=0$$

Series representations

$$\frac{1}{165}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times 3\times 12}}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)=-\frac{1}{55}\times 2^{-1/3-3/(2\pi)}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{d^9\,\pi\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}\left(-34\sqrt[3]{2}+2309\times 2^{2/3}\sqrt[3]{\frac{1}{133\,553+165\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\,463\,837-z_0)^k\,z_0^{-k}}{k!}}}-\sqrt[3]{133\,553+165\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\,463\,837-z_0)^k\,z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{1}{165}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times 3\times 12}}\left(34-2309\sqrt[3]{\frac{2}{133553+165\sqrt{2463837}}}+\sqrt[3]{\frac{1}{2}\left(133553+165\sqrt{2463837}\right)}\right)=-\frac{1}{55}\times 2^{-1/3-3/(2\pi)}\times 3^{-1-3/(2\pi)}$$

$$\sqrt[2\pi]{d^9\,\pi\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\left(-34\sqrt[3]{2}+2309\times 2^{2/3}\left(1\Big/\left(133553+165\exp\left(i\pi\left\lfloor\frac{\arg(2463837-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2463837-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/3)-\left(133553+165\exp\left(i\pi\left\lfloor\frac{\arg(2463837-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2463837-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\wedge(1/3)\right)\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$\frac{1}{165}\sqrt[2\pi]{\frac{\left(4\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\,d^3\right)\left(\sqrt{2}\,d^3\right)}{3\times 3\times 12}}\left(34-2309\sqrt[3]{\frac{2}{133553+165\sqrt{2463837}}}+\sqrt[3]{\frac{1}{2}\left(133553+165\sqrt{2463837}\right)}\right)=-\frac{1}{55}\times 2^{-1/3-3/(2\pi)}\times 3^{-1-3/(2\pi)}$$

$$\sqrt[2\pi]{d^9\,\pi\left(\frac{1}{z_0}\right)^{\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}$$

$$\left(-34\sqrt[3]{2}+2309\times 2^{2/3}\left(1\Big/\left(133553+165\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2463837-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2463837-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2463837-z_0)^k\,z_0^{-k}}{k!}\right)\right)\text{^}(1/3)-\left(133553+165\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2463837-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2463837-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2463837-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Dividing the two exact results

$$\frac{1}{11}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\left(34-2309\sqrt[3]{\frac{2}{133553+165\sqrt{2463837}}}+\sqrt[3]{\frac{1}{2}\left(133553+165\sqrt{2463837}\right)}\right)\sqrt[\pi]{\pi}\sqrt[2\pi]{d^9}$$

and

$$\frac{1}{55}\times 2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}$$

we obtain for d = 2 :

1/11 3^(-1 - 3/(2 π)) 4^(-1/π) 5^(1/(2 π) - 1) (3 + sqrt(5))^(1/(2 π)) (34 - 2309 (2/(133553 + 165 sqrt(2463837)))^(1/3) + (1/2 (133553 + 165 sqrt(2463837)))^(1/3)) π^(1/π) (2^9)^(1/(2 π))

Input

$$\frac{1}{11}\times 3^{-1-3/(2\pi)}\times 4^{-1/\pi}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[\pi]{\pi}\,\sqrt[2\pi]{2^9}$$

Exact result

$$\frac{1}{11}\times 2^{5/(2\pi)}\times 3^{-1-3/(2\pi)}\times 5^{1/(2\pi)-1}\sqrt[2\pi]{3+\sqrt{5}}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[\pi]{\pi}$$

Decimal approximation

0.789545456680319389807076097568174960370691754982444367460552827 9

…

0.78954545668….

Alternate forms

$$2^{5/(2\pi)-1} \times 3^{-1-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi}$$

root of $11x^3 - 204x^2 + 3780x - 27000$ near $x = 9.52422$

$$-\frac{1}{11} \times 2^{5/(2\pi)-1/3} \times 3^{-1-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \left(-34\sqrt[3]{2} + \frac{2309 \times 2^{2/3}}{\sqrt[3]{133\,553 + 165\sqrt{2\,463\,837}}} - \sqrt[3]{133\,553 + 165\sqrt{2\,463\,837}}\right) \sqrt[\pi]{\pi}$$

Expanded form

$$\frac{17}{11} \times 2^{1+5/(2\pi)} \times 3^{-1-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}}\ \sqrt[\pi]{\pi} - \frac{2309}{11} \times 2^{5/(2\pi)} \times 3^{-1-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \sqrt[3]{\frac{165\sqrt{2\,463\,837}}{24\,620\,779\,258} - \frac{133\,553}{24\,620\,779\,258}}\ \sqrt[\pi]{\pi} + \frac{1}{11} \times 2^{5/(2\pi)} \times 3^{-1-3/(2\pi)} \times 5^{1/(2\pi)-1} \sqrt[2\pi]{3+\sqrt{5}} \sqrt[3]{\frac{133\,553}{2} + \frac{165\sqrt{2\,463\,837}}{2}}\ \sqrt[\pi]{\pi}$$

Series representations

$$\frac{1}{11}\left(3^{-1-3/(2\pi)}\times 4^{-1/\pi}\right)5^{1/(2\pi)-1}\left(\sqrt[2\pi]{3+\sqrt{5}}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[\pi]{\pi}\,\sqrt[2\pi]{2^9}\right)=$$

$$-\frac{1}{11}\times 2^{-1/3+5/(2\pi)}\times 3^{-1-3/(2\pi)}\times 5^{-1+1/(2\pi)}\sqrt[\pi]{\pi}\,\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}\left(-34\sqrt[3]{2}+2309\times 2^{2/3}\sqrt[3]{\frac{1}{133\,553+165\sqrt{2\,463\,836}\sum_{k=0}^{\infty}2\,463\,836^{-k}\binom{\frac{1}{2}}{k}}}-\sqrt[3]{133\,553+165\sqrt{2\,463\,836}\sum_{k=0}^{\infty}2\,463\,836^{-k}\binom{\frac{1}{2}}{k}}\right)$$

$$\frac{1}{11}\left(3^{-1-3/(2\pi)}\times 4^{-1/\pi}\right)5^{1/(2\pi)-1}\left(\sqrt[2\pi]{3+\sqrt{5}}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[\pi]{\pi}\,\sqrt[2\pi]{2^9}\right)=$$

$$-\frac{1}{11}\times 2^{-1/3+5/(2\pi)}\times 3^{-1-3/(2\pi)}\times 5^{-1+1/(2\pi)}\sqrt[\pi]{\pi}\,\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\left(-34\sqrt[3]{2}+2309\times 2^{2/3}\sqrt[3]{\frac{1}{133\,553+165\sqrt{2\,463\,836}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2\,463\,836}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}-\sqrt[3]{133\,553+165\sqrt{2\,463\,836}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2\,463\,836}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$\frac{1}{11}\left(3^{-1-3/(2\pi)}\times 4^{-1/\pi}\right)5^{1/(2\pi)-1}$$
$$\left(\sqrt[2\pi]{3+\sqrt{5}}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[\pi]{\pi}\ \sqrt[2\pi]{2^9}\right)=$$
$$-\frac{1}{11}\times 2^{-1/3+5/(2\pi)}\times 3^{-1-3/(2\pi)}\times 5^{-1+1/(2\pi)}\ \sqrt[\pi]{\pi}$$
$$\sqrt[2\pi]{3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}}\left(-34\sqrt[3]{2}+\right.$$
$$2309\times 2^{2/3}\sqrt[3]{\frac{1}{133\,553+165\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\,463\,837-z_0)^k\,z_0^{-k}}{k!}}}-$$
$$\left.\sqrt[3]{133\,553+165\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2\,463\,837-z_0)^k\,z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

0.78954545668/((1/55 2^(-1/π) 3^(-1 - 3/(2 π)) (34 - 2309 (2/(133553 + 165 sqrt(2463837)))^(1/3) + (1/2 (133553 + 165 sqrt(2463837)))^(1/3)) π^(1/(2 π)) (2^9)^(1/(2 π))))

Input interpretation

$$0.78954545668 \Big/ \left(\frac{1}{55}\times 2^{-1/\pi}\times 3^{-1-3/(2\pi)}\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{2^9}\right)$$

Result

1.618008545899452535383488979077885884205766047524200992850407415 7…

1.618008545899…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Series representations

$$0.789545456680000 \Bigg/ \left(\frac{1}{55}\left(2^{-1/\pi}\times 3^{-1-3/(2\pi)}\right)\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\left(\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{2^9}\right)\right)=$$

$$-\left(\left(0.0447810031624599\times 2^{-7/(2\pi)}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right)\Bigg/\left(-0.0116872316515632+1.00000000000000\sqrt[3]{\frac{1}{133\,553+165\sqrt{2\,463\,836}\ \sum_{k=0}^{\infty}2\,463\,836^{-k}\binom{\frac{1}{2}}{k}}}-0.000272828291445403\sqrt[3]{133\,553+165\sqrt{2\,463\,836}\ \sum_{k=0}^{\infty}2\,463\,836^{-k}\binom{\frac{1}{2}}{k}}\right)\right)$$

$$0.789545456680000 \Big/ \left(\frac{1}{55}\left(2^{-1/\pi} \times 3^{-1-3/(2\pi)}\right)\left(34 - 2309\sqrt[3]{\frac{2}{133\,553 + 165\sqrt{2\,463\,837}}} + \sqrt[3]{\frac{1}{2}\left(133\,553 + 165\sqrt{2\,463\,837}\right)}\right)\left(\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{2^9}\right)\right) =$$

$$-\left(\left(0.0447810031624599 \times 2^{-7/(2\pi)} \times 3^{3/(2\pi)}\ \pi^{-1/(2\pi)}\right) \Big/ \left(\left(-0.0116872316515632 + 1.00000000000000\right.\right.\right.$$

$$\left.\left.\left.\sqrt[3]{\frac{1}{133\,553 + 165\sqrt{2\,463\,836}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2\,463\,836}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}} - 0.000272828291445403\right.\right.\right.$$

$$\left.\left.\left.\sqrt[3]{133\,553 + 165\sqrt{2\,463\,836}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2\,463\,836}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right)\right)\right)$$

$$0.789545456680000 \Big/ \left(\frac{1}{55}\left(2^{-1/\pi}\times 3^{-1-3/(2\pi)}\right)\left(34-2309\sqrt[3]{\frac{2}{133\,553+165\sqrt{2\,463\,837}}}+\sqrt[3]{\frac{1}{2}\left(133\,553+165\sqrt{2\,463\,837}\right)}\right)\left(\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{2^9}\right)\right)=$$

$$-\left(\left(0.0447810031624599\times 2^{-7/(2\pi)}\times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right)\Big/\left(-0.0116872316515632+1.00000000000000\sqrt[3]{\frac{1}{133\,553+165\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2\,463\,837-z_0)^k\,z_0^{-k}}{k!}}}-0.000272828291445403\sqrt[3]{133\,553+165\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2\,463\,837-z_0)^k\,z_0^{-k}}{k!}}\right)\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

From the two previous results

1.6180085459001070581002623979536005212943435960226956084921288971
…

1.6180085458994525353834889790778858842057660475242009928504074157
…

we obtain also:

(4(1.61800854589)^(2π))/(5(3+sqrt(5)))

Input interpretation

$$\frac{4\times 1.61800854589^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

Result

3.1415926534664902245699984079827970021559527485027503023189424414
…

3.14159265346649…≈ π (Ramanujan Recurring Number)

Series representations

$$\frac{4\times 1.6180085458900000^{2\pi}}{5\left(3+\sqrt{5}\right)} = \frac{4\, e^{0.962392200767571\,\pi}}{5\left(3+\sqrt{4}\,\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{4\times 1.6180085458900000^{2\pi}}{5\left(3+\sqrt{5}\right)} = \frac{4\, e^{0.962392200767571\,\pi}}{5\left(3+\sqrt{4}\,\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\frac{4\times 1.6180085458900000^{2\pi}}{5\left(3+\sqrt{5}\right)}=\frac{8\,e^{0.962392200767571\,\pi}\sqrt{\pi}}{5\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

1/6((4(1.61800854589)^(2π))/(5(3+sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.61800854589^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.6449340667…

1.6449340667…≈ ζ(2) = π²/6 = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.6180085458900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.924784401535141\,\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.6180085458900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.924784401535141\,\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.6180085458900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\, e^{1.924784401535141\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

We obtain also:

sqrt(1/(1/6((4(1.61800854589)^(2π))/(5(3+sqrt(5))))^2)*(4/3))

Input interpretation

$$\sqrt{\frac{1}{\frac{1}{6}\left(\frac{4\times 1.61800854589^{2\,\pi}}{5\left(3+\sqrt{5}\right)}\right)^2}\times\frac{4}{3}}$$

Result

0.90031631619…

0.90031631619…≈ 0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed form

$$\frac{2\sqrt{2}}{\pi} \approx 0.900316316157106$$

All 2nd roots of 0.8105694692

$0.9003163162\, e^{0} \approx 0.90032$ (real, principal root)

$0.9003163162\, e^{i\pi} \approx -0.90032$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3}{6}\left(\frac{4\times 1.6180085458900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(-1+\frac{25}{2}\, e^{-1.924784401535141\,\pi}\left(3+\sqrt{5}\right)^2\right)^k}{k!}$$

$$\sqrt{\frac{4}{\frac{3}{6}\left(\frac{4\times 1.6180085458900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{25}{2}\, e^{-1.924784401535141\,\pi}\left(3+\sqrt{5}\right)^2 - z_0\right)^k z_0^{-k}}{k!}$$

for $\left(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0)\right)$

$$\sqrt{\frac{4}{\frac{3}{6}\left(\frac{4\times 1.6180085458900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2}} = \exp\left(i\pi\left\lfloor \frac{\arg\left(-x+\frac{25}{2}\, e^{-1.924784401535141\,\pi}\left(3+\sqrt{5}\right)^2\right)}{2\pi}\right\rfloor\right)\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k x^{-k} \left(-\frac{1}{2}\right)_k \left(-x+\frac{25}{2}\, e^{-1.924784401535141\,\pi}\left(3+\sqrt{5}\right)^2\right)^k}{k!}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

Fig. 1

The solutions of the cubic equations $47x^3 - 29x^2 + 18x - 4 = 0$ and $55x^3 - 34x^2 + 21x - 5 = 0$ and the following formula related to the solutions of cubic equations (((54 - sqrt(7013 /3))^(1/3) + (54 + sqrt(7013/3))^(1/3)))+((-2/3 + (440/9 - sqrt(81483/14))^ (1/3) + (440/9 + sqrt(81483/14))^(1/3))) have been considered a part of the geometry and mathematics of the bubble universe, which is just one of the infinite bubbles of a universe with eternal inflation, where the octahedron is comparable to the pre big-bang universe in which the symmetry was very high and the entropy extremely low and the sphere, the bubble universe that emerges from the quantum vacuum. Even proceeding in this way, the solutions obtained from these connections are Pigreco, the golden ratio and the value of $\zeta(2) = \pi^2/6$. We also derive the Recurring Numbers of Ramanujan 4096 and 1729 and $(2\sqrt{2})/\pi$, i.e. the Del Gaudio-Nardelli constant.

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

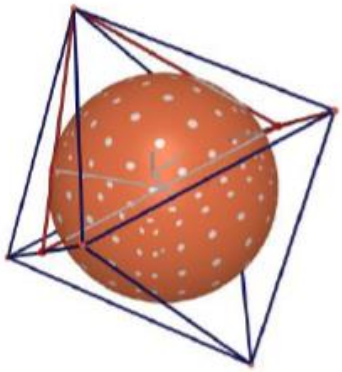


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\ z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\ x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^{0}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$
$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

Indeed:

From:

https://www.academia.edu/75787512/The_Theory_of_String_A_Candidate_for_a_Generalized_Unification_Model

The Einstein's field equation and the String Theory.

The Einstein's field equation which includes the cosmological constant is:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (8)$$

where $R_{\mu\nu}$ is the Ricci tensor, R its trace, λ the cosmological constant, $g_{\mu\nu}$ the metric tensor of the space geometry, G the Newton's gravitational constant and $T_{\mu\nu}$ the tensor representing the properties of energy, matter and momentum.

The left hand-side of (8) represents the gravitational field and, consequently, the warped space-time, while the right hand-side represents the matter, i.e. the sources of the gravitational field.

In string theory the gravity is related to the gravitons which are bosons, whereas the matter is related to fermions. It follows that the left and right hand of (8) may be respectively related to the action of bosonic and of superstrings.

The actions of bosonic string and superstring (also containing fermions) are connected by the Palumbo-Nardelli relation (Palumbo et al. 2005):

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\varphi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right]=$$

$$=\int_0^\infty \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|\widetilde{H}_3|^2-\frac{\kappa_{10}^2}{g_{10}^2}Tr_\nu\left(|F_2|^2\right)\right]$$

The sign minus in the above equation comes from the inversion of any relationship.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^8\right)^{-k}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^8\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

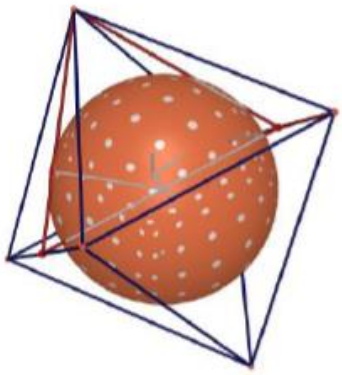
$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard to a sphere inscribed in an octahedron



we have the following formulas:

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\,2}{\pi} \quad (\text{for } l \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\,\pi)\rfloor)}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we have the following convex regular polyhedrons volumes:

$\frac{\sqrt{2}}{12}d^3$ Tetrahedron Volume

$\frac{\sqrt{2}}{3}d^3$ Octahedron Volume

$\frac{5}{12}(3+\sqrt{5})d^3$ Icosahedron Volume

Regarding the Sphere the volume is

$$V = \frac{4}{3}\pi r^3$$

With regard the sphere inscribed in a Tetrahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_T}{V_S} = \frac{\frac{\sqrt{2}}{12}d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(((√2 d^3)/12))*1/(4/3*π*(d/2)^3)

Input

$$\left(\frac{1}{12}\left(\sqrt{2}\ d^3\right)\right) \times \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.225079079039276517388799797751685145666143537488805156392859367 8

…

0.225079079….

Property

$\frac{1}{\sqrt{2}\ \pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Octahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_O}{V_S} = \frac{\frac{\sqrt{2}}{3}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

((1/3*√2*d^3)/(4/3*π*(d/2)^3))

Input

$$\frac{\frac{1}{3}\sqrt{2}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\ 2}{\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\, d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\,\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\,\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\,\pi)\rfloor)}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Icosahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_I}{V_S} = \frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3)

Input

$$\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{5\left(3+\sqrt{5}\right)}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

4.166730504921372682725163141597223093105782245731637719697502271 9

…

4.166730504921….

Property

$\frac{5\left(3+\sqrt{5}\right)}{2\pi}$ is a transcendental number

Expanded form

$$\frac{15}{2\pi}+\frac{5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

Alternate forms

$$\frac{15+5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

$$\frac{\frac{15}{2}+\frac{5\sqrt{5}}{2}}{\pi} \quad (\text{for } d \neq 0)$$

Series representations

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\pi\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}{3\times3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

Indeed, we obtain:

$$\frac{\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)\left(d^3\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2\right)}}{\sqrt[2\pi]{\frac{4}{3}\left(\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\right)\sqrt{2}\left(d^3 \times \frac{\sqrt{2}}{12}\right)d^3}}$$

= 1.6180085459001 07…

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4 \times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535…. = π (Ramanujan Recurring Number)

In conclusion, from the following further formula concerning the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron volumes and their ratios (and/or the inverses) with the sphere volume, raised the expression

$$\left(2\log(2)log^{2/3}(3^2)\right)/\left(3e^{7/3}\right)$$

$$\left(\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\frac{\frac{1}{4}\left(\left(15+7\sqrt{5}\right)d^3\right)}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\right)^{\left(2\log(2)\log^{2/3}(3^2)\right)/\left(3e^{7/3}\right)}$$

i.e.

$$\left(\frac{3}{2}\right)^{\left(4\log(2)\log^{2/3}(9)\right)/\left(3e^{7/3}\right)}\left(\frac{5\left(3+\sqrt{5}\right)\left(15+7\sqrt{5}\right)}{\pi}\right)^{\left(2\log(2)\log^{2/3}(9)\right)/\left(3e^{7/3}\right)}$$

we obtain:

1.61803817115191398574568170847431346739030216149893707129177220 71
…

1.6180381711…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

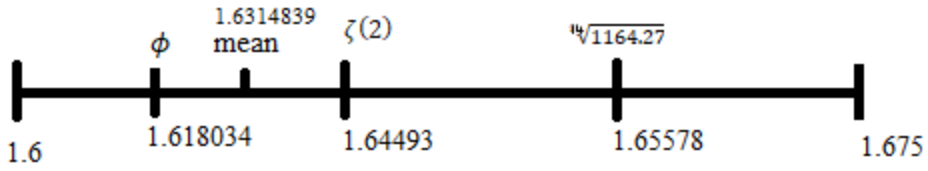
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

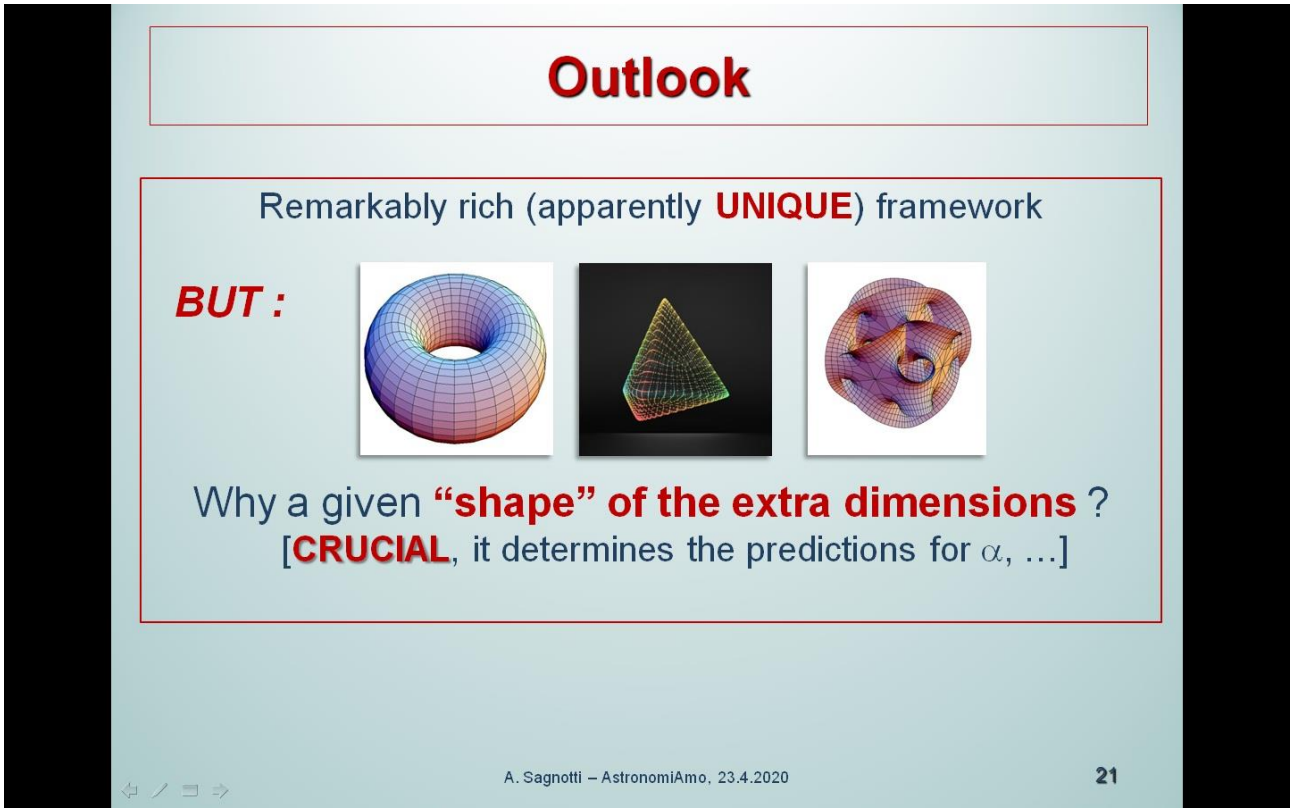
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix


Outlook
Remarkably rich (apparently UNIQUE) framework
BUT :
Why a given "shape" of the extra dimensions ?
[CRUCIAL, it determines the predictions for α, …]
A. Sagnotti – AstronomiAmo, 23.4.2020
21

From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

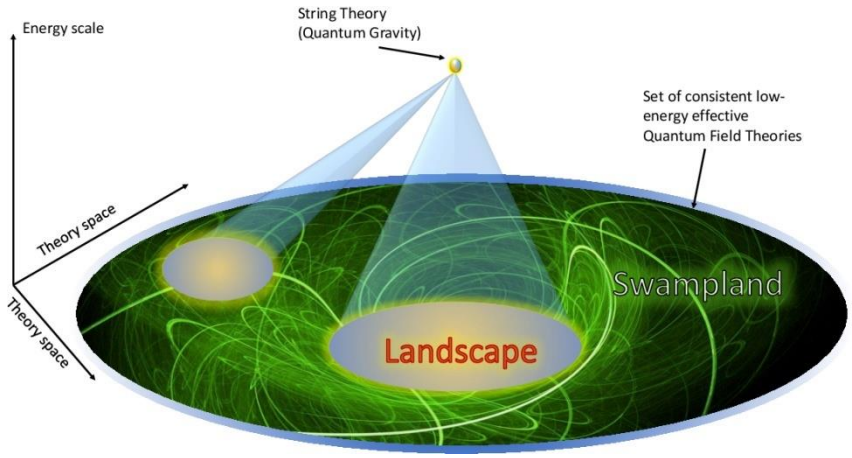


Fig. 1

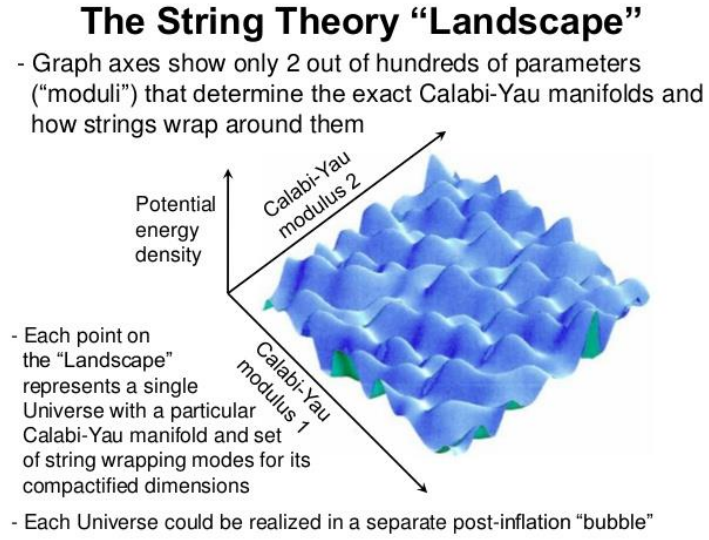

The String Theory "Landscape"
- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them
Potential energy density
Calabi-Yau modulus 2
Calabi-Yau modulus 1
- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

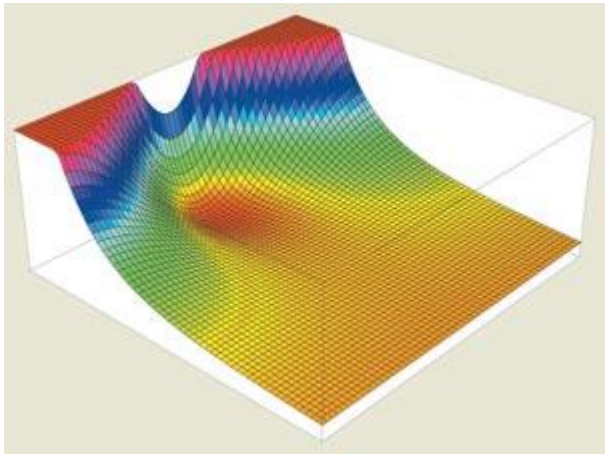


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

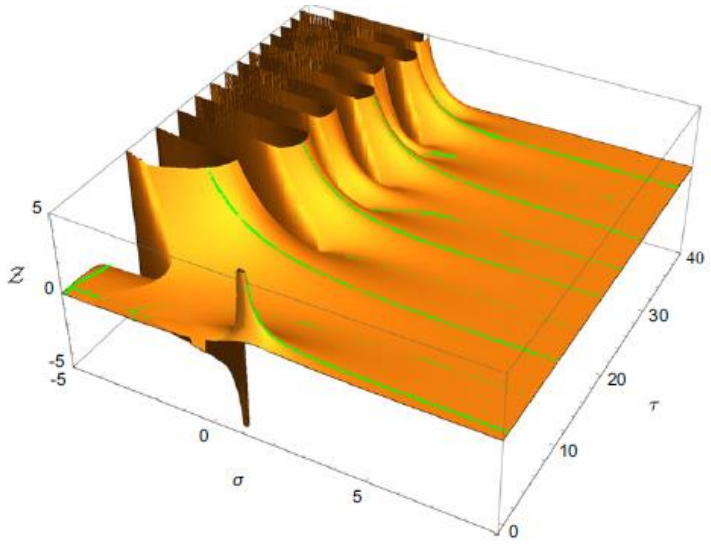


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

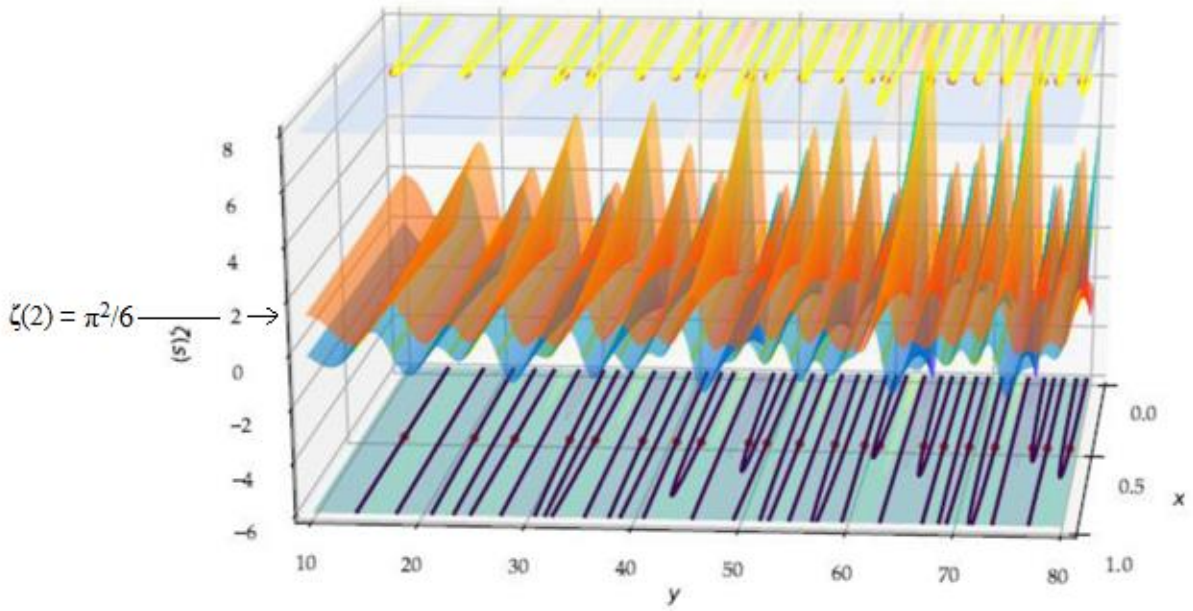


Figure 1. $C(x,y)$ and $S(x,y)$ surfaces of the Riemann $\zeta(x,y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

3D plot | $\zeta(2 + i\,t)$

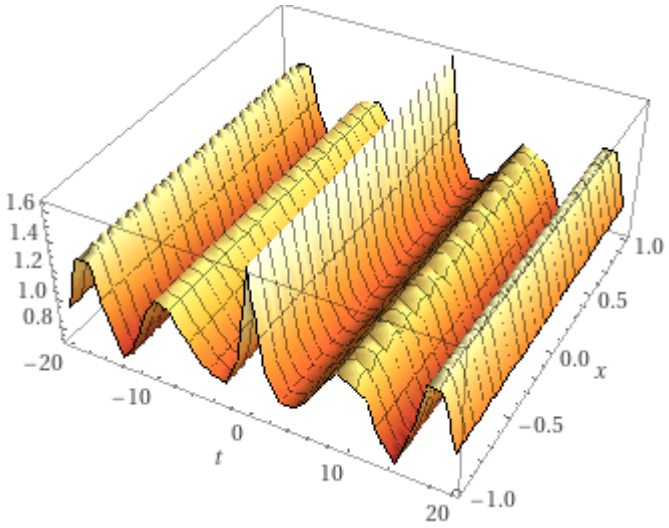


Fig. 6

Where ζ(2+it) :

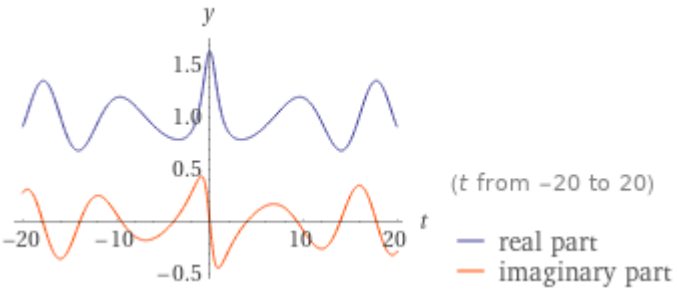
Input

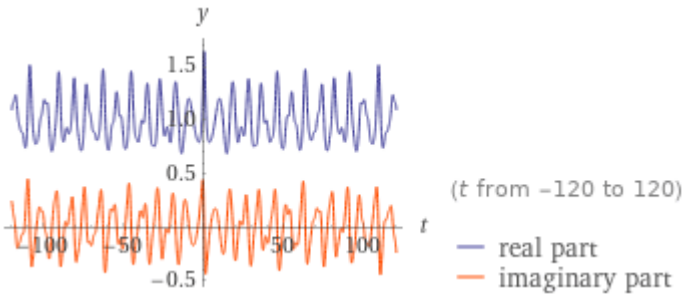
$$\zeta(2 + i\,t)$$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Plots





Roots

$$t = 2\,i\,(n+1), \quad n \in \mathbb{Z}, \quad n \geq 1$$

$$t = -i\,(\rho_n - 2), \quad n \neq 0, \quad n \in \mathbb{Z}$$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\,t\,\zeta'(2) - \frac{1}{2}\,t^2\,\zeta''(2) - \frac{1}{6}\,i\,\zeta^{(3)}(2)\,t^3 + \frac{1}{24}\,\zeta^{(4)}(2)\,t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2 + i\,t) = \zeta(2 + i\,t,\, 1)$$

$$\zeta(2 + i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2 + i\,t) = \frac{\zeta\left(2 + i\,t,\, \frac{1}{2}\right)}{-1 + 2^{2+i\,t}}$$

$\zeta(s,\, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2 + i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = \frac{\sum_{k=0}^{\infty} (1 + 2\,k)^{-2-i\,t}}{1 - 2^{-2-i\,t}} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \mathrm{Im}(t) < 1$$

$\mathrm{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^\infty \frac{\tau^{1+i\,t}}{-1+e^\tau}\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^\infty \tau^{2+i\,t}\,\mathrm{csch}^2(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^\infty e^{-\tau}\,\tau^{1+i\,t}\,\mathrm{csch}(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\mathrm{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+i\,t) = -i\,2^{2+i\,t}\,\pi^{1+i\,t}\,\Gamma(-1-i\,t)\sinh\left(\frac{\pi\,t}{2}\right)\zeta(-1-i\,t)$$

$$\zeta(2+i\,t) = \frac{\pi^{3/2+i\,t}\,\Gamma\left(-\frac{1}{2}-\frac{i\,t}{2}\right)\zeta(-1-i\,t)}{\Gamma\left(1+\frac{i\,t}{2}\right)}$$

$$\zeta(2+i\,t) = -\frac{i\sum_{k=0}^\infty \frac{\Gamma\left(k-\frac{i\,t}{2}\right)\sum_{j=0}^k (-1)^j\,(1+2\,j)\binom{k}{j}\zeta(2+2\,j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{i\,t}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

we obtain:

2Pi/(ln(2))

Input:

$$2\times\frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\,\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372033095 8…

9.06472028365….

Alternative representations:

$$\frac{2\,\pi}{\log(2)}=\frac{2\,\pi}{\log_e(2)}$$

$$\frac{2\,\pi}{\log(2)}=\frac{2\,\pi}{\log(a)\log_a(2)}$$

$$\frac{2\,\pi}{\log(2)}=\frac{2\,\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

1.64381856858…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\right/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$
for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^-}6$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right)=0.054323\,(0.0814845\,b+1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M}=-\frac{\frac{226\,802\,245}{18\,480\,874}+b}{M}$$

$$\frac{\partial M(b,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b}=-\frac{18\,480\,874\,M}{226\,802\,245+18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M}=\frac{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b}=\frac{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right\}=0 \text{ at } (b,M)=(-16,0)$$

Global minima:

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

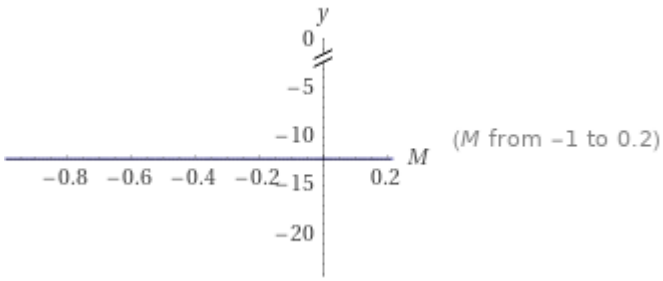
Input interpretation:

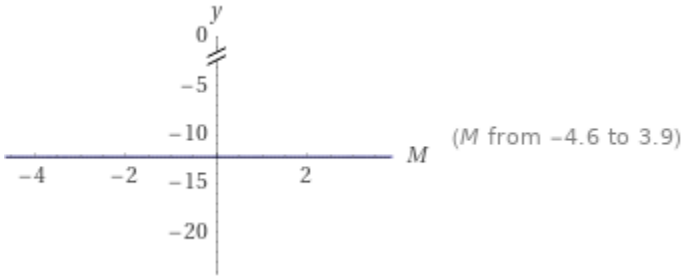
$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

Result:

$$\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:





Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323\, M^2\right)}{M^2}\right) = \frac{3.55271 \times 10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM = \frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2} = -12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM = 0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

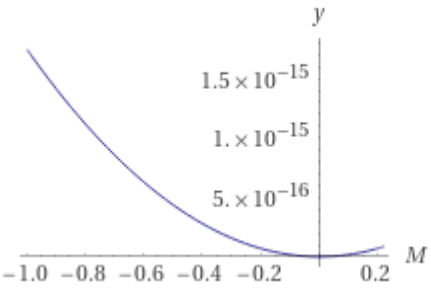
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

Result:

0

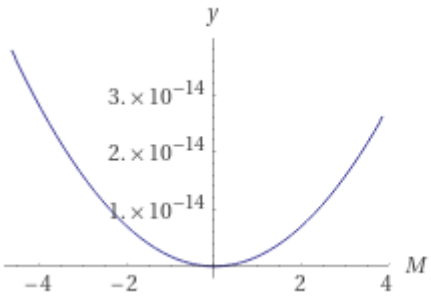
Plots: **(possible mathematical connection with an open string)**



(*M* from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$$O\left(M^{62194}\right)$$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times10^{-15}\,M^2+O\left(\left(\frac{1}{M}\right)^{62194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}\,M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.388513449474645453489707833780880208333333333333333333\ldots\times10^{-16}$$

-4.38851344947*10^{-16}

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^2+6.58545\times10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times0.2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots \times 10^{-17}$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875 \times 10^{-14}$

1.57986484181*10^-14

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-15}$

7.021621519*10^-15

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times10^{-15}+1.57986484181\times10^{-14}+7.021621519\times10^{-17}-4.38851344947\times10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots\times10^{-8}$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_P^E=\mathbf{E}_P l_P^2=\phi_P l_P=\sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left(\left(1/\left(7.021621519\times10^{-15}+1.57986484181\times10^{-14}+7.021621519\times10^{-17}-4.38851344947\times10^{-16}\right)\right)\text{ ^ }(1/7)-\frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

$1.6181818182\ldots$

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

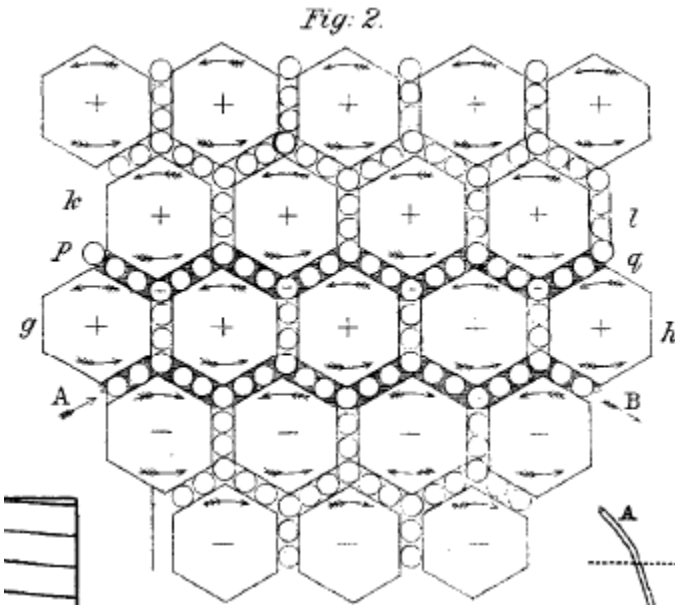
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

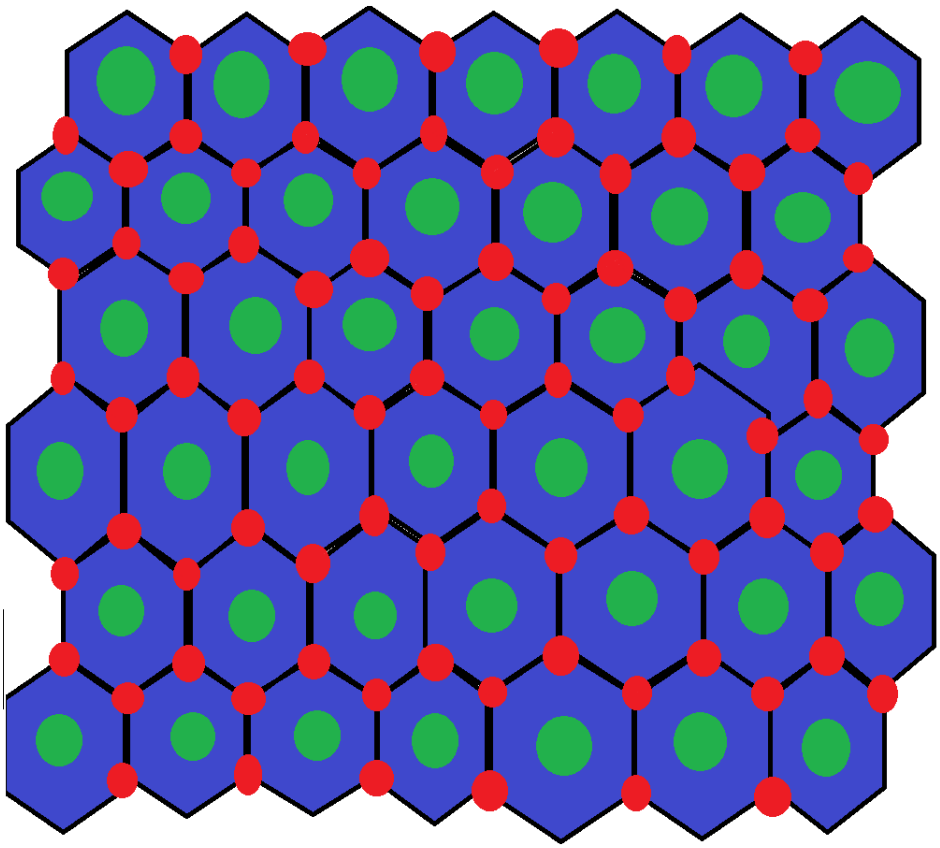
Fig. 2



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 3

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Fig. 4

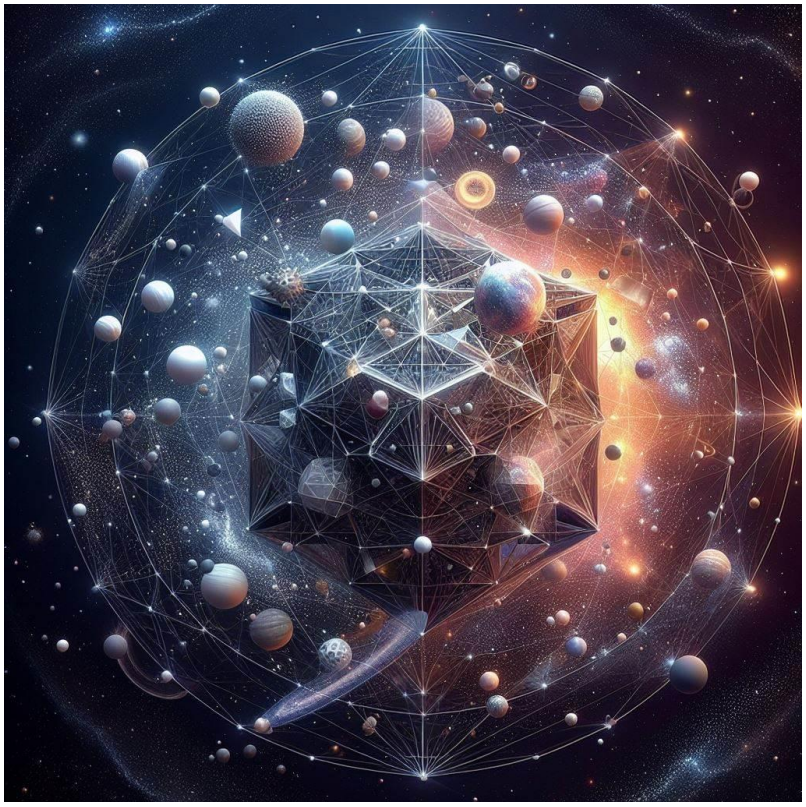


Image representing the solutions of the cubic equations in relation to the boundary conditions of the Plateau problem useful for understanding the formation of the bubbles of a universe with eternal inflation and of an octahedron as a mathematical model of the primordial universe before the Big Bang, therefore highly symmetrical and with extremely low entropy. As for the cubic equations, they represent the geometric characteristics of the quantum vacuum, while the spherical surface would represent the geometry of the bubble of the universe.

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability. I would like to thank also my son **Francesco Nardelli**, student of Educational Sciences, majoring in History and Philosophy, for his useful suggestions regarding the intriguing image at the end of this work

References

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

See also:

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *Emilian Dudas, Sergio Ferrara, Alex Kehagias and Augusto Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the analysis of various Cardano's Formulas for Cubic Equations and of some equations concerning the Superconformal Gravity and the Topology of Diffeomorphism Groups. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory VI

Michele Nardelli[1], Antonio Nardelli

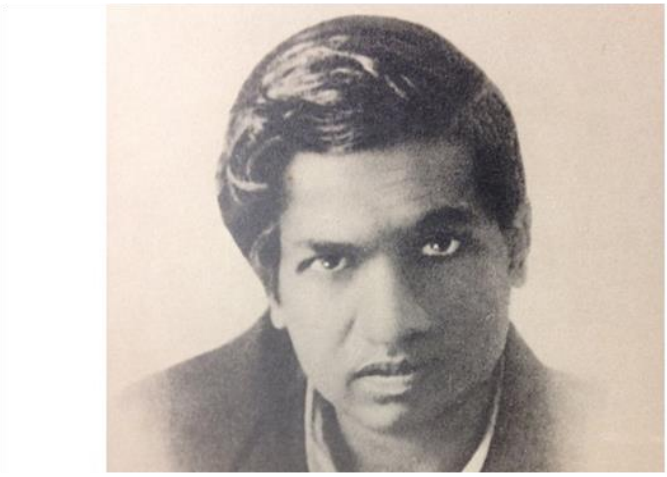
Abstract

In this paper (part VI), we analyze various Cubic Equations that can be solved with Cardano's formula and some equations of Superconformal Gravity and the Topology of Diffeomorphism Groups. We obtain new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Introduction

Cardano's method for cubic equations involves cube roots and introduces symmetry concepts. Similarly, string theory and M-theory involve symmetries and transformation groups that play a fundamental role in describing particles and interactions. Solutions of complete cubic equations via Cardano's method involve complex cube roots. Similarly, equations and solutions in superstring theory can often involve advanced algebraic and mathematical complexities. Solutions of cubic equations can be understood geometrically through topology and algebraic geometry. In string theory, the geometric and topological configurations of strings and branes are also central. Duality is an important concept in string theory, where different mathematical descriptions can be equivalent. Similarly, in solving cubic equations, duality links exist through cube roots.

Thus, in string theory, duality refers to the fact that different mathematical formulations or physical descriptions of a theory can be equivalent. For example, two seemingly different theories can be dual to each other, meaning they are two different ways of describing the same physical reality. This concept of duality has been very useful for understanding profound aspects of string theory and field theories.

Similarly, in solving cubic equations, duality can emerge through cube roots. The cubic equation $ax^3+bx^2+cx+d=0$ can be solved using cube roots, as in the Cardano method I described above. However, cube roots involve the concept of complex numbers, and it is sometimes possible to express the solutions in ways that highlight a duality between the solutions.

For example, if we consider complex cube roots, we can see a duality between them, since there are connections via the cube roots of unity. This duality is similar to the concept of duality in string theory, where different descriptions can be connected by similar transformations.

In both cases, duality provides a deeper way of understanding the underlying mathematical or physical structure, allowing you to explore hidden connections between different concepts.

Cubic surfaces can be associated with geometric and topological concepts. In string theory, for example, some models require spaces with particular shapes, and the geometry of the spaces can influence the properties of strings and particles. Some

mathematical and physical models involving surfaces might be described by cubic or complex equations. The geometry of surfaces and the associated mathematical structure can influence the properties of universe bubbles in the multiverse. Cubic surfaces can represent certain topological spaces, and their properties can affect the structure of the associated string theory.

The connection between the surface of a D-brane and the roots of a cubic equation can be explored in specific contexts within string theory, especially when it comes to studying the geometric and topological properties of D-branes. D-branes are extended objects on whose surface open strings can terminate. The geometry of the D-brane surface can interfere with the physics of surrounding strings. In some cases, geometry can be described mathematically through objects known in algebraic surface theory, such as cubic equations.

D-branes can host open fields, which describe particles interacting with the D-brane. Field equations on a D-brane can involve surface geometry, and in some situations, these equations can be associated with higher degree algebraic equations, such as cubic equations.

The compactification of extra dimensions, which is an important part of string theory, can be associated with the surface geometry of a D-brane. Compactification can lead to a reduction in dimensions and a geometric description that can involve algebraic equations, including cubic equations.

The topological invariants associated with the surface of a D-brane, such as the Betti number or the Chern number, can be related to geometric characteristics that can be described via algebraic equations, including cubic equations.

In some situations of duality, the description of a D-brane in a context can be dualistic to a description based on algebraic objects such as curves or surfaces, which can be represented via algebraic equations, including cubic equations.

In this paper, we will hypothesize that the solutions of the cubic equations and some solutions of various equations of Twisted Superconformal Gravity, can be used as boundary conditions to further the understanding of the formation of universe bubbles in eternal inflation. Furthermore, an octahedron could serve as a mathematical or conceptual model of the universe in the pre-Big Bang phase. Cubic equations, in this context, could represent the geometric characteristics of the quantum vacuum, while the spherical surface could be used to describe the geometry of the universe bubble.

The results $(2\sqrt{2})/\pi$, the golden ratio φ and π obtained, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and universe bubbles in eternal inflation. It may reflect a fundamental relationship between the geometry of solids and the mathematical characteristics of solutions to cubic equations.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and universe bubbles.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of universe bubbles and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

Finally, it is also possible to hypothesize the solutions of the cubic equations solvable with the Cardano method, as boundary conditions to deepen the understanding of the formation of inflationary universe bubbles eternal and of an octahedron which could represent a mathematical or conceptual model of the primordial universe before the Big Bang. Furthermore, regarding the cubic equations, they can represent the geometric characteristics of the quantum vacuum, while the spherical surface represents the geometry of the universe bubble. The results above $(2\sqrt{2})/\pi$, the golden ratio φ and π which are obtained, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

These mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe. The cubic equations, in this context, could reflect the geometric and dynamic characteristics of the quantum vacuum, while the spherical surface could represent the geometry of the bubbles of the universe.

Regarding fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the primordial universe and universe bubbles.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

From

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

Cubic equations and Cardano's Formulas

$$\sqrt[3]{42 + \sqrt{1700}} + \sqrt[3]{42 - \sqrt{1700}} - 2.$$

We obtain:

(42+sqrt(1700))^1/3 + (42-sqrt(1700))^1/3 - 2

Input

$$\sqrt[3]{42 + \sqrt{1700}} + \sqrt[3]{42 - \sqrt{1700}} - 2$$

Result

$$\sqrt[3]{42 + 10\sqrt{17}} + \sqrt[3]{42 - 10\sqrt{17}} - 2$$

Decimal approximation

3.282260997562922360344611230521558106729668907533546012575644317 8

…

3.28226099756….

Alternate forms

root of $x^3 + 6x^2 - 100$ near $x = 3.28226$

$$-2 + \sqrt[3]{2\left(21 - 5\sqrt{17}\right)} + \sqrt[3]{2\left(21 + 5\sqrt{17}\right)}$$

$$-2+\frac{1}{3}\sqrt[3]{1134-270\sqrt{17}}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}$$

Minimal polynomial

x^3+6x^2-100

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

Now, we analyze the following formula:

$$\begin{aligned}9 &+ \sqrt[3]{4846\tfrac{1}{2}+\sqrt{23{,}487{,}833\tfrac{1}{4}}}+\sqrt[3]{4846\tfrac{1}{2}-\sqrt{23{,}487{,}833\tfrac{1}{4}}}\\ &-\sqrt[3]{256\tfrac{1}{2}+\sqrt{65{,}063\tfrac{1}{4}}}-\sqrt[3]{256\tfrac{1}{2}-\sqrt{65{,}063\tfrac{1}{4}}}\\ &-\sqrt[3]{256\tfrac{1}{2}+\sqrt{65{,}063\tfrac{1}{4}}}-\sqrt[3]{256\tfrac{1}{2}-\sqrt{65{,}063\tfrac{1}{4}}}.\end{aligned}$$

we obtain:

9+(4846*1/2+sqrt(23487833*1/4))^1/3+(4846*1/2-sqrt(23487833*1/4))^1/3-(256*1/2+sqrt(65063*1/4))^1/3-(256*1/2-sqrt(65063*1/4))^1/3-(256*1/2+sqrt(65063*1/4))^1/3-(256*1/2-sqrt(65063*1/4))^1/3

Input

$$9+\sqrt[3]{4846\times\frac{1}{2}+\sqrt{23\,487\,833\times\frac{1}{4}}}+\sqrt[3]{4846\times\frac{1}{2}-\sqrt{23\,487\,833\times\frac{1}{4}}}-\sqrt[3]{256\times\frac{1}{2}+\sqrt{65\,063\times\frac{1}{4}}}-\sqrt[3]{256\times\frac{1}{2}-\sqrt{65\,063\times\frac{1}{4}}}-\sqrt[3]{256\times\frac{1}{2}+\sqrt{65\,063\times\frac{1}{4}}}-\sqrt[3]{256\times\frac{1}{2}-\sqrt{65\,063\times\frac{1}{4}}}$$

Result

$$9-2\sqrt[3]{128-\frac{\sqrt{65\,063}}{2}}-2\sqrt[3]{128+\frac{\sqrt{65\,063}}{2}}+\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}+\sqrt[3]{2423+\frac{\sqrt{23\,487\,833}}{2}}$$

Decimal approximation

11.982419366220698639695898747860934765502524771639649365064267 4…

+

0.516697994570876924313494146280425143515036589865954357329753537…

i

Alternate complex forms

$$\left(9-2\sqrt[3]{128-\frac{\sqrt{65\,063}}{2}}-2\sqrt[3]{128+\frac{\sqrt{65\,063}}{2}}+\frac{1}{2}\sqrt[3]{-2423+\frac{\sqrt{23\,487\,833}}{2}}+\sqrt[3]{2423+\frac{\sqrt{23\,487\,833}}{2}}\right)+\frac{1}{2}\left(\sqrt{3}\sqrt[3]{-2423+\frac{\sqrt{23\,487\,833}}{2}}\right)i$$

$$\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}+\left(9-2\sqrt[3]{128-\frac{\sqrt{65\,063}}{2}}-2\sqrt[3]{128+\frac{\sqrt{65\,063}}{2}}+\frac{1}{2}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)^2\right)}\left(\cos\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}}-2423\right)\Big/\left(2\left(9-2\sqrt[3]{128-\frac{\sqrt{65\,063}}{2}}-2\sqrt[3]{128+\frac{\sqrt{65\,063}}{2}}+\frac{1}{2}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}+\sqrt[3]{2423+\frac{\sqrt{23\,487\,833}}{2}}\right)\right)\right)\right)+i\sin\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}}-2423\right)\Big/\left(2\left(9-2\sqrt[3]{128-\frac{\sqrt{65\,063}}{2}}-2\sqrt[3]{128+\frac{\sqrt{65\,063}}{2}}+\frac{1}{2}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}+\sqrt[3]{2423+\frac{\sqrt{23\,487\,833}}{2}}\right)\right)\right)\right)$$

$$\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}+\left(9-2\sqrt[3]{128-\frac{\sqrt{65\,063}}{2}}-2\sqrt[3]{128+\frac{\sqrt{65\,063}}{2}}+\frac{1}{2}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)^2\right)}$$

$$\exp\left(i\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}}-2423\right)\Big/\left(2\left(9-2\sqrt[3]{128-\frac{\sqrt{65\,063}}{2}}-2\sqrt[3]{128+\frac{\sqrt{65\,063}}{2}}+\frac{1}{2}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}+\sqrt[3]{2423+\frac{\sqrt{23\,487\,833}}{2}}\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r = \sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}+\left(9-2\sqrt[3]{128-\frac{\sqrt{65\,063}}{2}}-2\sqrt[3]{128+\frac{\sqrt{65\,063}}{2}}+\frac{1}{2}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)^2\right)}$$ (radius), $\theta =$ $$\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}}-2423\right)\Big/\left(2\left(9-2\sqrt[3]{128-\frac{\sqrt{65\,063}}{2}}-2\sqrt[3]{128+\frac{\sqrt{65\,063}}{2}}+\frac{1}{2}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}+\sqrt[3]{2423+\frac{\sqrt{23\,487\,833}}{2}}\right)\right)\right)$$ (angle)

$r \approx 11.994$ (radius), $\theta \approx 0.043095$ (angle)

11.994

Alternate forms

$$\frac{1}{2}\left(-2\times 2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2\times 2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+2^{2/3}\sqrt[3]{4846-\sqrt{23\,487\,833}}+2^{2/3}\sqrt[3]{4846+\sqrt{23\,487\,833}}+18\right)$$

$$9-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{\frac{1}{2}\left(4846-\sqrt{23\,487\,833}\right)}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}$$

$$-\frac{1}{\sqrt[3]{2}}\left(-9\sqrt[3]{2}+2\sqrt[3]{256-\sqrt{65\,063}}+2\sqrt[3]{256+\sqrt{65\,063}}-\sqrt[3]{4846-\sqrt{23\,487\,833}}-\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)$$

From the sum of the two above alternate forms

$$-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}$$

and

$$-\frac{1}{\sqrt[3]{2}}\left(-9\sqrt[3]{2}+2\sqrt[3]{256-\sqrt{65\,063}}+2\sqrt[3]{256+\sqrt{65\,063}}-\sqrt[3]{4846-\sqrt{23\,487\,833}}-\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)$$

we obtain:

(-2+(2 (21 - 5 √(17)))^(1/3) + (2(21 + 5 √(17)))^(1/3))+((-(-9 2^(1/3) + 2(256 - √(65063))^(1/3) + 2(256 + √(65063))^(1/3) - (4846 - √(23487833))^(1/3) - (4846 + √(23487833))^(1/3))/2^(1/3)))

Input

$$\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}\right)-\frac{1}{\sqrt[3]{2}}\left(-9\sqrt[3]{2}+2\sqrt[3]{256-\sqrt{65\,063}}+2\sqrt[3]{256+\sqrt{65\,063}}-\sqrt[3]{4846-\sqrt{23\,487\,833}}-\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)$$

Result

$$-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{4846-\sqrt{23\,487\,833}}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)$$

Decimal approximation

$$15.264680363783621000040509978382492872232193679173195377639911 7\ldots + 0.516697994570876924313494146280425143515036589865954357329753537\ldots\, i$$

Alternate complex forms

$$\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{-4846+\sqrt{23\,487\,833}}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)+\frac{1}{2}\left(\sqrt{3}\,\sqrt[3]{\frac{1}{2}\left(-4846+\sqrt{23\,487\,833}\right)}\right)i$$

$$\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}+\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)^2\right)}\left(\cos\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}\right)\Big/\left(2\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)\right)\right)\right)+i\sin\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}\right)\Big/\left(2\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)\right)\right)\right)$$

$$\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}+\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)^2\right)}$$

$$\exp\left(i\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}\right)\Big/\left(2\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}+\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)^2\right)}$$ (radius),

$$\theta=\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}\right)\Big/\left(2\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)\right)\right)$$ (angle)

$r \approx 15.273$ (radius), $\theta \approx 0.033836$ (angle)

15.273

Alternate forms

$$\frac{1}{2}\left(2\sqrt[3]{2\left(21-5\sqrt{17}\right)}+2\sqrt[3]{2\left(21+5\sqrt{17}\right)}+2^{2/3}\left(-2\sqrt[3]{256-\sqrt{65\,063}}+9\sqrt[3]{2}-2\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{4846-\sqrt{23\,487\,833}}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)-4\right)$$

$$7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}$$

$$\frac{1}{\sqrt[3]{2}}\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{17}}+2^{2/3}\sqrt[3]{21+5\sqrt{17}}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{4846-\sqrt{23\,487\,833}}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)$$

Expanded form

$$7+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{\frac{1}{2}\left(4846-\sqrt{23\,487\,833}\right)}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}$$

From

$$\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}\right)-\frac{1}{\sqrt[3]{2}}\left(-9\sqrt[3]{2}+2\sqrt[3]{256-\sqrt{65063}}+2\sqrt[3]{256+\sqrt{65063}}-\sqrt[3]{4846-\sqrt{23487833}}-\sqrt[3]{4846+\sqrt{23487833}}\right)$$

after some calculations, we obtain:

(123-Pi^2)((-2+(2(21-5√(17)))^(1/3)+(2(21+5√(17)))^(1/3))+((-(-9 2^(1/3)+2(256-√(65063))^(1/3) + 2(256 + √(65063))^(1/3) - (4846 - √(23487833))^(1/3) - (4846 + √(23487833))^(1/3))/2^(1/3))))+e-φ

Input

$$(123-\pi^2)\left(\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}\right)-\frac{1}{\sqrt[3]{2}}\left(-9\sqrt[3]{2}+2\sqrt[3]{256-\sqrt{65063}}+2\sqrt[3]{256+\sqrt{65063}}-\sqrt[3]{4846-\sqrt{23487833}}-\sqrt[3]{4846+\sqrt{23487833}}\right)\right)+e-\phi$$

ϕ is the golden ratio

Result

$$-\phi+e+\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65063}}-2\sqrt[3]{256+\sqrt{65063}}+\sqrt[3]{4846-\sqrt{23487833}}+\sqrt[3]{4846+\sqrt{23487833}}\right)\right)(123-\pi^2)$$

Decimal approximation

1727.99957608547339580416490951394233889668836375612329674445514 3…

+

58.45424853096708927277656937154707324466572955610620504586056687…

i

Alternate complex forms

$$\left(e-\phi+\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65063}}-2\sqrt[3]{256+\sqrt{65063}}+\frac{1}{2}\sqrt[3]{-4846+\sqrt{23487833}}+\sqrt[3]{4846+\sqrt{23487833}}\right)\right)\left(123-\pi^2\right)\right)+\frac{1}{2}\left(\sqrt{3}\sqrt[3]{\frac{1}{2}\left(-4846+\sqrt{23487833}\right)}\left(123-\pi^2\right)\right)i$$

$$\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}\left(\pi^2-123\right)^2+\left(\frac{1}{2}\left(-1-\sqrt{5}\right)+e+\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)\left(123-\pi^2\right)\right)^2\right)}$$

$$\left(\cos\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}\left(123-\pi^2\right)\right)\Big/\left(2\left(-\phi+e+\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)\left(123-\pi^2\right)\right)\right)\right)+i\sin\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}\left(123-\pi^2\right)\right)\Big/\left(2\left(-\phi+e+\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)\left(123-\pi^2\right)\right)\right)\right)$$

$$\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}\left(\pi^2-123\right)^2+\left(\frac{1}{2}\left(-1-\sqrt{5}\right)+e+\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\left(123-\pi^2\right)\right)^2\right)\right)}$$

$$\exp\left(i\tan^{-1}\left(\left(\sqrt{3}\sqrt{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}\left(123-\pi^2\right)\right)\Big/\left(2\left(-\phi+e+\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\left(123-\pi^2\right)\right)\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r = \sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}\left(\pi^2-123\right)^2+\left(\frac{1}{2}\left(-1-\sqrt{5}\right)+e+\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)\left(123-\pi^2\right)\right)^2\right)}\ \text{(radius)},$$

$$\theta = \tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}\left(123-\pi^2\right)\right)\Big/\left(2\left(-\phi+e+\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\sqrt{23\,487\,833}-4846}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\right)\left(123-\pi^2\right)\right)\right)\right)\ \text{(angle)}$$

$r \approx 1729.0$ (radius), $\theta \approx 0.033815$ (angle)

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron.

As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$\frac{1}{2}\left(\left(2\sqrt[3]{2\left(21-5\sqrt{17}\right)}+2\sqrt[3]{2\left(21+5\sqrt{17}\right)}+2^{2/3}\left(-2\sqrt[3]{256-\sqrt{65\,063}}+9\sqrt[3]{2}-2\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{4846-\sqrt{23\,487\,833}}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)-4\right)\left(123-\pi^2\right)-\sqrt{5}+2e-1\right)$$

$$-\phi + e + \left(7 + \sqrt[3]{42 - 10\sqrt{17}} + \sqrt[3]{42 + 10\sqrt{17}} - 2^{2/3}\sqrt[3]{256 - \sqrt{65063}} - 2^{2/3}\sqrt[3]{256 + \sqrt{65063}} + \sqrt[3]{2423 - \frac{\sqrt{23487833}}{2}} + \sqrt[3]{\frac{1}{2}\left(4846 + \sqrt{23487833}\right)}\right)\left(123 - \pi^2\right)$$

$$\frac{1}{\sqrt[3]{2}}\left(-\sqrt[3]{2}\,\phi + \sqrt[3]{2}\,e - \left(7\sqrt[3]{2} + 2^{2/3}\sqrt[3]{21 - 5\sqrt{17}} + 2^{2/3}\sqrt[3]{21 + 5\sqrt{17}} - 2\sqrt[3]{256 - \sqrt{65063}} - 2\sqrt[3]{256 + \sqrt{65063}} + \sqrt[3]{4846 - \sqrt{23487833}} + \sqrt[3]{4846 + \sqrt{23487833}}\right)\left(\pi^2 - 123\right)\right)$$

Expanded forms

$$-\frac{1}{2} - \frac{\sqrt{5}}{2} + 123\left(7 + \sqrt[3]{42 - 10\sqrt{17}} + \sqrt[3]{42 + 10\sqrt{17}} - 2^{2/3}\sqrt[3]{256 - \sqrt{65063}} - 2^{2/3}\sqrt[3]{256 + \sqrt{65063}} + \sqrt[3]{\frac{1}{2}\left(4846 - \sqrt{23487833}\right)} + \sqrt[3]{\frac{1}{2}\left(4846 + \sqrt{23487833}\right)}\right) + e - \left(7 + \sqrt[3]{42 - 10\sqrt{17}} + \sqrt[3]{42 + 10\sqrt{17}} - 2^{2/3}\sqrt[3]{256 - \sqrt{65063}} - 2^{2/3}\sqrt[3]{256 + \sqrt{65063}} + \sqrt[3]{\frac{1}{2}\left(4846 - \sqrt{23487833}\right)} + \sqrt[3]{\frac{1}{2}\left(4846 + \sqrt{23487833}\right)}\right)\pi^2$$

$$\frac{1721}{2} - \frac{\sqrt{5}}{2} + 123\sqrt[3]{2\left(21 - 5\sqrt{17}\right)} + 123\sqrt[3]{2\left(21 + 5\sqrt{17}\right)} - 123 \times 2^{2/3}\sqrt[3]{256 - \sqrt{65\,063}} - 123 \times 2^{2/3}\sqrt[3]{256 + \sqrt{65\,063}} + 123\sqrt[3]{\frac{1}{2}\left(4846 - \sqrt{23\,487\,833}\right)} + 123\sqrt[3]{\frac{1}{2}\left(4846 + \sqrt{23\,487\,833}\right)} + e - 7\pi^2 - \sqrt[3]{2\left(21 - 5\sqrt{17}\right)}\,\pi^2 - \sqrt[3]{2\left(21 + 5\sqrt{17}\right)}\,\pi^2 + 2^{2/3}\sqrt[3]{256 - \sqrt{65\,063}}\,\pi^2 + 2^{2/3}\sqrt[3]{256 + \sqrt{65\,063}}\,\pi^2 - \sqrt[3]{\frac{1}{2}\left(4846 - \sqrt{23\,487\,833}\right)}\,\pi^2 - \sqrt[3]{\frac{1}{2}\left(4846 + \sqrt{23\,487\,833}\right)}\,\pi^2$$

(1/27(((123-Pi^2)((-2+(2(21-5√(17)))^(1/3)+(2(21+5√(17)))^(1/3))+((-(-9 2^(1/3)+2(256-√(65063))^(1/3)+2(256+√(65063))^(1/3)-(4846-√(23487833))^(1/3)-(4846+√(23487833))^(1/3))/2^(1/3))))+e-φ)-1))^2

Input

$$\left(\frac{1}{27}\left(\left((123 - \pi^2)\left(\left(-2 + \sqrt[3]{2\left(21 - 5\sqrt{17}\right)} + \sqrt[3]{2\left(21 + 5\sqrt{17}\right)}\right) - \frac{1}{\sqrt[3]{2}}\left(-9\sqrt[3]{2} + 2\sqrt[3]{256 - \sqrt{65\,063}} + 2\sqrt[3]{256 + \sqrt{65\,063}} - \sqrt[3]{4846 - \sqrt{23\,487\,833}} - \sqrt[3]{4846 + \sqrt{23\,487\,833}}\right)\right) + e - \phi\right) - 1\right)\right)^2$$

ϕ is the golden ratio

Exact result

$$\frac{1}{729}\left(-\phi - 1 + e + \left(-2 + \sqrt[3]{2\left(21 - 5\sqrt{17}\right)} + \sqrt[3]{2\left(21 + 5\sqrt{17}\right)} + \frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2} - 2\sqrt[3]{256 - \sqrt{65\,063}} - 2\sqrt[3]{256 + \sqrt{65\,063}} + \sqrt[3]{4846 - \sqrt{23\,487\,833}} + \sqrt[3]{4846 + \sqrt{23\,487\,833}}\right)\right)\left(123 - \pi^2\right)\right)^2$$

Decimal approximation

4086.571518008346700300613943154371865172718441313207184514529926…

+

276.956001188957665748042959894154203119577113179583874479094281…

i

Alternate complex forms

$$\frac{1}{729}\left(-\frac{3}{4}\left(\frac{1}{2}\left(-4846+\sqrt{23\,487\,833}\right)\right)^{2/3}\left(123-\pi^2\right)^2+\left(-1+e-\phi+\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{-4846+\sqrt{23\,487\,833}}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\left(123-\pi^2\right)\right)^2\right)+\frac{1}{243\sqrt{3}}\left(\sqrt[3]{\frac{1}{2}\left(-4846+\sqrt{23\,487\,833}\right)}\left(123-\pi^2\right)\left(-1+e-\phi+\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}+\frac{1}{\sqrt[3]{2}}\left(9\sqrt[3]{2}-2\sqrt[3]{256-\sqrt{65\,063}}-2\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{-4846+\sqrt{23\,487\,833}}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\left(123-\pi^2\right)\right)\right)i$$

Polar coordinates

$r \approx 4095.9$ (radius), $\theta \approx 0.067669$ (angle)

$4095.9 \approx 4096$

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$2\pi\sqrt{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

From the alternate form

$$7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}$$
$$\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}$$

we obtain, for d = 2:

(7+(42-10√(17))^(1/3)+(42+10√(17))^(1/3)-2^(2/3)(256-√(65063))^(1/3)-2^(2/3)
(256 +√(65063))^(1/3)+(2423-
√(23487833)/2)^(1/3)+(1/2(4846+√(23487833)))^(1/3))((5/12(3+√5)2^3)((4/3π(2/2)^
3)^2))^(1/(2Pi))

Input

$$\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)\times 2^3\right)\left(\frac{4}{3}\pi\left(\frac{2}{2}\right)^3\right)^2}$$

Exact result

$$2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\sqrt[\pi]{\pi}$$

Decimal approximation

37.962661278440319566528899789302745046959791570189596634642203 45…

+

1.285007644030458687637347785370611074005548769817280708019180877…

i

Alternate complex forms

$$2^{5/(2\pi)} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-\right.$$
$$2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{-2423+\frac{\sqrt{23\,487\,833}}{2}}+$$
$$\left.\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\sqrt[\pi]{\pi}+$$
$$\left(2^{-1+5/(2\pi)} \times 3^{1/2-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{-2423+\frac{\sqrt{23\,487\,833}}{2}}\ \sqrt[\pi]{\pi}\right) i$$

$$2^{5/(2\pi)} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$

$$\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}+\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-\right.\right.}$$

$$2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)^2\right)$$

$$\sqrt[\pi]{\pi}\left(\cos\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}\right)\Big/\right.\right.\right.$$

$$\left.\left.\left(2\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-\right.\right.\right.\right.$$

$$2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+$$

$$\left.\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\right)\right)\right)+$$

$$i\sin\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}\right)\Big/\right.\right.$$

$$\left.\left.\left(2\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-\right.\right.\right.\right.$$

$$\left.\left.\left.\left.2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\right)\right)\right)$$

$$2^{5/(2\pi)} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$

$$\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}+\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-\right.\right.}$$

$$\left.2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\right.$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)^2\right)$$

$$\sqrt[\pi]{\pi}\,\exp\left(i\tan^{-1}\left(\left(\sqrt{3}\,\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}\right)\Big/\right.\right.$$

$$\left.\left(2\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-\right.\right.\right.$$

$$\left.\left.\left.2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\right.\right.\right.$$

$$\left.\left.\left.\frac{1}{2}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 37.984$ (radius), $\theta \approx 0.033836$ (angle)

37.984

Expanded form

$$7\times 2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{15+5\sqrt{5}}\ \sqrt[\pi]{\pi}\ +$$
$$2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{15+5\sqrt{5}}\ \sqrt[3]{42-10\sqrt{17}}\ \sqrt[\pi]{\pi}\ +$$
$$2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{15+5\sqrt{5}}\ \sqrt[3]{42+10\sqrt{17}}\ \sqrt[\pi]{\pi}\ -$$
$$2^{2/3+5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{15+5\sqrt{5}}\ \sqrt[3]{256-\sqrt{65\,063}}\ \sqrt[\pi]{\pi}\ -$$
$$2^{2/3+5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{15+5\sqrt{5}}\ \sqrt[3]{256+\sqrt{65\,063}}\ \sqrt[\pi]{\pi}\ +$$
$$2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{15+5\sqrt{5}}\ \sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}\ \sqrt[\pi]{\pi}\ +$$
$$2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{15+5\sqrt{5}}\ \sqrt[3]{2423+\frac{\sqrt{23\,487\,833}}{2}}\ \sqrt[\pi]{\pi}$$

Alternate forms

$$2^{5/(2\pi)-4/3}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\left(14\sqrt[3]{2}+2\times 2^{2/3}\sqrt[3]{21-5\sqrt{17}}+2\times 2^{2/3}\sqrt[3]{21+5\sqrt{17}}-\right.$$
$$4\sqrt[3]{256-\sqrt{65\,063}}-4\sqrt[3]{256+\sqrt{65\,063}}+$$
$$\left.\sqrt[3]{\sqrt{23\,487\,833}-4846}+2\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\sqrt[\pi]{\pi}\ +$$
$$i\,2^{5/(2\pi)-4/3}\times 3^{(\pi-3)/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{\sqrt{23\,487\,833}-4846}\ \sqrt[\pi]{\pi}$$

$$2^{5/(2\pi)-1/3}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{17}}+2^{2/3}\sqrt[3]{21+5\sqrt{17}}-2\sqrt[3]{256-\sqrt{65\,063}}-\right.$$
$$\left.2\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{4846-\sqrt{23\,487\,833}}+\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\sqrt[\pi]{\pi}$$

Series representations

$$\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{2}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)2^3\right)}=$$

$$2^{-1/3+5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\left(\frac{\frac{1}{2}}{k}\right)\right)}$$

$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{16}\sum_{k=0}^{\infty}16^{-k}\left(\frac{\frac{1}{2}}{k}\right)}+2^{2/3}\sqrt[3]{21+5\sqrt{16}\sum_{k=0}^{\infty}16^{-k}\left(\frac{\frac{1}{2}}{k}\right)}-2\sqrt[3]{256-\sqrt{65\,062}\sum_{k=0}^{\infty}65\,062^{-k}\left(\frac{\frac{1}{2}}{k}\right)}-2\sqrt[3]{256+\sqrt{65\,062}\sum_{k=0}^{\infty}65\,062^{-k}\left(\frac{\frac{1}{2}}{k}\right)}+\sqrt[3]{4846-\sqrt{23\,487\,832}\sum_{k=0}^{\infty}23\,487\,832^{-k}\left(\frac{\frac{1}{2}}{k}\right)}+\sqrt[3]{4846+\sqrt{23\,487\,832}\sum_{k=0}^{\infty}23\,487\,832^{-k}\left(\frac{\frac{1}{2}}{k}\right)}\right)$$

$$\left(7+\sqrt[3]{42-10\sqrt{17}}\ +\sqrt[3]{42+10\sqrt{17}}\ -2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}\ -\right.$$

$$2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}\ +\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}\ +$$

$$\left.\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{2}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)2^3\right)}\ =$$

$$2^{-1/3+5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi^2\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{16}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\ +\right.$$

$$2^{2/3}\sqrt[3]{21+5\sqrt{16}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\ -$$

$$2\sqrt[3]{256-\sqrt{65\,062}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{65062}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\ -$$

$$2\sqrt[3]{256+\sqrt{65\,062}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{65062}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\ +$$

$$\sqrt[3]{4846-\sqrt{23\,487\,832}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{23487832}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\ +$$

$$\left.\sqrt[3]{4846+\sqrt{23\,487\,832}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{23487832}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-\right.$$

$$2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}+$$

$$\left.\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\sqrt[2\pi]{\frac{1}{12}\left(\frac{4}{3}\pi\left(\frac{2}{2}\right)^3\right)^2 5\left(\left(3+\sqrt{5}\right)2^3\right)}=$$

$$2^{-1/3+5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}\sqrt[2\pi]{\pi^2\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}$$

$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k z_0^{-k}}{k!}}+\right.$$

$$2^{2/3}\sqrt[3]{21+5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k z_0^{-k}}{k!}}-$$

$$2\sqrt[3]{256-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(65\,063-z_0)^k z_0^{-k}}{k!}}-$$

$$2\sqrt[3]{256+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(65\,063-z_0)^k z_0^{-k}}{k!}}+$$

$$\sqrt[3]{4846-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(23\,487\,833-z_0)^k z_0^{-k}}{k!}}+$$

$$\left.\sqrt[3]{4846+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(23\,487\,833-z_0)^k z_0^{-k}}{k!}}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

And:

(7+(42-10√(17))^(1/3)+(42+10√(17))^(1/3)-2^(2/3)(256-√65063)^(1/3)-2^(2/3)(256+√65063)^(1/3)+(2423-√(23487833)/2)^(1/3)+(1/2(4846+√(23487833)))^(1/3))((4/3π(2/2)^3)(1/3√2 2^3)(√2/12 2^3))^(1/(2Pi))

Input

$$\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\sqrt[2\pi]{\left(\frac{4}{3}\pi\left(\frac{2}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\times 2^3\right)\left(\frac{\sqrt{2}}{12}\times 2^3\right)}$$

Exact result

$$2^{7/(2\pi)}\times 3^{-3/(2\pi)}\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)^{2\pi\sqrt{\pi}}$$

Decimal approximation

23.462583911954236425932612169313361477496763114725919253900641 7…

+

0.794190888105354143092490769700586312770913672775154631863696707…

i

Alternate complex forms

$$2^{7/(2\pi)} \times 3^{-3/(2\pi)} \left(7 + \sqrt[3]{42 - 10\sqrt{17}} + \sqrt[3]{42 + 10\sqrt{17}} - 2^{2/3} \sqrt[3]{256 - \sqrt{65\,063}} - 2^{2/3} \sqrt[3]{256 + \sqrt{65\,063}} + \frac{1}{2} \sqrt{-2423 + \frac{\sqrt{23\,487\,833}}{2}} + \sqrt[3]{\frac{1}{2}\left(4846 + \sqrt{23\,487\,833}\right)} \right)^{2\pi} \sqrt{\pi} + \left(2^{-1+7/(2\pi)} \times 3^{1/2-3/(2\pi)} \sqrt{-2423 + \frac{\sqrt{23\,487\,833}}{2}} \right)^{2\pi} \sqrt{\pi} \, i$$

$$2^{7/(2\pi)} \times 3^{-3/(2\pi)}$$

$$\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}+\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-\right.\right.}$$

$$\left.2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\right.$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)^2\right)$$

$$\sqrt[2\pi]{\pi}\left(\cos\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}\right)\Big/\right.\right.\right.$$

$$\left.\left.\left(2\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-\right.\right.\right.\right.$$

$$\left.\left.\left.\left.2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\right)\right)\right)\right)+$$

$$i\sin\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}\right)\Big/\right.\right.$$

$$\left.\left.\left(2\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-\right.\right.\right.\right.$$

$$\left.\left.\left.\left.2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\frac{1}{2}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\right)\right)\right)\right)$$

$$2^{7/(2\pi)} \times 3^{-3/(2\pi)}$$
$$\sqrt{\left(\frac{3}{4}\left(\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)\right)^{2/3}+\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-\right.\right.}$$
$$2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+$$
$$\left.\left.\frac{1}{2}\sqrt[3]{\frac{1}{2}\left(\sqrt{23\,487\,833}-4846\right)}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)^2\right)$$
$$\sqrt[2\pi]{\pi}\,\exp\left(i\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}\right)\Big/\right.\right.$$
$$\left.\left(2\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-\right.\right.\right.$$
$$2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+$$
$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\frac{\sqrt{23\,487\,833}}{2}-2423}+\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 23.476$ (radius), $\theta \approx 0.033836$ (angle)

23.476

Expanded form

$$7\times 2^{7/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}+2^{7/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[3]{42-10\sqrt{17}}\sqrt[2\pi]{\pi}+$$
$$2^{7/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[3]{42+10\sqrt{17}}\sqrt[2\pi]{\pi}-$$
$$2^{2/3+7/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[3]{256-\sqrt{65\,063}}\sqrt[2\pi]{\pi}-$$
$$2^{2/3+7/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[3]{256+\sqrt{65\,063}}\sqrt[2\pi]{\pi}+$$
$$2^{7/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}\sqrt[2\pi]{\pi}+$$
$$2^{7/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[3]{2423+\frac{\sqrt{23\,487\,833}}{2}}\sqrt[2\pi]{\pi}$$

Alternate forms

$$2^{7/(2\pi)-4/3} \times 3^{-3/(2\pi)} \left(14 \sqrt[3]{2} + 2 \times 2^{2/3} \sqrt[3]{21 - 5\sqrt{17}} + 2 \times 2^{2/3} \sqrt[3]{21 + 5\sqrt{17}} - 4 \sqrt[3]{256 - \sqrt{65\,063}} - 4 \sqrt[3]{256 + \sqrt{65\,063}} + \sqrt[3]{\sqrt{23\,487\,833} - 4846} + 2 \sqrt[3]{4846 + \sqrt{23\,487\,833}}\right)^{2\pi}\sqrt{\pi} + i\, 2^{7/(2\pi)-4/3} \times 3^{(\pi-3)/(2\pi)} \sqrt[3]{\sqrt{23\,487\,833} - 4846}^{\,2\pi}\sqrt{\pi}$$

$$2^{7/(2\pi)-1/3} \times 3^{-3/(2\pi)} \left(7 \sqrt[3]{2} + 2^{2/3} \sqrt[3]{21 - 5\sqrt{17}} + 2^{2/3} \sqrt[3]{21 + 5\sqrt{17}} - 2 \sqrt[3]{256 - \sqrt{65\,063}} - 2 \sqrt[3]{256 + \sqrt{65\,063}} + \sqrt[3]{4846 - \sqrt{23\,487\,833}} + \sqrt[3]{4846 + \sqrt{23\,487\,833}}\right)^{2\pi}\sqrt{\pi}$$

Series representations

$$\left(7+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}-2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}-\right.$$

$$2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}+\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}+$$

$$\left.\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\;2^3\right)\left(\sqrt{2}\;2^3\right)\right)4\left(\pi\left(\frac{2}{2}\right)^3\right)}{(3\times 12)\,3}}=$$

$$2^{-1/3+3/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}$$

$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k\,z_0^{-k}}{k!}}+\right.$$

$$2^{2/3}\sqrt[3]{21+5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k\,z_0^{-k}}{k!}}-$$

$$2\sqrt[3]{256-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(65\,063-z_0)^k\,z_0^{-k}}{k!}}-$$

$$2\sqrt[3]{256+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(65\,063-z_0)^k\,z_0^{-k}}{k!}}+$$

$$\sqrt[3]{4846-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(23\,487\,833-z_0)^k\,z_0^{-k}}{k!}}+$$

$$\left.\sqrt[3]{4846+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(23\,487\,833-z_0)^k\,z_0^{-k}}{k!}}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\left(7+\sqrt[3]{42-10\sqrt{17}}\ +\sqrt[3]{42+10\sqrt{17}}\ -\right.$$

$$2^{2/3}\sqrt[3]{256-\sqrt{65\,063}}\ -2^{2/3}\sqrt[3]{256+\sqrt{65\,063}}\ +$$

$$\left.\sqrt[3]{2423-\frac{\sqrt{23\,487\,833}}{2}}\ +\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23\,487\,833}\right)}\right)$$

$$\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ 2^3\right)\left(\sqrt{2}\ 2^3\right)\right)4\left(\pi\left(\frac{2}{2}\right)^3\right)}{(3\times 12)\,3}}=2^{-1/3+3/\pi}\times 3^{-3/(2\pi)}$$

$$\sqrt[2\pi]{\pi\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}\left(7\sqrt[3]{2}\ +\right.$$

$$2^{2/3}\sqrt[3]{21-5\exp\left(i\,\pi\left\lfloor\frac{\arg(17-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(17-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\ +$$

$$2^{2/3}\sqrt[3]{21+5\exp\left(i\,\pi\left\lfloor\frac{\arg(17-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(17-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\ -$$

$$2\left(256-\exp\left(i\,\pi\left\lfloor\frac{\arg(65\,063-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(65\,063-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\wedge(1/3)\ -$$

$$2\left(256+\exp\left(i\,\pi\left\lfloor\frac{\arg(65\,063-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(65\,063-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\wedge(1/3)\ +$$

$$\left(4846-\exp\left(i\,\pi\left\lfloor\frac{\arg(23\,487\,833-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(23\,487\,833-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\wedge(1/3)\ +$$

$$\left(4846+\exp\left(i\,\pi\left\lfloor\frac{\arg(23\,487\,833-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\,(23\,487\,833-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\wedge$$

$$\left.(1/3)\right)\ \text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$\left(7+\sqrt[3]{42-10\sqrt{17}}\ +\sqrt[3]{42+10\sqrt{17}}\ -\right.$$

$$\left.2^{2/3}\sqrt[3]{256-\sqrt{65063}}\ -2^{2/3}\sqrt[3]{256+\sqrt{65063}}\ +\sqrt[3]{2423-\frac{\sqrt{23487833}}{2}}\ +\sqrt[3]{\frac{1}{2}\left(4846+\sqrt{23487833}\right)}\right)$$

$$\sqrt[2\pi]{\frac{\left(\left(\sqrt{2}\ 2^3\right)\left(\sqrt{2}\ 2^3\right)\right)4\left(\pi\left(\frac{2}{2}\right)^3\right)}{(3\times 12)\,3}}=2^{-1/3+3/\pi}\times 3^{-3/(2\pi)}$$

$$\sqrt[2\pi]{\pi\left(\frac{1}{z_0}\right)^{\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1+\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}$$

$$\left(7\sqrt[3]{2}+2^{2/3}\left(21-5\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(17-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(17-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)+\right.$$

$$2^{2/3}\left(21+5\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(17-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(17-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)-$$

$$2\left(256-\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(65063-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(65063-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(65063-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)-$$

$$2\left(256+\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(65063-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(65063-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(65063-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)+$$

$$\left(4846-\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(23487833-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(23487833-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(23487833-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)+$$

$$\left(4846+\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(23487833-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(23487833-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(23487833-z_0)^k\,z_0^{-k}}{k!}\right)\text{^}(1/3)\Bigg)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Dividing the two alternate forms, we obtain:

(2^(5/(2π)-1/3)3^(-3/(2π))(5(3+√5))^(1/(2π))(7 2^1/3+2^(2/3)(21-5√17)^1/3+2^(2/3)(21+5√17)^1/3-(4846+√23487833)^1/3)π^(1/π))

Input

$$2^{5/(2\pi)-1/3} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{17}}+2^{2/3}\sqrt[3]{21+5\sqrt{17}}-\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\sqrt[\pi]{\pi}$$

Exact result

$$2^{5/(2\pi)-1/3} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)} \left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{17}}+2^{2/3}\sqrt[3]{21+5\sqrt{17}}-\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\sqrt[\pi]{\pi}$$

Decimal approximation

−11.540379044312035971219326181336586064929309883611623915664922 18

…

-11.540379044312….

Expanded form

$$7\times 2^{5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ +$$
$$2^{1/3+5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{21-5\sqrt{17}}\ \sqrt[\pi]{\pi}\ +$$
$$2^{1/3+5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{21+5\sqrt{17}}\ \sqrt[\pi]{\pi}\ -$$
$$2^{5/(2\pi)-1/3}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[3]{4846+\sqrt{23\,487\,833}}\ \sqrt[\pi]{\pi}$$

Series representations

$$2^{5/(2\pi)-1/3}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{17}}+2^{2/3}\sqrt[3]{21+5\sqrt{17}}-\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)$$
$$\sqrt[\pi]{\pi}=2^{-1/3+5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\ \sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$
$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{16}\ \sum_{k=0}^{\infty}16^{-k}\binom{\frac{1}{2}}{k}}+2^{2/3}\sqrt[3]{21+5\sqrt{16}\ \sum_{k=0}^{\infty}16^{-k}\binom{\frac{1}{2}}{k}}-\right.$$
$$\left.\sqrt[3]{4846+\sqrt{23\,487\,832}\ \sum_{k=0}^{\infty}23\,487\,832^{-k}\binom{\frac{1}{2}}{k}}\right)$$

$$2^{5/(2\pi)-1/3}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{17}}+2^{2/3}\sqrt[3]{21+5\sqrt{17}}-\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)$$
$$\sqrt[\pi]{\pi}=2^{-1/3+5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}\,\sqrt[\pi]{\pi}\sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$
$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{16}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\right.$$
$$2^{2/3}\sqrt[3]{21+5\sqrt{16}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}-$$
$$\left.\sqrt[3]{4846+\sqrt{23\,487\,832}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{23\,487\,832}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$2^{5/(2\pi)-1/3}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}$$
$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{17}}+2^{2/3}\sqrt[3]{21+5\sqrt{17}}-\sqrt[3]{4846+\sqrt{23\,487\,833}}\right)$$
$$\sqrt[\pi]{\pi}=2^{-1/3+5/(2\pi)}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5}\,\sqrt[\pi]{\pi}$$
$$\sqrt[2\pi]{3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}$$
$$\left(7\sqrt[3]{2}+2^{2/3}\sqrt[3]{21-5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k z_0^{-k}}{k!}}+\right.$$
$$2^{2/3}\sqrt[3]{21+5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(17-z_0)^k z_0^{-k}}{k!}}-$$
$$\left.\sqrt[3]{4846+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(23\,487\,833-z_0)^k z_0^{-k}}{k!}}\right)$$

for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

-11.54037904431203597/(2^(7/(2π)-1/3)3^(-3/(2π))(7 2^1/3+2^(2/3)(21-5√17)^1/3+2^(2/3)(21+5√17)^1/3-(4846+√23487833)^1/3)π^(1/(2π)))

Input interpretation

$$-\left(11.54037904431203597 \Big/ \left(2^{7/(2\pi)-1/3} \times 3^{-3/(2\pi)} \left(7\sqrt[3]{2} + 2^{2/3}\sqrt[3]{21-5\sqrt{17}} + 2^{2/3}\sqrt[3]{21+5\sqrt{17}} - \sqrt[3]{4846+\sqrt{23\,487\,833}}\right)^{2\pi}\sqrt{\pi}\right)\right)$$

Result

1.618008545900107058…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Series representations

$$-\left(11.540379044312035970000 \Big/ \left(2^{7/(2\pi)-1/3} \times 3^{-3/(2\pi)} \left(7\sqrt[3]{2} + 2^{2/3}\sqrt[3]{21-5\sqrt{17}} + 2^{2/3}\sqrt[3]{21+5\sqrt{17}} - \sqrt[3]{4846+\sqrt{23\,487\,833}}\right)\sqrt[2\pi]{\pi}\right)\right) =$$

$$-\left(\left(9.159604917526345200 \times 2^{-7/(2\pi)} \times 3^{3/(2\pi)}\, \pi^{-1/(2\pi)}\right) \Big/ \left(5.559503681888698162 + 1.0000000000000000000 \sqrt[3]{21-5\sqrt{16}\sum_{k=0}^{\infty} 16^{-k}\binom{\frac{1}{2}}{k}} + 1.0000000000000000000 \sqrt[3]{21+5\sqrt{16}\sum_{k=0}^{\infty} 16^{-k}\binom{\frac{1}{2}}{k}} - 0.6299605249474365824 \sqrt[3]{4846+\sqrt{23\,487\,832}\sum_{k=0}^{\infty} 23\,487\,832^{-k}\binom{\frac{1}{2}}{k}}\right)\right)$$

$$-\left(11.540379044312035970000 \Big/ \left(2^{7/(2\pi)-1/3} \times 3^{-3/(2\pi)} \left(7\sqrt[3]{2} + 2^{2/3}\sqrt[3]{21-5\sqrt{17}} + 2^{2/3}\sqrt[3]{21+5\sqrt{17}} - \sqrt[3]{4846+\sqrt{23\,487\,833}}\right)^{2\pi\sqrt{\pi}}\right)\right) =$$

$$-\left(\left(9.159604917526345200 \times 2^{-7/(2\pi)} \times 3^{3/(2\pi)}\, \pi^{-1/(2\pi)}\right) \Big/ \left(5.555903681888698162 + 1.0000000000000000000 \sqrt[3]{21-5\sqrt{16}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}} + 1.0000000000000000000 \sqrt[3]{21+5\sqrt{16}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{16}\right)^k\left(-\frac{1}{2}\right)_k}{k!}} - 0.6299605249474365824 \sqrt[3]{4846+\sqrt{23\,487\,832}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{23\,487\,832}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)\right)$$

$$-\left(11.540379044312035970000 \Big/ \left(2^{7/(2\pi)-1/3} \times 3^{-3/(2\pi)} \left(7\sqrt[3]{2} + 2^{2/3}\sqrt[3]{21-5\sqrt{17}} + 2^{2/3}\sqrt[3]{21+5\sqrt{17}} - \sqrt[3]{4846+\sqrt{23\,487\,833}}\right)^{2\pi}\sqrt{\pi}\right)\right) =$$

$$-\left(\left(9.159604917526345200 \times 2^{-7/(2\pi)} \times 3^{3/(2\pi)}\,\pi^{-1/(2\pi)}\right) \Big/ \left(5.555903681888698162 + 1.0000000000000000000\right.\right.$$

$$\sqrt[3]{21-5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (17-z_0)^k\, z_0^{-k}}{k!}} +$$

$$1.0000000000000000000$$

$$\sqrt[3]{21+5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (17-z_0)^k\, z_0^{-k}}{k!}} -$$

$$0.6299605249474365824$$

$$\left.\left.\sqrt[3]{4846+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (23\,487\,833-z_0)^k\, z_0^{-k}}{k!}}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535884…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298884450469 20
…

1.64493406684…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.618008545900002^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.618008545900000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.92478440155986\pi}\sqrt{\pi}^2}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Suppose we have a cubic equation. We use the solutions of this cubic equation as part of the geometry of the bubble universe

Suppose one of the solutions of the cubic equation is

$$\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2$$

That is equal to 3.28226099756...

x_1 = (42+sqrt(1700))^1/3 + (42-sqrt(1700))^1/3 - 2

We can associate this solution with a certain length, for example, we could consider $a=x_1$ as the length of the radius of a hypersphere, which represents the bubble universe. Next, we can calculate the volume of this hypersphere using the radius determined by solving the cubic equation. Now, let's consider a related equation involving the ratio of the volume of the octahedron to the volume of this hypersphere. This equation may reflect a broader relationship between the geometry of the bubble universe and other geometric shapes, such as the octahedron. Using this broader relationship, we can further explore how solutions to the cubic equation affect the geometry of the bubble universe and its relationships with other geometric shapes.

In this way, we can see how solutions to cubic equations, along with other geometric relationships, can provide a completer and more in-depth picture of the mathematics and geometry of the bubble universe within a broader cosmological context.

V = 1/3*√2*a^3 (octahedron volume) ; $V = \frac{1}{3}\sqrt{2}a^3$

$$V_n(r) = \frac{\pi^{n/2}}{\Gamma(\frac{n}{2}+1)} r^n,$$ (hypersphere volume)

With $r = \left(\frac{a}{2}\right)$

(1/3·√2·a^3)/(π^(n/2)/Γ(n/2+1) (a/2)^n)

Input

$$\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{\pi^{n/2}}{\Gamma(\frac{n}{2}+1)}\left(\frac{a}{2}\right)^n}$$

$\Gamma(x)$ is the gamma function

Exact result

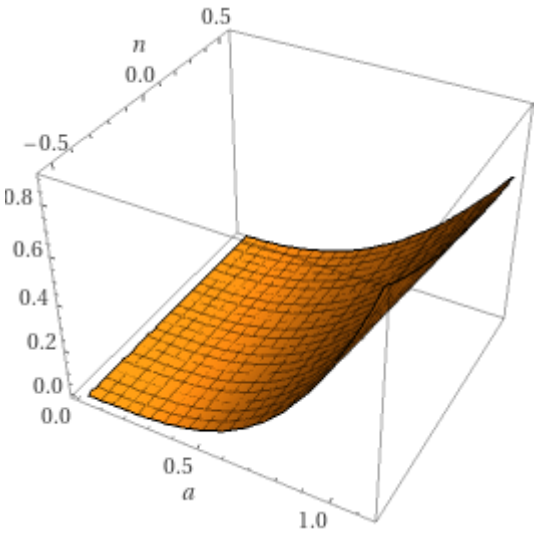
$$\frac{1}{3} \times 2^{n+1/2}\,\pi^{-n/2}\,a^{3-n}\,\Gamma\left(\frac{n}{2}+1\right)$$

Values

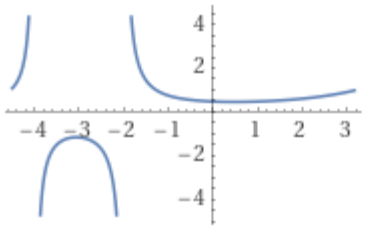
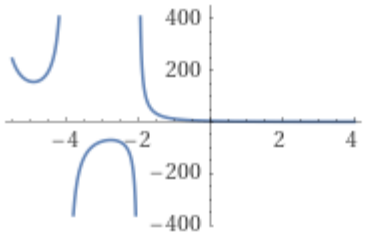
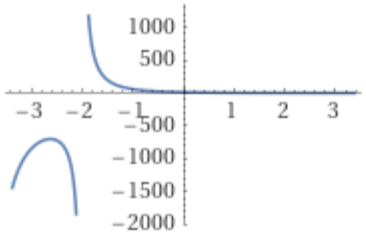
a	
0	$\frac{1}{3}\times 0^{3-n}\times 2^{n+1/2}\,\pi^{-n/2}\,\Gamma\left(\frac{n}{2}+1\right)$
1	$\frac{1}{3}\times 2^{n+1/2}\,\pi^{-n/2}\,\Gamma\left(\frac{n}{2}+1\right)$
2	$\frac{8}{3}\sqrt{2}\,\pi^{-n/2}\,\Gamma\left(\frac{n}{2}+1\right)$
3	$2^{n+1/2}\times 3^{2-n}\,\pi^{-n/2}\,\Gamma\left(\frac{n}{2}+1\right)$

3D plot (figure that can be related to a D-brane/Instanton)

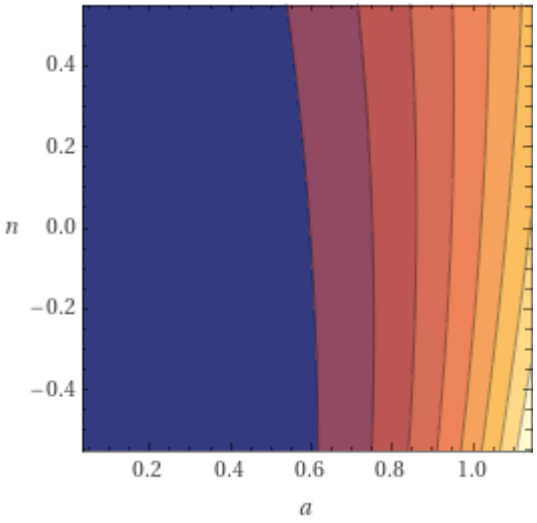
The key observation from the above plots and is that at $a = 1$, which is taken as the energy density of the universe at the Big Bang, with $n = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.



Plots (figures that can be related to the open strings)

a	
1	 (n from −4.5 to 3.2)
2	 (n from −5.5 to 4)
3	 (n from −3.4 to 3.4)

Contour plot



Alternate form

$$\frac{2^{n+3/2}\,\pi^{-n/2}\,\frac{n+2}{2}!\,a^{3-n}}{3\,(n+2)}$$

$n!$ is the factorial function

Root

$a = 0, \quad \mathrm{Re}(n) < 3$

$\mathrm{Re}(z)$ is the real part of z

Integer roots

$a = 0, \quad n = \pm 1$

$a = 0, \quad n = -9$

$a = 0, \quad n = -7$

$a = 0, \quad n = -5$

$a = 0, \quad n = -3$

Series expansion at n=0

$$\frac{\sqrt{2}\,a^3}{3} - \frac{n\left(a^3\left(2\log(a)+\gamma+\log\left(\frac{\pi}{4}\right)\right)\right)}{3\sqrt{2}} + \frac{1}{72\sqrt{2}}$$
$$a^3 n^2\left(24\log^2(a) + 24\left(\gamma+\log\left(\frac{\pi}{4}\right)\right)\log(a) + \pi^2 + 6\gamma^2 + 6\log^2(\pi) + 24\log^2(2) - 24\log(2)\log(\pi) + 12\gamma\log\left(\frac{\pi}{4}\right)\right) -$$
$$\frac{1}{144\sqrt{2}} n^3\left(a^3\left(16\log^3(a) + 24\left(\gamma+\log\left(\frac{\pi}{4}\right)\right)\log^2(a) + 2\left(6\gamma^2+\pi^2+6\log^2\left(\frac{4}{\pi}\right)+12\gamma\log\left(\frac{\pi}{4}\right)\right)\log(a) + 2\gamma^3 + 2\log^3(\pi) - 16\log^3(2) - 12\log(2)\log^2(\pi) + 24\log^2(2)\log(\pi) + \gamma\left(\pi^2+6\log^2\left(\frac{4}{\pi}\right)\right) + \pi^2\log(\pi) + 6\gamma^2\log\left(\frac{\pi}{4}\right) - \pi^2\log(4) - 2\psi^{(2)}(1)\right)\right) +$$
$$\frac{1}{11520\sqrt{2}} a^3 n^4\left(320\log^4(a) + 640\left(\gamma+\log\left(\frac{\pi}{4}\right)\right)\log^3(a) + 80\left(6\gamma^2+\pi^2+6\log^2\left(\frac{4}{\pi}\right)+12\gamma\log\left(\frac{\pi}{4}\right)\right)\log^2(a) + 80\log(a)\left(2\gamma^3+\gamma\left(\pi^2+6\log^2\left(\frac{4}{\pi}\right)\right)+6\gamma^2\log\left(\frac{\pi}{4}\right)+\pi^2\log\left(\frac{\pi}{4}\right) - 2\left(8\log^3(2) - \log^3(\pi) - 12\log^2(2)\log(\pi) + \log(64)\log^2(\pi) + \psi^{(2)}(1)\right)\right) + 3\pi^4 + 20\gamma^4 + 20\log^4(\pi) + 320\log^4(2) - 160\log(2)\log^3(\pi) - 640\log^3(2)\log(\pi) + 480\log^2(2)\log^2(\pi) + 20\pi^2\log^2(\pi) + 20\gamma^2\left(\pi^2+6\log^2\left(\frac{4}{\pi}\right)\right) + 80\pi^2\log^2(2) - 80\pi^2\log(2)\log(\pi) - 80\gamma^3\log\left(\frac{4}{\pi}\right) - 40\gamma\left(\pi^2\log\left(\frac{4}{\pi}\right) + 2\left(8\log^3(2) - \log^3(\pi) - 12\log^2(2)\log(\pi) + \log(64)\log^2(\pi) + \psi^{(2)}(1)\right)\right) - 80\log(\pi)\psi^{(2)}(1) + 160\log(2)\psi^{(2)}(1)\right) + O\left(n^5\right)$$

(Taylor series)

$\log(x)$ is the natural logarithm

$\psi^{(n)}(x)$ is the n^{th} derivative of the digamma function

γ is the Euler-Mascheroni constant

Series expansion at n=∞

$$2^{(n+1)/2}\, n^{n/2}\, (e\,\pi)^{-n/2}\, a^{3-n}$$
$$\left(\frac{\sqrt{\pi}\,\sqrt{n}}{3}+\frac{1}{18}\sqrt{\pi}\,\sqrt{\frac{1}{n}}+\frac{1}{216}\sqrt{\pi}\left(\frac{1}{n}\right)^{3/2}-\frac{139\sqrt{\pi}\left(\frac{1}{n}\right)^{5/2}}{19440}-\right.$$
$$\left.\frac{571\sqrt{\pi}\left(\frac{1}{n}\right)^{7/2}}{466560}+\frac{163879\sqrt{\pi}\left(\frac{1}{n}\right)^{9/2}}{19595520}+O\left(\left(\frac{1}{n}\right)^{11/2}\right)\right)$$

Derivative

$$\frac{\partial}{\partial n}\left(\frac{\sqrt{2}\,a^3}{\frac{3\left(\pi^{n/2}\left(\frac{a}{2}\right)^n\right)}{\Gamma\left(\frac{n}{2}+1\right)}}\right)=\frac{1}{3}\times 2^{n-1/2}\,a^{3-n}\,\pi^{-n/2}\,\Gamma\left(\frac{n}{2}+1\right)\left(-2\log(a)+\psi^{(0)}\left(\frac{n}{2}+1\right)+\log\left(\frac{4}{\pi}\right)\right)$$

Limit

$$\lim_{n\to-\infty}\frac{1}{3}\times 2^{1/2+n}\,a^{3-n}\,\pi^{-n/2}\,\Gamma\left(1+\frac{n}{2}\right)=0$$

For a = (1/3·√2·a^3)/(π^(n/2)/Γ(n/2+1) (a/2)^n) and n = 11:

(1/3·√2·(((42+sqrt(1700))^1/3 + (42-sqrt(1700))^1/3 - 2))^3)/(π^(11/2)/Γ(11/2+1) ((((42+sqrt(1700))^1/3 + (42-sqrt(1700))^1/3 - 2))/2)^11)

Input

$$\frac{\frac{1}{3}\sqrt{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)^3}{\frac{\pi^{11/2}}{\Gamma\left(\frac{11}{2}+1\right)}\left(\frac{1}{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)\right)^{11}}$$

$\Gamma(x)$ is the gamma function

Exact result

$$\frac{110\,880\sqrt{2}}{\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5}$$

Decimal approximation

0.038039425950802565717795035068022590360997029049862771906203488 3

…

0.0380394259508….

Property

$$\frac{110\,880\sqrt{2}}{\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5}$$ is a transcendental number

Alternate forms

$$\frac{110\,880\sqrt{2}}{\pi^5\left(\text{root of } x^3+6x^2-100 \text{ near } x=3.28226\right)^8}$$

$$\frac{1}{78\,125\,\pi^5}693\sqrt{\left(573\,792+\sqrt[3]{17\left(11\,136\,138\,675\,626\,417-925\,340\,430\,455\sqrt{17}\right)}+\sqrt[3]{17\left(11\,136\,138\,675\,626\,417+925\,340\,430\,455\sqrt{17}\right)}\right)}$$

$$\frac{110\,880\sqrt{2}}{\left(-2+\sqrt[3]{2\left(21-5\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\sqrt{17}\right)}\right)^8\pi^5}$$

Alternative representations

$$\frac{\sqrt{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)^3}{\frac{\left(\pi^{11/2}\left(\frac{1}{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)\right)^{11}\right)3}{\Gamma\left(\frac{11}{2}+1\right)}} = \frac{\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)^3\sqrt{2}}{\frac{3\left(\pi^{11/2}\left(\frac{1}{2}\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)\right)^{11}\right)}{\frac{11}{2}!}}$$

$$\frac{\sqrt{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)^3}{\frac{\left(\pi^{11/2}\left(\frac{1}{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)\right)^{11}\right)3}{\Gamma\left(\frac{11}{2}+1\right)}} = \frac{\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)^3\sqrt{2}}{\frac{3\left(\pi^{11/2}\left(\frac{1}{2}\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)\right)^{11}\right)}{(1)_{\frac{11}{2}}}}$$

$$\frac{\sqrt{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)^3}{\frac{\left(\pi^{11/2}\left(\frac{1}{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)\right)^{11}\right)3}{\Gamma\left(\frac{11}{2}+1\right)}} = \frac{\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)^3\sqrt{2}}{\frac{3\left(\pi^{11/2}\left(\frac{1}{2}\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)\right)^{11}\right)}{\Gamma\left(\frac{13}{2},0\right)}}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(a,\ x)$ is the incomplete gamma function

Series representations

$$\frac{\sqrt{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)^3}{\frac{\left(\pi^{11/2}\left(\frac{1}{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)\right)^{11}\right)3}{\Gamma\left(\frac{11}{2}+1\right)}} = \frac{2048\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(\frac{13}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}{3\pi^{11/2}\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)^8} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{\sqrt{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)^3}{\frac{\left(\pi^{11/2}\left(\frac{1}{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)\right)^{11}\right)3}{\Gamma\left(\frac{11}{2}+1\right)}} = \left(2048\sqrt{2}\right)\Big/\left(3\pi^{9/2}\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)^8 \sum_{k=0}^{\infty}\left(\frac{13}{2}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)$$

$$\frac{\sqrt{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)^3}{\frac{\left(\pi^{11/2}\left(\frac{1}{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)\right)^{11}\right)3}{\Gamma\left(\frac{11}{2}+1\right)}} =$$

$$\left(2048\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{13}{2}\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/$$

$$\left(3\pi^{11/2}\left(-2+\left(42-\exp\left(i\pi\left\lfloor\frac{\arg(1700-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.\right.\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k(1700-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{\wedge}$$

$$(1/3)+\left(42+\exp\left(i\pi\left\lfloor\frac{\arg(1700-x)}{2\pi}\right\rfloor\right)\sqrt{x}\right.$$

$$\left.\left.\left.\sum_{k=0}^{\infty}\frac{(-1)^k(1700-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^{\wedge}\right.\right.$$

$$\left.(1/3)\right)^8\Bigg) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$\mathbb{Z}$ is the set of integers

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

i is the imaginary unit

$\mathbb{R}$ is the set of real numbers

Integral representations

$$\frac{\sqrt{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)^3}{\frac{\left(\pi^{11/2}\left(\frac{1}{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)\right)^{11}\right)3}{\Gamma\left(\frac{11}{2}+1\right)}} =$$

$$\frac{2048\sqrt{2}}{3\pi^{11/2}\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)^8}\int_0^{\infty}e^{-t}t^{11/2}\,dt$$

$$\frac{\sqrt{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)^3}{\frac{\left(\pi^{11/2}\left(\frac{1}{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)\right)^{11}\right)3}{\Gamma\left(\frac{11}{2}+1\right)}} = \frac{2048\sqrt{2}}{3\pi^{11/2}\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)^8}\int_0^1 \log^{11/2}\left(\frac{1}{t}\right)dt$$

$$\frac{\sqrt{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)^3}{\frac{\left(\pi^{11/2}\left(\frac{1}{2}\left(\sqrt[3]{42+\sqrt{1700}}+\sqrt[3]{42-\sqrt{1700}}-2\right)\right)^{11}\right)3}{\Gamma\left(\frac{11}{2}+1\right)}} = \frac{2048\exp\left(\int_0^1 \frac{\frac{11}{2}-\frac{13x}{2}+x^{13/2}}{(-1+x)\log(x)}\,dx\right)\sqrt{2}}{3\pi^{11/2}\left(-2+\sqrt[3]{42-\sqrt{1700}}+\sqrt[3]{42+\sqrt{1700}}\right)^8}$$

$\log(x)$ is the natural logarithm

Multiplying the result

$$\frac{110880\sqrt{2}}{\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5}$$

by

$$\frac{27\sqrt[3]{\frac{3}{e}}}{4\log^{10/3}(2)} \approx 23.667978\,18359$$

we obtain:

((110880 sqrt(2))/((-2 + (42 - 10 sqrt(17))^(1/3) + (42 + 10 sqrt(17))^(1/3))^8 π^5))*((27 (3/e)^(1/3))/(4 log^(10/3)(2)))

Input

$$\frac{110880\sqrt{2}}{\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5}\times\frac{27\sqrt[3]{\frac{3}{e}}}{4\log^{10/3}(2)}$$

$\log(x)$ is the natural logarithm

Exact result

$$\frac{748440\sqrt{2}\sqrt[3]{\frac{3}{e}}}{\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\log^{10/3}(2)}$$

Decimal approximation

0.900316303520071672167006207996848279879584766814023520667758924 3

…

$0.9003163052\ldots \approx 0.9003163161571\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed form

$$\frac{2\sqrt{2}}{\pi} \approx 0.900316316157$$

Alternate forms

$$\frac{748440\sqrt{2}\sqrt[3]{\frac{3}{e}}}{\pi^5\log^{10/3}(2)\left(\text{root of } x^3+6x^2-100 \text{ near } x=3.28226\right)^8}$$

$$\frac{1}{312\,500\,\pi^5\,\log^{10/3}(2)}18\,711\,\sqrt{\left(573\,792+\sqrt[3]{17\left(11\,136\,138\,675\,626\,417-925\,340\,430\,455\,\sqrt{17}\right)}+\sqrt[3]{17\left(11\,136\,138\,675\,626\,417+925\,340\,430\,455\,\sqrt{17}\right)}\right)\sqrt[3]{\frac{3}{e}}}$$

$$\frac{748\,440\,\sqrt{2}\,\sqrt[3]{\frac{3}{e}}}{\left(-2+\sqrt[3]{2\left(21-5\,\sqrt{17}\right)}+\sqrt[3]{2\left(21+5\,\sqrt{17}\right)}\right)^8\pi^5\log^{10/3}(2)}$$

Reduced logarithmic form

$$\frac{748\,440\,\sqrt{2}\,\sqrt[3]{\frac{3}{e}}}{\left(-2+\sqrt[3]{42-10\,\sqrt{17}}+\sqrt[3]{42+10\,\sqrt{17}}\right)^8\pi^5\log^{10/3}(2)}$$

Alternative representations

$$\frac{\left(27\,\sqrt[3]{\frac{3}{e}}\right)(110\,880\,\sqrt{2})}{(4\log^{10/3}(2))\left(\left(-2+\sqrt[3]{42-10\,\sqrt{17}}+\sqrt[3]{42+10\,\sqrt{17}}\right)^8\pi^5\right)}=\frac{2\,993\,760\,\sqrt[3]{\frac{3}{e}}\,\sqrt{2}}{(4\log_e^{10/3}(2))\left(\pi^5\left(-2+\sqrt[3]{42-10\,\sqrt{17}}+\sqrt[3]{42+10\,\sqrt{17}}\right)^8\right)}$$

$$\frac{\left(27\sqrt[3]{\frac{3}{e}}\right)\left(110\,880\sqrt{2}\right)}{\left(4\log^{10/3}(2)\right)\left(\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\right)} = \frac{2\,993\,760\sqrt[3]{\frac{3}{e}}\sqrt{2}}{\left(4\left(2\coth^{-1}(3)\right)^{10/3}\right)\left(\pi^5\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\right)}$$

$$\frac{\left(27\sqrt[3]{\frac{3}{e}}\right)\left(110\,880\sqrt{2}\right)}{\left(4\log^{10/3}(2)\right)\left(\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\right)} = \frac{2\,993\,760\sqrt[3]{\frac{3}{e}}\sqrt{2}}{\left(4\left(\log(a)\log_a(2)\right)^{10/3}\right)\left(\pi^5\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\right)}$$

$\log_b(x)$ is the base- b logarithm

$\coth^{-1}(x)$ is the inverse hyperbolic cotangent function

Series representations

$$\frac{\left(27\sqrt[3]{\frac{3}{e}}\right)\left(110\,880\sqrt{2}\right)}{\left(4\log^{10/3}(2)\right)\left(\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\right)} = \left(748\,440\sqrt{2}\sqrt[3]{\frac{3}{e}}\right)\Big/\left(\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\left(2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)^{10/3}\right) \quad \text{for } x<0$$

$$\frac{\left(27\sqrt[3]{\frac{3}{e}}\right)\left(110\,880\sqrt{2}\right)}{\left(4\log^{10/3}(2)\right)\left(\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\right)}=$$
$$\left(748\,440\sqrt{2}\sqrt[3]{\frac{3}{e}}\right)\Big/\left(\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\left(\log(z_0)+\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{10/3}\right)$$

$$\frac{\left(27\sqrt[3]{\frac{3}{e}}\right)\left(110\,880\sqrt{2}\right)}{\left(4\log^{10/3}(2)\right)\left(\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\right)}=$$
$$\left(748\,440\sqrt{2}\sqrt[3]{\frac{3}{e}}\right)\Big/\left(\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-z_0)^k z_0^{-k}}{k}\right)^{10/3}\right)$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Integral representations

$$\frac{\left(27\sqrt[3]{\frac{3}{e}}\right)\left(110\,880\sqrt{2}\right)}{\left(4\log^{10/3}(2)\right)\left(\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\right)}=$$
$$\frac{748\,440\sqrt{2}\sqrt[3]{\frac{3}{e}}}{\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\left(\int_1^2\frac{1}{t}\,dt\right)^{10/3}}$$

$$\frac{\left(27\sqrt[3]{\frac{3}{e}}\right)(110880\sqrt{2})}{(4\log^{10/3}(2))\left(\left(-2+\sqrt[3]{42-10\sqrt{17}}+\sqrt[3]{42+10\sqrt{17}}\right)^8\pi^5\right)}=$$
$$\frac{5987520\times 2^{5/6}\sqrt[3]{\frac{3}{e}}\left(-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{2/3}}{\left(-2+\sqrt[3]{2(21-5\sqrt{17})}+\sqrt[3]{2(21+5\sqrt{17})}\right)^8\pi^{5/3}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^4}$$

for $-1<\gamma<0$

$\Gamma(x)$ is the gamma function

From

SUPERCONFORMAL GRAVITY AND THE TOPOLOGY OF DIFFEOMORPHISM GROUPS - *Jay Cushing, Gregory W. Moore, Martin Roček, Vivek Saxena* - arXiv:2311.08394v2 [hep-th] 17 Nov 2023

We analyze the following equation:

$$\delta A_\mu^{(\mathrm{R})}=\partial_\mu\Theta^{(\mathrm{R})}-\frac{i}{2}\left(\frac{i}{2\sqrt{2}}\epsilon\,S_{\mu,A}{}^{A}+\epsilon^{A\dot{A}}S_{\mu,A\dot{A}}\right)+\frac{3i\sqrt{2}}{4}\left(\frac{i}{2\sqrt{2}}\epsilon\,\Xi_{C\dot{C}}+\epsilon_{A\dot{C}}\Xi^{A}{}_{C}\right)e_\mu{}^{C\dot{C}}=0$$

We consider:

(δ*Θ^(R)-i/2*(i/(2√2)*ε*S^A+ε^A*S)+1/4*(3i√2)*(i/(2√2)*ε*Ξ+ε*Ξ^A)*e^(C*C))

Input

$$\delta\Theta^R-\frac{i}{2}\left(\frac{i}{2\sqrt{2}}\varepsilon S^A+\varepsilon^A S\right)+\frac{1}{4}(3i\sqrt{2})\left(\frac{i}{2\sqrt{2}}\varepsilon\,\Xi+\varepsilon\,\Xi^A\right)e^{CC}$$

i is the imaginary unit

Exact result

$$\frac{3i\,e^{C^2}\left(\varepsilon\,\Xi^A+\frac{i\varepsilon\Xi}{2\sqrt{2}}\right)}{2\sqrt{2}}-\frac{1}{2}i\left(S\,\varepsilon^A+\frac{i\varepsilon S^A}{2\sqrt{2}}\right)+\delta\Theta^R$$

Expanded form

$$\frac{3\,i\,e^{C^2}\,\varepsilon\,\Xi^A}{2\sqrt{2}}+\frac{\varepsilon\,S^A}{4\sqrt{2}}-\frac{1}{2}\,i\,S\,\varepsilon^A-\frac{3}{8}\,e^{C^2}\,\varepsilon\,\Xi+\delta\,\Theta^R$$

Alternate forms

$$\frac{1}{8}\left(6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\,\Xi^A+\sqrt{2}\;\varepsilon\,S^A-4\,i\,S\,\varepsilon^A-3\,e^{C^2}\,\varepsilon\,\Xi+8\,\delta\,\Theta^R\right)$$

$$\frac{1}{8}\left(3\,i\,e^{C^2}\,\varepsilon\left(2\sqrt{2}\;\Xi^A+i\,\Xi\right)+\sqrt{2}\;\varepsilon\,S^A-4\,i\,S\,\varepsilon^A+8\,\delta\,\Theta^R\right)$$

$$\frac{3\,i\,e^{C^2}\,\varepsilon\,\Xi^A}{2\sqrt{2}}+\frac{1}{8}\left(\sqrt{2}\;\varepsilon\,S^A-4\,i\,S\,\varepsilon^A-3\,e^{C^2}\,\varepsilon\,\Xi+8\,\delta\,\Theta^R\right)$$

Series expansion at A=0

$$\begin{aligned}&\frac{1}{8}\left(\varepsilon\left(\sqrt{2}+e^{C^2}\left(-3\,\Xi+6\,i\,\sqrt{2}\right)\right)+8\,\delta\,\Theta^R-4\,i\,S\right)+\\&\frac{1}{8}\,A\left(6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\log(\Xi)+\sqrt{2}\;\varepsilon\log(S)-4\,i\,S\log(\varepsilon)\right)+\\&\frac{1}{16}\,A^2\left(6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\log^2(\Xi)+\sqrt{2}\;\varepsilon\log^2(S)-4\,i\,S\log^2(\varepsilon)\right)+\\&\frac{1}{48}\,A^3\left(6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\log^3(\Xi)+\sqrt{2}\;\varepsilon\log^3(S)-4\,i\,S\log^3(\varepsilon)\right)+\\&\frac{1}{192}\,A^4\left(6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\log^4(\Xi)+\sqrt{2}\;\varepsilon\log^4(S)-4\,i\,S\log^4(\varepsilon)\right)+O\left(A^5\right)\end{aligned}$$

(Taylor series)

$\log(x)$ is the natural logarithm

Derivative

$$\begin{aligned}&\frac{\partial}{\partial A}\left(\delta\,\Theta^R-\frac{1}{2}\,i\left(\frac{i\,\varepsilon\,S^A}{2\sqrt{2}}+\varepsilon^A\,S\right)+\frac{1}{4}\left(3\,i\,\sqrt{2}\right)\left(\frac{i\,\varepsilon\,\Xi}{2\sqrt{2}}+\varepsilon\,\Xi^A\right)e^{C\,C}\right)=\\&\frac{1}{8}\left(6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\,\Xi^A\log(\Xi)+\sqrt{2}\;\varepsilon\,S^A\log(S)-4\,i\,S\,\varepsilon^A\log(\varepsilon)\right)\end{aligned}$$

Indefinite integral

$$\int\left(-\frac{1}{2}\,i\left(\frac{i\,S^A\,\varepsilon}{2\sqrt{2}}+S\,\varepsilon^A\right)+\delta\,\Theta^R+\frac{3\,i\,e^{C^2}\left(\frac{i\,\varepsilon\,\Xi}{2\sqrt{2}}+\varepsilon\,\Xi^A\right)}{2\sqrt{2}}\right)dA=$$
$$\frac{1}{8}\left(\frac{6\,i\,\sqrt{2}\;e^{C^2}\;\varepsilon\,\Xi^A}{\log(\Xi)}-3\,A\,e^{C^2}\;\varepsilon\,\Xi+8\,A\,\delta\,\Theta^R+\frac{\sqrt{2}\;\varepsilon\,S^A}{\log(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log(\varepsilon)}\right)+\text{constant}$$

From the indefinite integral result

$$\int\left(-\frac{1}{2}\,i\left(\frac{i\,S^A\,\varepsilon}{2\sqrt{2}}+S\,\varepsilon^A\right)+\delta\,\Theta^R+\frac{3\,i\,e^{C^2}\left(\frac{i\,\varepsilon\,\Xi}{2\sqrt{2}}+\varepsilon\,\Xi^A\right)}{2\sqrt{2}}\right)dA=$$
$$\frac{1}{8}\left(\frac{6\,i\,\sqrt{2}\;e^{C^2}\;\varepsilon\,\Xi^A}{\log(\Xi)}-3\,A\,e^{C^2}\;\varepsilon\,\Xi+8\,A\,\delta\,\Theta^R+\frac{\sqrt{2}\;\varepsilon\,S^A}{\log(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log(\varepsilon)}\right)+\text{constant}$$

we obtain:

1/8 (8 A δ Θ^R - 3 A e^(C^2) ε Ξ + (sqrt(2) S^A ε)/log(S) - (4 i S ε^A)/log(ε) + (6 i sqrt(2) e^(C^2) ε Ξ^A)/log(Ξ))

Input

$$\frac{1}{8}\left(8\,A\,\delta\,\Theta^R-3\,A\,e^{C^2}\;\varepsilon\,\Xi+\frac{\sqrt{2}\;S^A\,\varepsilon}{\log(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log(\varepsilon)}+\frac{6\,i\,\sqrt{2}\;e^{C^2}\;\varepsilon\,\Xi^A}{\log(\Xi)}\right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Expanded form

$$\frac{3\,i\,e^{C^2}\;\varepsilon\,\Xi^A}{2\sqrt{2}\;\log(\Xi)}-\frac{3}{8}\,A\,e^{C^2}\;\varepsilon\,\Xi+A\,\delta\,\Theta^R+\frac{\varepsilon\,S^A}{4\sqrt{2}\;\log(S)}-\frac{i\,S\,\varepsilon^A}{2\log(\varepsilon)}$$

Alternate form

$$\frac{1}{8\log(\varepsilon)\log(\Xi)\log(S)}\left(6i\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A\log(\varepsilon)\log(S)-3A\,e^{C^2}\,\varepsilon\,\Xi\log(\varepsilon)\log(\Xi)\log(S)+8A\delta\log(\varepsilon)\log(\Xi)\Theta^R\log(S)+\sqrt{2}\,\varepsilon\,S^A\log(\varepsilon)\log(\Xi)-4iS\,\varepsilon^A\log(\Xi)\log(S)\right)$$

Series expansion at A=0

$$\frac{1}{8}\left(\frac{6i\sqrt{2}\,e^{C^2}\,\varepsilon}{\log(\Xi)}-\frac{4iS}{\log(\varepsilon)}+\frac{\sqrt{2}\,\varepsilon}{\log(S)}\right)+\frac{1}{8}A\left(\varepsilon\left(\sqrt{2}+e^{C^2}\left(-3\Xi+6i\sqrt{2}\right)\right)+8\delta\Theta^R-4iS\right)+\frac{1}{16}A^2\left(6i\sqrt{2}\,e^{C^2}\,\varepsilon\log(\Xi)+\sqrt{2}\,\varepsilon\log(S)-4iS\log(\varepsilon)\right)+\frac{1}{48}A^3\left(6i\sqrt{2}\,e^{C^2}\,\varepsilon\log^2(\Xi)+\sqrt{2}\,\varepsilon\log^2(S)-4iS\log^2(\varepsilon)\right)+\frac{1}{192}A^4\left(6i\sqrt{2}\,e^{C^2}\,\varepsilon\log^3(\Xi)+\sqrt{2}\,\varepsilon\log^3(S)-4iS\log^3(\varepsilon)\right)+O\left(A^5\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial A}\left(\frac{1}{8}\left(8A\delta\Theta^R-3A\,e^{C^2}\,\varepsilon\,\Xi+\frac{\sqrt{2}\,S^A\,\varepsilon}{\log(S)}-\frac{4iS\,\varepsilon^A}{\log(\varepsilon)}+\frac{6i\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log(\Xi)}\right)\right)=\frac{1}{8}\left(3i\,e^{C^2}\,\varepsilon\left(2\sqrt{2}\,\Xi^A+i\Xi\right)+\sqrt{2}\,\varepsilon\,S^A-4iS\,\varepsilon^A+8\delta\Theta^R\right)$$

Indefinite integral

$$\int\frac{1}{8}\left(8A\delta\Theta^R-3A\,e^{C^2}\,\varepsilon\,\Xi+\frac{\sqrt{2}\,S^A\,\varepsilon}{\log(S)}-\frac{4iS\,\varepsilon^A}{\log(\varepsilon)}+\frac{6i\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log(\Xi)}\right)dA=\frac{1}{8}\left(-\frac{3}{2}A^2\,e^{C^2}\,\varepsilon\,\Xi+4A^2\delta\Theta^R+\frac{6i\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log^2(\Xi)}+\frac{\sqrt{2}\,\varepsilon\,S^A}{\log^2(S)}-\frac{4iS\,\varepsilon^A}{\log^2(\varepsilon)}\right)+\text{constant}$$

Again, from the indefinite integral result

$$\int \frac{1}{8}\left(8\,A\,\delta\,\Theta^R - 3\,A\,e^{C^2}\,\varepsilon\,\Xi + \frac{\sqrt{2}\,S^A\,\varepsilon}{\log(S)} - \frac{4\,i\,S\,\varepsilon^A}{\log(\varepsilon)} + \frac{6\,i\,\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log(\Xi)}\right)dA =$$
$$\frac{1}{8}\left(-\frac{3}{2}\,A^2\,e^{C^2}\,\varepsilon\,\Xi + 4\,A^2\,\delta\,\Theta^R + \frac{6\,i\,\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log^2(\Xi)} + \frac{\sqrt{2}\,\varepsilon\,S^A}{\log^2(S)} - \frac{4\,i\,S\,\varepsilon^A}{\log^2(\varepsilon)}\right) + \text{constant}$$

we obtain:

1/8 (4 A^2 δ Θ^R - 3/2 A^2 e^(C^2) ε Ξ + (sqrt(2) S^A ε)/(log^2(S)) - (4 i S ε^A)/(log^2(ε)) + (6 i sqrt(2) e^(C^2) ε Ξ^A)/(log^2(Ξ)))

Input

$$\frac{1}{8}\left(4\,A^2\,\delta\,\Theta^R - \frac{3}{2}\,A^2\,e^{C^2}\,\varepsilon\,\Xi + \frac{\sqrt{2}\,S^A\,\varepsilon}{\log^2(S)} - \frac{4\,i\,S\,\varepsilon^A}{\log^2(\varepsilon)} + \frac{6\,i\,\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log^2(\Xi)}\right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Expanded form

$$-\frac{3}{16}\,A^2\,e^{C^2}\,\varepsilon\,\Xi + \frac{1}{2}\,A^2\,\delta\,\Theta^R + \frac{3\,i\,e^{C^2}\,\varepsilon\,\Xi^A}{2\,\sqrt{2}\,\log^2(\Xi)} + \frac{\varepsilon\,S^A}{4\,\sqrt{2}\,\log^2(S)} - \frac{i\,S\,\varepsilon^A}{2\log^2(\varepsilon)}$$

Alternate form assuming A, C, R, S, δ, ε, Θ, and Ξ are positive

$$\frac{1}{16}\left(-3\,A^2\,e^{C^2}\,\varepsilon\,\Xi + 8\,A^2\,\delta\,\Theta^R + \frac{12\,i\,\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log^2(\Xi)} + \frac{2\,\sqrt{2}\,\varepsilon\,S^A}{\log^2(S)} - \frac{8\,i\,S\,\varepsilon^A}{\log^2(\varepsilon)}\right)$$

Alternate forms

$$\left(A^2\log^2(\varepsilon)\log^2(\Xi)\log^2(S)\left(8\,\delta\,\Theta^R - 3\,e^{C^2}\,\varepsilon\,\Xi\right) + 2\left(\sqrt{2}\,\varepsilon\,S^A\log^2(\varepsilon)\log^2(\Xi) - 2\,i\right.\right.$$
$$\left.\left.\log^2(S)\left(2\,S\,\varepsilon^A\log^2(\Xi) - 3\,\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A\log^2(\varepsilon)\right)\right)\right) \Big/ \left(16\log^2(\varepsilon)\log^2(\Xi)\log^2(S)\right)$$

$$\left(-3\,A^2\,e^{C^2}\,\varepsilon\,\Xi\log^2(\varepsilon)\log^2(\Xi)\log^2(S)+8\,A^2\,\delta\log^2(\varepsilon)\log^2(\Xi)\,\Theta^R\log^2(S)+12\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\,\Xi^A\log^2(\varepsilon)\log^2(S)+2\,\sqrt{2}\;\varepsilon\,S^A\log^2(\varepsilon)\log^2(\Xi)-8\,i\,S\,\varepsilon^A\log^2(\Xi)\log^2(S)\right)\Big/\left(16\log^2(\varepsilon)\log^2(\Xi)\log^2(S)\right)$$

Series expansion at A=0

$$\frac{1}{8}\left(\frac{6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon}{\log^2(\Xi)}-\frac{4\,i\,S}{\log^2(\varepsilon)}+\frac{\sqrt{2}\;\varepsilon}{\log^2(S)}\right)+\frac{1}{8}\,A\left(\frac{6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon}{\log(\Xi)}-\frac{4\,i\,S}{\log(\varepsilon)}+\frac{\sqrt{2}\;\varepsilon}{\log(S)}\right)+\frac{1}{16}\,A^2\left(\varepsilon\left(\sqrt{2}+e^{C^2}\left(-3\,\Xi+6\,i\,\sqrt{2}\right)\right)+8\,\delta\,\Theta^R-4\,i\,S\right)+\frac{1}{48}\,A^3\left(6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\log(\Xi)+\sqrt{2}\;\varepsilon\log(S)-4\,i\,S\log(\varepsilon)\right)+\frac{1}{192}\,A^4\left(6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\log^2(\Xi)+\sqrt{2}\;\varepsilon\log^2(S)-4\,i\,S\log^2(\varepsilon)\right)+O\left(A^5\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial A}\left(\frac{1}{8}\left(4\,A^2\,\delta\,\Theta^R-\frac{3}{2}\,A^2\,e^{C^2}\,\varepsilon\,\Xi+\frac{\sqrt{2}\;S^A\,\varepsilon}{\log^2(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log^2(\varepsilon)}+\frac{6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\,\Xi^A}{\log^2(\Xi)}\right)\right)=\frac{1}{8}\left(\frac{6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\,\Xi^A}{\log(\Xi)}-3\,A\,e^{C^2}\,\varepsilon\,\Xi+8\,A\,\delta\,\Theta^R+\frac{\sqrt{2}\;\varepsilon\,S^A}{\log(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log(\varepsilon)}\right)$$

Indefinite integral

$$\int\frac{1}{8}\left(4\,A^2\,\delta\,\Theta^R-\frac{3}{2}\,A^2\,e^{C^2}\,\varepsilon\,\Xi+\frac{\sqrt{2}\;S^A\,\varepsilon}{\log^2(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log^2(\varepsilon)}+\frac{6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\,\Xi^A}{\log^2(\Xi)}\right)dA=\frac{1}{8}\left(-\frac{1}{2}\,A^3\,e^{C^2}\,\varepsilon\,\Xi+\frac{4}{3}\,A^3\,\delta\,\Theta^R+\frac{6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\,\Xi^A}{\log^3(\Xi)}+\frac{\sqrt{2}\;\varepsilon\,S^A}{\log^3(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log^3(\varepsilon)}\right)+\text{constant}$$

In conclusion, we have the indefinite integral result

$$\int \frac{1}{8}\left(4\,A^2\,\delta\,\Theta^R - \frac{3}{2}\,A^2\,e^{C^2}\,\varepsilon\,\Xi + \frac{\sqrt{2}\,S^A\,\varepsilon}{\log^2(S)} - \frac{4\,i\,S\,\varepsilon^A}{\log^2(\varepsilon)} + \frac{6\,i\,\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log^2(\Xi)}\right) dA =$$
$$\frac{1}{8}\left(-\frac{1}{2}\,A^3\,e^{C^2}\,\varepsilon\,\Xi + \frac{4}{3}\,A^3\,\delta\,\Theta^R + \frac{6\,i\,\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log^3(\Xi)} + \frac{\sqrt{2}\,\varepsilon\,S^A}{\log^3(S)} - \frac{4\,i\,S\,\varepsilon^A}{\log^3(\varepsilon)}\right) + \text{constant}$$

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

Thence, we obtain:

((1/8 (4/3 A^3 δ Θ^R - 1/2 A^3 e^(C^2) ε Ξ + (sqrt(2) S^A ε)/(log^3(S)) - (4 i S ε^A)/(log^3(ε)) + (6 i sqrt(2) e^(C^2) ε Ξ^A)/(log^3(Ξ))))) ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

Input

$$\left(\frac{1}{8}\left(\frac{4}{3}A^3\,\delta\,\Theta^R-\frac{1}{2}A^3\,e^{C^2}\,\varepsilon\,\Xi+\frac{\sqrt{2}\,S^A\,\varepsilon}{\log^3(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log^3(\varepsilon)}+\frac{6\,i\,\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log^3(\Xi)}\right)\right)$$
$$\sqrt[2\pi]{\left(\frac{5}{12}\left(3+\sqrt{5}\right)d^3\right)\left(\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3\right)^2}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result

$$2^{-3-2/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\sqrt[\pi]{\pi}\;\sqrt[2\pi]{d^9}$$
$$\left(-\frac{1}{2}A^3\,e^{C^2}\,\varepsilon\,\Xi+\frac{4}{3}A^3\,\delta\,\Theta^R+\frac{6\,i\,\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log^3(\Xi)}+\frac{\sqrt{2}\,\varepsilon\,S^A}{\log^3(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log^3(\varepsilon)}\right)$$

((1/8 (4/3 A^3 δ Θ^R - 1/2 A^3 e^(C^2) ε Ξ + (sqrt(2) S^A ε)/(log^3(S)) - (4 i S ε^A)/(log^3(ε)) + (6 i sqrt(2) e^(C^2) ε Ξ^A)/(log^3(Ξ))))) ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

Input

$$\left(\frac{1}{8}\left(\frac{4}{3}A^3\,\delta\,\Theta^R-\frac{1}{2}A^3\,e^{C^2}\,\varepsilon\,\Xi+\frac{\sqrt{2}\,S^A\,\varepsilon}{\log^3(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log^3(\varepsilon)}+\frac{6\,i\,\sqrt{2}\,e^{C^2}\,\varepsilon\,\Xi^A}{\log^3(\Xi)}\right)\right)$$
$$\sqrt[2\pi]{\left(\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3\right)\left(\frac{1}{3}\sqrt{2}\,d^3\right)\left(\frac{\sqrt{2}}{12}\,d^3\right)}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result

$$2^{-3-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}\left(-\frac{1}{2}A^3 e^{C^2}\varepsilon\,\Xi+\frac{4}{3}A^3\delta\,\Theta^R+\frac{6i\sqrt{2}\ e^{C^2}\varepsilon\,\Xi^A}{\log^3(\Xi)}+\frac{\sqrt{2}\ \varepsilon\,S^A}{\log^3(S)}-\frac{4i\,S\,\varepsilon^A}{\log^3(\varepsilon)}\right)$$

Expanded form

$$-2^{-4-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ A^3 e^{C^2}\sqrt[2\pi]{d^9}\ \varepsilon\,\Xi+2^{-1-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{\pi}\ A^3\sqrt[2\pi]{d^9}\ \delta\,\Theta^R+\frac{i\,2^{-3/2-1/\pi}\times 3^{1-3/(2\pi)}\sqrt[2\pi]{\pi}\ e^{C^2}\sqrt[2\pi]{d^9}\ \varepsilon\,\Xi^A}{\log^3(\Xi)}+\frac{2^{-5/2-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}\ \varepsilon\,S^A}{\log^3(S)}-\frac{i\,2^{-1-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}\ S\,\varepsilon^A}{\log^3(\varepsilon)}$$

Alternate form assuming A, C, d, R, S, δ, ε, Θ, and Ξ are positive

$$\left(2^{-9/2-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{\pi}\ d^{9/(2\pi)}\left(12\,\varepsilon\,S^A\log^3(\varepsilon)\log^3(\Xi)+\log^3(S)\left(\log^3(\varepsilon)\left(\sqrt{2}\ A^3\log^3(\Xi)\left(8\,\delta\,\Theta^R-3\,e^{C^2}\varepsilon\,\Xi\right)+72\,i\,e^{C^2}\varepsilon\,\Xi^A\right)-24\,i\sqrt{2}\ S\,\varepsilon^A\log^3(\Xi)\right)\right)\right)\Big/\left(\log^3(\varepsilon)\log^3(\Xi)\log^3(S)\right)$$

Alternate forms

$$\left(2^{-4-1/\pi}\times 3^{-1-3/(2\pi)}\sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9}\left(-3\,A^3 e^{C^2}\varepsilon\,\Xi\log^3(\varepsilon)\log^3(\Xi)\log^3(S)+8\,A^3\delta\log^3(\varepsilon)\log^3(\Xi)\,\Theta^R\log^3(S)+36\,i\sqrt{2}\ e^{C^2}\varepsilon\,\Xi^A\log^3(\varepsilon)\log^3(S)+6\sqrt{2}\ \varepsilon\,S^A\log^3(\varepsilon)\log^3(\Xi)-24\,i\,S\,\varepsilon^A\log^3(\Xi)\log^3(S)\right)\right)\Big/\left(\log^3(\varepsilon)\log^3(\Xi)\log^3(S)\right)$$

$$\frac{2^{-5/2-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\;\varepsilon\,S^A}{\log^3(S)}+\frac{1}{\log^3(\varepsilon)\log^3(\Xi)}$$

$$2^{-4-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\,\sqrt[2\pi]{d^9}\left(-3\,A^3\,e^{C^2}\,\varepsilon\,\Xi\log^3(\varepsilon)\log^3(\Xi)+8\,A^3\,\delta\log^3(\varepsilon)\log^3(\Xi)\,\Theta^R+36\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\,\Xi^A\log^3(\varepsilon)-24\,i\,S\,\varepsilon^A\log^3(\Xi)\right)$$

$$\sqrt[2\pi]{d^9}\left(A^3\left(2^{-1-1/\pi}\times 3^{-1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;\delta\,\Theta^R-2^{-4-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;e^{C^2}\;\varepsilon\,\Xi\right)+\varepsilon\left(\frac{2^{-5/2-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;S^A}{\log^3(S)}+\frac{i\,2^{-3/2-1/\pi}\times 3^{1-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;e^{C^2}\;\Xi^A}{\log^3(\Xi)}\right)-\frac{i\,2^{-1-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{\pi}\;S\,\varepsilon^A}{\log^3(\varepsilon)}\right)$$

Derivative

$$\frac{\partial}{\partial A}\left(\frac{1}{8}\left(\frac{4}{3}\,A^3\,\delta\,\Theta^R-\frac{1}{2}\,A^3\,e^{C^2}\,\varepsilon\,\Xi+\frac{\sqrt{2}\;S^A\,\varepsilon}{\log^3(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log^3(\varepsilon)}+\frac{6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\,\Xi^A}{\log^3(\Xi)}\right)\sqrt[2\pi]{\frac{\left(4\,\pi\left(\frac{d}{2}\right)^3\right)\left(\sqrt{2}\;d^3\right)\left(\sqrt{2}\;d^3\right)}{3\times 3\times 12}}\right)=$$

$$2^{-3-1/\pi}\times 3^{-3/(2\pi)}\,\sqrt[2\pi]{d^9}\,\sqrt[2\pi]{\pi}\left(-\frac{3}{2}\,A^2\,e^{C^2}\,\varepsilon\,\Xi+4\,A^2\,\delta\,\Theta^R+\frac{6\,i\,\sqrt{2}\;e^{C^2}\,\varepsilon\,\Xi^A}{\log^2(\Xi)}+\frac{\sqrt{2}\;\varepsilon\,S^A}{\log^2(S)}-\frac{4\,i\,S\,\varepsilon^A}{\log^2(\varepsilon)}\right)$$

Dividing the two exact results

$$2^{-3-2/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\ \sqrt[2\pi]{d^9}$$

$$\left(-\frac{1}{2} A^3 e^{C^2} \varepsilon \Xi + \frac{4}{3} A^3 \delta \Theta^R + \frac{6 i \sqrt{2}\ e^{C^2} \varepsilon \Xi^A}{\log^3(\Xi)} + \frac{\sqrt{2}\ \varepsilon S^A}{\log^3(S)} - \frac{4 i S \varepsilon^A}{\log^3(\varepsilon)}\right)$$

and

$$2^{-3-1/\pi} \times 3^{-3/(2\pi)}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{d^9} \left(-\frac{1}{2} A^3 e^{C^2} \varepsilon \Xi + \frac{4}{3} A^3 \delta \Theta^R + \frac{6 i \sqrt{2}\ e^{C^2} \varepsilon \Xi^A}{\log^3(\Xi)} + \frac{\sqrt{2}\ \varepsilon S^A}{\log^3(S)} - \frac{4 i S \varepsilon^A}{\log^3(\varepsilon)}\right)$$

we obtain:

((2^(-3 - 2/π) 3^(-3/(2 π)) (5 (3 + sqrt(5)))^(1/(2 π)) π^(1/π)))/((2^(-3 - 1/π) 3^(-3/(2 π)) π^(1/(2 π))))

Input

$$\frac{2^{-3-2/\pi} \times 3^{-3/(2\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}}{2^{-3-1/\pi} \times 3^{-3/(2\pi)}\ \sqrt[2\pi]{\pi}}$$

Exact result

$$2^{-1/\pi} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\frac{2^{-3-2/\pi}\left(3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\right)}{2^{-3-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}}=2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{2^{-3-2/\pi}\left(3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\right)}{2^{-3-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}}=$$
$$2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{2^{-3-2/\pi}\left(3^{-3/(2\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \sqrt[\pi]{\pi}\right)}{2^{-3-1/\pi}\times 3^{-3/(2\pi)}\sqrt[2\pi]{\pi}}=$$
$$2^{-1/\pi}\sqrt[2\pi]{5}\ \sqrt[2\pi]{\pi}\ \sqrt[2\pi]{3+\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535884…. ≈ π (Ramanujan Recurring Number)

And again:

1/6((4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))))^2

Input interpretation

$$\frac{1}{6}\left(\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684685871684117952387042047633606941698820298888445046920
…

1.64493406684…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{8\,e^{1.92478440155986\,\pi}}{75\left(3+\sqrt{4}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.92478440155986\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Now, we analyze the following equation:

$$\mathbf{D}_\mu\Omega^{iA} = D_\mu\Omega^{iA} + \tfrac{i}{\sqrt{2}}\left(\mathbf{D}^{A\dot{A}}\lambda\right)\Psi_\mu{}^i{}_{\dot{A}} - \tfrac{i}{4\sqrt{2}}\left(\widehat{F}^A{}_B + 4\,\phi\,T^A{}_B\right)\Psi_\mu{}^{iB} - \tfrac{i}{4\sqrt{2}}D^i{}_j\Psi_\mu{}^{jA} - \tfrac{i}{2\sqrt{2}}[\lambda,\phi]\Psi_\mu{}^{iA} - \tfrac{i}{2}\lambda\,S_\mu{}^{iA}\ ,$$

From

$$D_\mu\Omega^{iA} + \tfrac{i}{\sqrt{2}}\left(\mathbf{D}^{A\dot{A}}\lambda\right)\Psi_\mu{}^i{}_{\dot{A}} - \tfrac{i}{4\sqrt{2}}\left(\widehat{F}^A{}_B + 4\,\phi\,T^A{}_B\right)\Psi_\mu{}^{iB} - \tfrac{i}{4\sqrt{2}}D^i{}_j\Psi_\mu{}^{jA} - \tfrac{i}{2\sqrt{2}}[\lambda,\phi]\Psi_\mu{}^{iA} - \tfrac{i}{2}\lambda\,S_\mu{}^{iA}\ ,$$

we obtain:

(D*Ω^(i*A)+(i/(√2))*(D^(A*A)*λ)*Ψ^i-(i/(4√2))*(F^A+4*φ*T^A)*Ψ^(i*B)-(i/(4√2))*D^i*Ψ^(j*A)-(i/(2√2))*λ*Ψ^(i*A)-i/2*λ*S^(i*A))

Input

$$D\,\Omega^{i\,A}+\frac{i}{\sqrt{2}}\left(D^{A\,A}\,\lambda\right)\Psi^{i}-\frac{i}{4\,\sqrt{2}}\left(F^{A}+4\,\phi\,T^{A}\right)\Psi^{i\,B}-\frac{i}{4\,\sqrt{2}}\,D^{i}\,\Psi^{j\,A}-\frac{i}{2\,\sqrt{2}}\,\lambda\,\Psi^{i\,A}-\frac{i}{2}\,\lambda\,S^{i\,A}$$

i is the imaginary unit

ϕ is the golden ratio

Exact result

$$\frac{i\,\lambda\,\Psi^{i}\,D^{A^2}}{\sqrt{2}}-\frac{i\,\Psi^{i\,B}\left(F^{A}+4\,\phi\,T^{A}\right)}{4\,\sqrt{2}}-\frac{i\,D^{i}\,\Psi^{A\,j}}{4\,\sqrt{2}}+D\,\Omega^{i\,A}-\frac{i\,\lambda\,\Psi^{i\,A}}{2\,\sqrt{2}}-\frac{1}{2}\,i\,\lambda\,S^{i\,A}$$

Alternate forms

$$D\,\Omega^{i\,A}-\frac{1}{8}\,i\left(\sqrt{2}\,\left(-4\,\lambda\,\Psi^{i}\,D^{A^2}+F^{A}\,\Psi^{i\,B}+D^{i}\,\Psi^{A\,j}+2\,\lambda\,\Psi^{i\,A}\right)+4\,\sqrt{3+\sqrt{5}}\;T^{A}\,\Psi^{i\,B}+4\,\lambda\,S^{i\,A}\right)$$

$$\frac{1}{4\,\sqrt{2}}\,i\left(4\,\lambda\,\Psi^{i}\,D^{A^2}-F^{A}\,\Psi^{i\,B}-2\left(2\,\phi\,T^{A}\,\Psi^{i\,B}+\lambda\,\Psi^{i\,A}+\sqrt{2}\;\lambda\,S^{i\,A}\right)-D^{i}\,\Psi^{A\,j}-4\,i\,\sqrt{2}\;D\,\Omega^{i\,A}\right)$$

$$\frac{1}{8}\left(4\,i\,\sqrt{2}\;\lambda\,\Psi^{i}\,D^{A^2}-i\,\sqrt{2}\,\left(\Psi^{i\,B}\left(F^{A}+2\left(1+\sqrt{5}\right)T^{A}\right)+2\,\lambda\,\Psi^{i\,A}\right)-i\,\sqrt{2}\;D^{i}\,\Psi^{A\,j}+8\,D\,\Omega^{i\,A}-4\,i\,\lambda\,S^{i\,A}\right)$$

Expanded form

$$\frac{i\,\lambda\,\Psi^{i}\,D^{A^2}}{\sqrt{2}}-\frac{i\,F^{A}\,\Psi^{i\,B}}{4\,\sqrt{2}}-\frac{1}{2}\,i\,\sqrt{\frac{5}{2}}\;T^{A}\,\Psi^{i\,B}-\frac{i\,T^{A}\,\Psi^{i\,B}}{2\,\sqrt{2}}-\frac{i\,D^{i}\,\Psi^{A\,j}}{4\,\sqrt{2}}+D\,\Omega^{i\,A}-\frac{i\,\lambda\,\Psi^{i\,A}}{2\,\sqrt{2}}-\frac{1}{2}\,i\,\lambda\,S^{i\,A}$$

Derivative

$$\frac{\partial}{\partial A}\left(D\,\Omega^{i\,A}+\frac{i\left(D^{A\,A}\,\lambda\right)\Psi^{i}}{\sqrt{2}}-\frac{i\left(F^{A}+4\,\phi\,T^{A}\right)\Psi^{i\,B}}{4\sqrt{2}}-\frac{i\,D^{i}\,\Psi^{j\,A}}{4\sqrt{2}}-\frac{i\,\lambda\,\Psi^{i\,A}}{2\sqrt{2}}-\frac{1}{2}\,i\,\lambda\,S^{i\,A}\right)=$$
$$\frac{1}{8}\left(8\,i\,\sqrt{2}\;A\,\lambda\,\Psi^{i}\,D^{A^2}\log(D)-\right.$$
$$i\,\sqrt{2}\;\Psi^{i\,B}\left(F^{A}\log(F)+2\left(1+\sqrt{5}\right)T^{A}\log(T)\right)-i\,\sqrt{2}\;D^{i}\,j\log(\Psi)\,\Psi^{A\,j}+$$
$$\left.8\,i\,D\,\Omega^{i\,A}\log(\Omega)+2\,\sqrt{2}\;\lambda\,\Psi^{i\,A}\log(\Psi)+4\,\lambda\,S^{i\,A}\log(S)\right)$$

$\log(x)$ is the natural logarithm

Indefinite integral

$$\int\left(-\frac{1}{2}\,i\,S^{i\,A}\,\lambda+\frac{i\,D^{A^2}\,\lambda\,\Psi^{i}}{\sqrt{2}}-\frac{i\,\lambda\,\Psi^{i\,A}}{2\sqrt{2}}-\frac{i\left(F^{A}+4\,\phi\,T^{A}\right)\Psi^{i\,B}}{4\sqrt{2}}-\frac{i\,D^{i}\,\Psi^{A\,j}}{4\sqrt{2}}+D\,\Omega^{i\,A}\right)$$
$$dA=\frac{1}{8}\left(-\frac{i\,\sqrt{2}\;F^{A}\,\Psi^{i\,B}}{\log(F)}-\frac{2\,i\,\sqrt{2}\,\left(1+\sqrt{5}\right)T^{A}\,\Psi^{i\,B}}{\log(T)}-\right.$$
$$\frac{i\,\sqrt{2}\;D^{i}\,\Psi^{A\,j}}{j\log(\Psi)}+\frac{2\,i\,\sqrt{2\,\pi}\;\lambda\,\Psi^{i}\,\operatorname{erfi}\left(A\sqrt{\log(D)}\right)}{\sqrt{\log(D)}}-$$
$$\left.\frac{8\,i\,D\,\Omega^{i\,A}}{\log(\Omega)}-\frac{2\,\sqrt{2}\;\lambda\,\Psi^{i\,A}}{\log(\Psi)}-\frac{4\,\lambda\,S^{i\,A}}{\log(S)}\right)+\text{constant}$$

$\operatorname{erfi}(x)$ is the imaginary error function

From the expanded form

$$\frac{i\,\lambda\,\Psi^{i}\,D^{A^2}}{\sqrt{2}}-\frac{i\,F^{A}\,\Psi^{i\,B}}{4\sqrt{2}}-\frac{1}{2}\,i\,\sqrt{\frac{5}{2}}\;T^{A}\,\Psi^{i\,B}-$$
$$\frac{i\,T^{A}\,\Psi^{i\,B}}{2\sqrt{2}}-\frac{i\,D^{i}\,\Psi^{A\,j}}{4\sqrt{2}}+D\,\Omega^{i\,A}-\frac{i\,\lambda\,\Psi^{i\,A}}{2\sqrt{2}}-\frac{1}{2}\,i\,\lambda\,S^{i\,A}$$

we obtain:

Input

$$\frac{i\,\lambda\,\Psi^{i}\,D^{A^2}}{\sqrt{2}}-\frac{i\,F^{A}\,\Psi^{i\,B}}{4\sqrt{2}}-\frac{1}{2}\,i\,\sqrt{\frac{5}{2}}\;T^{A}\,\Psi^{i\,B}-$$
$$\frac{i\,T^{A}\,\Psi^{i\,B}}{2\sqrt{2}}-\frac{i\,D^{i}\,\Psi^{A\,j}}{4\sqrt{2}}+D\,\Omega^{i\,A}-\frac{i\,\lambda\,\Psi^{i\,A}}{2\sqrt{2}}-\frac{1}{2}\,i\,\lambda\,S^{i\,A}$$

i is the imaginary unit

Alternate forms

$$D\,\Omega^{i\,A} - \frac{1}{8}\,i\left(\sqrt{2}\,\left(-4\,\lambda\,\Psi^{i}\,D^{A^2} + F^{A}\,\Psi^{i\,B} + D^{i}\,\Psi^{A\,j} + 2\,\lambda\,\Psi^{i\,A}\right) + 4\,\sqrt{3+\sqrt{5}}\;T^{A}\right.$$
$$\left.\Psi^{i\,B} + 4\,\lambda\,S^{i\,A}\right)$$

$$\frac{1}{8}\left(4\,i\,\sqrt{2}\;\lambda\,\Psi^{i}\,D^{A^2} - i\,\sqrt{2}\,\left(\Psi^{i\,B}\left(F^{A} + 2\left(1+\sqrt{5}\right)T^{A}\right) + 2\,\lambda\,\Psi^{i\,A}\right) - i\,\sqrt{2}\;D^{i}\,\Psi^{A\,j} + 8\right.$$
$$\left.D\,\Omega^{i\,A} - 4\,i\,\lambda\,S^{i\,A}\right)$$

$$\frac{1}{4\,\sqrt{2}}\,i\left(4\,\lambda\,\Psi^{i}\,D^{A^2} - F^{A}\,\Psi^{i\,B} - 2\left(\left(1+\sqrt{5}\right)T^{A}\,\Psi^{i\,B} + \lambda\,\Psi^{i\,A} + \sqrt{2}\;\lambda\,S^{i\,A}\right) - D^{i}\,\Psi^{A\,j} - \right.$$
$$\left.4\,i\,\sqrt{2}\;D\,\Omega^{i\,A}\right)$$

Series expansion at A=0

$$\frac{1}{8}\left(-i\,\sqrt{2}\,\left(2\,\lambda + \left(3+2\,\sqrt{5}\right)\Psi^{i\,B}\right) - i\,\sqrt{2}\;D^{i} + 8\,D + 4\,i\,\sqrt{2}\;\lambda\,\Psi^{i} - 4\,i\,\lambda\right) +$$
$$\frac{1}{8}\,A\left(-i\,\sqrt{2}\,\left(\Psi^{i\,B}\left(\log(F) + 2\left(1+\sqrt{5}\right)\log(T)\right) + 2\,i\,\lambda\,\log(\Psi)\right) - \right.$$
$$\left.i\,\sqrt{2}\;D^{i}\;j\log(\Psi) + 8\,i\,D\log(\Omega) + 4\,\lambda\log(S)\right) +$$
$$\frac{1}{8}\,A^2\left(-i\,\sqrt{2}\,\left(-\lambda\log^2(\Psi) + \Psi^{i\,B}\left(\frac{\log^2(F)}{2} + \left(1+\sqrt{5}\right)\log^2(T)\right)\right) - \right.$$
$$\left.\frac{i\,D^{i}\;j^2\log^2(\Psi)}{\sqrt{2}} + 4\,i\,\sqrt{2}\;\lambda\,\Psi^{i}\log(D) - 4\,D\log^2(\Omega) + 2\,i\,\lambda\log^2(S)\right) +$$
$$\frac{1}{8}\,A^3\left(-i\,\sqrt{2}\,\left(\Psi^{i\,B}\left(\frac{\log^3(F)}{6} + \frac{1}{3}\left(1+\sqrt{5}\right)\log^3(T)\right) - \frac{1}{3}\,i\,\lambda\log^3(\Psi)\right) - \right.$$
$$\left.\frac{i\,D^{i}\;j^3\log^3(\Psi)}{3\,\sqrt{2}} - \frac{4}{3}\,i\,D\log^3(\Omega) - \frac{2}{3}\,\lambda\log^3(S)\right) +$$
$$\frac{1}{8}\,A^4\left(-i\,\sqrt{2}\,\left(\frac{1}{12}\,\lambda\log^4(\Psi) + \Psi^{i\,B}\left(\frac{\log^4(F)}{24} + \frac{1}{12}\left(1+\sqrt{5}\right)\log^4(T)\right)\right) - \right.$$
$$\left.\frac{i\,D^{i}\;j^4\log^4(\Psi)}{12\,\sqrt{2}} + 2\,i\,\sqrt{2}\;\lambda\,\Psi^{i}\log^2(D) + \frac{1}{3}\,D\log^4(\Omega) - \frac{1}{6}\,i\,\lambda\log^4(S)\right) + O\left(A^5\right)$$

(Taylor series)

$\log(x)$ is the natural logarithm

Derivative

$$\frac{\partial}{\partial A}\left(\frac{i\,\lambda\,\Psi^{i}\,D^{A^2}}{\sqrt{2}}-\frac{i\,F^{A}\,\Psi^{i\,B}}{4\sqrt{2}}-\frac{1}{2}\,i\,\sqrt{\frac{5}{2}}\;T^{A}\,\Psi^{i\,B}-\frac{i\,T^{A}\,\Psi^{i\,B}}{2\sqrt{2}}-\frac{i\,D^{i}\,\Psi^{A\,j}}{4\sqrt{2}}+D\,\Omega^{i\,A}-\frac{i\,\lambda\,\Psi^{i\,A}}{2\sqrt{2}}-\frac{1}{2}\,i\,\lambda\,S^{i\,A}\right)=$$
$$\frac{1}{8}\left(8\,i\,\sqrt{2}\;A\,\lambda\,\Psi^{i}\,D^{A^2}\log(D)-i\,\sqrt{2}\;F^{A}\,\Psi^{i\,B}\log(F)-2\,i\,\sqrt{10}\;T^{A}\,\Psi^{i\,B}\log(T)-2\,i\,\sqrt{2}\;T^{A}\,\Psi^{i\,B}\log(T)-i\,\sqrt{2}\;D^{i}\,j\log(\Psi)\,\Psi^{A\,j}+8\,i\,D\,\Omega^{i\,A}\log(\Omega)+2\,\sqrt{2}\;\lambda\,\Psi^{i\,A}\log(\Psi)+4\,\lambda\,S^{i\,A}\log(S)\right)$$

Indefinite integral

$$\int\left(\frac{i\,\lambda\,\Psi^{i}\,D^{A^2}}{\sqrt{2}}-\frac{i\,F^{A}\,\Psi^{i\,B}}{4\sqrt{2}}-\frac{1}{2}\,i\,\sqrt{\frac{5}{2}}\;T^{A}\,\Psi^{i\,B}-\frac{i\,T^{A}\,\Psi^{i\,B}}{2\sqrt{2}}-\frac{i\,D^{i}\,\Psi^{A\,j}}{4\sqrt{2}}+D\,\Omega^{i\,A}-\frac{i\,\lambda\,\Psi^{i\,A}}{2\sqrt{2}}-\frac{1}{2}\,i\,\lambda\,S^{i\,A}\right)dA=$$
$$\frac{1}{8}\left(-\frac{i\,\sqrt{2}\;F^{A}\,\Psi^{i\,B}}{\log(F)}-\frac{2\,i\,\sqrt{2}\,\left(1+\sqrt{5}\right)T^{A}\,\Psi^{i\,B}}{\log(T)}-\frac{i\,\sqrt{2}\;D^{i}\,\Psi^{A\,j}}{j\log(\Psi)}+\frac{2\,i\,\sqrt{2\,\pi}\;\lambda\,\Psi^{i}\,\operatorname{erfi}\left(A\,\sqrt{\log(D)}\right)}{\sqrt{\log(D)}}-\frac{8\,i\,D\,\Omega^{i\,A}}{\log(\Omega)}-\frac{2\,\sqrt{2}\;\lambda\,\Psi^{i\,A}}{\log(\Psi)}-\frac{4\,\lambda\,S^{i\,A}}{\log(S)}\right)+\text{constant}$$

$\operatorname{erfi}(x)$ is the imaginary error function

From the indefinite integral result

$$\int\left(\frac{i\,\lambda\,\Psi^{i}\,D^{A^2}}{\sqrt{2}}-\frac{i\,F^{A}\,\Psi^{i\,B}}{4\sqrt{2}}-\frac{1}{2}\,i\sqrt{\frac{5}{2}}\;T^{A}\,\Psi^{i\,B}-\frac{i\,T^{A}\,\Psi^{i\,B}}{2\sqrt{2}}-\frac{i\,D^{i}\,\Psi^{A\,j}}{4\sqrt{2}}+D\,\Omega^{i\,A}-\frac{i\,\lambda\,\Psi^{i\,A}}{2\sqrt{2}}-\frac{1}{2}\,i\,\lambda\,S^{i\,A}\right)dA=$$
$$\frac{1}{8}\left(-\frac{i\sqrt{2}\;F^{A}\,\Psi^{i\,B}}{\log(F)}-\frac{2\,i\sqrt{2}\,\left(1+\sqrt{5}\right)T^{A}\,\Psi^{i\,B}}{\log(T)}-\frac{i\sqrt{2}\;D^{i}\,\Psi^{A\,j}}{j\log(\Psi)}+\frac{2\,i\sqrt{2\,\pi}\;\lambda\,\Psi^{i}\,\operatorname{erfi}\left(A\sqrt{\log(D)}\right)}{\sqrt{\log(D)}}-\frac{8\,i\,D\,\Omega^{i\,A}}{\log(\Omega)}-\frac{2\sqrt{2}\;\lambda\,\Psi^{i\,A}}{\log(\Psi)}-\frac{4\,\lambda\,S^{i\,A}}{\log(S)}\right)+\text{constant}$$

for j = 1, A = λ = 2, B = F = 4, Ψ = Ω = 8 and D = S = 16 , T = 32:

1/8(-(i√(2)16 8^(i 4))/ln(4)-(2i√(2)(1+√(5))32^2 8^(4i))/ln(32)-(i√(2) 16^i 8^(2))/(1ln(8))+(2i√(2π)2 8^i erfi(2√(ln(16))))/√(ln(16))-(8 i 16 8^(2i))/ln(8)-(2√(2)2 8^(2i))/ln(8)-(4*2 16^(2i))/ln(16))

Input

$$\frac{1}{8}\left(-\frac{i\sqrt{2}\times16\times8^{i\times4}}{\log(4)}-\frac{2\,i\sqrt{2}\,\left(1+\sqrt{5}\right)\times32^{2}\times8^{4\,i}}{\log(32)}-\frac{\left(i\sqrt{2}\right)\times16^{i}\times8^{2}}{1\log(8)}+\frac{\left(2\,i\sqrt{2\,\pi}\times2\right)\times8^{i}\,\operatorname{erfi}\left(2\sqrt{\log(16)}\right)}{\sqrt{\log(16)}}-\frac{8\,i\times16\times8^{2\,i}}{\log(8)}-\frac{2\sqrt{2}\times2\times8^{2\,i}}{\log(8)}-\frac{4\times2\times16^{2\,i}}{\log(16)}\right)$$

$\log(x)$ is the natural logarithm

$\operatorname{erfi}(x)$ is the imaginary error function

i is the imaginary unit

Exact result

$$\frac{1}{8}\left(i\,2^{5/2+3i}\sqrt{\frac{\pi}{\log(16)}}\,\operatorname{erfi}\left(2\sqrt{\log(16)}\right)-\frac{i\,2^{9/2+12i}}{\log(4)}-\frac{2^{5/2+6i}}{\log(8)}-\frac{i\,2^{13/2+4i}}{\log(8)}-\frac{i\,2^{7+6i}}{\log(8)}-\frac{2^{3+8i}}{\log(16)}-\frac{i\,2^{23/2+12i}\left(1+\sqrt{5}\right)}{\log(32)}\right)$$

Decimal approximation

$$-7389.265150066663364000422541504459760549698784931239014327998 83\ldots$$
$$-$$
$$4125.323022312409993337650869542332480760491892633872804238778 31\ldots\,i$$

(using the principal branch of the logarithm for complex exponentiation)

Alternate complex forms

$$-7389.3-4125.3\,i$$

$$\frac{1}{8}\sqrt{\left(\left(\frac{1}{\log(8)\log(16)}4\left(-\log(8)\sqrt{\pi\log(256)}\,\operatorname{erfi}\left(2\sqrt{\log(16)}\right)\cos(\log(8))+\sqrt{2}\,\log(16)\sin(\log(64))+\log(64)\sin(\log(256))+16\sqrt{2}\,\log(16)\cos(\log(16))+32\log(16)\cos(\log(64))\right)+\frac{16\sqrt{2}\left(128\left(1+\sqrt{5}\right)\log(4)+\log(32)\right)\cos(\log(4096))}{\log(4)\log(32)}\right)^2+16\left(\sqrt{\frac{2\pi}{\log(16)}}\,\operatorname{erfi}\left(2\sqrt{\log(16)}\right)\sin(\log(8))-\frac{16\sqrt{2}\,\sin(\log(16))}{\log(8)}-\frac{32\sin(\log(64))}{\log(8)}-\frac{4\sqrt{2}\,\sin(\log(4096))}{\log(4)}-\frac{512\sqrt{2}\left(1+\sqrt{5}\right)\sin(\log(4096))}{\log(32)}+\frac{\sqrt{2}\,\cos(\log(64))}{\log(8)}+\frac{2\cos(\log(256))}{\log(16)}\right)^2\right)}$$

$$(\cos(-2.63241)+i\sin(-2.63241))$$

Polar coordinates

$$r = \frac{1}{8}\sqrt{\left(\left(\frac{1}{\log(8)\log(16)}4\left(-\log(8)\sqrt{\pi\log(256)}\ \operatorname{erfi}\left(2\sqrt{\log(16)}\right)\cos(\log(8)) + \sqrt{2}\ \log(16)\sin(\log(64)) + \log(64)\sin(\log(256)) + 16\sqrt{2}\ \log(16)\cos(\log(16)) + 32\log(16)\cos(\log(64))\right) + \frac{16\sqrt{2}\left(128\left(1+\sqrt{5}\right)\log(4)+\log(32)\right)\cos(\log(4096))}{\log(4)\log(32)}\right)^2 + 16\left(\sqrt{\frac{2\pi}{\log(16)}}\ \operatorname{erfi}\left(2\sqrt{\log(16)}\right)\sin(\log(8)) - \frac{16\sqrt{2}\ \sin(\log(16))}{\log(8)} - \frac{32\sin(\log(64))}{\log(8)} - \frac{4\sqrt{2}\ \sin(\log(4096))}{\log(4)} - \frac{512\sqrt{2}\left(1+\sqrt{5}\right)\sin(\log(4096))}{\log(32)} + \frac{\sqrt{2}\ \cos(\log(64))}{\log(8)} + \frac{2\cos(\log(256))}{\log(16)}\right)^2\right)}$$

(radius), $\theta = -2.63241$ (angle)

$r \approx 8462.8$ (radius), $\theta = -2.63241$ (angle)

8462.8

From which:

1/40((-(i√(2)16 8^(4i))/ln(4)-(2i√(2)(1+√(5))32^2 8^(4i))/ln(32)-(i√(2) 16^i 64)/(1ln(8))+(2i√(2π)2 8^i erfi(2√(ln(16))))/√(ln(16))-(8i 16 8^(2i))/ln(8)-(2√(2)2 8^(2i))/ln(8)-(8 16^(2i))/ln(16)))-41-Φ

Input

$$\frac{1}{40}\left(-\frac{i\sqrt{2}\times 16\times 8^{4i}}{\log(4)}-\frac{2i\sqrt{2}\left(1+\sqrt{5}\right)\times 32^2\times 8^{4i}}{\log(32)}-\frac{\left(i\sqrt{2}\right)\times 16^i\times 64}{1\log(8)}+\frac{\left(2i\sqrt{2\pi}\times 2\right)\times 8^i\,\text{erfi}\left(2\sqrt{\log(16)}\right)}{\sqrt{\log(16)}}-\frac{(8i)\times 16\times 8^{2i}}{\log(8)}-\frac{2\sqrt{2}\times 2\times 8^{2i}}{\log(8)}-\frac{8\times 16^{2i}}{\log(16)}\right)-41-\Phi$$

$\log(x)$ is the natural logarithm

$\text{erfi}(x)$ is the imaginary error function

i is the imaginary unit

Φ is the golden ratio conjugate

Exact result

$$\frac{1}{40}\left(i\,2^{5/2+3i}\sqrt{\frac{\pi}{\log(16)}}\,\text{erfi}\left(2\sqrt{\log(16)}\right)-\frac{i\,2^{9/2+12i}}{\log(4)}-\frac{2^{5/2+6i}}{\log(8)}-\frac{i\,2^{13/2+4i}}{\log(8)}-\frac{i\,2^{7+6i}}{\log(8)}-\frac{2^{3+8i}}{\log(16)}-\frac{i\,2^{23/2+12i}\left(1+\sqrt{5}\right)}{\log(32)}\right)-\Phi-41$$

Decimal approximation

$$-1519.47106400208256764828909513525759022766006616605356572773521\ldots$$
$$-$$
$$825.06460446248199866753017390846649615209837852677456084775566 2\ldots\,i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates

$r = 1729.023977819987541236340705735024565673737099289504215104308 69$ (radius), $\theta = -2.64414376264584625402287612535307256679434088980585584801910492 2$ (angle)

1729.023977819987…

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Fig.1

We will hypothesize that the solutions of the cubic equations and some solutions of various equations of Twisted Superconformal Gravity, can be used as boundary conditions to further the understanding of the formation of universe bubbles in eternal inflation. Furthermore, an octahedron could serve as a mathematical or conceptual model of the universe in the pre-Big Bang phase. Cubic equations, in this context, could represent the geometric characteristics of the quantum vacuum, while the spherical surface could be used to describe the geometry of the universe bubble.

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

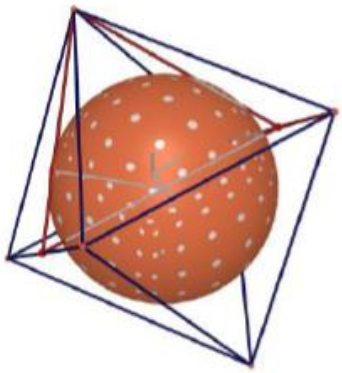


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

Indeed:

From:

https://www.academia.edu/75787512/The_Theory_of_String_A_Candidate_for_a_Generalized_Unification_Model

The Einstein's field equation and the String Theory.

The Einstein's field equation which includes the cosmological constant is:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (8)$$

where $R_{\mu\nu}$ is the Ricci tensor, R its trace, λ the cosmological constant, $g_{\mu\nu}$ the metric tensor of the space geometry, G the Newton's gravitational constant and $T_{\mu\nu}$ the tensor representing the properties of energy, matter and momentum.

The left hand-side of (8) represents the gravitational field and, consequently, the warped space-time, while the right hand-side represents the matter, i.e. the sources of the gravitational field.

In string theory the gravity is related to the gravitons which are bosons, whereas the matter is related to fermions. It follows that the left and right hand of (8) may be respectively related to the action of bosonic and of superstrings.

The actions of bosonic string and superstring (also containing fermions) are connected by the Palumbo-Nardelli relation (Palumbo et al. 2005):

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\varphi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right]=$$

$$=\int_0^\infty \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|\widetilde{H}_3|^2-\frac{\kappa_{10}^2}{g_{10}^2}Tr_\nu\left(|F_2|^2\right)\right]$$

The sign minus in the above equation comes from the inversion of any relationship.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^{\,8}\right)^{-k}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^{\,8}\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4(-1)^k\, 1195^{-1-2k}\left(5^{1+2k} - 4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1 \sqrt{1-t^2}\, dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

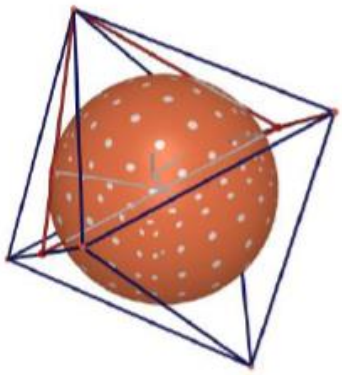
$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard to a sphere inscribed in an octahedron



we have the following formulas:

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\,2}{\pi} \quad (\text{for } l \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\,\pi)\rfloor)}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we have the following convex regular polyhedrons volumes:

$\frac{\sqrt{2}}{12}d^3$ Tetrahedron Volume

$\frac{\sqrt{2}}{3}d^3$ Octahedron Volume

$\frac{5}{12}(3+\sqrt{5})d^3$ Icosahedron Volume

Regarding the Sphere the volume is

$$V = \frac{4}{3}\pi r^3$$

With regard the sphere inscribed in a Tetrahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_T}{V_S} = \frac{\frac{\sqrt{2}}{12}d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(((√2 d^3)/12))*1/(4/3*π*(d/2)^3)

Input

$$\left(\frac{1}{12}\left(\sqrt{2}\ d^3\right)\right) \times \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.225079079039276517388799797751685145666143537488805156392859367 8

…

0.225079079….

Property

$\frac{1}{\sqrt{2}\ \pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Octahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_O}{V_S} = \frac{\frac{\sqrt{2}}{3}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

((1/3*√2*d^3)/(4/3*π*(d/2)^3))

Input

$$\frac{\frac{1}{3}\sqrt{2}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\ 2}{\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Icosahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_I}{V_S} = \frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3)

Input

$$\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{5\left(3+\sqrt{5}\right)}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

4.166730504921372682725163141597223093105782245731637719697502271 9

…

4.166730504921….

Property

$\frac{5\left(3+\sqrt{5}\right)}{2\pi}$ is a transcendental number

Expanded form

$$\frac{15}{2\pi} + \frac{5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

Alternate forms

$$\frac{15+5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

$$\frac{\frac{15}{2}+\frac{5\sqrt{5}}{2}}{\pi} \quad (\text{for } d \neq 0)$$

Series representations

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\pi\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\ a^3\right)\left(\sqrt{2}\ d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

Indeed, we obtain:

$$\frac{\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)\left(d^3\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2\right)}}{\sqrt[2\pi]{\frac{4}{3}\left(\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\right)\sqrt{2}\left(d^3 \times \frac{\sqrt{2}}{12}\right)d^3}}$$

= 1.6180085459001 07…

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4 \times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358487162422193250985871527893102518924557617007810061 3

…

3.1415926535…. = π (Ramanujan Recurring Number)

In conclusion, from the following further formula concerning the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron volumes and their ratios (and/or the inverses) with the sphere volume, raised the expression

$$(2\log(2)log^{2/3}(3^2))/(3e^{7/3})$$

$$\left(\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\frac{\frac{1}{4}\left(\left(15+7\sqrt{5}\right)d^3\right)}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\right)^{(2\log(2)\log^{2/3}(3^2))/(3e^{7/3})}$$

i.e.

$$\left(\frac{3}{2}\right)^{(4\log(2)\log^{2/3}(9))/(3e^{7/3})}\left(\frac{5\left(3+\sqrt{5}\right)\left(15+7\sqrt{5}\right)}{\pi}\right)^{(2\log(2)\log^{2/3}(9))/(3e^{7/3})}$$

we obtain:

1.6180381711519139857456817084743134673903021614989370712917722071

…

1.6180381711…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

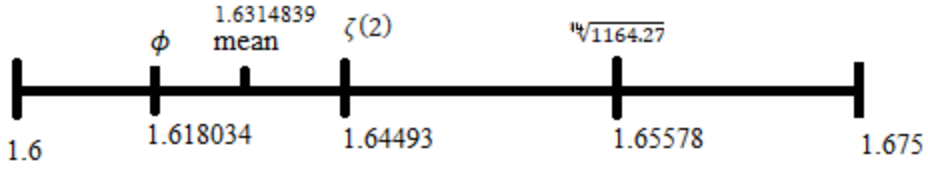
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

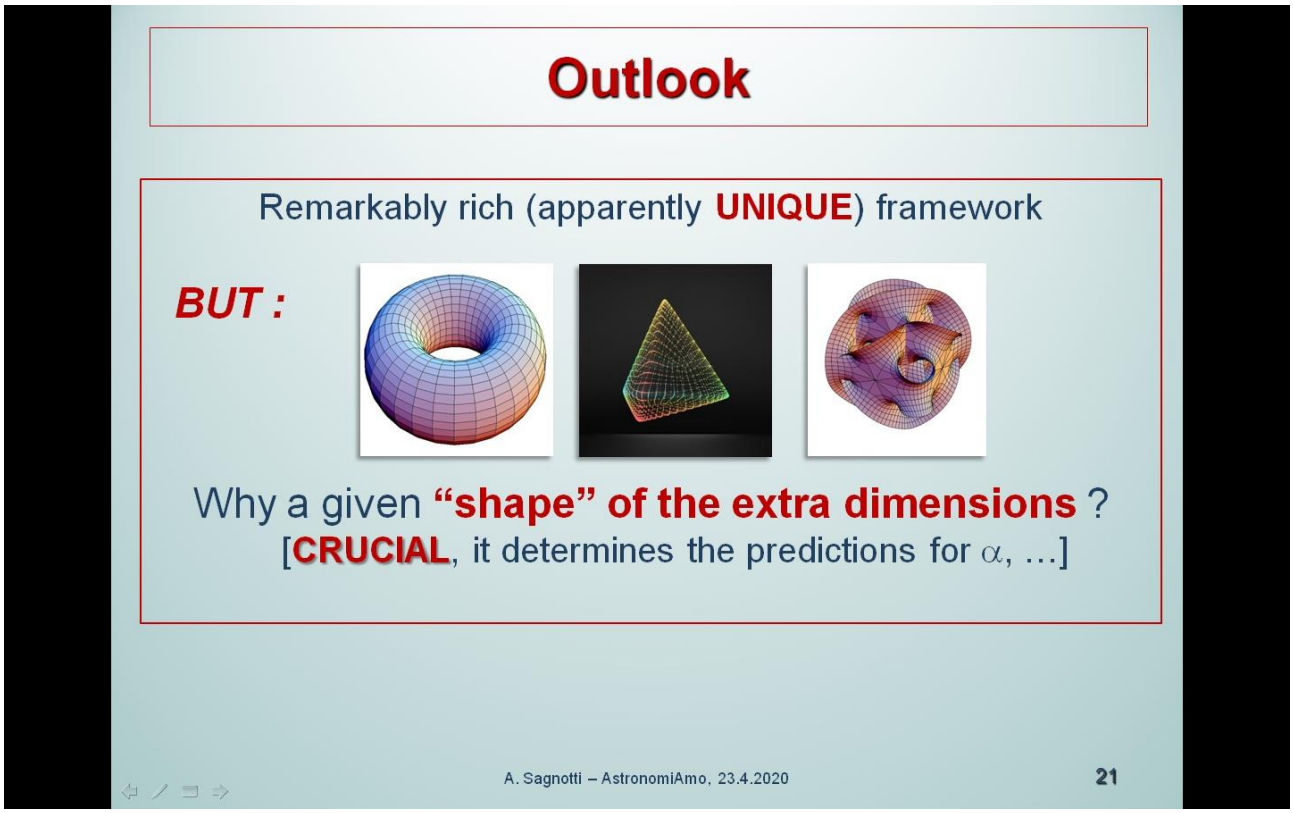
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix


Outlook
Remarkably rich (apparently UNIQUE) framework
BUT :
Why a given "shape" of the extra dimensions ?
[CRUCIAL, it determines the predictions for α, …]
A. Sagnotti – AstronomiAmo, 23.4.2020
21

From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

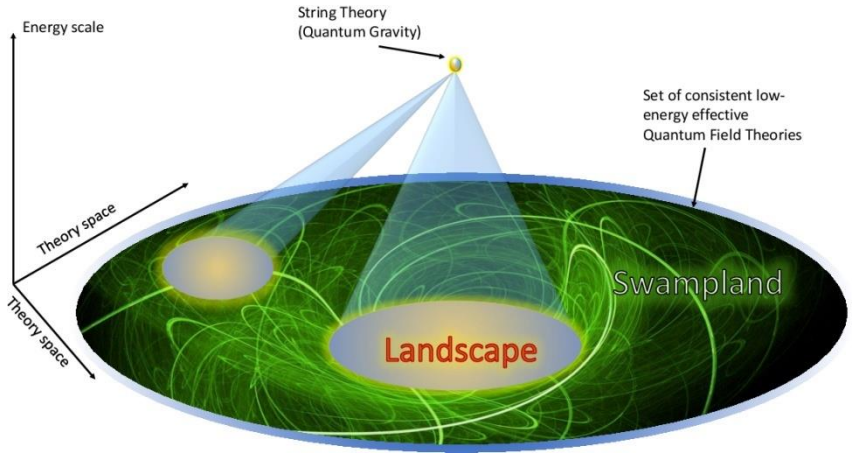


Fig. 1

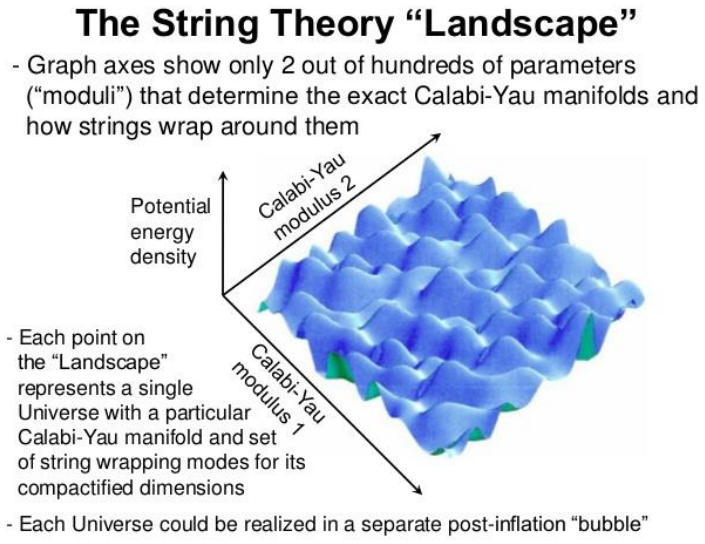

The String Theory "Landscape"
- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them
Potential energy density
Calabi-Yau modulus 2
Calabi-Yau modulus 1
- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

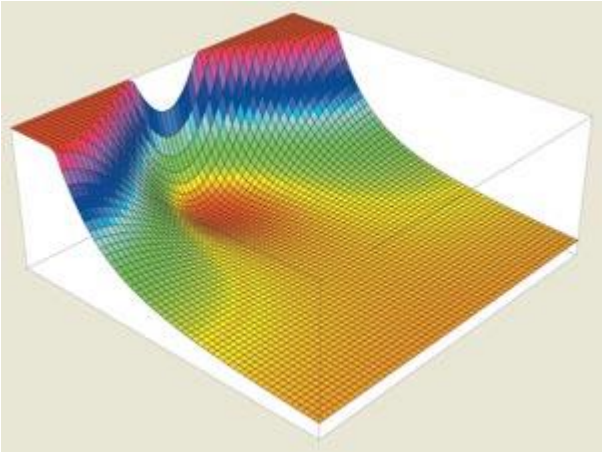


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

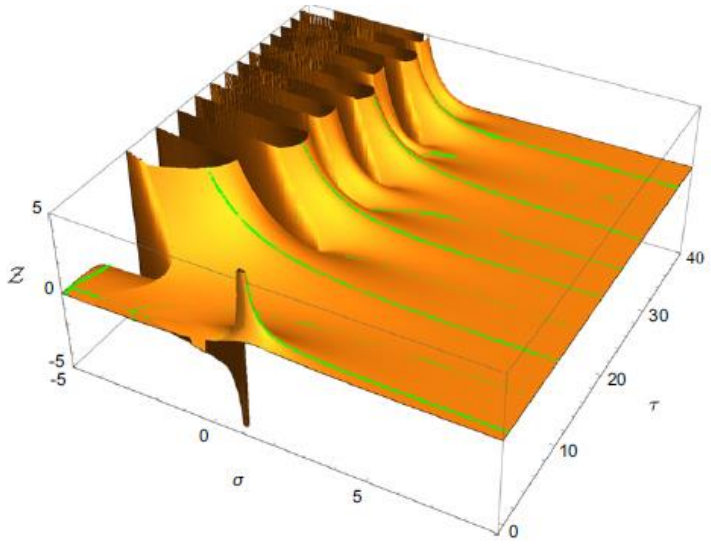


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

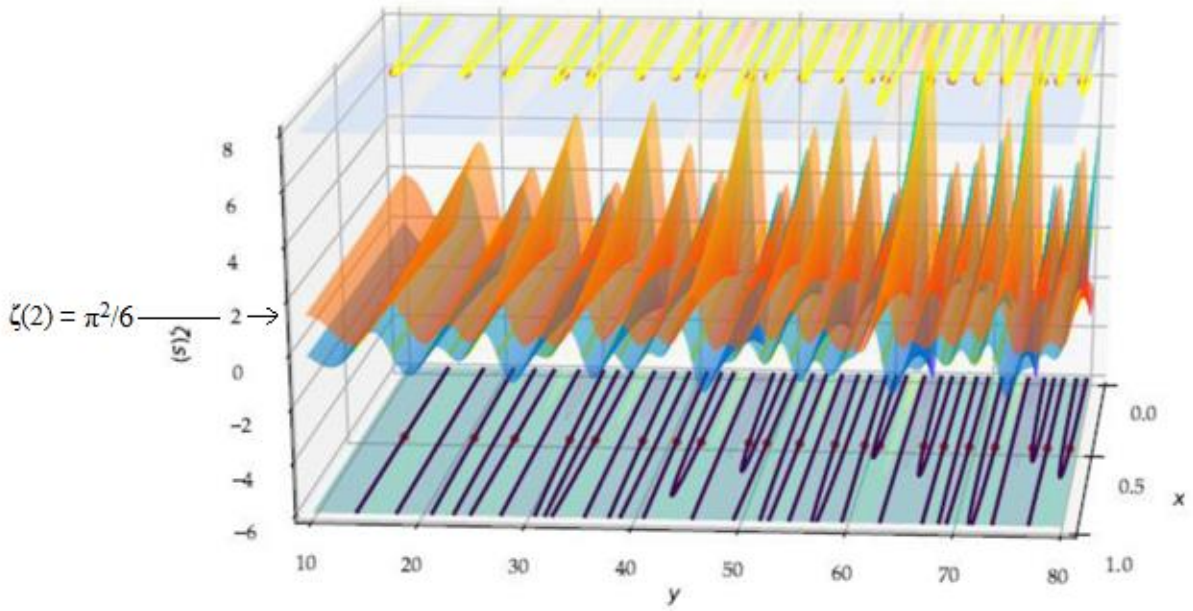


Figure 1. $C(x,y)$ and $S(x,y)$ surfaces of the Riemann $\zeta(x,y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

3D plot | $\zeta(2 + i\,t)$

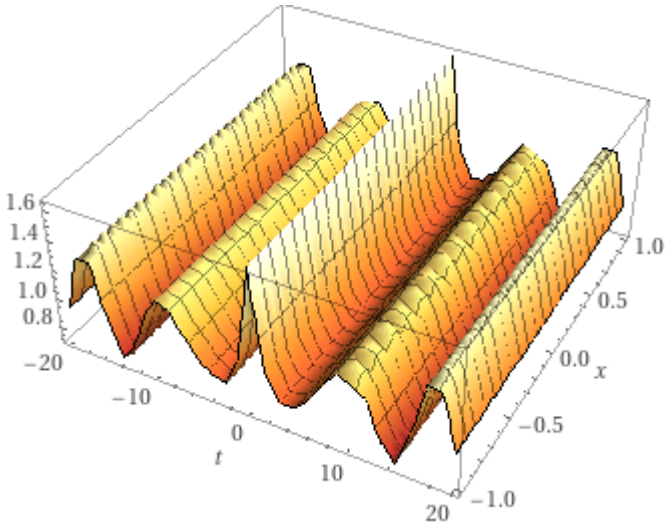


Fig. 6

Where ζ(2+it) :

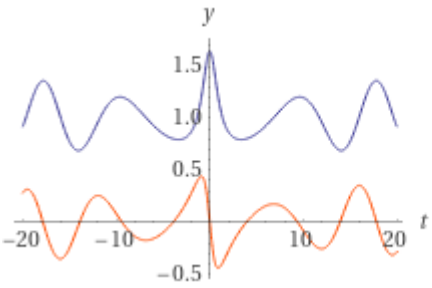
Input

$\zeta(2 + i\,t)$

$\zeta(s)$ is the Riemann zeta function

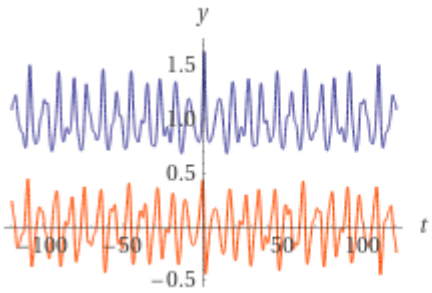
i is the imaginary unit

Plots



(t from −20 to 20)

real part
imaginary part



(t from −120 to 120)

real part
imaginary part

Roots

$t = 2\,i\,(n+1), \quad n \in \mathbb{Z}, \quad n \geq 1$

$t = -i\,(\rho_n - 2), \quad n \neq 0, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\,t\,\zeta'(2) - \frac{1}{2}\,t^2\,\zeta''(2) - \frac{1}{6}\,i\,\zeta^{(3)}(2)\,t^3 + \frac{1}{24}\,\zeta^{(4)}(2)\,t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2 + i\,t) = \zeta(2 + i\,t,\, 1)$$

$$\zeta(2 + i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2 + i\,t) = \frac{\zeta\left(2 + i\,t,\, \frac{1}{2}\right)}{-1 + 2^{2+i\,t}}$$

$\zeta(s,\, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2 + i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = \frac{\sum_{k=0}^{\infty} (1 + 2\,k)^{-2-i\,t}}{1 - 2^{-2-i\,t}} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \mathrm{Im}(t) < 1$$

$\mathrm{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^\infty \frac{\tau^{1+i\,t}}{-1+e^\tau}\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^\infty \tau^{2+i\,t}\,\mathrm{csch}^2(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^\infty e^{-\tau}\,\tau^{1+i\,t}\,\mathrm{csch}(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\mathrm{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+i\,t) = -i\,2^{2+i\,t}\,\pi^{1+i\,t}\,\Gamma(-1-i\,t)\sinh\left(\frac{\pi\,t}{2}\right)\zeta(-1-i\,t)$$

$$\zeta(2+i\,t) = \frac{\pi^{3/2+i\,t}\,\Gamma\left(-\frac{1}{2}-\frac{i\,t}{2}\right)\zeta(-1-i\,t)}{\Gamma\left(1+\frac{i\,t}{2}\right)}$$

$$\zeta(2+i\,t) = -\frac{i\sum_{k=0}^\infty \frac{\Gamma\left(k-\frac{i\,t}{2}\right)\sum_{j=0}^k (-1)^j\,(1+2\,j)\binom{k}{j}\zeta(2+2\,j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{i\,t}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372033095 8…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

$1.64381856858\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\right/\right.$$
$$\left.\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\text{^}(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right/\right.$$
$$\left.\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^-}6$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right)=0.054323\,(0.0814845\,b+1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M}=-\frac{\frac{226\,802\,245}{18\,480\,874}+b}{M}$$

$$\frac{\partial M(b,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b}=-\frac{18\,480\,874\,M}{226\,802\,245+18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M}=\frac{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b}=\frac{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right\}=0 \text{ at } (b,M)=(-16,0)$$

Global minima:

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

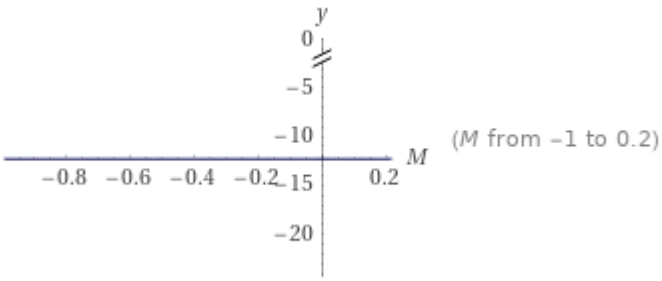
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

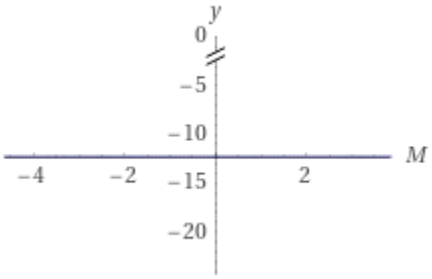
Result:

$$\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323\, M^2\right)}{M^2}\right) = \frac{3.55271 \times 10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM = \frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2} = -12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM = 0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

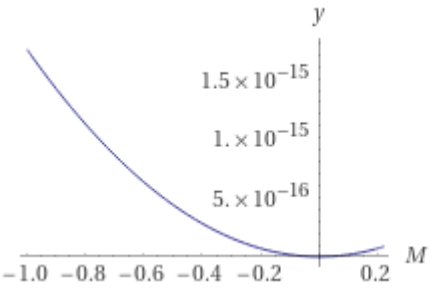
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

Result:

0

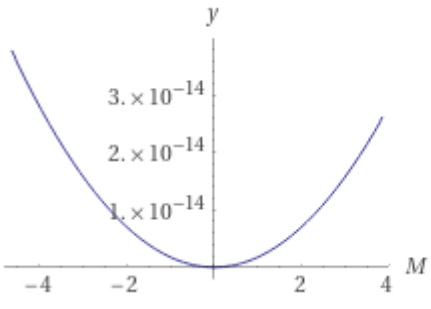
Plots: **(possible mathematical connection with an open string)**



(*M* from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O\left(M^{62194}\right)$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times10^{-15}\,M^2+O\left(\left(\frac{1}{M}\right)^{62\,194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}\,M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.38851344947464545348970783378088020833333333333333333333\ldots \times 10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 0.2^2 + 6.58545 \times 10^{-10}\sqrt{0.2^4}\right)}{0.2^2} + 1\right)^2 \times 0.2^2$$

Result:

$$7.02162151915943272558353253404940833333333333333333333333\ldots \times 10^{-17}$$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875\times10^{-14}$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots \times 10^{-8}$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left((1/(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}))\ \hat{}\ (1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\ \sqrt[4]{3}\ e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

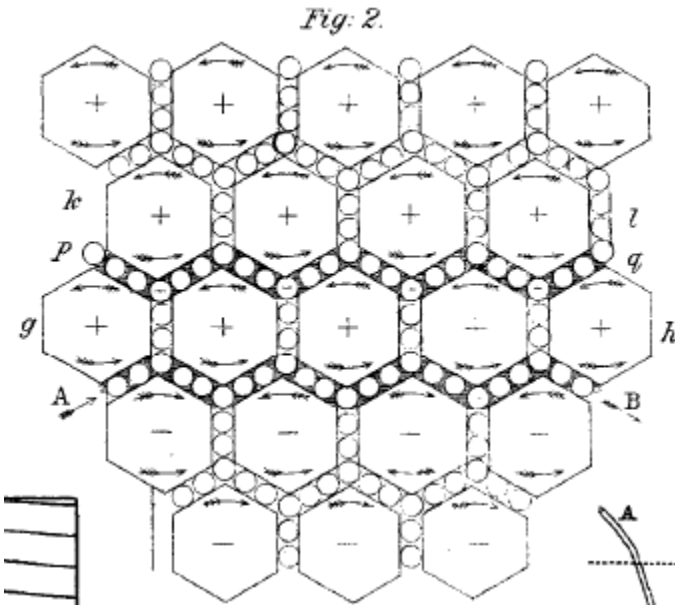
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

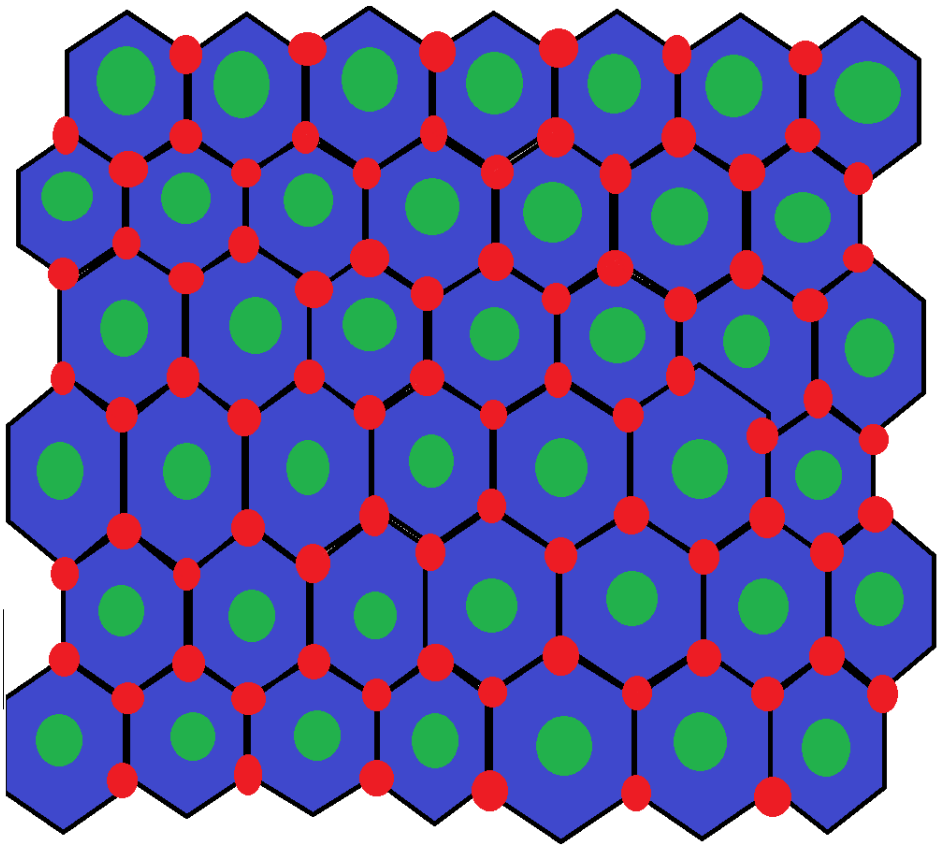
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Fig. 3

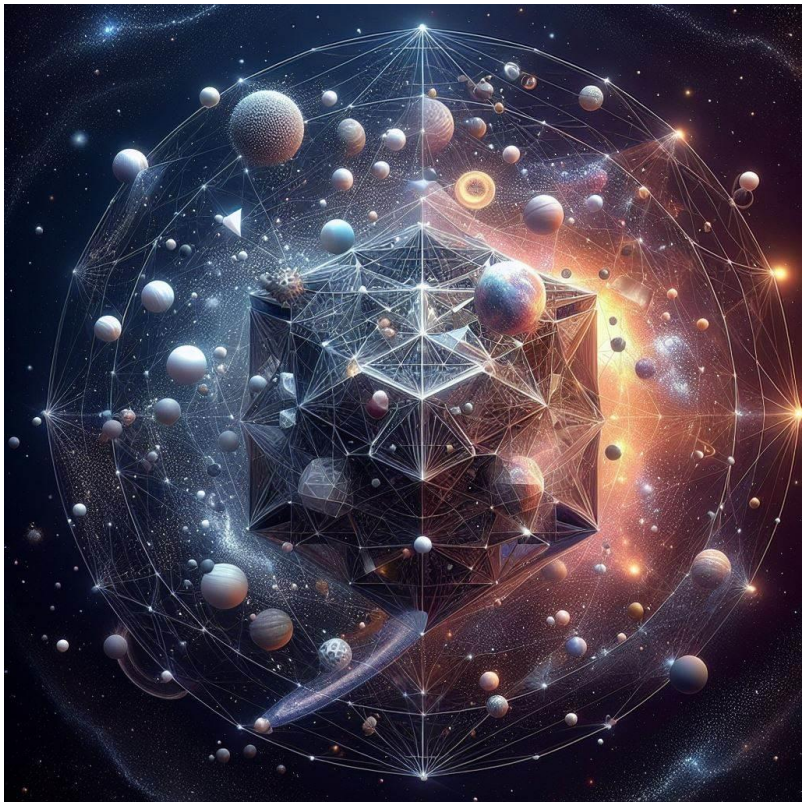


Image representing the solutions of the cubic equations in relation to the boundary conditions of the Plateau problem useful for understanding the formation of the bubbles of a universe with eternal inflation and of an octahedron as a mathematical model of the primordial universe before the Big Bang, therefore highly symmetrical and with extremely low entropy. As for the cubic equations, they represent the geometric characteristics of the quantum vacuum, while the spherical surface would represent the geometry of the bubble of the universe.

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability. I would like to thank also my son **Francesco Nardelli**, student of Educational Sciences, majoring in History and Philosophy, for his useful suggestions regarding the intriguing image at the end of this work

References

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

SUPERCONFORMAL GRAVITY AND THE TOPOLOGY OF DIFFEOMORPHISM GROUPS - *Jay Cushing, Gregory W. Moore, Martin Roček, Vivek Saxena* - arXiv:2311.08394v2 [hep-th] 17 Nov 2023

See also:

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *Emilian Dudas, Sergio Ferrara, Alex Kehagias and Augusto Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the analysis of various Cardano's Formulas and Cubic Equations. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory VII

Michele Nardelli[1], Antonio Nardelli

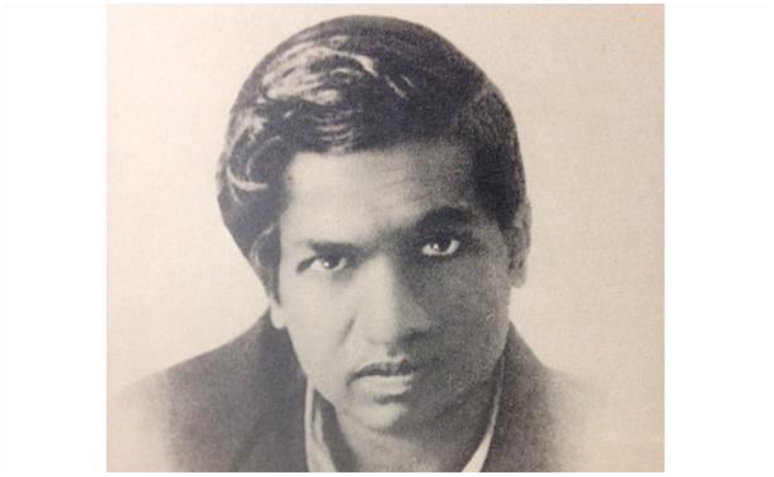
Abstract

In this paper (part VII), we analyze various Cardano's Formulas and Cubic Equations. We obtain new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan (1887-1920)



https://www.moduscc.it/ramanujan-il-grande-matematico-indiano-13453-131115/

Introduction

Cardano's method for cubic equations involves cube roots and introduces symmetry concepts. Similarly, string theory and M-theory involve symmetries and transformation groups that play a fundamental role in describing particles and interactions. Solutions of complete cubic equations via Cardano's method involve complex cube roots. Similarly, equations and solutions in superstring theory can often involve advanced algebraic and mathematical complexities. Solutions of cubic equations can be understood geometrically through topology and algebraic geometry. In string theory, the geometric and topological configurations of strings and branes are also central. Duality is an important concept in string theory, where different mathematical descriptions can be equivalent. Similarly, in solving cubic equations, duality links exist through cube roots.

Thus, in string theory, duality refers to the fact that different mathematical formulations or physical descriptions of a theory can be equivalent. For example, two seemingly different theories can be dual to each other, meaning they are two different ways of describing the same physical reality. This concept of duality has been very useful for understanding profound aspects of string theory and field theories.

Similarly, in solving cubic equations, duality can emerge through cube roots. The cubic equation $ax^3+bx^2+cx+d=0$ can be solved using cube roots, as in the Cardano method I described above. However, cube roots involve the concept of complex numbers, and it is sometimes possible to express the solutions in ways that highlight a duality between the solutions.

For example, if we consider complex cube roots, we can see a duality between them, since there are connections via the cube roots of unity. This duality is similar to the concept of duality in string theory, where different descriptions can be connected by similar transformations.

In both cases, duality provides a deeper way of understanding the underlying mathematical or physical structure, allowing you to explore hidden connections between different concepts.

Cubic surfaces can be associated with geometric and topological concepts. In string theory, for example, some models require spaces with particular shapes, and the geometry of the spaces can influence the properties of strings and particles. Some

mathematical and physical models involving surfaces might be described by cubic or complex equations. The geometry of surfaces and the associated mathematical structure can influence the properties of universe bubbles in the multiverse. Cubic surfaces can represent certain topological spaces, and their properties can affect the structure of the associated string theory.

The connection between the surface of a D-brane and the roots of a cubic equation can be explored in specific contexts within string theory, especially when it comes to studying the geometric and topological properties of D-branes. D-branes are extended objects on whose surface open strings can terminate. The geometry of the D-brane surface can interfere with the physics of surrounding strings. In some cases, geometry can be described mathematically through objects known in algebraic surface theory, such as cubic equations.

D-branes can host open fields, which describe particles interacting with the D-brane. Field equations on a D-brane can involve surface geometry, and in some situations, these equations can be associated with higher degree algebraic equations, such as cubic equations.

The compactification of extra dimensions, which is an important part of string theory, can be associated with the surface geometry of a D-brane. Compactification can lead to a reduction in dimensions and a geometric description that can involve algebraic equations, including cubic equations.

The topological invariants associated with the surface of a D-brane, such as the Betti number or the Chern number, can be related to geometric characteristics that can be described via algebraic equations, including cubic equations.

In some situations of duality, the description of a D-brane in a context can be dualistic to a description based on algebraic objects such as curves or surfaces, which can be represented via algebraic equations, including cubic equations.

In this paper, we will hypothesize that the solutions of the cubic equations, can be used as boundary conditions to further the understanding of the formation of universe bubbles in eternal inflation. Furthermore, an octahedron could serve as a mathematical or conceptual model of the universe in the quantum phase. Cubic equations, in this context, could represent the geometric characteristics of the quantum vacuum, while the spherical surface could be used to describe the geometry of the bubble-universe.

The results $(2\sqrt{2})/\pi$, the golden ratio φ and π obtained, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and bubbles universe in eternal inflation. It may reflect a fundamental relationship between the geometry of solids and the mathematical characteristics of solutions to cubic equations.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and bubbles universe.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of bubbles universe and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

Finally, it is also possible to hypothesize the solutions of the cubic equations solvable with the Cardano method, as boundary conditions to deepen the understanding of the formation of inflationary universe bubbles eternal and of an octahedron which could represent a mathematical or conceptual model of the quantum universe before the Big Bang. Furthermore, regarding the cubic equations, they can represent the geometric characteristics of the quantum vacuum, while the spherical surface represents the geometry of the bubble universe. The results above $(2\sqrt{2})/\pi$, the golden ratio φ and π which are obtained, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

These mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe. The cubic equations, in this context, could reflect the geometric and dynamic characteristics of the quantum vacuum, while the spherical surface could represent the geometry of the bubbles universe.

Regarding fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the quantum universe and bubbles universe.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

From

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

Cubic equations and Cardano's Formulas

From

$$\sqrt[3]{444\tfrac{1}{2} + \sqrt{175{,}628\tfrac{1}{4}}} + \sqrt[3]{444\tfrac{1}{2} - \sqrt{175{,}628\tfrac{1}{4}}} + 2.$$

we obtain:

(444*1/2+sqrt(175628*1/4))^(1/3)+(444*1/2-sqrt(175628*1/4))^(1/3) + 2

Input

$$\sqrt[3]{444 \times \frac{1}{2} + \sqrt{175\,628 \times \frac{1}{4}}} + \sqrt[3]{444 \times \frac{1}{2} - \sqrt{175\,628 \times \frac{1}{4}}} + 2$$

Result

$$\sqrt[3]{222 + 23\sqrt{83}} + \sqrt[3]{222 - 23\sqrt{83}} + 2$$

Decimal approximation

11.875159993887280987220498054890267220246684529438320111739178945

…

11.87515999388728…. ≈ 11.8477 (S_{BH} Entropy)

Alternate form

$$2+\sqrt[3]{3\left(148+\sqrt[3]{5377\left(222-23\sqrt{83}\right)}+\sqrt[3]{5377\left(222+23\sqrt{83}\right)}\right)}$$

Minimal polynomial

$$x^9-18x^8+144x^7-2004x^6+18000x^5-83952x^4+664725x^3-3001662x^2+5612796x-91183976$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

From

$$\sqrt[3]{165\frac{8}{27}+\sqrt{27{,}161\frac{3}{7}}}+\sqrt[3]{165\frac{8}{27}-\sqrt{27{,}161\frac{3}{7}}}-\frac{1}{3}.$$

we obtain:

(165*8/27+sqrt(27161*3/14))^(1/3)+(165*8/27-sqrt(27161*3/14))^(1/3) – 1/3

Input

$$\sqrt[3]{165\times\frac{8}{27}+\sqrt{27\,161\times\frac{3}{14}}}+\sqrt[3]{165\times\frac{8}{27}-\sqrt{27\,161\times\frac{3}{14}}}-\frac{1}{3}$$

Result

$$-\frac{1}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}$$

Decimal approximation

6.176452628495648529834407551973779922203846551626100551426685839…

+

2.610889113030967045476170307559989203233160825467758590799343312…

i

Alternate complex forms

$$\left(-\frac{1}{3}+\frac{1}{2}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\frac{1}{2}\left(\sqrt{3}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)i$$

$$\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{1}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)}\left(\cos\left(\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}}{2\left(-\frac{1}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}}{2\left(-\frac{1}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)}\right)\right)\right)$$

$$\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{1}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)}\exp\left(i\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}}{2\left(-\frac{1}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)}\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(-\frac{1}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)}\ \text{(radius)},\quad \theta=\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}}{2\left(-\frac{1}{3}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)}\right)\ \text{(angle)}$$

$r \approx 6.7056$ (radius), $\theta \approx 0.39994$ (angle)

6.7056

Alternate forms

$$\frac{1}{42}\left(14^{2/3}\sqrt[3]{-3\left(9\sqrt{1\,140\,762}-6160\right)}+14^{2/3}\sqrt[3]{3\left(6160+9\sqrt{1\,140\,762}\right)}-14\right)$$

$$-\frac{1}{3}+\frac{\sqrt[3]{\frac{1}{14}\left(6160-9\sqrt{1\,140\,762}\right)}}{3^{2/3}}+\frac{\sqrt[3]{\frac{1}{14}\left(6160+9\sqrt{1\,140\,762}\right)}}{3^{2/3}}$$

$$\frac{-3^{2/3}+3\sqrt[3]{440-9\sqrt{\frac{81\,483}{14}}}+3\sqrt[3]{440+9\sqrt{\frac{81\,483}{14}}}}{3\times 3^{2/3}}$$

Minimal polynomial

$$275\,562\,x^9+826\,686\,x^8+1\,102\,248\,x^7-79\,974\,216\,x^6-161\,234\,388\,x^5-134\,576\,316\,x^4+33\,364\,155\,555\,x^3+33\,409\,034\,739\,x^2+11\,139\,337\,539\,x-256\,358\,600\,977$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

From the algebraic sum of the two above results

$$\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{222-23\sqrt{83}}+2$$

and

$$-\frac{1}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}$$

after some calculations:

89(((222 + 23 sqrt(83))^(1/3) + (222 - 23 sqrt(83))^(1/3) + 2)+(-1/3 + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3)))+123-18+π-√2

Input

$$89\left(\left(\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{222-23\sqrt{83}}+2\right)+\left(-\frac{1}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)+123-18+\pi-\sqrt{2}$$

Result

$$105-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\pi$$

Decimal approximation

1713.320902483297425207547553669970000483724763738731596769560230…

+

232.369131059756067047379157372839039087751313466630514581141554 8…

i

Property

$$105-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\pi \text{ is a transcendental number}$$

Alternate complex forms

$$\left(105 - \sqrt{2} + 89\left(\frac{5}{3} + \sqrt[3]{222 - 23\sqrt{83}} + \sqrt[3]{222 + 23\sqrt{83}} + \frac{1}{2}\sqrt[3]{-\frac{440}{9} + \sqrt{\frac{81\,483}{14}}} + \sqrt[3]{\frac{440}{9} + \sqrt{\frac{81\,483}{14}}}\right) + \pi\right) + \frac{1}{2}\left(89\sqrt{3}\,\sqrt[3]{-\frac{440}{9} + \sqrt{\frac{81\,483}{14}}}\right) i$$

$$\sqrt{\left(\frac{23763}{4}\left(\sqrt{\frac{81483}{14}}-\frac{440}{9}\right)^{2/3}+\left(105-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)+\pi\right)^2\right)}$$

$$\left(\cos\left(\tan^{-1}\left(\left(89\sqrt{3}\sqrt[3]{\sqrt{\frac{81483}{14}}-\frac{440}{9}}\right)\Big/\left(2\left(105-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)+\pi\right)\right)\right)\right)+i\sin\left(\tan^{-1}\left(\left(89\sqrt{3}\sqrt[3]{\sqrt{\frac{81483}{14}}-\frac{440}{9}}\right)\Big/\left(2\left(105-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)+\pi\right)\right)\right)\right)$$

$$\sqrt{\left(\frac{23\,763}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(105-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\pi\right)^2\right)}$$

$$\exp\left(i\tan^{-1}\left(\left(89\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\left(2\left(105-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\pi\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r = \sqrt{\left(\frac{23\,763}{4}\left(\sqrt{\frac{81\,483}{14}} - \frac{440}{9}\right)^{2/3} + \left(105 - \sqrt{2} + 89\left(\frac{5}{3} + \sqrt[3]{222 - 23\sqrt{83}} + \sqrt[3]{222 + 23\sqrt{83}} + \frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}} - \frac{440}{9}} + \sqrt[3]{\frac{440}{9} + \sqrt{\frac{81\,483}{14}}}\right) + \pi\right)^2\right)}$$

(radius), $\theta = \tan^{-1}\left(\left(89\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}} - \frac{440}{9}}\right) \Big/ \left(2\left(105 - \sqrt{2} + 89\left(\frac{5}{3} + \sqrt[3]{222 - 23\sqrt{83}} + \sqrt[3]{222 + 23\sqrt{83}} + \frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}} - \frac{440}{9}} + \sqrt[3]{\frac{440}{9} + \sqrt{\frac{81\,483}{14}}}\right) + \pi\right)\right)\right)$ (angle)

$r \approx 1729$ (radius), $\theta \approx 0.13480$ (angle)

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron.

As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$\frac{1}{42}\left(3738\sqrt[3]{222-23\sqrt{83}}-42\sqrt{2}+3738\sqrt[3]{222+23\sqrt{83}}+89\times 14^{2/3}\sqrt[3]{-3\left(9\sqrt{1\,140\,762}-6160\right)}+89\times 14^{2/3}\sqrt[3]{3\left(6160+9\sqrt{1\,140\,762}\right)}+42\pi+10\,640\right)$$

$$\frac{760}{3} - \sqrt{2} + 89\sqrt[3]{222 - 23\sqrt{83}} + 89\sqrt[3]{222 + 23\sqrt{83}} + \frac{89\sqrt[3]{\frac{1}{14}\left(6160 - 9\sqrt{1\,140\,762}\right)}}{3^{2/3}} + \frac{89\sqrt[3]{\frac{1}{14}\left(6160 + 9\sqrt{1\,140\,762}\right)}}{3^{2/3}} + \pi$$

$$\frac{1}{42}\left(10\,640 - 42\sqrt{2} + 3738\sqrt[3]{222 - 23\sqrt{83}} + 3738\sqrt[3]{222 + 23\sqrt{83}} + 89\times 14^{2/3}\sqrt[3]{-3\left(9\sqrt{1\,140\,762} - 6160\right)} + 89\times 14^{2/3}\sqrt[3]{3\left(6160 + 9\sqrt{1\,140\,762}\right)}\right) + \pi$$

Series representations

$$89\left(\left(\sqrt[3]{222 + 23\sqrt{83}} + \sqrt[3]{222 - 23\sqrt{83}} + 2\right) + \left(-\frac{1}{3} + \sqrt[3]{\frac{440}{9} - \sqrt{\frac{81\,483}{14}}} + \sqrt[3]{\frac{440}{9} + \sqrt{\frac{81\,483}{14}}}\right)\right) + 123 -$$

$$18 + \pi - \sqrt{2} = \frac{1}{3}\left(760 + 3\pi - 3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2 - z_0)^k z_0^{-k}}{k!} + 267\sqrt[3]{222 - 23\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(83 - z_0)^k z_0^{-k}}{k!}} + 267\sqrt[3]{222 + 23\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(83 - z_0)^k z_0^{-k}}{k!}} + 89\sqrt[3]{3}\sqrt[3]{440 - 9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{81\,483}{14} - z_0\right)^k z_0^{-k}}{k!}} + 89\sqrt[3]{3}\sqrt[3]{440 + 9\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{81\,483}{14} - z_0\right)^k z_0^{-k}}{k!}}\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$89\left(\left(\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{222-23\sqrt{83}}+2\right)+\left(-\frac{1}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)+123-18+\pi-\sqrt{2}=$$

$$\frac{1}{3}\left(760+3\pi-3\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+267\sqrt[3]{222-23\exp\left(i\pi\left\lfloor\frac{\arg(83-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(83-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+267\sqrt[3]{222+23\exp\left(i\pi\left\lfloor\frac{\arg(83-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(83-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}+89\sqrt[3]{3}\left(440-9\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{81\,483}{14}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{81\,483}{14}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\wedge(1/3)+89\sqrt[3]{3}\left(440+9\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{81\,483}{14}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{81\,483}{14}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\wedge(1/3)\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$89\left(\left(\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{222-23\sqrt{83}}+2\right)+\left(-\frac{1}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)+$$

$$123-18+\pi-\sqrt{2}=\frac{1}{3}\left(760+3\pi-3\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}+267\left(222-23\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(83-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(83-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(83-z_0)^k z_0^{-k}}{k!}\right)\text{^}(1/3)+267\left(222+23\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(83-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(83-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(83-z_0)^k z_0^{-k}}{k!}\right)\text{^}(1/3)+89\sqrt[3]{3}\left(440-9\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{81\,483}{14}-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(\frac{81\,483}{14}-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{81\,483}{14}-z_0\right)^k z_0^{-k}}{k!}\right)\text{^}(1/3)+89\sqrt[3]{3}\left(440+9\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{81\,483}{14}-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(\frac{81\,483}{14}-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{81\,483}{14}-z_0\right)^k z_0^{-k}}{k!}\right)\text{^}(1/3)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

(1/27((89(((222 + 23 sqrt(83))^(1/3) + (222 - 23 sqrt(83))^(1/3) + 2)+(-1/3 + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3)))+123-18+π-√2)-1))^2

Input

$$\left(\frac{1}{27}\left(\left(89\left(\left(\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{222-23\sqrt{83}}+2\right)+\left(-\frac{1}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)+123-18+\pi-\sqrt{2}\right)-1\right)\right)^2$$

Exact result

$$\frac{1}{729}\left(104-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\pi\right)^2$$

Decimal approximation

3947.938902622425347898191880695348875712819497975572870430526632…

+

1091.606365447191021578699666723331711411822807119813979028105142…

i

Property

$$\frac{1}{729}\left(104-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\pi\right)^2 \text{ is a transcendental number}$$

Alternate complex forms

$$\frac{1}{729}\left(-\frac{23\,763}{4}\left(-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}\right)^{2/3}+\left(104-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{1}{2}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\pi\right)^2\right)+\frac{1}{243\sqrt{3}}\left(89\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\left(104-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{1}{2}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\pi\right)\right)i$$

$$\frac{1}{91854}\Bigg(8022938-63588\sqrt{2}+1996092\sqrt[3]{5377}+7921\times 42^{2/3}\sqrt[3]{3889723}-$$
$$7476\left(3\sqrt{2}-757\right)\sqrt[3]{222-23\sqrt{83}}+998046\left(222-23\sqrt{83}\right)^{2/3}+$$
$$5659332\sqrt[3]{222+23\sqrt{83}}-22428\sqrt{2}\sqrt[3]{222+23\sqrt{83}}+$$
$$998046\left(222+23\sqrt{83}\right)^{2/3}-534\sqrt[6]{2}\,7^{2/3}\sqrt{3\left(9\sqrt{1140762}-6160\right)}+$$
$$67373\times 14^{2/3}\sqrt{3\left(9\sqrt{1140762}-6160\right)}+$$
$$7921\sqrt[3]{14}\left(3\left(9\sqrt{1140762}-6160\right)\right)^{2/3}+$$
$$23763\times 14^{2/3}\sqrt{3\left(222-23\sqrt{83}\right)\left(9\sqrt{1140762}-6160\right)}+$$
$$23763\times 14^{2/3}\sqrt{3\left(222+23\sqrt{83}\right)\left(9\sqrt{1140762}-6160\right)}-$$
$$1068\sqrt[6]{2}\,7^{2/3}\sqrt{3\left(6160+9\sqrt{1140762}\right)}+$$
$$134746\times 14^{2/3}\sqrt{3\left(6160+9\sqrt{1140762}\right)}+$$
$$7921\sqrt[3]{14}\left(3\left(6160+9\sqrt{1140762}\right)\right)^{2/3}+$$
$$47526\times 14^{2/3}\sqrt{3\left(222-23\sqrt{83}\right)\left(6160+9\sqrt{1140762}\right)}+$$
$$47526\times 14^{2/3}\sqrt{3\left(222+23\sqrt{83}\right)\left(6160+9\sqrt{1140762}\right)}+$$
$$3\Bigg(21196-84\sqrt{2}+7476\sqrt[3]{222-23\sqrt{83}}+$$
$$7476\sqrt[3]{222+23\sqrt{83}}+89\times 14^{2/3}\sqrt{3\left(9\sqrt{1140762}-6160\right)}+$$
$$178\times 14^{2/3}\sqrt{3\left(6160+9\sqrt{1140762}\right)}\Bigg)\pi+126\pi^2\Bigg)$$
$$\exp\Bigg(i\tan^{-1}\Bigg(\Bigg(\Bigg(89\sqrt{3}\sqrt{\sqrt{\frac{81483}{14}}-\frac{440}{9}}$$
$$\left(104-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\right.\right.$$
$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)+\pi\right)\Bigg)\Big/$$
$$\Bigg(\Bigg(\left(104-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\right.\right.$$
$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)+\pi\right)^2-$$
$$\left.\left.\frac{23763}{4}\left(\sqrt{\frac{81483}{14}}-\frac{440}{9}\right)^{2/3}\right)\right)\Bigg)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$r \approx 4096.1$ (radius), $\theta \approx 0.26976$ (angle)

$4096.1 \approx 4096$

The number $4096 = 64^2$, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

(89(((222 + 23 sqrt(83))^(1/3) + (222 - 23 sqrt(83))^(1/3) + 2)+(-1/3 + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3)))+123-18+π-√2)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\left(89\left(\left(\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{222-23\sqrt{83}}+2\right)+\left(-\frac{1}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)+123-18+\pi-\sqrt{2}\right)\wedge(1/15)+{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Exact result

$${C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\left(105-\sqrt{2}+89\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\pi\right)\wedge(1/15)$$

Decimal approximation

1.64487206495953345904597657815585050513012843245163088741507 40… +

0.01477249632824918570178842024452184594947032262062235770777 7806…

i

Alternate complex forms

$1.6449 + 0.0148\,i$

$1.6449\,(\cos(0.0089807) + i\sin(0.0089807))$

$$1.6449\, e^{0.0089807\, i}$$

Polar coordinates

$r = 1.64497$ (radius), $\theta = 0.00899726$ (angle)

1.64497 ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \left(\frac{1}{42}\left(10\,640 - 42\sqrt{2} + 3738\sqrt[3]{222 - 23\sqrt{83}} + 3738\sqrt[3]{222 + 23\sqrt{83}} + 89\times 14^{2/3}\sqrt[3]{-3\left(9\sqrt{1\,140\,762} - 6160\right)} + 89\times 14^{2/3}\sqrt[3]{3\left(6160 + 9\sqrt{1\,140\,762}\right)} + 42\pi\right)\right)\text{^}(1/15)$$

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi} + \frac{1}{\sqrt[9]{3}}\left(\left(760\times 3^{2/3} - 3\sqrt{2}\;3^{2/3} + 267\times 3^{2/3}\sqrt[3]{222 - 23\sqrt{83}} + 267\times 3^{2/3}\sqrt[3]{222 + 23\sqrt{83}} + 267\sqrt[3]{440 - 9\sqrt{\frac{81\,483}{14}}} + 267\sqrt[3]{440 + 9\sqrt{\frac{81\,483}{14}}} + 3\times 3^{2/3}\pi\right)\text{^}(1/15)\right)$$

$$\frac{1}{42}\,{C_{\text{MRB}}}^{-1/(4\pi)}\left(42\,{C_{\text{MRB}}}^{1+\pi}+42^{14/15}\left(10640-42\sqrt{2}+3738\sqrt[3]{222-23\sqrt{83}}+3738\sqrt[3]{222+23\sqrt{83}}+89\times 14^{2/3}\sqrt[3]{-3\left(9\sqrt{1140762}-6160\right)}+89\times 14^{2/3}\sqrt[3]{3\left(6160+9\sqrt{1140762}\right)}+42\pi\right)\text{^}\,(1/15)\sqrt[4\pi]{C_{\text{MRB}}}\right)$$

Always, from the algebraic sum of the two results

$$\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{222-23\sqrt{83}}+2$$

and

$$-\frac{1}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}$$

we obtain:

((222 + 23 sqrt(83))^(1/3) + (222 - 23 sqrt(83))^(1/3) + 2)+((-1/3 + (440/9 - sqrt(81483/14))^(1/3) + (440/9 + sqrt(81483/14))^(1/3)))

Input

$$\left(\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{222-23\sqrt{83}}+2\right)+\left(-\frac{1}{3}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81483}{14}}}\right)$$

Result

$$\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\sqrt[3]{\frac{440}{9}-\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}$$

Decimal approximation

18.051612622382929517054905606864047142450531081064420663165864 78…

+

2.610889113030967045476170307559989203233160825467758590799343312…

i

Alternate complex forms

$$\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{1}{2}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)+\frac{1}{2}\left(\sqrt{3}\sqrt[3]{-\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)i$$

$$\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\right.\right.}$$

$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)$$

$$\left(\cos\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\right.\right.\right.$$

$$\left.\left.\left.\left(2\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\right.\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)+$$

$$i\sin\left(\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\right.\right.$$

$$\left.\left.\left(2\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\right.\right.\right.$$

$$\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)$$

$$\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\right.\right.}$$
$$\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)$$
$$\exp\left(i\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\right.\right.$$
$$\left.\left.\left(2\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)\right)$$

i is the imaginary unit

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=\sqrt{\left(\frac{3}{4}\left(\sqrt{\frac{81\,483}{14}}-\frac{440}{9}\right)^{2/3}+\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{1}{2}\right.\right.}$$
$$\left.\left.\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)^2\right)$$
$$\text{(radius)},\quad \theta=\tan^{-1}\left(\left(\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}\right)\Big/\left(2\left(\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\right.\right.\right.\text{(angle)}$$
$$\left.\left.\left.\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}\right)\right)\right)$$

$r\approx 18.239$ (radius), $\theta\approx 0.14364$ (angle)

18.239

Alternate forms

$$\frac{1}{42}\left(42\sqrt[3]{222-23\sqrt{83}}+42\sqrt[3]{222+23\sqrt{83}}+14^{2/3}\sqrt[3]{-3\left(9\sqrt{1\,140\,762}-6160\right)}+14^{2/3}\sqrt[3]{3\left(6160+9\sqrt{1\,140\,762}\right)}+70\right)$$

$$\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{\sqrt[3]{\frac{1}{14}\left(6160-9\sqrt{1\,140\,762}\right)}}{3^{2/3}}+\frac{\sqrt[3]{\frac{1}{14}\left(6160+9\sqrt{1\,140\,762}\right)}}{3^{2/3}}$$

$$\frac{5}{3}+\sqrt[3]{222-23\sqrt{83}}+\sqrt[3]{222+23\sqrt{83}}+\frac{1}{2}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\frac{1}{2}i\sqrt{3}\sqrt[3]{\sqrt{\frac{81\,483}{14}}-\frac{440}{9}}+\sqrt[3]{\frac{440}{9}+\sqrt{\frac{81\,483}{14}}}$$

Now, we have that for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , thus:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

for a = 18.239 :

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot 18.239^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{18.239}{2}\right)^3}$$

(1/3·√2·18.239^3)/(4/3·π·(18.239/2)^3)

Input interpretation

$$\frac{\frac{1}{3}\sqrt{2}\times 18.239^3}{\frac{4}{3}\pi\left(\frac{18.239}{2}\right)^3}$$

Result

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Series representations

$$\frac{\sqrt{2}\ 18.239^3}{\frac{1}{3}\left(4\,\pi\left(\frac{18.239}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{\sqrt{2}\ 18.239^3}{\frac{1}{3}\left(4\,\pi\left(\frac{18.239}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k (2-x)^k\, x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\sqrt{2}\ 18.239^3}{\frac{1}{3}\left(4\,\pi\left(\frac{18.239}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\ z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k\ z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function
$(a)_n$ is the Pochhammer symbol (rising factorial)
$\mathbb{R}$ is the set of real numbers
$\arg(z)$ is the complex argument
$\lfloor x \rfloor$ is the floor function
i is the imaginary unit

Now, we have the following equation:

$$z^3 - 6Sz^2 + 12Sz - 8T^2 = 0$$

z^3-6S*z^2+12S*z-8T^2 = 0

Input

$$z^3 - 6\,S\,z^2 + 12\,S\,z - 8\,T^2 = 0$$

Alternate forms

$$-6\,S\,(z-2)\,z - 8\,T^2 + z^3 = 0$$

$$S\left(12\,z - 6\,z^2\right) = 8\,T^2 - z^3$$

Solutions

$$z = 2^{2/3}\sqrt[3]{2\,S^3 - 3\,S^2 + \sqrt{-3\,S^4 + 4\,S^3\,T^2 + 4\,S^3 - 6\,S^2\,T^2 + T^4}\ + T^2}\ - \frac{36\,S - 36\,S^2}{9\times 2^{2/3}\sqrt[3]{2\,S^3 - 3\,S^2 + \sqrt{-3\,S^4 + 4\,S^3\,T^2 + 4\,S^3 - 6\,S^2\,T^2 + T^4}\ + T^2}} + 2\,S$$

$$z = -\frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{2S^3 - 3S^2 + \sqrt{-3S^4 + 4S^3T^2 + 4S^3 - 6S^2T^2 + T^4} + T^2}}{\sqrt[3]{2}} +$$

$$\frac{\left(1 + i\sqrt{3}\right)\left(36S - 36S^2\right)}{18 \times 2^{2/3}\sqrt[3]{2S^3 - 3S^2 + \sqrt{-3S^4 + 4S^3T^2 + 4S^3 - 6S^2T^2 + T^4} + T^2}} + 2S$$

$$z = -\frac{\left(1 + i\sqrt{3}\right)\sqrt[3]{2S^3 - 3S^2 + \sqrt{-3S^4 + 4S^3T^2 + 4S^3 - 6S^2T^2 + T^4} + T^2}}{\sqrt[3]{2}} +$$

$$\frac{\left(1 - i\sqrt{3}\right)\left(36S - 36S^2\right)}{18 \times 2^{2/3}\sqrt[3]{2S^3 - 3S^2 + \sqrt{-3S^4 + 4S^3T^2 + 4S^3 - 6S^2T^2 + T^4} + T^2}} + 2S$$

Solutions for the variable z

$$z = 2^{2/3}\sqrt[3]{2S^3 - 3S^2 + \sqrt{-3S^4 + 4S^3T^2 + 4S^3 - 6S^2T^2 + T^4} + T^2} -$$

$$\frac{36S - 36S^2}{9 \times 2^{2/3}\sqrt[3]{2S^3 - 3S^2 + \sqrt{-3S^4 + 4S^3T^2 + 4S^3 - 6S^2T^2 + T^4} + T^2}} + 2S$$

$$z = -\frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{2S^3 - 3S^2 + \sqrt{-3S^4 + 4S^3T^2 + 4S^3 - 6S^2T^2 + T^4} + T^2}}{\sqrt[3]{2}} +$$

$$\frac{\left(1 + i\sqrt{3}\right)\left(36S - 36S^2\right)}{18 \times 2^{2/3}\sqrt[3]{2S^3 - 3S^2 + \sqrt{-3S^4 + 4S^3T^2 + 4S^3 - 6S^2T^2 + T^4} + T^2}} + 2S$$

$$z = -\frac{\left(1+i\sqrt{3}\right)\sqrt[3]{2S^3-3S^2+\sqrt{-3S^4+4S^3T^2+4S^3-6S^2T^2+T^4}+T^2}}{\sqrt[3]{2}}+$$

$$\frac{\left(1-i\sqrt{3}\right)\left(36S-36S^2\right)}{18\times 2^{2/3}\sqrt[3]{2S^3-3S^2+\sqrt{-3S^4+4S^3T^2+4S^3-6S^2T^2+T^4}+T^2}}+2S$$

Implicit derivatives

$$\frac{\partial S(T,\,z)}{\partial z} = \frac{-4S(-1+z)+z^2}{2(-2+z)z}$$

$$\frac{\partial S(T,\,z)}{\partial T} = \frac{8T}{6z-3z^2}$$

$$\frac{\partial T(S,\,z)}{\partial z} = \frac{3\left(-4S(-1+z)+z^2\right)}{16T}$$

$$\frac{\partial T(S,\,z)}{\partial S} = -\frac{3(-2+z)z}{8T}$$

$$\frac{\partial z(S,\,T)}{\partial T} = \frac{16T}{3\left(-4S(-1+z)+z^2\right)}$$

$$\frac{\partial z(S,\,T)}{\partial S} = \frac{2(-2+z)z}{-4S(-1+z)+z^2}$$

From the first solution

$$z = 2^{2/3}\sqrt[3]{2S^3 - 3S^2 + \sqrt{-3S^4 + 4S^3T^2 + 4S^3 - 6S^2T^2 + T^4}\ + T^2}\ - \frac{36S - 36S^2}{9\times 2^{2/3}\sqrt[3]{2S^3 - 3S^2 + \sqrt{-3S^4 + 4S^3T^2 + 4S^3 - 6S^2T^2 + T^4}\ + T^2}} + 2S$$

we consider S = 2 and T = 4 :

2^(2/3) (2 2^3 - 3 2^2 + √(-3 2^4 + 4 2^3 4^2 + 4 2^3 - 6 2^2 4^2 + 4^4) + 4^2)^(1/3) - (36 2 - 36 2^2)/(9 2^(2/3) (2 2^3 - 3 2^2 + √(-3 2^4 + 4 2^3 4^2 + 4 2^3 - 6 2^2 4^2 + 4^4) + 4^2)^(1/3)) + 2 2

Input

$$2^{2/3}\sqrt[3]{2\times 2^3 - 3\times 2^2 + \sqrt{-3\times 2^4 + 4\times 2^3\times 4^2 + 4\times 2^3 - 6\times 2^2\times 4^2 + 4^4}\ + 4^2}\ - (36\times 2 - 36\times 2^2)\Big/\left(9\times 2^{2/3}\sqrt[3]{2\times 2^3 - 3\times 2^2 + \sqrt{-3\times 2^4 + 4\times 2^3\times 4^2 + 4\times 2^3 - 6\times 2^2\times 4^2 + 4^4}\ + 4^2}\right) + 2\times 2$$

Exact result

$$4 + 4\sqrt[3]{\frac{2}{20 + 4\sqrt{23}}}\ + 2^{2/3}\sqrt[3]{20 + 4\sqrt{23}}$$

Decimal approximation

10.875414481128341059693703890828005786743539465885233666261949039

…

10.875414481128341….

Alternate forms

2 root of $x^3 - 6x^2 + 6x - 16$ near $x = 5.43771$

$$2\left(2+\frac{2^{2/3}}{\sqrt[3]{5+\sqrt{23}}}+\sqrt[3]{2\left(5+\sqrt{23}\right)}\right)$$

$$4+\frac{1}{3}\sqrt[3]{2160-432\sqrt{23}}+2\sqrt[3]{2\left(5+\sqrt{23}\right)}$$

Minimal polynomial

$x^3-12x^2+24x-128$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

From the second solution

$$z=-\frac{\left(1-i\sqrt{3}\right)\sqrt[3]{2S^3-3S^2+\sqrt{-3S^4+4S^3T^2+4S^3-6S^2T^2+T^4}+T^2}}{\sqrt[3]{2}}+$$

$$\frac{\left(1+i\sqrt{3}\right)\left(36S-36S^2\right)}{18\times 2^{2/3}\sqrt[3]{2S^3-3S^2+\sqrt{-3S^4+4S^3T^2+4S^3-6S^2T^2+T^4}+T^2}}+2S$$

always for S = 2 and T = 4:

-((1-i √(3))(2 8-3 4+√(-3 16+4 8 16+4 8-6 4 16+256)+ 16)^(1/3))/2^(1/3)+((1+i √(3))(36 2-36 4))/(18 2^(2/3)(2 8-3 4+√(-3 16+4 8 16+4 8-6 4 16+256)+16)^(1/3))+4

Input

$$-\frac{1}{\sqrt[3]{2}}\left(1-i\sqrt{3}\right)\sqrt[3]{2\times 8-3\times 4+\sqrt{-3\times 16+4\times 8\times 16+4\times 8-6\times 4\times 16+256}+16}+\frac{\left(1+i\sqrt{3}\right)(36\times 2-36\times 4)}{18\times 2^{2/3}\sqrt[3]{2\times 8-3\times 4+\sqrt{-3\times 16+4\times 8\times 16+4\times 8-6\times 4\times 16+256}+16}}+4$$

i is the imaginary unit

Exact result

$$4-2\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}$$

Decimal approximation

0.562292759435829470153148054585997106628230267057383166869025480…

+

3.384300993629551781987950954365016155815813990693284835757406 92… i

Alternate complex forms

$$\left(4-2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}\right)+\left(-2\sqrt{3}\sqrt[3]{\frac{2}{20+4\sqrt{23}}}+\sqrt{3}\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}\right)i$$

$$\left(2\left(\cos\left(\tan^{-1}\left(\frac{\sqrt{3}\ \sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}-2\sqrt{3}\ \sqrt[3]{\frac{2}{20+4\sqrt{23}}}}{4-2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\sqrt{3}\ \sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}-2\sqrt{3}\ \sqrt[3]{\frac{2}{20+4\sqrt{23}}}}{4-2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}}\right)\right)\right)\right)\Bigg/\left(\sqrt{\frac{5+\sqrt{23}}{2\left(5+\sqrt{23}\right)-2\left(4+\sqrt{23}\right)\sqrt[3]{2\left(5+\sqrt{23}\right)}+\left(3+\sqrt{23}\right)\left(2\left(5+\sqrt{23}\right)\right)^{2/3}}}\right)$$

$$\frac{2\exp\left(i\tan^{-1}\left(\frac{\sqrt{3}\ \sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}-2\sqrt{3}\ \sqrt[3]{\frac{2}{20+4\sqrt{23}}}}{4-2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}}\right)\right)}{\sqrt{\frac{5+\sqrt{23}}{2\left(5+\sqrt{23}\right)-2\left(4+\sqrt{23}\right)\sqrt[3]{2\left(5+\sqrt{23}\right)}+\left(3+\sqrt{23}\right)\left(2\left(5+\sqrt{23}\right)\right)^{2/3}}}}$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=\frac{2}{\sqrt{\frac{5+\sqrt{23}}{2\left(5+\sqrt{23}\right)-2\left(4+\sqrt{23}\right)\sqrt[3]{2\left(5+\sqrt{23}\right)}+\left(3+\sqrt{23}\right)\left(2\left(5+\sqrt{23}\right)\right)^{2/3}}}}\ \text{(radius)}$$

$$,\quad \theta=\tan^{-1}\left(\frac{\sqrt{3}\ \sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}-2\sqrt{3}\ \sqrt[3]{\frac{2}{20+4\sqrt{23}}}}{4-2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}}\right)\ \text{(angle)}$$

$r\approx 3.4307$ (radius), $\theta\approx 1.4062$ (angle)

3.4307

Expanded form

$$4-2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-2i\sqrt{3}\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}+i\sqrt{3}\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}$$

Alternate forms

root of $x^3-12x^2+24x-128$ near $x=0.562293+3.3843i$

$$4+\frac{-2-2i\sqrt{3}}{\sqrt[3]{2\left(5+\sqrt{23}\right)}}+i\left(\sqrt{3}+i\right)\sqrt[3]{2\left(5+\sqrt{23}\right)}$$

$$4+\frac{2^{2/3}\left(-1-i\sqrt{3}\right)}{\sqrt[3]{5+\sqrt{23}}}+i\left(\sqrt{3}+i\right)\sqrt[3]{2\left(5+\sqrt{23}\right)}$$

Minimal polynomial

$x^3-12x^2+24x-128$

From the algebraic sum of the two

$$4+4\sqrt[3]{\frac{2}{20+4\sqrt{23}}}+2^{2/3}\sqrt[3]{20+4\sqrt{23}}$$

and

$$4-2\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}$$

we obtain:

(4 + 4 (2/(20 + 4 sqrt(23)))^(1/3) + 2^(2/3) (20 + 4 sqrt(23))^(1/3))+(4 - 2 (1 + i sqrt(3)) (2/(20 + 4 sqrt(23)))^(1/3) - (1 - i sqrt(3)) (1/2 (20 + 4 sqrt(23)))^(1/3))

Input

$$\left(4+4\sqrt[3]{\frac{2}{20+4\sqrt{23}}}+2^{2/3}\sqrt[3]{20+4\sqrt{23}}\right)+\left(4-2\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}\right)$$

i is the imaginary unit

Result

$$8+4\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-2\left(1+i\sqrt{3}\right)\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\left(1-i\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}+2^{2/3}\sqrt[3]{20+4\sqrt{23}}$$

Decimal approximation

11.437707240564170529846851945414002893371769732942616833130974 51…

\+

3.384300993629551781987950954365016155815813990693284835757406924…

i

Alternate complex forms

$$\left(8+2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}+2^{2/3}\sqrt[3]{20+4\sqrt{23}}\right)+\left(-2\sqrt{3}\sqrt[3]{\frac{2}{20+4\sqrt{23}}}+\sqrt{3}\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}\right)i$$

$$2\sqrt{14+\frac{2\sqrt[3]{2}}{\left(5+\sqrt{23}\right)^{2/3}}+\frac{4\times 2^{2/3}}{\sqrt[3]{5+\sqrt{23}}}+4\sqrt[3]{2\left(5+\sqrt{23}\right)}+\left(2\left(5+\sqrt{23}\right)\right)^{2/3}}\left(\cos\left(\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}-2\sqrt{3}\sqrt[3]{\frac{2}{20+4\sqrt{23}}}}{8+2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}+2^{2/3}\sqrt[3]{20+4\sqrt{23}}}\right)\right)+i\sin\left(\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}-2\sqrt{3}\sqrt[3]{\frac{2}{20+4\sqrt{23}}}}{8+2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}+2^{2/3}\sqrt[3]{20+4\sqrt{23}}}\right)\right)\right)$$

$$2\sqrt{14+\frac{2\sqrt[3]{2}}{\left(5+\sqrt{23}\right)^{2/3}}+\frac{4\times 2^{2/3}}{\sqrt[3]{5+\sqrt{23}}}+4\sqrt[3]{2\left(5+\sqrt{23}\right)}+\left(2\left(5+\sqrt{23}\right)\right)^{2/3}}\exp\left(i\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}-2\sqrt{3}\sqrt[3]{\frac{2}{20+4\sqrt{23}}}}{8+2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}+2^{2/3}\sqrt[3]{20+4\sqrt{23}}}\right)\right)$$

$\tan^{-1}(x)$ is the inverse tangent function

Polar coordinates

$$r=2\sqrt{14+\frac{2\sqrt[3]{2}}{\left(5+\sqrt{23}\right)^{2/3}}+\frac{4\times 2^{2/3}}{\sqrt[3]{5+\sqrt{23}}}+4\sqrt[3]{2\left(5+\sqrt{23}\right)}+\left(2\left(5+\sqrt{23}\right)\right)^{2/3}}$$ (radius),

$$\theta=\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}-2\sqrt{3}\sqrt[3]{\frac{2}{20+4\sqrt{23}}}}{8+2\sqrt[3]{\frac{2}{20+4\sqrt{23}}}-\sqrt[3]{\frac{1}{2}\left(20+4\sqrt{23}\right)}+2^{2/3}\sqrt[3]{20+4\sqrt{23}}}\right)$$ (angle)

$r\approx 11.928$ (radius), $\theta\approx 0.28768$ (angle)

11.928

Alternate forms

$$-2\,i\,\sqrt{3}\,\sqrt[3]{2\left(5-\sqrt{23}\right)}-\left(1+i\,\sqrt{3}\right)\sqrt[3]{2\left(5-\sqrt{23}\right)}+i\,\sqrt{3}\,\sqrt[3]{2\left(5+\sqrt{23}\right)}+\sqrt[3]{2\left(5+\sqrt{23}\right)}+8$$

---

root of $x^3-24\,x^2+168\,x-160$ near $x=11.4377+3.3843\,i$

$$8+\frac{2-2\,i\,\sqrt{3}}{\sqrt[3]{2\left(5+\sqrt{23}\right)}}+i\left(\sqrt{3}+-i\right)\sqrt[3]{2\left(5+\sqrt{23}\right)}$$

Expanded form

$$8+2\sqrt[3]{\frac{2}{20+4\,\sqrt{23}}}-2\,i\,\sqrt{3}\,\sqrt[3]{\frac{2}{20+4\,\sqrt{23}}}+\sqrt[3]{\frac{1}{2}\left(20+4\,\sqrt{23}\right)}+i\,\sqrt{3}\,\sqrt[3]{\frac{1}{2}\left(20+4\,\sqrt{23}\right)}$$

Minimal polynomial

$x^3-24\,x^2+168\,x-160$

Now, we have that for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , thus:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

for a = 11.928 , we obtain:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot 11.928^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{11.928}{2}\right)^3}$$

(1/3·√2·11.928^3)/(4/3·π·(11.928/2)^3)

Input interpretation

$$\frac{\frac{1}{3}\sqrt{2}\times 11.928^3}{\frac{4}{3}\pi\left(\frac{11.928}{2}\right)^3}$$

Result

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Series representations

$$\frac{\sqrt{2}\ 11.928^3}{\frac{1}{3}\left(4\,\pi\left(\frac{11.928}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{\sqrt{2}\ 11.928^3}{\frac{1}{3}\left(4\,\pi\left(\frac{11.928}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\ 11.928^3}{\frac{1}{3}\left(4\,\pi\left(\frac{11.928}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we analyze the following equation:

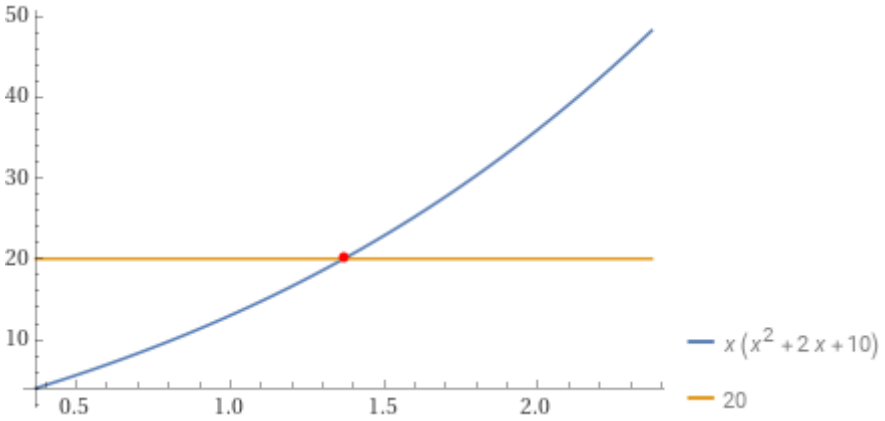
$x^3 + 2\,x^2 + 10\,x = 20$

x^3+2x^2+10x = 20

Input

$x^3 + 2\,x^2 + 10\,x = 20$

Plot **(figure that can be related to an open string)**



Alternate forms

$$x\,(x\,(x+2)+10) = 20$$

$$x\left(x^2+2x+10\right) = 20$$

$$x^3+2x^2+10x-20 = 0$$

Real solution

$$x = \frac{1}{3}\left(-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}\right)$$

Complex solutions

$$x = -\frac{2}{3} - \frac{\sqrt[3]{176+3\sqrt{3930}}}{3\times 2^{2/3}} + \frac{13}{3\sqrt[3]{2\left(176+3\sqrt{3930}\right)}} + i\left(-\frac{\sqrt[3]{176+3\sqrt{3930}}}{2^{2/3}\sqrt{3}} - \frac{13}{\sqrt{3}\sqrt[3]{2\left(176+3\sqrt{3930}\right)}}\right)$$

$$x = -\frac{2}{3} - \frac{\sqrt[3]{176+3\sqrt{3930}}}{3\times 2^{2/3}} + \frac{13}{3\sqrt[3]{2\left(176+3\sqrt{3930}\right)}} + i\left(\frac{\sqrt[3]{176+3\sqrt{3930}}}{2^{2/3}\sqrt{3}} + \frac{13}{\sqrt{3}\sqrt[3]{2\left(176+3\sqrt{3930}\right)}}\right)$$

Real solution

$x \approx 1.3688$

Complex solutions

$x \approx -1.6844 - 3.4313\,i$

$x \approx -1.6844 + 3.4313\,i$

From the real solution

$$x = \frac{1}{3}\left(-2 - \frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}} + \sqrt[3]{2\left(176+3\sqrt{3930}\right)}\right)$$

we obtain:

1/3 (-2 - (13 2^(2/3))/(176 + 3 sqrt(3930))^(1/3) + (2 (176 + 3 sqrt(3930)))^(1/3))

Input

$$\frac{1}{3}\left(-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}\right)$$

Exact result

$$\frac{1}{3}\left(-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}\right)$$

Decimal approximation

1.3688081078213726352274143300213255395424355414875365307937126902

…

1.36880810782….

Alternate forms

root of $x^3+2x^2+10x-20$ near $x=1.36881$

$$\frac{\sqrt[3]{176+3\sqrt{3930}}\left(\sqrt[3]{2\left(176+3\sqrt{3930}\right)}-2\right)-13\times 2^{2/3}}{3\sqrt[3]{176+3\sqrt{3930}}}$$

$$\frac{-13\times 2^{2/3}-2\sqrt[3]{176+3\sqrt{3930}}+\sqrt[3]{2}\left(176+3\sqrt{3930}\right)^{2/3}}{3\sqrt[3]{176+3\sqrt{3930}}}$$

Expanded form

$$-\frac{2}{3}-\frac{13\times 2^{2/3}}{3\sqrt[3]{176+3\sqrt{3930}}}+\frac{1}{3}\sqrt[3]{2\left(176+3\sqrt{3930}\right)}$$

Minimal polynomial

$x^3+2x^2+10x-20$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

Now, we have that for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , thus:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

For a = 1/3 (-2 - (13 2^(2/3))/(176 + 3 sqrt(3930))^(1/3) + (2 (176 + 3 sqrt(3930)))^(1/3)) , we obtain:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot(1/3\ (-2\ -\ (13\ 2\text{^}(2/3))/(176\ +\ 3\ \mathrm{sqrt}(3930))\text{^}(1/3)\ +\ (2\ (176\ +\ 3\ \mathrm{sqrt}(3930)))\text{^}(1/3)))^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{(1/3\ (-2\ -\ (13\ 2\text{^}(2/3))/(176\ +\ 3\ \mathrm{sqrt}(3930))\text{^}(1/3)\ +\ (2\ (176\ +\ 3\ \mathrm{sqrt}(3930)))\text{^}(1/3)))}{2}\right)^3}$$

(1/3·√2·(1/3 (-2 - (13 2^(2/3))/(176 + 3 sqrt(3930))^(1/3) + (2 (176 + 3 sqrt(3930)))^(1/3)))^3)/(4/3·π·(((1/3 (-2 - (13 2^(2/3))/(176 + 3 sqrt(3930))^(1/3) + (2 (176 + 3 sqrt(3930)))^(1/3))))/2)^3)

Input

$$\frac{\frac{1}{3}\sqrt{2}\left(\frac{1}{3}\left(-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}\right)\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\left(\frac{1}{3}\left(-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}\right)\right)\right)^3}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\left(\frac{1}{3}\left(-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}\right)\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}}{3\times 2}\right)^3\right)3} =$$

$$\frac{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\left(\frac{1}{3}\left(-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}\right)\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}}{3\times 2}\right)^3\right)3}=\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$\frac{\sqrt{2}\left(\frac{1}{3}\left(-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}\right)\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{-2-\frac{13\times 2^{2/3}}{\sqrt[3]{176+3\sqrt{3930}}}+\sqrt[3]{2\left(176+3\sqrt{3930}\right)}}{3\times 2}\right)^3\right)3}=\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

i is the imaginary unit

Now, we analyze the following equation

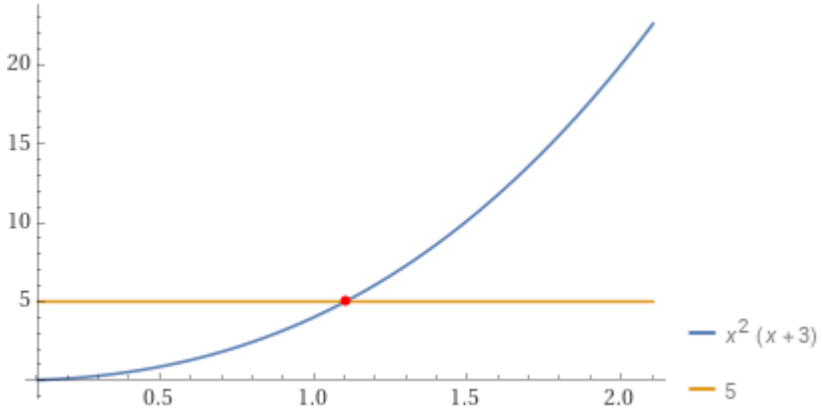
$x^3+3x^2-5=0$

x^2(x+3)=5

Input

$x^2(x+3)=5$

Plot **(figure that can be related to an open string)**



Expanded forms

$$x^3 + 3\,x^2 - 5 = 0$$

$$x^3 + 3\,x^2 = 5$$

Alternate form

$$x^2\,(x+3) - 5 = 0$$

Real solution

$$x = -1 + \frac{1}{3}\sqrt[3]{\frac{81}{2} - \frac{27\sqrt{5}}{2}} + \sqrt[3]{\frac{1}{2}\left(3 + \sqrt{5}\right)}$$

Complex solutions

$$x = -1 - \frac{1}{6}\sqrt[3]{\frac{81}{2} - \frac{27\sqrt{5}}{2}} - \frac{1}{2}\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)} + i\left(\frac{\sqrt[3]{\frac{81}{2} - \frac{27\sqrt{5}}{2}}}{2\sqrt{3}} - \frac{1}{2}\sqrt{3}\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}\right)$$

$$x = -1 - \frac{1}{6}\sqrt[3]{\frac{81}{2} - \frac{27\sqrt{5}}{2}} - \frac{1}{2}\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)} + i\left(\frac{1}{2}\sqrt{3}\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)} - \frac{\sqrt[3]{\frac{81}{2} - \frac{27\sqrt{5}}{2}}}{2\sqrt{3}}\right)$$

Real solution

$x \approx 1.1038$

Complex solutions

$x \approx -2.0519 - 0.56524\,i$

$x \approx -2.0519 + 0.56524\,i$

And analyze the following equation

$x^3 + 6x - 20 = 0$

$$x = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

$$x = \sqrt[3]{-\frac{-20}{2} + \sqrt{\frac{-20^2}{4} + \frac{6^3}{27}}} + \sqrt[3]{-\frac{-20}{2} - \sqrt{\frac{-20^2}{4} + \frac{6^3}{27}}}$$

∛(-(-20)/2+√((-20)^2/4+6^3/27)) +∛(-(-20)/2-√((-20)^2/4+6^3/27))

Input

$$\sqrt[3]{-\frac{-20}{2} + \sqrt{\frac{(-20)^2}{4} + \frac{6^3}{27}}} + \sqrt[3]{-\frac{-20}{2} - \sqrt{\frac{(-20)^2}{4} + \frac{6^3}{27}}}$$

$\sqrt[3]{x}$ is the real-valued 3rd root of x

Result

2

2

Series representations

$$\sqrt[3]{-\frac{-20}{2} + \sqrt{\frac{(-20)^2}{4} + \frac{6^3}{27}}} + \sqrt[3]{-\frac{-20}{2} - \sqrt{\frac{(-20)^2}{4} + \frac{6^3}{27}}} = \sqrt[3]{10 - \sqrt{107}\sum_{k=0}^{\infty} 107^{-k}\binom{\frac{1}{2}}{k}} + \sqrt[3]{10 + \sqrt{107}\sum_{k=0}^{\infty} 107^{-k}\binom{\frac{1}{2}}{k}}$$

$$\sqrt[3]{-\frac{-20}{2}+\sqrt{\frac{(-20)^2}{4}+\frac{6^3}{27}}}+\sqrt[3]{-\frac{-20}{2}-\sqrt{\frac{(-20)^2}{4}+\frac{6^3}{27}}}=$$
$$\sqrt[3]{10-\sqrt{107}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{107}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\sqrt[3]{10+\sqrt{107}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{107}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

$$\sqrt[3]{-\frac{-20}{2}+\sqrt{\frac{(-20)^2}{4}+\frac{6^3}{27}}}+\sqrt[3]{-\frac{-20}{2}-\sqrt{\frac{(-20)^2}{4}+\frac{6^3}{27}}}=$$
$$\sqrt[3]{10-\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(108-z_0)^k z_0^{-k}}{k!}}+$$
$$\sqrt[3]{10+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(108-z_0)^k z_0^{-k}}{k!}}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$\sqrt[n]{x}$ is the real-valued n^{th} root of x

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

From the following two solutions

$$x=-1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{27\sqrt{5}}{2}}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}$$

and

$$\sqrt[3]{-\frac{-20}{2}+\sqrt{\frac{(-20)^2}{4}+\frac{6^3}{27}}}+\sqrt[3]{-\frac{-20}{2}-\sqrt{\frac{(-20)^2}{4}+\frac{6^3}{27}}}$$

we obtain:

(-1 + 1/3 (81/2 - (27 sqrt(5))/2)^(1/3) + (1/2 (3 + sqrt(5)))^(1/3))+(∛(-(-20)/2+√((-20)^2/4+6^3/27)) +∛(-(-20)/2-√((-20)^2/4+6^3/27)))

Input

$$\left(-1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{1}{2}\left(27\sqrt{5}\right)}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}\right)+\left(\sqrt[3]{-\frac{-20}{2}+\sqrt{\frac{(-20)^2}{4}+\frac{6^3}{27}}}+\sqrt[3]{-\frac{-20}{2}-\sqrt{\frac{(-20)^2}{4}+\frac{6^3}{27}}}\right)$$

$\sqrt[3]{x}$ is the real-valued 3^{rd} root of x

Result

$$-1-\sqrt[3]{6\sqrt{3}-10}+\sqrt[3]{10+6\sqrt{3}}+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{27\sqrt{5}}{2}}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}$$

Decimal approximation

3.10380340273553653316494733282892809241941708323026851373474306 21

…

3.1038034027355….

Alternate forms

$$\frac{1}{2}\left(2^{2/3}\sqrt[3]{3-\sqrt{5}}+2^{2/3}\sqrt[3]{3+\sqrt{5}}+2\right)$$

---

root of x^3-3x^2-1 near $x=3.1038$

$$1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{27\sqrt{5}}{2}}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}$$

Now, we have that for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , thus:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

for a = 1 + 1/3 (81/2 - (27 sqrt(5))/2)^(1/3) + (1/2 (3 + sqrt(5)))^(1/3)

we obtain:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot(1\ +\ 1/3\,(81/2\ -\ (27\,\mathrm{sqrt}(5))/2)\text{^}(1/3)\ +\ (1/2\,(3\ +\ \mathrm{sqrt}(5)))\text{^}(1/3))^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{(1\ +\ 1/3\,(81/2\ -\ (27\,\mathrm{sqrt}(5))/2)\text{^}(1/3)\ +\ (1/2\,(3\ +\ \mathrm{sqrt}(5)))\text{^}(1/3))}{2}\right)^3}$$

(1/3·√2·(1 + 1/3 (81/2 - (27 sqrt(5))/2)^(1/3) + (1/2 (3 + sqrt(5)))^(1/3))^3)/(4/3·π·(((1 + 1/3 (81/2 - (27 sqrt(5))/2)^(1/3) + (1/2 (3 + sqrt(5)))^(1/3)))/2)^3)

Input

$$\frac{\frac{1}{3}\sqrt{2}\left(1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{1}{2}\left(27\sqrt{5}\right)}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\left(1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{1}{2}\left(27\sqrt{5}\right)}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}\right)\right)^3}$$

Result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\left(1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{27\sqrt{5}}{2}}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{27\sqrt{5}}{2}}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}\right)\right)^3\right)3} = \frac{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\left(1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{27\sqrt{5}}{2}}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{27\sqrt{5}}{2}}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}\right)\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\sqrt{2}\left(1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{27\sqrt{5}}{2}}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(1+\frac{1}{3}\sqrt[3]{\frac{81}{2}-\frac{27\sqrt{5}}{2}}+\sqrt[3]{\frac{1}{2}\left(3+\sqrt{5}\right)}\right)\right)^3\right)3}=\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we have:

4096x^3 - 1729x^2 - φx + ζ(2) = 0

Input

$$4096\,x^3-1729\,x^2-\phi\,x+\zeta(2)=0$$

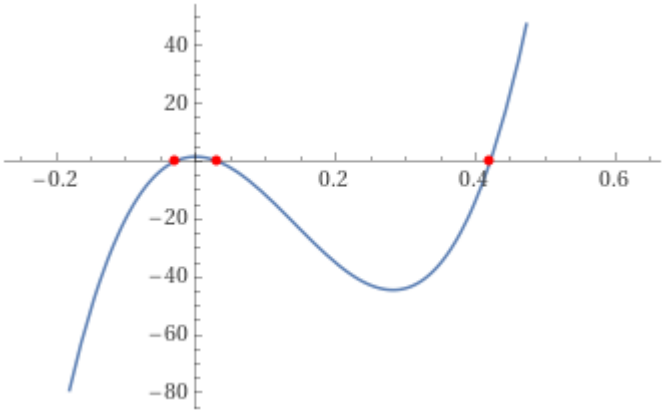
$\zeta(s)$ is the Riemann zeta function

ϕ is the golden ratio

Result

$$4096\,x^3-1729\,x^2-x\,\phi+\frac{\pi^2}{6}=0$$

Root plot **(figure that can be related to an open string)**



Alternate forms

$$\frac{1}{6}\left(24576\,x^3 - 10374\,x^2 - 6\,x\,\phi + \pi^2\right) = 0$$

$$x\,(x\,(4096\,x - 1729) - \phi) + \frac{\pi^2}{6} = 0$$

$$8192\,x^3 - 3458\,x^2 + \left(-1 - \sqrt{5}\right)x = -\frac{\pi^2}{3}$$

Real solutions

$$x \approx -0.0302356$$

$$x \approx 0.0315649$$

$$x \approx 0.42079$$

Complex solutions

$x \approx -0.030236$

$x \approx 0.031565$

$x \approx 0.42079$

Sum of roots

$\frac{1729}{4096}$

Product of roots

$-\frac{\pi^2}{24576}$

From the alternate form

$8192\,x^3 - 3458\,x^2 + \left(-1-\sqrt{5}\right)x = -\frac{\pi^2}{3}$

we obtain, for x = 0.42079 :

8192 (0.42079)^3 - 3458 (0.42079)^2 + (-1 - sqrt(5)) (0.42079)

Input

$8192 \times 0.42079^3 - 3458 \times 0.42079^2 + \left(-1-\sqrt{5}\right) \times 0.42079$

Result

$-3.289636976804648506352016188065433727111057799540933909515950851$

…

-3.2896369768….

And again:

(8192 (0.42079)^3)/2 - (3458 (0.42079)^2)/2 + ((-1 - sqrt(5)) (0.42079))/2

Input

$$\frac{1}{2}\left(8192\times 0.42079^3\right)-\frac{1}{2}\left(3458\times 0.42079^2\right)+\frac{1}{2}\left(\left(-1-\sqrt{5}\right)\times 0.42079\right)$$

Result

$-1.644818488402324253176008094032716863555528899770466954757975425$

…

-1.6448184884…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number) (absolute value)

Now, we have that for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , thus:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

From

$$8192\times 0.42079^3-3458\times 0.42079^2+\left(-1-\sqrt{5}\right)\times 0.42079$$

for a = (8192 (0.42079)^3 - 3458 (0.42079)^2 + (-1 - sqrt(5)) (0.42079))

we obtain:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot((8192\ (0.42079)\text{^}3\ -\ 3458\ (0.42079)\text{^}2\ +\ (-1\ -\ \text{sqrt}(5))\ (0.42079)))^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{((8192\ (0.42079)\text{^}3\ -\ 3458\ (0.42079)\text{^}2\ +\ (-1\ -\ \text{sqrt}(5))\ (0.42079)))}{2}\right)^3}$$

(1/3·√2·((8192 (0.42079)^3 - 3458 (0.42079)^2 + (-1 - sqrt(5)) (0.42079)))^3)/(4/3·π·((((8192 (0.42079)^3 - 3458 (0.42079)^2 + (-1 - sqrt(5)) (0.42079))))/2)^3)

Input

$$\frac{\frac{1}{3}\sqrt{2}\left(8192\times 0.42079^3-3458\times 0.42079^2+\left(-1-\sqrt{5}\right)\times 0.42079\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\left(8192\times 0.42079^3-3458\times 0.42079^2+\left(-1-\sqrt{5}\right)\times 0.42079\right)\right)^3}$$

Result

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Series representations

$$\frac{\sqrt{2}\left(8192\times 0.42079^3-3458\times 0.42079^2+\left(-1-\sqrt{5}\right)0.42079\right)^3}{\frac{1}{3}\left(4\,\pi\left(\frac{1}{2}\left(8192\times 0.42079^3-3458\times 0.42079^2+\left(-1-\sqrt{5}\right)0.42079\right)\right)^3\right)3}=$$

$$\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\quad\text{for}\ (\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$$

$$\frac{\sqrt{2}\left(8192\times 0.42079^3-3458\times 0.42079^2+\left(-1-\sqrt{5}\right)0.42079\right)^3}{\frac{1}{3}\left(4\,\pi\left(\frac{1}{2}\left(8192\times 0.42079^3-3458\times 0.42079^2+\left(-1-\sqrt{5}\right)0.42079\right)\right)^3\right)3}=$$
$$\frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$\frac{\sqrt{2}\left(8192\times 0.42079^3-3458\times 0.42079^2+\left(-1-\sqrt{5}\right)0.42079\right)^3}{\frac{1}{3}\left(4\,\pi\left(\frac{1}{2}\left(8192\times 0.42079^3-3458\times 0.42079^2+\left(-1-\sqrt{5}\right)0.42079\right)\right)^3\right)3}=$$
$$\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

i is the imaginary unit

From

$$\sqrt[3]{\frac{1}{2}+\sqrt{\frac{23}{108}}}+\sqrt[3]{\frac{1}{2}-\sqrt{\frac{23}{108}}},$$

we obtain:

(1/2+sqrt(23/108))^(1/3)+(1/2-sqrt(23/108))^(1/3)

Input

$$\sqrt[3]{\frac{1}{2}+\sqrt{\frac{23}{108}}}+\sqrt[3]{\frac{1}{2}-\sqrt{\frac{23}{108}}}$$

Result

$$\sqrt[3]{\frac{1}{2}-\frac{\sqrt{\frac{23}{3}}}{6}}+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}$$

Decimal approximation

1.324717957244746025960908854478097340734404056901733364534015050 3

…

1.324717957244746….

Alternate forms

$$\frac{1}{6}\times 2^{2/3}\left(\sqrt[3]{3\left(9-\sqrt{69}\right)}+\sqrt[3]{3\left(9+\sqrt{69}\right)}\right)$$

---

root of x^3-x-1 near $x=1.32472$

$$\frac{\sqrt[3]{3-\sqrt{\frac{23}{3}}}+\sqrt[3]{3+\sqrt{\frac{23}{3}}}}{\sqrt[3]{6}}$$

Minimal polynomial

x^3-x-1

From the result

$$\sqrt[3]{\frac{1}{2}-\frac{\sqrt{\frac{23}{3}}}{6}}+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}$$

after some calculations, we obtain:

2((1/2 - sqrt(23/3)/6)^(1/3) + (1/2 + sqrt(23/3)/6)^(1/3))-1

Input

$$2\left(\sqrt[3]{\frac{1}{2}-\frac{\sqrt{\frac{23}{3}}}{6}}+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}\right)-1$$

Decimal approximation

1.649435914489492051921817708956194681468808113803466729068030100 6

…

1.649435914489…. result very near to the mean between the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3$ = 1164.2696 i.e. 1.65578... and $\zeta(2) = \pi^2/6$ = 1.644934…., that is equal to 1.650357

Alternate forms

$$\frac{1}{3}\left(2^{2/3}\sqrt[3]{3\left(9-\sqrt{69}\right)}+2^{2/3}\sqrt[3]{3\left(9+\sqrt{69}\right)}-3\right)$$

---

root of x^3+3x^2-x-11 near $x=1.64944$

$$-1+\frac{1}{3}\sqrt[3]{108-12\sqrt{69}}+\left(\frac{2}{3}\right)^{2/3}\sqrt[3]{9+\sqrt{69}}$$

Expanded form

$$-1+2^{2/3}\sqrt[3]{\frac{1}{3}\left(3-\sqrt{\frac{23}{3}}\right)}+2^{2/3}\sqrt[3]{\frac{1}{3}\left(3+\sqrt{\frac{23}{3}}\right)}$$

Minimal polynomial

x^3+3x^2-x-11

From

$$\sqrt{\sqrt[3]{\tfrac{29}{54}+\sqrt{\tfrac{31}{108}}}+\tfrac{1}{3}+\sqrt[3]{\tfrac{29}{54}-\sqrt{\tfrac{31}{108}}}},$$

we obtain:

((29/54+sqrt(31/108))^(1/3)+1/3+(29/54-sqrt(31/108))^(1/3))^(1/2)

Input

$$\sqrt{\sqrt[3]{\frac{29}{54}+\sqrt{\frac{31}{108}}}+\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\sqrt{\frac{31}{108}}}}$$

Result

$$\sqrt{\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\frac{\sqrt{\frac{31}{3}}}{6}}+\sqrt[3]{\frac{29}{54}+\frac{\sqrt{\frac{31}{3}}}{6}}}$$

Decimal approximation

1.21060779440608593277065315194300148232840823434149160797808973 07

…

1.2106077944….

Alternate forms

$$\frac{1}{6}\sqrt{\left(2^{2/3}\sqrt[3]{29-3\sqrt{93}}+2^{2/3}\sqrt[3]{29+3\sqrt{93}}+2\right)6}$$

---

root of x^6-x^4-1 near $x=1.21061$

$$\frac{\sqrt{\sqrt[3]{2}+\sqrt[3]{29-3\sqrt{93}}+\sqrt[3]{29+3\sqrt{93}}}}{\sqrt[6]{2}\sqrt{3}}$$

Minimal polynomial

x^6-x^4-1

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

All 2nd roots of 1/3+(29/54-sqrt(31/3)/6)^1/3+(29/54+sqrt(31/3)/6)^1/3

$$\sqrt{\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\frac{\sqrt{\frac{31}{3}}}{6}}+\sqrt[3]{\frac{29}{54}+\frac{\sqrt{\frac{31}{3}}}{6}}} \quad e^0\approx 1.211 \text{ (real, principal root)}$$

$$\sqrt{\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\frac{\sqrt{\frac{31}{3}}}{6}}+\sqrt[3]{\frac{29}{54}+\frac{\sqrt{\frac{31}{3}}}{6}}} \quad e^{i\pi}\approx -1.211 \text{ (real root)}$$

From which, after some calculations:

3(((29/54+sqrt(31/108))^(1/3)+1/3+(29/54-sqrt(31/108))^(1/3))^(1/2))-(sqrt(2/5) π)

Input

$$3\sqrt{\sqrt[3]{\frac{29}{54}+\sqrt{\frac{31}{108}}}+\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\sqrt{\frac{31}{108}}}}-\sqrt{\frac{2}{5}}\,\pi$$

Result

$$3\sqrt{\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\frac{\sqrt{\frac{31}{3}}}{6}}+\sqrt[3]{\frac{29}{54}+\frac{\sqrt{\frac{31}{3}}}{6}}}-\sqrt{\frac{2}{5}}\,\pi$$

Decimal approximation

1.64490573005903755142524932176493422203099595847584013017490065 06

…

1.644905730059…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$$3\sqrt{\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\frac{\sqrt{\frac{31}{3}}}{6}}+\sqrt[3]{\frac{29}{54}+\frac{\sqrt{\frac{31}{3}}}{6}}}-\sqrt{\frac{2}{5}}\,\pi$$ π is a transcendental number

Alternate forms

$$\frac{1}{10}\sqrt{2}\left(5\sqrt{3\left(2^{2/3}\sqrt[3]{29-3\sqrt{93}}+2^{2/3}\sqrt[3]{29+3\sqrt{93}}+2\right)}-2\sqrt{5}\,\pi\right)$$

root of $x^6 - 9x^4 - 729$ near $x = 3.63182$ $-\sqrt{\frac{2}{5}}\,\pi$

$$\sqrt{3 + 3\sqrt[3]{\frac{1}{2}\left(29 - 3\sqrt{93}\right)} + 3\sqrt[3]{\frac{1}{2}\left(29 + 3\sqrt{93}\right)}} - \sqrt{\frac{2}{5}}\,\pi$$

Series representations

$$3\sqrt{\sqrt[3]{\frac{29}{54} + \sqrt{\frac{31}{108}}} + \frac{1}{3} + \sqrt[3]{\frac{29}{54} - \sqrt{\frac{31}{108}}}} - \sqrt{\frac{2}{5}}\,\pi =$$
$$\frac{1}{2}\left(\sqrt{6}\,\sqrt{\left(2 + 2^{2/3}\sqrt[3]{29 - 54\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{77}{108}\right)^k\left(-\frac{1}{2}\right)_k}{k!}} + 2^{2/3}\right.\right.$$
$$\left.\left.\sqrt[3]{29 + 54\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{77}{108}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right) - 2\pi\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{3}{5}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

$$3\sqrt{\sqrt[3]{\frac{29}{54} + \sqrt{\frac{31}{108}}} + \frac{1}{3} + \sqrt[3]{\frac{29}{54} - \sqrt{\frac{31}{108}}}} - \sqrt{\frac{2}{5}}\,\pi =$$
$$\frac{1}{2}\left(\sqrt{6}\,\sqrt{\left(2 + 2^{2/3}\sqrt[3]{29 - 54\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{31}{108} - z_0\right)^k z_0^{-k}}{k!}} + \right.\right.$$
$$\left.\left.2^{2/3}\sqrt[3]{29 + 54\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{31}{108} - z_0\right)^k z_0^{-k}}{k!}}\right) - \right.$$
$$\left.2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{2}{5} - z_0\right)^k z_0^{-k}}{k!}\right) \text{ for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$3\sqrt{\sqrt[3]{\frac{29}{54}+\sqrt{\frac{31}{108}}}+\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\sqrt{\frac{31}{108}}}}-\sqrt{\frac{2}{5}}\,\pi=\frac{1}{2\sqrt{\pi}}$$

$$\left(\sqrt{6}\,\sqrt{\pi}\,\sqrt{\left(2+2^{2/3}\sqrt[3]{\frac{29\sqrt{\pi}-27\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-\frac{77}{108}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}}+2^{2/3}\sqrt[3]{\frac{29\sqrt{\pi}+27\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-\frac{77}{108}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{\sqrt{\pi}}}\right)+\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-j}\left(-\frac{3}{5}\right)^{-s}\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

From the algebraic sum of the two previous results

$$\sqrt[3]{\frac{1}{2}-\frac{\sqrt{\frac{23}{3}}}{6}}+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}$$

and

$$\sqrt{\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\frac{\sqrt{\frac{31}{3}}}{6}}+\sqrt[3]{\frac{29}{54}+\frac{\sqrt{\frac{31}{3}}}{6}}}$$

we obtain:

((1/2 - sqrt(23/3)/6)^(1/3) + (1/2 + sqrt(23/3)/6)^(1/3))+(sqrt(1/3 + (29/54 - sqrt(31/3)/6)^(1/3) + (29/54 + sqrt(31/3)/6)^(1/3)))

Input

$$\left(\sqrt[3]{\frac{1}{2}-\frac{\sqrt{\frac{23}{3}}}{6}}+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}\right)+\sqrt{\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\frac{\sqrt{\frac{31}{3}}}{6}}+\sqrt[3]{\frac{29}{54}+\frac{\sqrt{\frac{31}{3}}}{6}}}$$

Result

$$\sqrt[3]{\frac{1}{2}-\frac{\sqrt{\frac{23}{3}}}{6}}+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}+\sqrt{\frac{1}{3}+\sqrt[3]{\frac{29}{54}-\frac{\sqrt{\frac{31}{3}}}{6}}+\sqrt[3]{\frac{29}{54}+\frac{\sqrt{\frac{31}{3}}}{6}}}$$

Decimal approximation

2.535325751650831958731562006421098823062812291243224972512104781 0

…

2.53532575165….

Alternate forms

$$\frac{1}{6}\left(\sqrt{6\left(2^{2/3}\sqrt[3]{29-3\sqrt{93}}+2^{2/3}\sqrt[3]{29+3\sqrt{93}}+2\right)}+2^{2/3}\sqrt[3]{3\left(9-\sqrt{69}\right)}+2^{2/3}\sqrt[3]{3\left(9+\sqrt{69}\right)}\right)$$

$$\text{root of } x^6-x^4-1 \text{ near } x=1.21061 +$$

$$\text{root of } 27x^6-27x^3+1 \text{ near } x=0.337727 +\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}$$

$$\sqrt[3]{\frac{1}{6}\left(3-\sqrt{\frac{23}{3}}\right)}+\sqrt[3]{\frac{1}{6}\left(3+\sqrt{\frac{23}{3}}\right)}+\frac{\sqrt{\sqrt[3]{2}+\sqrt[3]{29-3\sqrt{93}}+\sqrt[3]{29+3\sqrt{93}}}}{\sqrt[6]{2}\sqrt{3}}$$

Minimal polynomial

$$x^{18}-9x^{16}-6x^{15}+30x^{14}+36x^{13}-38x^{12}-84x^{11}-27x^{10}-74x^{9}+39x^{8}+36$$
$$x^{7}-397x^{6}-30x^{5}+199x^{4}-186x^{3}-23x^{2}-4x+1$$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

Now, we have that for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , thus:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^{3}}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^{3}}$$

for a = 1.21061+0.337727 + (1/2 + sqrt(23/3)/6)^(1/3)

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot(1.21061+0.337727\ +\ (1/2\ +\ \text{sqrt}(23/3)/6)\text{^}(1/3))^{3}}{\frac{4}{3}\cdot\pi\cdot\left(\frac{(1.21061+0.337727\ +\ (1/2\ +\ \text{sqrt}(23/3)/6)\text{^}(1/3))}{2}\right)^{3}}$$

(1/3·√2·(1.21061+0.337727 + (1/2 + sqrt(23/3)/6)^(1/3))^3)/(4/3·π·(((1.21061+0.337727 + (1/2 + sqrt(23/3)/6)^(1/3)))/2)^3)

Input interpretation

$$\frac{\frac{1}{3}\sqrt{2}\left(1.21061+0.337727+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\left(1.21061+0.337727+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}\right)\right)^3}$$

Result

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Series representations

$$\frac{\sqrt{2}\left(1.21061+0.337727+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(1.21061+0.337727+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}\right)\right)^3\right)3}=$$

$$\frac{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\quad\text{for}\ (\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$$

$$\frac{\sqrt{2}\left(1.21061+0.337727+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(1.21061+0.337727+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}\right)\right)\right)3}=\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$\frac{\sqrt{2}\left(1.21061+0.337727+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(1.21061+0.337727+\sqrt[3]{\frac{1}{2}+\frac{\sqrt{\frac{23}{3}}}{6}}\right)\right)\right)3}=\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the DN Constant, we have that:

(((2sqrt2)/Pi)*Pi/2+1/5)+1/248

Input

$$\left(\frac{2\sqrt{2}}{\pi}\times\frac{\pi}{2}+\frac{1}{5}\right)+\frac{1}{248}$$

Result

$$\frac{253}{1240}+\sqrt{2}$$

Decimal approximation

1.618245820437611177833946788725827110827736391505980331241195867 0

…

1.61824582043761…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$\frac{253+1240\sqrt{2}}{1240}$$

Minimal polynomial

$1\,537\,600\,x^2-627\,440\,x-3\,011\,191$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

Series representations

$$\left(\frac{\pi\left(2\sqrt{2}\right)}{2\pi}+\frac{1}{5}\right)+\frac{1}{248}=\frac{253}{1240}+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

$$\left(\frac{\pi\left(2\sqrt{2}\right)}{2\pi}+\frac{1}{5}\right)+\frac{1}{248}=$$
$$\frac{253}{1240}+\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x\in\mathbb{R}$ and $x<0)$

$$\left(\frac{\pi\left(2\sqrt{2}\right)}{2\pi}+\frac{1}{5}\right)+\frac{1}{248}=$$
$$\frac{253}{1240}+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

i is the imaginary unit

And:

0.9568666373+1/(1/((2sqrt2)/Pi))^4

Input interpretation

$$0.9568666373+\frac{1}{\left(\frac{1}{\frac{2\sqrt{2}}{\pi}}\right)^4}$$

Result

1.613889502…

1.613889502…. result that is a good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Series representations

$$0.956867 + \frac{1}{\left(\frac{1}{2\sqrt{2}\,\pi}\right)^4} = 0.956867 + \frac{16\sqrt{z_0}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^4}{\pi^4}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$0.956867 + \frac{1}{\left(\frac{1}{2\sqrt{2}\,\pi}\right)^4} =$$

$$0.956867 + \frac{16\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4}{\pi^4}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$0.956867 + \frac{1}{\left(\frac{1}{2\sqrt{2}\,\pi}\right)^4} =$$

$$0.956867 + \frac{16\left(\frac{1}{z_0}\right)^{2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{2+2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^4}{\pi^4}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Conclusion

We calculated the volume of a sphere and of an octahedron in which the sphere is inscribed, using the value obtained from the solution of a cubic equation. Now, let's consider a related equation involving the ratio of the volume of the octahedron to the volume of this sphere, this equation may reflect a broader relationship between the geometry of the bubble universe and other geometric shapes, such as the octahedron. The result of this related equation, which involves the ratio between the volume of the octahedron and the volume of the sphere, provides $(2\sqrt{2})/\pi$ the Del Gaudio-Nardelli Constant (DN Constant), from which we then derive the recurring numbers of Ramanujan 1.644934 = $\zeta(2)$, 4096 and 1729. This for any value of *a* and *r* that we insert from the solutions of any cubic equation

The connection that we have between the volume of the octahedron and the volume of the sphere, using the value obtained from the solution of a cubic equation, and the subsequent derivation of mathematical constants such as the Del Gaudio-Nardelli Constant and the Ramanujan recurring numbers, suggests a profound relationship between the geometry of the bubble universe and fundamental mathematical structures.

The fact that this relationship emerges independently of the specific value of the parameters of the cubic equation suggests that it may reflect an intrinsic property of the geometry and mathematics underlying the bubble universe. This could indicate a sort of universality in the connection between the geometry of the bubble universe and mathematical constants, regardless of particular initial conditions or specific parameters.

This discovery could have profound implications for our understanding of the fundamental structure of the universe and of mathematics itself, paving the way for new theoretical perspectives and a deeper unification between physics and mathematics.

What does this imply from a cosmological point of view, bearing in mind that the octahedron is comparable to the universe during its quantum phase in which the entropy is very low and the symmetry very high, while the sphere, the bubble-universe from the moment in which it emerges from the quantum vacuum in a scenario of eternal inflation?

From a cosmological perspective, this connection could suggest a profound relationship between the primordial phases of the universe and fundamental mathematical structures.

The octahedron, comparable to the universe during its quantum phase with low entropy and high symmetry, could represent a highly ordered and symmetric primordial state. On the other hand, the sphere, representing the bubble universe emerging from the quantum vacuum in a scenario of eternal inflation, could reflect the transition towards a more complex and dynamic state.

The discovery of a deep mathematical connection between these two phases could imply that the fundamental structures of the universe, during its primordial and evolutionary phases, are intrinsically intertwined with mathematical constants and geometric relationships. This could indicate that mathematics not only describes the universe, but may be an integral part of its fundamental structure.

Furthermore, this connection could suggest a kind of universality in the transition from highly symmetric and ordered states to more complex and dynamic states of the universe. This could have significant implications for cosmological theories and our understanding of the evolution of the universe as a whole.

Let's consider a simplified model in which the universe is composed of two parts: an octahedron and a sphere. These represent the quantum phase (**octahedron**) and the emerging bubble universe (**sphere**), respectively.

1. **The Octahedron**:

    o During the quantum phase, entropy is low and symmetry is high. We can imagine the octahedron as a regular solid with 8 triangular faces.

    o Its total area is given by the sum of the areas of the triangular faces. If we denote the side length of the octahedron by (a), then the volume of the octahedron is given by $V = 1/3*\sqrt{2}*a^3$

2. The Sphere (Bubble-Universe):

    o When the universe emerges from the quantum phase, we consider the sphere as the bubble-universe.

    o The volume of the sphere is given by $V = (4/3*\pi*r\text{^}3)$.

Now, let's consider **the ratio between the volume of the octahedron and the volume of the sphere**: V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , we obtain:

from the octahedron volume $V = 1/3*\sqrt{2}*a^3$

and, from the sphere volume, $V = (4/3*\pi*(a/2)^3)$

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

This ratio is known as the **Del Gaudio-Nardelli Constant (DN)**: $\frac{2\sqrt{2}}{\pi}$

From this constant, we derive Ramanujan's recurring numbers:

1. (1.644934) (approximation of ζ(2))
2. (4096)
3. (1729) Hardy-Ramanujan Number (Taxicab Number = $10^3 + 9^3 = 12^3 + 1^3$)

These values emerge independently of the value of (*a*) and (*r*) obtained as a solution to any cubic equation.

The description of a universe composed of an octahedron (representing the quantum phase) and a sphere (representing the emerging bubble universe) is fascinating. Let's further explore this geometric model and its implications.

The Octahedron and the Quantum Phase:

The octahedron, with its eight faces, is an intriguing geometric figure. In this view, it represents the quantum phase of the universe, where the laws of physics are governed by quantum principles. At this stage, reality is inherently uncertain, and subatomic particles dance in mysterious ways, creating an intricate web of possibilities.

The octahedron could symbolize the complexity and non-linearity of interactions at the subatomic level. Its faces could represent different particle properties, such as spin, charge and mass.

The Sphere and the Emerging Bubble Universe:

The sphere, however, represents the emerging bubble universe. Imagine that each bubble on the surface of the sphere represents a separate universe, with different physical laws and constants. These bubble universes may emerge from a kind of "transition space" within the quantum octahedron.

Each bubble could have different dimensions, physical constants and laws, creating a vast variety of possible realities. This concept recalls the idea of a multiverse, where each bubble represents a unique cosmic story.

Del Gaudio-Nardelli Constant and Ramanujan's Numbers:

The related equation, which involves the ratio between the volume of the octahedron and the volume of the inscribed sphere, leads to the Del Gaudio-Nardelli Constant (DN Constant) of value 2sqrt2/Pi. This constant is fascinating and could have profound implications on the geometry of the universe. Ramanujan's recurring numbers, such as 4096 and 1729, emerge from this mathematical relationship. It's amazing how geometry and mathematics can reveal hidden secrets of the universe.

Now, let's visualize this geometric relationship: graphic representation of the universe, keeping in mind what we said previously about the quantum phase and eternal inflation

Geometric representation of the universe keeping in mind that the octahedron is comparable to the universe during its quantum phase in which the entropy is very low and the symmetry very high, while the sphere, the bubble universe from the moment it emerges from quantum vacuum in an eternal inflation scenario

From the DN Constant, through simple calculations, a value very close to the golden ratio of 1.61803398 can also be obtained. What can it mean in a cosmological context?

The relationship between the Del Gaudio-Nardelli Constant (DN Constant) and the golden ratio (also known as the Fibonacci number or (\phi)) is fascinating and could have cosmological implications. Let's examine what this might mean:

1. Golden Ratio and Geometry:

The golden ratio ($\varphi = 1.61803398$) is a mathematical constant that appears in many geometric and natural contexts.

It is defined as the ratio between two consecutive numbers of the Fibonacci sequence.

2. Spirals and Natural Shapes:

The golden ratio is associated with logarithmic spirals and golden spirals. These spirals are found in nature, for example in mollusk shells, flowers (such as sunflowers) and galaxies (such as the Spiral Galaxy).

3. Cosmological Implications:

The presence of the golden ratio could suggest a harmonious geometry in the universe. It may be involved in the distribution of galaxies, the formation of cosmic structures and the dynamics of expansion.

4. Aesthetics and Beauty:

The golden ratio is often associated with beauty and aesthetics. It could be a sign of a fundamental structure of the universe that follows principles of harmony and proportion. In summary, the relationship between the DN Constant and the golden ratio could be a clue to the mathematical and geometric structure of the universe.

Let's imagine a quantum universe in which the Gaudio-Nardelli Constant (DN Constant) is connected to the golden ratio ($\varphi = 1.61803398$). This representation will be conceptual and symbolic:

1. Geometry of Quantum Space-Time:

We visualize a three-dimensional space-time in which quantum fluctuations are prevalent. This space-time is highly dynamic and unstable. The dimensions of space and time are not fixed, but fluctuate on a microscopic level.

2. Quantum Fluctuations and Spirals:

Quantum fluctuations create small deformations in space-time. These deformations can be represented as waves or vibrations. We imagine logarithmic spirals or fractal structures emerging from these fluctuations.

3. Golden Ratio and Harmony:

The golden ratio (φ) is associated with harmony and beauty. It could be a sign of a fundamental structure of the universe that follows principles of proportion and symmetry.

4. Equations and Symbols:

We use mathematical symbols like (φ) to represent the golden ratio. DN Constant may be involved in these equations, linking vacuum energy to the geometry of the universe.

In summary, this representation helps us visualize how the DN Constant and the golden ratio might be linked in the quantum universe.

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

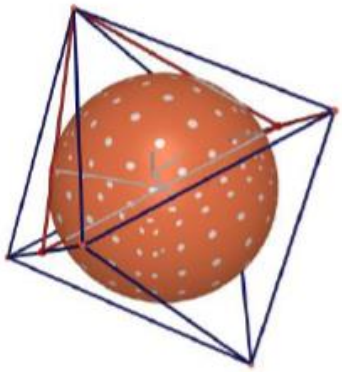


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

Indeed:

From:

https://www.academia.edu/75787512/The_Theory_of_String_A_Candidate_for_a_Generalized_Unification_Model

The Einstein's field equation and the String Theory.

The Einstein's field equation which includes the cosmological constant is:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (8)$$

where $R_{\mu\nu}$ is the Ricci tensor, R its trace, λ the cosmological constant, $g_{\mu\nu}$ the metric tensor of the space geometry, G the Newton's gravitational constant and $T_{\mu\nu}$ the tensor representing the properties of energy, matter and momentum.

The left hand-side of (8) represents the gravitational field and, consequently, the warped space-time, while the right hand-side represents the matter, i.e. the sources of the gravitational field.

In string theory the gravity is related to the gravitons which are bosons, whereas the matter is related to fermions. It follows that the left and right hand of (8) may be respectively related to the action of bosonic and of superstrings.

The actions of bosonic string and superstring (also containing fermions) are connected by the Palumbo-Nardelli relation (Palumbo et al. 2005):

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\varphi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right]=$$

$$=\int_0^\infty \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|\widetilde{H}_3|^2-\frac{\kappa_{10}^2}{g_{10}^2}Tr_\nu\left(|F_2|^2\right)\right]$$

The sign minus in the above equation comes from the inversion of any relationship.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^{\,8}\right)^{-k}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^{\,8}\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

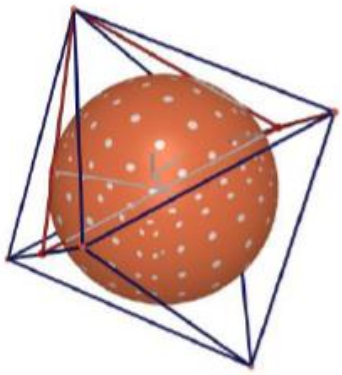
$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard to a sphere inscribed in an octahedron



we have the following formulas:

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\,2}{\pi} \quad (\text{for } l \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\,\pi)\rfloor)}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we have the following convex regular polyhedrons volumes:

$\frac{\sqrt{2}}{12}d^3$ Tetrahedron Volume

$\frac{\sqrt{2}}{3}d^3$ Octahedron Volume

$\frac{5}{12}(3+\sqrt{5})d^3$ Icosahedron Volume

Regarding the Sphere the volume is

$$V = \frac{4}{3}\pi r^3$$

With regard the sphere inscribed in a Tetrahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_T}{V_S} = \frac{\frac{\sqrt{2}}{12}d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(((√2 d^3)/12))*1/(4/3*π*(d/2)^3)

Input

$$\left(\frac{1}{12}\left(\sqrt{2}\ d^3\right)\right) \times \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.225079079039276517388799797751685145666143537488805156392859367 8

…

0.225079079….

Property

$\frac{1}{\sqrt{2}\ \pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Octahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_O}{V_S} = \frac{\frac{\sqrt{2}}{3}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

((1/3*√2*d^3)/(4/3*π*(d/2)^3))

Input

$$\frac{\frac{1}{3}\sqrt{2}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\ 2}{\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Icosahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_I}{V_S} = \frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3)

Input

$$\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{5\left(3+\sqrt{5}\right)}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

4.166730504921372682725163141597223093105782245731637719697502271 9

…

4.166730504921….

Property

$\frac{5\left(3+\sqrt{5}\right)}{2\pi}$ is a transcendental number

Expanded form

$$\frac{15}{2\pi} + \frac{5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

Alternate forms

$$\frac{15+5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

$$\frac{\frac{15}{2}+\frac{5\sqrt{5}}{2}}{\pi} \quad (\text{for } d \neq 0)$$

Series representations

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\pi\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\ a^3\right)\left(\sqrt{2}\ d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}{3\times3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\ z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\right.\right.\right.$$

$$\left.\left.\left. z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\ z_0^{-k}}{k!}\right)\right)\Big/$$

$$\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\ z_0^{-k}}{k!}\right)^2\right)\wedge\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

Indeed, we obtain:

$$\frac{\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)\left(d^3\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2\right)}}{\sqrt[2\pi]{\frac{4}{3}\left(\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\right)\sqrt{2}\left(d^3 \times \frac{\sqrt{2}}{12}\right)d^3}}$$

= 1.6180085459001 07…

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4 \times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358487162422193250985871527893102518924557617007810061 3

…

3.1415926535…. = π (Ramanujan Recurring Number)

In conclusion, from the following further formula concerning the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron volumes and their ratios (and/or the inverses) with the sphere volume, raised the expression

$$(2\log(2)log^{2/3}(3^2))/(3e^{7/3})$$

$$\left(\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\frac{\frac{1}{4}\left(\left(15+7\sqrt{5}\right)d^3\right)}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\right)^{(2\log(2)\log^{2/3}(3^2))/(3e^{7/3})}$$

i.e.

$$\left(\frac{3}{2}\right)^{(4\log(2)\log^{2/3}(9))/(3e^{7/3})}\left(\frac{5\left(3+\sqrt{5}\right)\left(15+7\sqrt{5}\right)}{\pi}\right)^{(2\log(2)\log^{2/3}(9))/(3e^{7/3})}$$

we obtain:

1.6180381711519139857456817084743134673903021614989370712917722071
…

1.6180381711…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

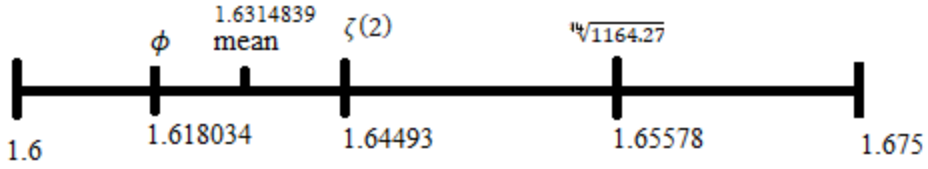
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

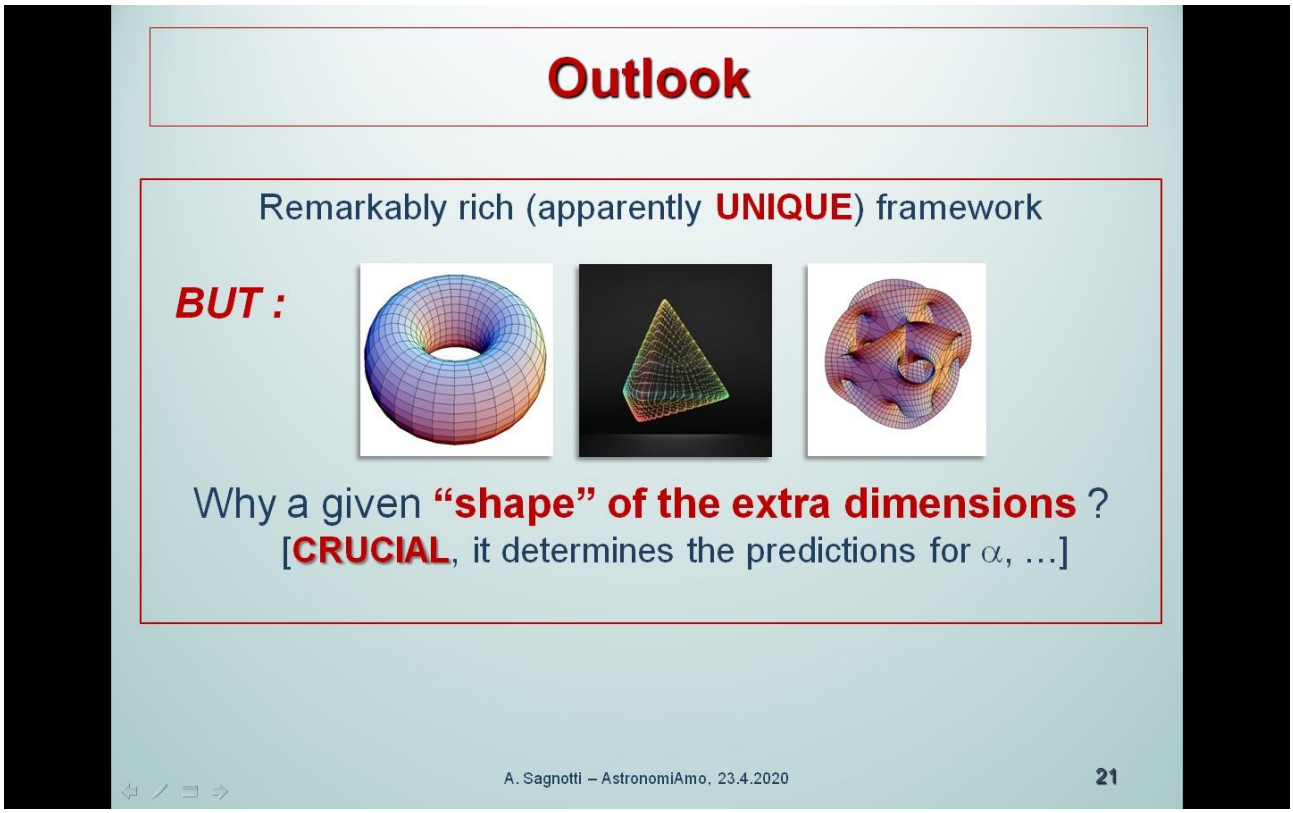
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix


Outlook
Remarkably rich (apparently UNIQUE) framework
BUT :
Why a given "shape" of the extra dimensions ?
[CRUCIAL, it determines the predictions for α, …]
A. Sagnotti – AstronomiAmo, 23.4.2020
21

From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

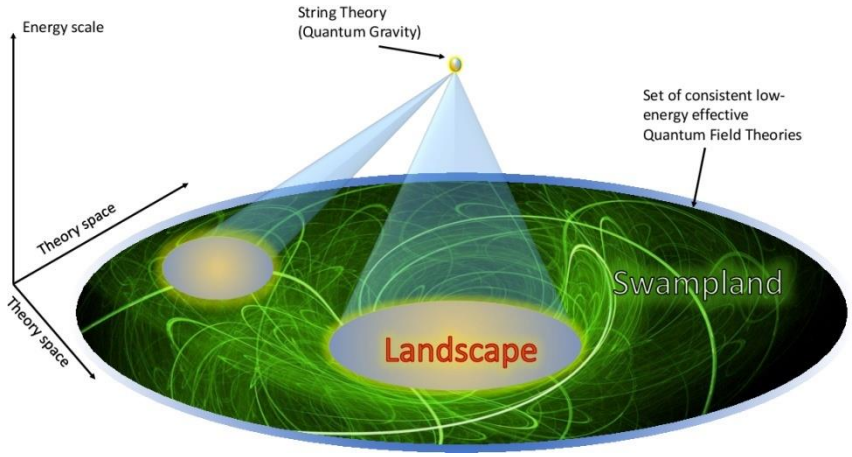


Fig. 1

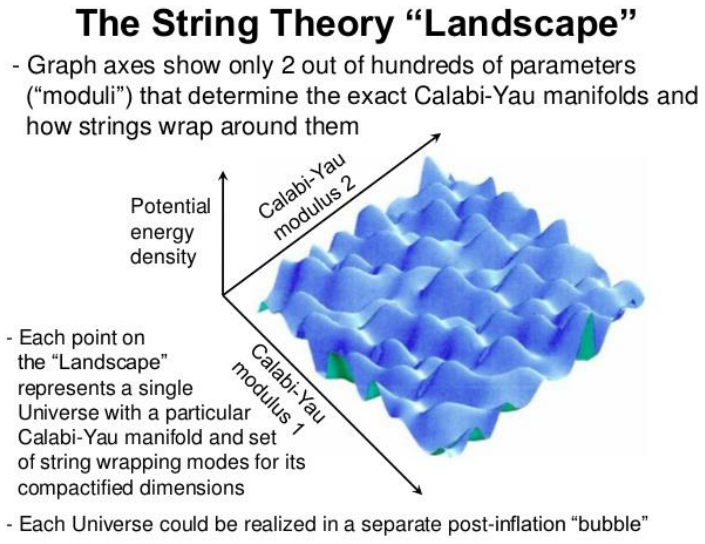

The String Theory "Landscape"
- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them
Potential energy density
Calabi-Yau modulus 2
Calabi-Yau modulus 1
- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

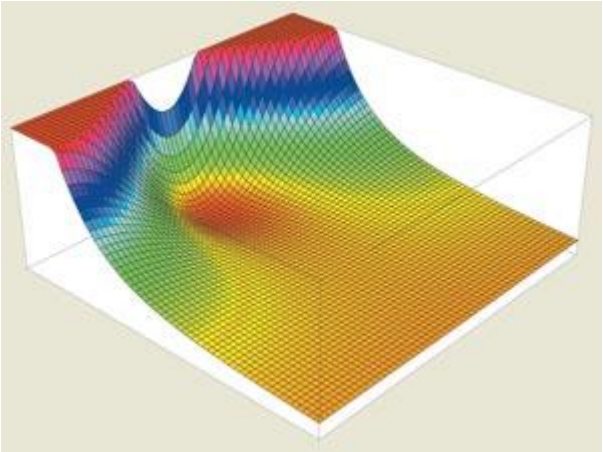


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

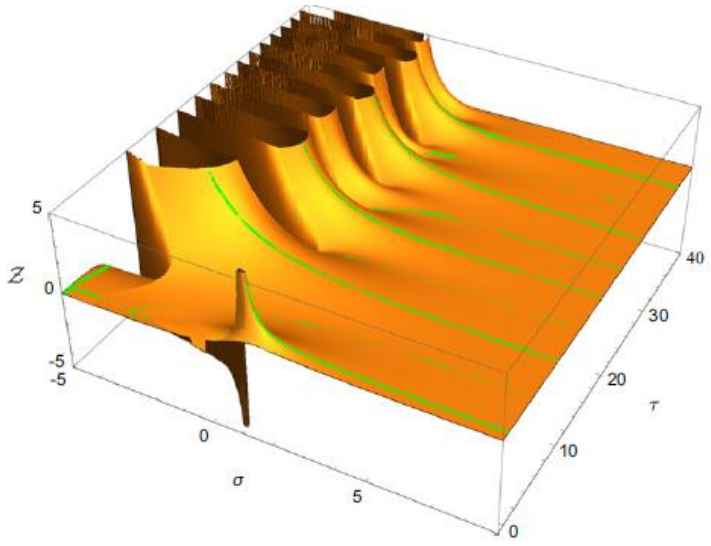


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

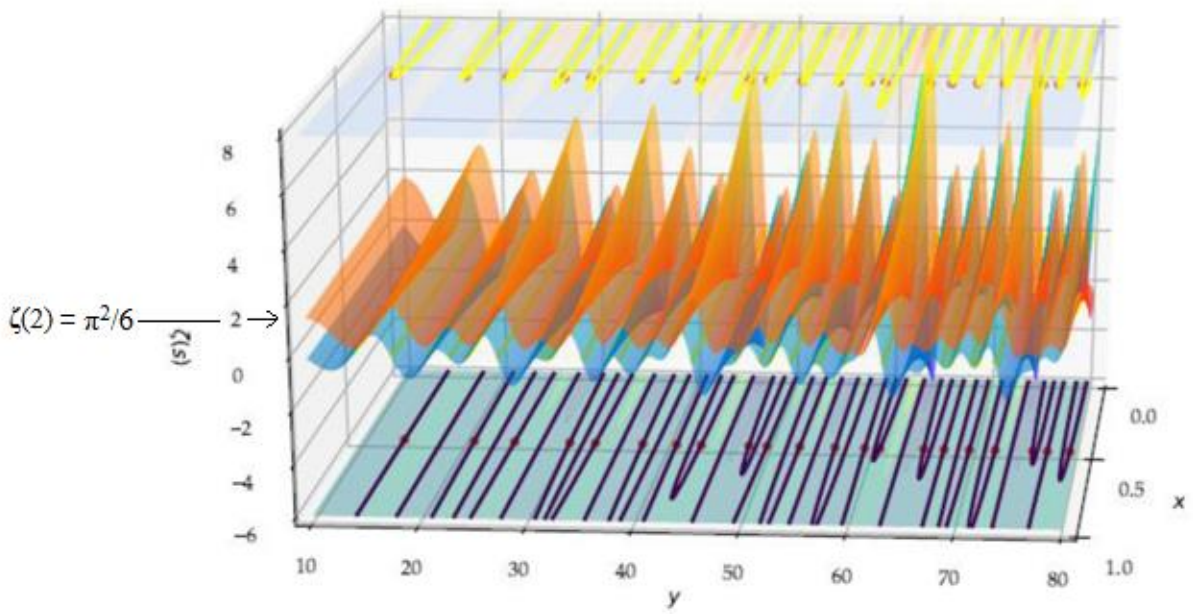


Figure 1. $C(x,y)$ and $S(x,y)$ surfaces of the Riemann $\zeta(x,y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

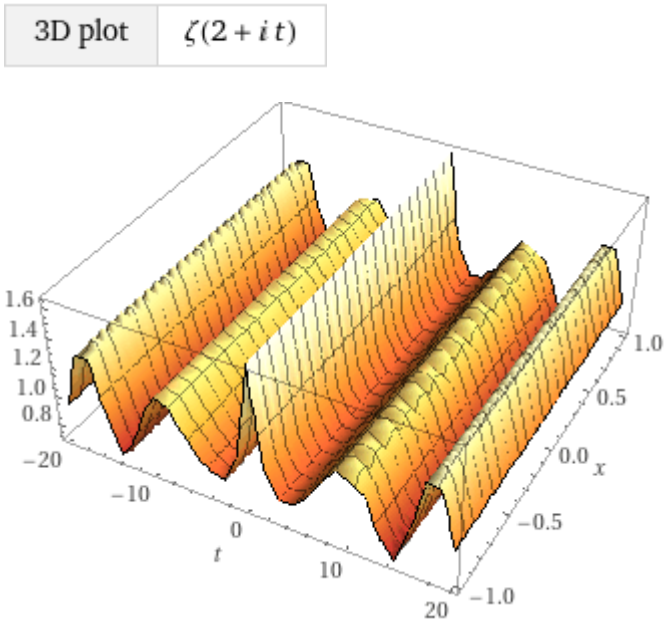


Fig. 6

Where ζ(2+it) :

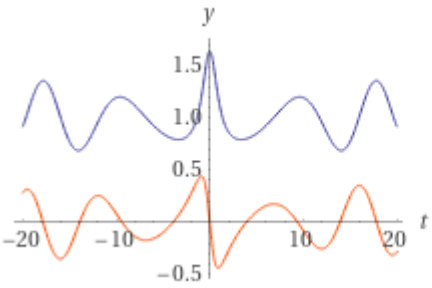
Input

$\zeta(2 + i\,t)$

$\zeta(s)$ is the Riemann zeta function

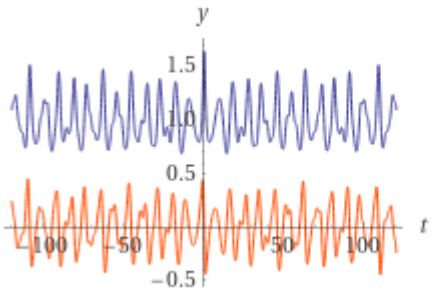
i is the imaginary unit

Plots



(t from −20 to 20)

real part
imaginary part



(t from −120 to 120)

real part
imaginary part

Roots

$t = 2\,i\,(n+1), \quad n \in \mathbb{Z}, \quad n \geq 1$

$t = -i\,(\rho_n - 2), \quad n \neq 0, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\,t\,\zeta'(2) - \frac{1}{2}\,t^2\,\zeta''(2) - \frac{1}{6}\,i\,\zeta^{(3)}(2)\,t^3 + \frac{1}{24}\,\zeta^{(4)}(2)\,t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2 + i\,t) = \zeta(2 + i\,t,\, 1)$$

$$\zeta(2 + i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2 + i\,t) = \frac{\zeta\left(2 + i\,t,\, \frac{1}{2}\right)}{-1 + 2^{2+i\,t}}$$

$\zeta(s,\, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2 + i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = \frac{\sum_{k=0}^{\infty} (1 + 2\,k)^{-2-i\,t}}{1 - 2^{-2-i\,t}} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \mathrm{Im}(t) < 1$$

$\mathrm{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^\infty \frac{\tau^{1+i\,t}}{-1+e^\tau}\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^\infty \tau^{2+i\,t}\,\mathrm{csch}^2(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^\infty e^{-\tau}\,\tau^{1+i\,t}\,\mathrm{csch}(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\mathrm{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+i\,t) = -i\,2^{2+i\,t}\,\pi^{1+i\,t}\,\Gamma(-1-i\,t)\sinh\left(\frac{\pi\,t}{2}\right)\zeta(-1-i\,t)$$

$$\zeta(2+i\,t) = \frac{\pi^{3/2+i\,t}\,\Gamma\left(-\frac{1}{2}-\frac{i\,t}{2}\right)\zeta(-1-i\,t)}{\Gamma\left(1+\frac{i\,t}{2}\right)}$$

$$\zeta(2+i\,t) = -\frac{i\sum_{k=0}^\infty \frac{\Gamma\left(k-\frac{i\,t}{2}\right)\sum_{j=0}^k (-1)^j\,(1+2\,j)\binom{k}{j}\zeta(2+2\,j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{i\,t}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372033095 8…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6} = 1.644934$ ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

1.64381856858…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\right/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right/\right.\right.$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$
for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^-}6$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

−5.010882647757...

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\;\phi\;e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\;\phi\;e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right)=0.054323\,(0.0814845\,b+1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M}=-\frac{\frac{226\,802\,245}{18\,480\,874}+b}{M}$$

$$\frac{\partial M(b,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b}=-\frac{18\,480\,874\,M}{226\,802\,245+18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M}=\frac{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b}=\frac{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right\}=0 \text{ at } (b,M)=(-16,0)$$

Global minima:

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

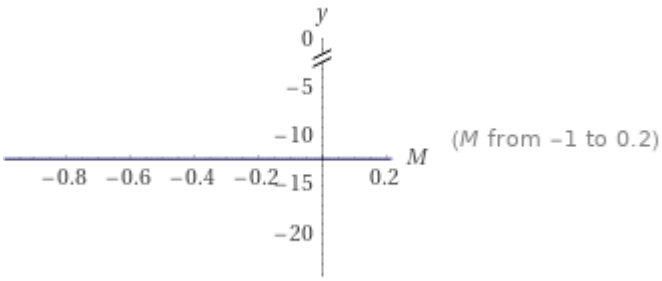
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

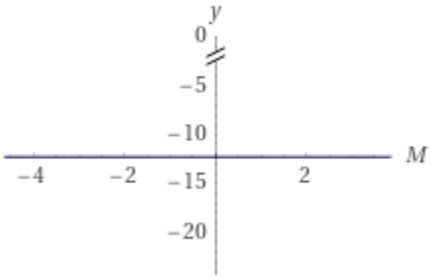
Result:

$$\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323\, M^2\right)}{M^2}\right) = \frac{3.55271 \times 10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V=\frac{1}{3}\left(0.0814845\,b+1\right)^2M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

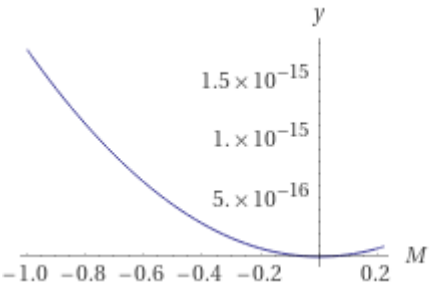
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

Result:

0

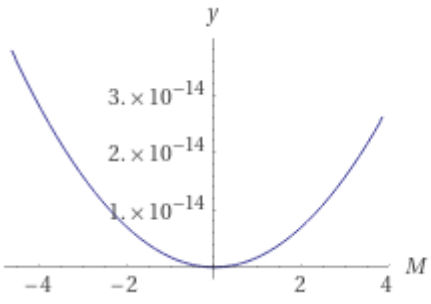
Plots: **(possible mathematical connection with an open string)**



(M from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(M from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O(M^{62194})$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times10^{-15}\,M^2+O\left(\left(\frac{1}{M}\right)^{62\,194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}\,M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.38851344947464545348970783378088020833333333333333333333\ldots \times 10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 0.2^2 + 6.58545 \times 10^{-10}\sqrt{0.2^4}\right)}{0.2^2} + 1\right)^2 \times 0.2^2$$

Result:

$$7.02162151915943272558353253404940833333333333333333333333\ldots \times 10^{-17}$$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875\times10^{-14}$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots \times 10^{-8}$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left(\left(1/\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)\text{ ^ }(1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

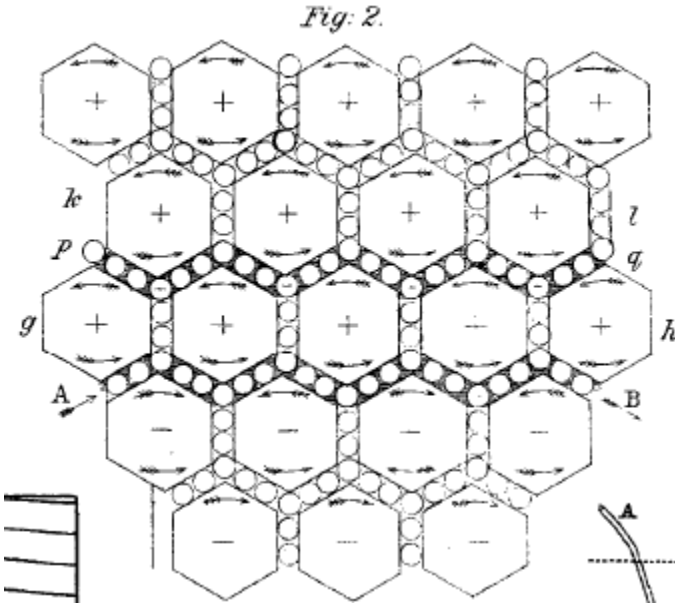
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

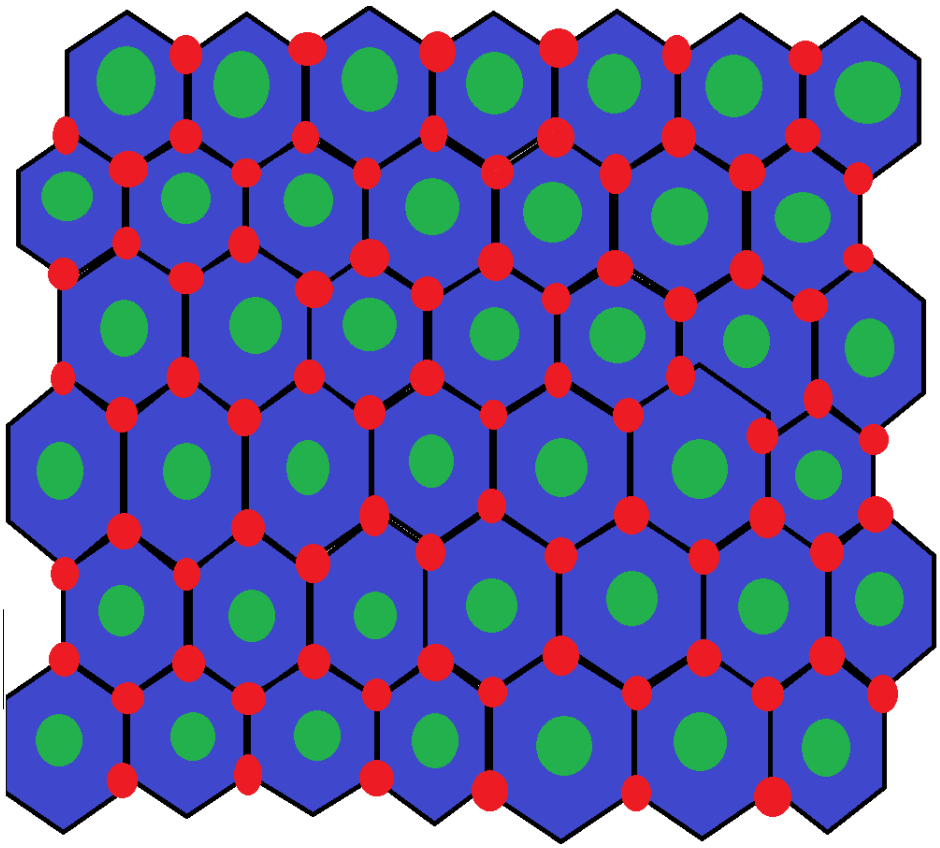
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Fig. 3

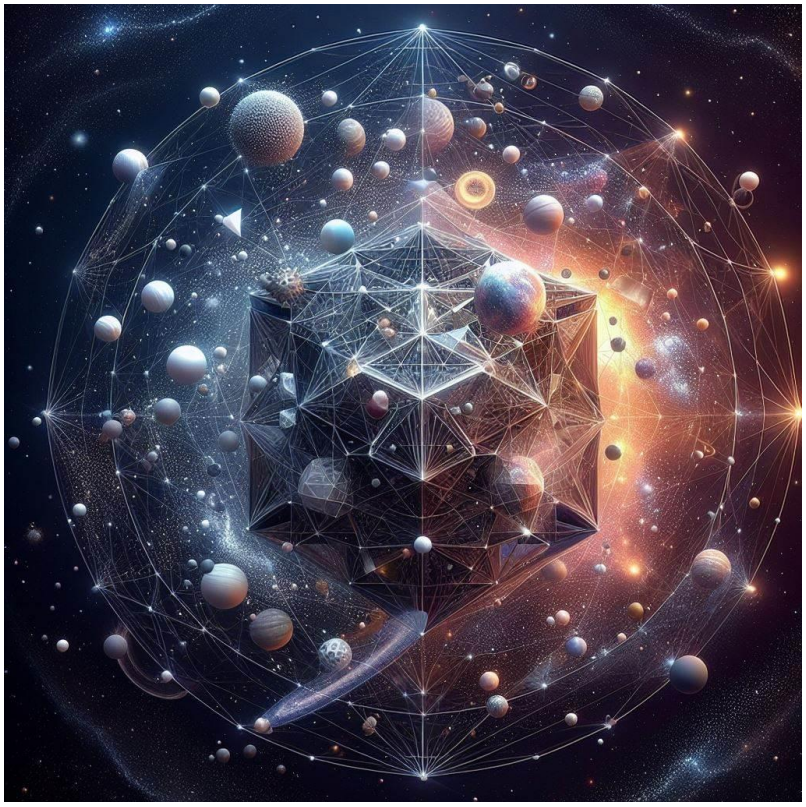


Image representing the solutions of the cubic equations in relation to the boundary conditions of the Plateau problem useful for understanding the formation of the bubbles of a universe with eternal inflation and of an octahedron as a mathematical model of the primordial universe before the Big Bang, therefore highly symmetrical and with extremely low entropy. As for the cubic equations, they represent the geometric characteristics of the quantum vacuum, while the spherical surface would represent the geometry of the bubble of the universe.

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability. I would like to thank also my son **Francesco Nardelli**, student of Educational Sciences, majoring in History and Philosophy, for his useful suggestions regarding the intriguing image at the end of this work

References

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

See also:

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *Emilian Dudas, Sergio Ferrara, Alex Kehagias and Augusto Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the analysis of various Cardano's Formulas and Cubic Equations. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory VIII

Michele Nardelli[1], Antonio Nardelli

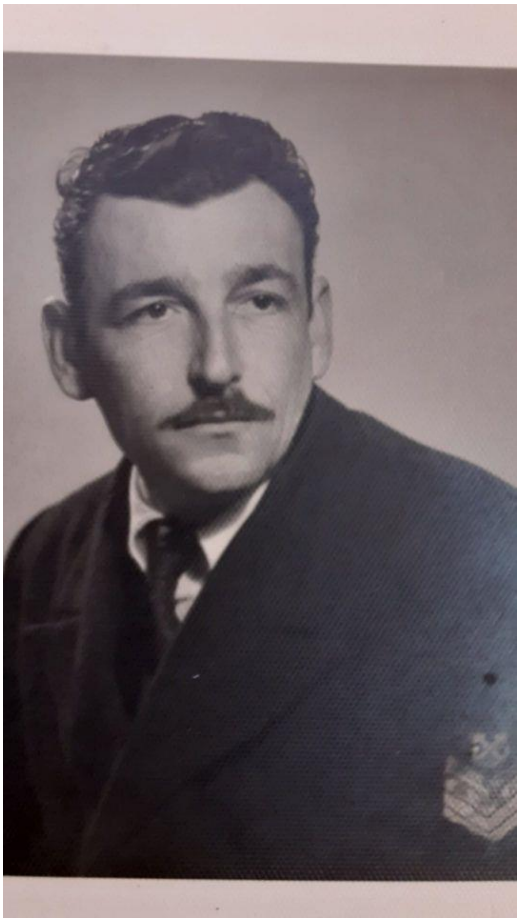
Abstract

In this paper (part VIII), we analyze various Cardano's Formulas and Cubic Equations. We obtain new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Prof. Antonio Grablovitz (1916-1991)



Dedicated to the memory of the excellent and brilliant mathematician Prof. Antonio Grablovitz, my beloved Master

(Dedicato alla memoria dell'eccelso e geniale matematico Prof. Antonio Grablovitz, mio amato Maestro)

Introduction

Cardano's method for cubic equations involves cube roots and introduces symmetry concepts. Similarly, string theory and M-theory involve symmetries and transformation groups that play a fundamental role in describing particles and interactions. Solutions of complete cubic equations via Cardano's method involve complex cube roots. Similarly, equations and solutions in superstring theory can often involve advanced algebraic and mathematical complexities. Solutions of cubic equations can be understood geometrically through topology and algebraic geometry. In string theory, the geometric and topological configurations of strings and branes are also central. Duality is an important concept in string theory, where different mathematical descriptions can be equivalent. Similarly, in solving cubic equations, duality links exist through cube roots.

Thus, in string theory, duality refers to the fact that different mathematical formulations or physical descriptions of a theory can be equivalent. For example, two seemingly different theories can be dual to each other, meaning they are two different ways of describing the same physical reality. This concept of duality has been very useful for understanding profound aspects of string theory and field theories.

Similarly, in solving cubic equations, duality can emerge through cube roots. The cubic equation $ax^3+bx^2+cx+d=0$ can be solved using cube roots, as in the Cardano method I described above. However, cube roots involve the concept of complex numbers, and it is sometimes possible to express the solutions in ways that highlight a duality between the solutions.

For example, if we consider complex cube roots, we can see a duality between them, since there are connections via the cube roots of unity. This duality is similar to the concept of duality in string theory, where different descriptions can be connected by similar transformations.

In both cases, duality provides a deeper way of understanding the underlying mathematical or physical structure, allowing you to explore hidden connections between different concepts.

Cubic surfaces can be associated with geometric and topological concepts. In string theory, for example, some models require spaces with particular shapes, and the geometry of the spaces can influence the properties of strings and particles. Some mathematical and physical models involving surfaces might be described by cubic or

complex equations. The geometry of surfaces and the associated mathematical structure can influence the properties of universe bubbles in the multiverse. Cubic surfaces can represent certain topological spaces, and their properties can affect the structure of the associated string theory.

The connection between the surface of a D-brane and the roots of a cubic equation can be explored in specific contexts within string theory, especially when it comes to studying the geometric and topological properties of D-branes. D-branes are extended objects on whose surface open strings can terminate. The geometry of the D-brane surface can interfere with the physics of surrounding strings. In some cases, geometry can be described mathematically through objects known in algebraic surface theory, such as cubic equations.

D-branes can host open fields, which describe particles interacting with the D-brane. Field equations on a D-brane can involve surface geometry, and in some situations, these equations can be associated with higher degree algebraic equations, such as cubic equations.

The compactification of extra dimensions, which is an important part of string theory, can be associated with the surface geometry of a D-brane. Compactification can lead to a reduction in dimensions and a geometric description that can involve algebraic equations, including cubic equations.

The topological invariants associated with the surface of a D-brane, such as the Betti number or the Chern number, can be related to geometric characteristics that can be described via algebraic equations, including cubic equations.

In some situations of duality, the description of a D-brane in a context can be dualistic to a description based on algebraic objects such as curves or surfaces, which can be represented via algebraic equations, including cubic equations.

In this paper, we will hypothesize that the solutions of the cubic equations, can be used as boundary conditions to further the understanding of the formation of universe bubbles in eternal inflation. Furthermore, an octahedron could serve as a mathematical or conceptual model of the universe in the quantistic phase. Cubic equations, in this context, could represent the geometric characteristics of the quantum vacuum, while the spherical surface could be used to describe the geometry of the bubble-universe.

The results $(2\sqrt{2})/\pi$, the golden ratio φ and π obtained, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and bubbles universe in eternal inflation. It may reflect a fundamental relationship between the geometry of solids and the mathematical characteristics of solutions to cubic equations.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and bubbles universe.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of bubbles universe and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

Finally, it is also possible to hypothesize the solutions of the cubic equations solvable with the Cardano method, as boundary conditions to deepen the understanding of the formation of inflationary universe bubbles eternal and of an octahedron which could represent a mathematical or conceptual model of the quantum universe before the Big Bang. Furthermore, regarding the cubic equations, they can represent the geometric characteristics of the quantum vacuum, while the spherical surface represents the geometry of the bubble universe. The results above $(2\sqrt{2})/\pi$, the golden ratio φ and π which are obtained, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

These mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe. The cubic equations, in this context, could reflect the geometric and dynamic characteristics of the quantum vacuum, while the spherical surface could represent the geometry of the bubbles universe.

Regarding fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the quantum universe and bubbles universe.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

1. *Cubic Equation*

The most general cubic equation (equation of third degree):

$$x^3+rx^2+px+q=0 \quad (1)$$

can be reduced in the following form:

$$x^3+px+q=0 \quad (2)$$

Indeed:

$$(x-u)(x-v)(x-z)=(x^2-vx-ux+uv)(x-z)=$$
$$=[x^2-(u+v)x+uv](x-z)=x^3-(u+v)x^2+uvx-zx^2+z(u+v)x-zuv=$$
$$=x^3-(u+v+z)x^2+[uv+z(u+v)]x-zuv. \quad (2b)$$

Thence:

$$x^3-(u+v+z)x^2+[uv+z(u+v)]x-zuv=0 \quad (3)$$

Putting:

$$\begin{cases} u+v+z=-r \\ z(u+v)+uv=p \\ uvz=-q \end{cases} \quad (4)$$

because the term r become null, is necessary and sufficient that $z=-(u+v)$. Thence:

$$x^3-(u+v+z)x^2+[uv+z(u+v)]x-zuv=0;$$

$$x^3-[u+v-(u+v)]x^2+[uv-(u+v)(u+v)]x-[-(u+v)uv]=0;$$

$$x^3-[uv-(u+v)^2]x-[-(u+v)uv]=0;$$

$$x^3-[uv-(u^2+2uv+v^2)]x+u^2v+uv^2=0;$$

$$x^3-[uv-u^2-2uv-v^2]x+u^2v+uv^2=0;$$

$$x^3-(-uv-u^2-v^2)x+uv(u+v)=0;$$

$$x^3-(u^2+uv+v^2)x+uv(u+v)=0 \quad (5)$$

Putting:

$$\begin{cases} u^2+uv+v^2=-p \\ uv(u+v)=q \end{cases} \quad (6)$$

the $x^3-(u^2+uv+v^2)x+uv(u+v)=0$, become:

$$x^3+px+q=0 \quad (7)$$

From the $u^2+uv+v^2=0$, we obtain two quadratic equations (of second degree):

1) $u^2+uv+v^2=0;$

$$u=\frac{-v\pm\sqrt{v^2-4v^2}}{2}=\frac{-v\pm\sqrt{-3v^2}}{2}=-\frac{1}{2}v\pm v\frac{\sqrt{3i^2}}{2}=-v\left(\frac{1}{2}\pm\frac{\sqrt{3i^2}}{2}\right)=-v\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right);$$

2) $v^2+uv+u^2=0$;

$$v=\frac{-u\pm\sqrt{u^2-4u^2}}{2}=\frac{-u\pm\sqrt{-3u^2}}{2}=-\frac{1}{2}u\pm u\frac{\sqrt{3i^2}}{2}=-u\left(\frac{1}{2}\pm\frac{\sqrt{3i^2}}{2}\right)=-u\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right).$$

Now, we take the eq. (6). We have that:

$$\begin{cases} u^2+uv+v^2=-p \\ u=-v\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right) \\ v=-u\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right) \end{cases} \qquad (7a) \qquad \begin{cases} uv=-uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2 \\ u^2=u\cdot u=-uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2 \\ v^2=v\cdot v=-uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2 \end{cases}$$

thence:

$$-\left[uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2+uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2+uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2\right]=-(uv+uv+uv)=p;$$

$$3uv=-p; \quad uv=-\frac{p}{3} \qquad (8)$$

Indeed, we have, for example:

$$-\left[uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2+uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2+uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\right]=p$$

$$-\left[uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\right]=$$

$$=-\left[uv\left(\frac{1}{4}+\frac{i\sqrt{3}}{2}-\frac{3}{4}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{4}+\frac{i\sqrt{3}}{2}-\frac{3}{4}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{4}+\frac{i\sqrt{3}}{2}-\frac{3}{4}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\right]=$$

$$=-\left[uv\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\right]=$$

$$=-\left[uv\left(\frac{1}{4}+\frac{3}{4}\right)+uv\left(\frac{1}{4}+\frac{3}{4}\right)+uv\left(\frac{1}{4}+\frac{3}{4}\right)\right]=$$

$$=-(uv+uv+uv)=p;\quad 3uv=-p;\quad uv=-\frac{p}{3}.$$

Now:

$$\begin{cases} u^2v+uv^2=q \\ u=-v\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right) \\ v=-u\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right) \end{cases} \quad (8b)$$

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q;$$

that for the (8b) and for sign + , for example, become:

$$v^2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\cdot v\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+u^2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\cdot u\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$$v^2\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-v)+u^2\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-u)=q$$

$$-v^3\left(\frac{1}{4}+\frac{3}{4}\right)-u^3\left(\frac{1}{4}+\frac{3}{4}\right)=q;\quad -v^3-u^3=q;$$

$$u^3+v^3=-q \quad (9)$$

Finally, we have:

$$\begin{cases} u^3+v^3=-q \\ uv=-\dfrac{p}{3} \end{cases} \qquad \text{thence:} \qquad \begin{cases} u^3+v^3=-q \\ u^3v^3=-\dfrac{p^3}{27} \end{cases} \quad (10)$$

We obtain the same result putting in the (2) $x=u+v$. Indeed, we have:

$$x^3+px+q=0;\qquad x=u+v$$

$$(u+v)^3+p(u+v)+q=0;\quad u^3+3u^2v+3uv^2+v^3+p(u+v)+q=0;$$
$$u^3+v^3+3uv(u+v)+p(u+v)+q=0;$$

from this equation, we obtain:

$$u^3+v^3=-q;\qquad 3uv(u+v)=-p(u+v);\qquad \text{thence, in conclusion:}$$

$$\begin{cases} u^3+v^3=-q \\ 3uv=-p \end{cases} \qquad \begin{cases} u^3+v^3=-q \\ uv=-\dfrac{p}{3} \end{cases} \qquad \begin{cases} u^3+v^3=-q \\ u^3v^3=-\dfrac{p^3}{27} \end{cases} \quad (11)$$

From the relations (10) and (11), we obtain the sum and the product of u^3 and v^3. We have that u^3 and v^3 must be roots of the following quadratic equation:

$$t^2+qt-\frac{p^3}{27}; \quad (12)$$

This equation is defined the "resolving" of the cubic equation (2). Thence, we have:

$$t^2+qt-\frac{p^3}{27}$$

$$\frac{-q\pm\sqrt{q^2-4(1)\left(-\frac{p^3}{27}\right)}}{2}=\frac{-q\pm\sqrt{q^2+\frac{4p^3}{27}}}{2}=-\frac{q}{2}\pm\sqrt{\frac{1}{4}q^2+\frac{1}{4}\cdot\frac{4p^3}{27}}=$$

$$=-\frac{q}{2}\pm\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}.$$

Thence, we have that:

$$\begin{aligned} t_1=u^3&=-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}} \\ t_2=v^3&=-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}} \end{aligned}; \quad (13)$$

from this, we obtain:

$$u=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}};$$

if $uv=-\frac{p}{3}$; $v=-\frac{p}{3u}$, we obtain:

$$v=-\frac{p}{3\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}}.$$

From the $x=u+v$, we obtain:

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}-\frac{p}{3\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}}. \quad (14)$$

The expression (14) is the "resolving" formula of the cubic equation (2), from this we have the three roots in correspondence of the three values of the cubic radical. If $p=0$ the eq. (2) become the binomial equation $x^3+q=0$, solved from the formula $x=\sqrt[3]{-q}$.

The expression (14) is written generally in the "Cardano" or "Tartaglia" form:

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}. \quad (15)$$

The solution that Cardano gives of the cubic equation of type $x^3+px=q$, leads to the following formula:

$$x=\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}+\frac{q}{2}}-\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}-\frac{q}{2}}; \quad (15b)$$

while for the cubic equation of type $x^3 + px^2 = q$, we have the following formula:

$$x = \sqrt[3]{\sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{p}{3}\right)^3} + \frac{q}{2}} - \sqrt[3]{\sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{p}{3}\right)^3} - \frac{q}{2}} \quad . \quad (15c)$$

Mathematical connections with some sectors of string theory

Now we show two examples of solutions of cubic equations and the possible mathematical connections with the "modes" that correspond to the physical vibrations of the bosonic strings by the Ramanujan function.

1) Let

$$x^3 - 8x = 3 \quad (16)$$

Putting

$$q = u - v = 3; \qquad \left(\frac{p}{3}\right)^3 = uv = \left(-\frac{8}{3}\right)^3 = -\frac{512}{27}; \quad \text{we apply the formula:}$$

$$x = \sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} + \frac{q}{2}} - \sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} - \frac{q}{2}} =$$

$$= \sqrt[3]{\sqrt{-\frac{512}{27} + \frac{9}{4}} + \frac{3}{2}} - \sqrt[3]{\sqrt{-\frac{512}{27} + \frac{9}{4}} - \frac{3}{2}} = \sqrt[3]{\sqrt{\frac{-2048 + 243}{108}} + \frac{3}{2}} - \sqrt[3]{\sqrt{\frac{-2048 + 243}{108}} - \frac{3}{2}} =$$

$$= \sqrt[3]{\sqrt{-\frac{1805}{108}} + \frac{3}{2}} - \sqrt[3]{\sqrt{-\frac{1805}{108}} - \frac{3}{2}} = \left(\sqrt[3]{\sqrt{\frac{1805}{108} i^2} + \frac{3}{2}}\right)^3 - \left(\sqrt[3]{\sqrt{\frac{1805}{108} i^2} - \frac{3}{2}}\right)^3 =$$

$$=\sqrt{\frac{1805}{108}i^2}+\frac{3}{2}-\sqrt{\frac{1805}{108}i^2}+\frac{3}{2}=\frac{3}{2}+\frac{3}{2}=\frac{6}{2}=3.$$

Indeed, for $x=3$, the eq. (16) become:

$$x^3-8x=3; \qquad 3^3-8\cdot 3=3; \qquad 27-24=3; \qquad 3=3$$

But, we have also that:

$$8x=x^3-3; \qquad 24=x^3-3; \qquad 24=27-3; \qquad 24=24$$

We know that the number 24 is related to the "modes" that correspond to the physical vibrations of the bosonic strings by the following Ramanujan function:

$$24=\frac{4\left[antilog\frac{\int_0^\infty \frac{\cos\pi txw'}{\cosh\pi x}e^{-\pi x^2w'}dx}{e^{-\frac{\pi^2}{4}w'}\phi_{w'}(itw')}\right]\cdot\frac{\sqrt{142}}{t^2w'}}{\log\left[\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)}+\sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right]}, \quad (a)$$

Thence, we have the following mathematical connection:

$$\frac{4\left[antilog\frac{\int_0^\infty \frac{\cos\pi txw'}{\cosh\pi x}e^{-\pi x^2w'}dx}{e^{-\frac{\pi^2}{4}w'}\phi_{w'}(itw')}\right]\cdot\frac{\sqrt{142}}{t^2w'}}{\log\left[\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)}+\sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right]}=x^3-3 \quad (17) \qquad \text{for} \quad x=3$$

A mathematical connection that link the cubic equation $x^3-8x=3$ with the number 24, i.e. with the "modes" that correspond to the physical vibrations of the bosonic strings.

We note also that 3 and 8, i.e. p and q are also Fibonacci's numbers.

2) Let

$$x^3+8x=24. \quad (18)$$

Putting

$$q=u-v=24; \qquad \left(\frac{p}{3}\right)^3=uv=\left(\frac{8}{3}\right)^3=\frac{512}{27}; \quad \text{for the formula:}$$

$$x=\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}+\frac{q}{2}}-\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}-\frac{q}{2}}\ ; \quad \text{we obtain:}$$

$$\sqrt[3]{\sqrt{\frac{512}{27}+\left(\frac{24}{2}\right)^2}+\frac{24}{2}}-\sqrt[3]{\sqrt{\frac{512}{27}+\left(\frac{24}{2}\right)^2}-\frac{24}{2}}=$$

$$=\sqrt[3]{\sqrt{\frac{512}{27}+144}+12}-\sqrt[3]{\sqrt{\frac{512}{27}+144}-12}=\sqrt[3]{\sqrt{162{,}962}+12}-\sqrt[3]{\sqrt{162{,}962}+12}=$$

$$=\sqrt[3]{12{,}765657+12}-\sqrt[3]{12{,}765657-12}=\sqrt[3]{24{,}765657}-\sqrt[3]{0{,}765657}\cong 2{,}914852-0{,}914852=2.$$

Thence, we have:

$$x^3+8x=24; \qquad 2^3+8\cdot 2=24; \qquad 8+16=24; \qquad 24=24;$$

$$24=x^3+8x;$$

$$\frac{4\left[anti\log\frac{\int_0^{\infty}\frac{\cos\pi txw'}{\cosh\pi x}e^{-\pi x^2w'}dx}{e^{-\frac{\pi t^2}{4}w'}\phi_{w'}(itw')}\right]\cdot\frac{\sqrt{142}}{t^2w'}}{\log\left[\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)}+\sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right]}=x^3+8x;\quad(19)\qquad\text{for } x=2$$

The further connection that link this cubic equation with the “modes” that correspond to the physical vibrations of the bosonic strings.

Creation and solution of cubic equation incomplete, without the first degree, defined “Premonica”.

Let u,v and z three roots of a same equation. Obviously, the equation is cubic and the factorization of it is:

$$x^3+px^2+rx+q=0\quad(20)$$

$$(x-u)(x-v)=x^2-ux-vx+uv=x^2-(u+v)x+uv;$$

$$\left[x^2-(u+v)x+uv\right](x-z)=x^3-(u+v)x^2+uvx-zx^2+z(u+v)x-zuv=$$

$$=x^3-(u+v+z)x^2+\left[uv+z(u+v)\right]x-uvz;\quad\text{thence:}$$

$$x^3-(u+v+z)x^2+\left[uv+z(u+v)\right]x-uvz=0;\quad(21)$$

from this, putting

$$\begin{cases} u+v+z=-p \\ z(u+v)+uv=r \\ uvz=-q \end{cases}$$

we obtain the eq. (20).

to cancel the term r, necessary and sufficient condition is that:

$$z(u+v)+uv=0; \qquad z(u+v)=-uv \quad \text{whence}$$

$$z=-\frac{uv}{u+v}. \quad (22)$$

The (22) is defined the "fundamental relation" . Substituting in the eq. (21) the value found of z, we obtain:

$$x^3-(u+v+z)x^2+[uv+z(u+v)]x-uvz=0$$

$$x^3-\left(u+v-\frac{uv}{u+v}\right)x^2+\left[uv-\frac{uv}{u+v}(u+v)\right]x-uv\left(-\frac{uv}{u+v}\right)=0;$$

$$x^3-\frac{(u+v)^2-uv}{u+v}x^2+\frac{u^2v^2}{u+v}=0; \quad \text{that we can rewrite also as follows:}$$

$$(u+v)x^3-\left[(u+v)^2-uv\right]x^2+u^2v^2=0. \quad (23)$$

With this relation we can solve a cubic equation in "Premonica" form, i.e. without the first degree.

$x^3+px^2+q=0$. For example, $3x^3-7x^2+4=0$, we will use the following system:

$$\begin{cases} u+v=3 \\ (u+v)^2-uv=7 \\ u^2v^2=4 \end{cases} \quad (23b) \quad \begin{cases} 9-uv=7 \\ u^2v^2=4 \end{cases} \quad \begin{cases} uv=2 \\ u^2v^2=4 \end{cases} \quad \begin{cases} uv=2 \\ uv=\pm 2 \end{cases}$$

The uv will be equal only if $uv=2$, thence:

$$\begin{cases} u+v=3 \\ uv=2 \end{cases} \quad \text{whence} \quad t^2-3t+2=0; \quad t=\frac{3\pm\sqrt{9-8}}{2}=\frac{3\pm 1}{2}=\begin{cases} t_1=2=u=x_1 \\ t_2=1=v=x_2 \end{cases}$$

Because the equation does not admit the term of first degree, the third root must be necessarily:

$$z=-\frac{uv}{u+v}=-\frac{1\cdot 2}{2+1}=-\frac{2}{3}=x_3$$

Now, we have the following cubic equation:

$$x^3-2x+1=0; \quad (24)$$

We want solve this equation by a system of relations between roots and coefficients. This procedure will be valid for each equation in form “Monica” form, i.e. without the second degree, whatever the coefficients.

Let u and v two of the roots of the equation, the third root will be necessarily $u+v$ and thence we have

$$x_1=u; \quad x_2=v; \quad x_3=u+v; \quad (25) \qquad \begin{cases} x_1=-(u+v)+u=-v \\ x_2=-(u+v)+v=-u \end{cases} \quad (25b)$$

Now:

$$(x-u)(x-v)=x^2-(u+v)x+uv;$$

$$[x^2-(u+v)x+uv][x+(u+v)]=x^3-(u+v)x^2+uvx+(u+v)x^2-(u+v)^2x+uv(u+v)=$$

$$=x^3-[(u+v)^2-uv]x+uv(u+v);$$ from this, we have:

$$x^3-[(u+v)^2-uv]x+uv(u+v)=0. \quad (26)$$

This equation is a cubic equation in the form "Monica", i.e. of type $x^3+px+q=0$, where we put:

$$\begin{cases}(u+v)^2-uv=-p\\ uv(u+v)=q\end{cases}$$

Now, we put $u+v=t$ and $uv=z$, we have:

$$\begin{cases}t^2-z=-p\\ tz=q\end{cases} \qquad \begin{cases}t^2=z-p\\ tz=q\end{cases} \qquad \begin{cases}t=\pm\sqrt{z-p}\\ tz=q\end{cases} \qquad tz=q; \quad \pm z\left(\sqrt{z-p}\right)=q;$$

we raise all to the square: $z^2(z-p)=q^2$; from which developing:

$$z^3-pz^2=q^2; \qquad z^3-pz^2-q^2=0. \quad (27)$$

This last equation is a "Premonica" and we can find u and v applying the Caccioppoli-Grablovitz method (comparison system between roots and coefficients). We resume the eq. (24)

$$x^3-2x+1=0$$

Decompose the equation. We have that:

$$\left(x^2+x-1\right)\left(x-1\right)=x^3-x^2+x^2-x-x+1=x^3-2x+1;\quad \text{thence:}$$

$$\left(x^2+x-1\right)\left(x-1\right)=0;\qquad x-1=0;\qquad x=1$$

$$x^2+x-1=0;\quad \frac{-1\pm\sqrt{1-4(1)(-1)}}{2}=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}\quad \begin{cases} x_1=\dfrac{-1+\sqrt{5}}{2} \\ x_2=\dfrac{-1-\sqrt{5}}{2} \end{cases}$$

The three roots are: $1;\ \frac{-1+\sqrt{5}}{2};\ \frac{-1-\sqrt{5}}{2}$.

Now, $x^3-2x+1=0$ applying the relation (27), we have:

$$z^3-pz^2-q^2=0;\quad z^3-2z^2-1=0\quad \text{whence, from (23b)}$$

$$\begin{cases} u+v=1 \\ (u+v)^2-uv=2 \\ u^2v^2=1 \end{cases}\qquad \begin{cases} u+v=1 \\ 1-uv=2 \\ uv=\pm 1 \end{cases}\qquad \begin{cases} u+v=1 \\ uv=-1 \\ uv=\pm 1 \end{cases}\qquad \begin{cases} u+v=1 \\ uv=-1 \end{cases}$$

Thence:

$$t^2-t-1=0;\qquad \frac{1\pm\sqrt{1+4}}{2}=\begin{cases} u=\dfrac{1+\sqrt{5}}{2} \\ v=\dfrac{1-\sqrt{5}}{2} \end{cases}$$

For the relations (25b), the third root will be $x_3 = u + v = \frac{1}{2} + \frac{\sqrt{5}}{2} + \frac{1}{2} - \frac{\sqrt{5}}{2} = 1$. The other two roots, will be: $x_1 = -u = \frac{-1-\sqrt{5}}{2}$; and $x_2 = -v = \frac{-1+\sqrt{5}}{2}$.

We note that the $x_2 = \frac{\sqrt{5}-1}{2} = 0{,}61803398...$ is equal to the aurea section.

2. "Pseudo-Moniche" cubic equations: Caccioppoli-Grablovitz method and applications

The easiest to solve equations are the "Pseudo-Moniche" (A. Grablovitz), that is, those in the form

$$x^3 + px^2 + q = 0$$

If two roots are u e v, the third root must be $-\frac{uv}{u+v}$.

Let one root $\sqrt{2}$ and another $\sqrt{3}$, we will have $x_3 = -\frac{\sqrt{2}\sqrt{3}}{\sqrt{2}+\sqrt{3}} = -\frac{\sqrt{6}}{\sqrt{2}+\sqrt{3}}$.

Rationalizing:

$$\frac{-\sqrt{6}(\sqrt{2}-\sqrt{3})}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})} = -\frac{\sqrt{12}-\sqrt{18}}{2-3} = -\frac{\sqrt{12}-\sqrt{18}}{-1} = 2\sqrt{3} - 3\sqrt{2}$$

Input:

$2\sqrt{3} - 3\sqrt{2}$

Decimal approximation:

−0.778539071981530559350173489617349501823405118510082963418425255

…

-0.77853907198…

Alternate form:

$$-\sqrt{6\left(5-2\sqrt{6}\right)}$$

Minimal polynomial:

$$x^4 - 60\,x^2 + 36$$

Indeed:

$$\left(x-\sqrt{3}\right)\cdot\left(x-\sqrt{2}\right) = x^2 - \left(\sqrt{3}+\sqrt{2}\right)x - \sqrt{6}$$

$$x^2 - \left(\sqrt{3}+\sqrt{2}\right)x - \sqrt{6}$$

$$x + \left(2\sqrt{3} - 3\sqrt{2}\right)$$

$$\left(x + 2\sqrt{3} - 3\sqrt{2}\right)\left(x^2 - \left(\sqrt{2}+\sqrt{3}\right)x - \sqrt{6}\right)$$

$$x^3 - \left(\sqrt{2}+\sqrt{3}\right)x^2 - \sqrt{6}x + 2\sqrt{3}x^2 + 2\sqrt{3}\cdot -\left(\sqrt{2}+\sqrt{3}\right)x + 2\sqrt{3}\cdot\left(-\sqrt{6}\right) - 3\sqrt{2}x^2 - 3\sqrt{2}\cdot -\left(\sqrt{2}+\sqrt{3}\right)x - 3\sqrt{2}\cdot\left(-\sqrt{6}\right)$$

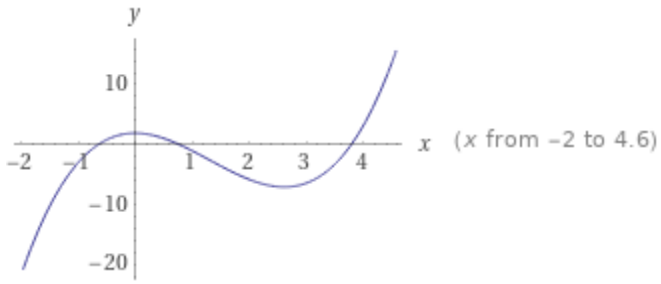
Input:

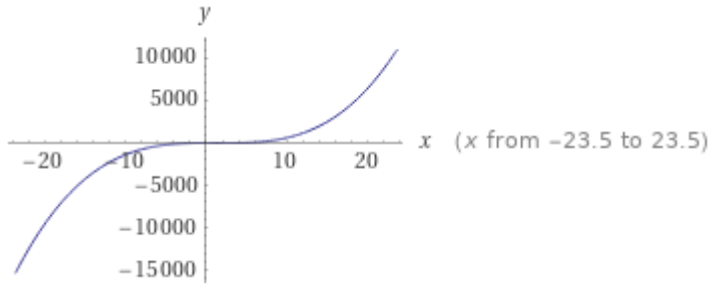
$$x^3 - \left(\sqrt{2}+\sqrt{3}\right)x^2 - \sqrt{6}\,x + 2\sqrt{3}\,x^2 + 2\sqrt{3}\times(-1)\left(\sqrt{2}+\sqrt{3}\right)x + 2\sqrt{3}\left(-\sqrt{6}\right) - \left(3\sqrt{2}\right)x^2 - 3\sqrt{2}\times(-1)\left(\sqrt{2}+\sqrt{3}\right)x - 3\sqrt{2}\left(-\sqrt{6}\right)$$

Exact result:

$$x^3 - \left(\sqrt{2}+\sqrt{3}\right)x^2 + 2\sqrt{3}\,x^2 - 3\sqrt{2}\,x^2 - 2\sqrt{3}\left(\sqrt{2}+\sqrt{3}\right)x + 3\sqrt{2}\left(\sqrt{2}+\sqrt{3}\right)x - \sqrt{6}\,x + 6\sqrt{3} - 6\sqrt{2}$$

Plots: **(figures that can be related to the open strings)**





Alternate forms:

$$\left(x+\sqrt{3}-4\sqrt{2}\right)x^2+6\sqrt{3}-6\sqrt{2}$$

$$\left(x+\sqrt{3}-4\sqrt{2}\right)x^2+6\sqrt{5-2\sqrt{6}}$$

$$x^3+\left(\sqrt{3}-4\sqrt{2}\right)x^2+\left(-6-\sqrt{6}+\sqrt{2}\left(3\sqrt{2}+\sqrt{3}\right)\right)x+6\sqrt{3}-6\sqrt{2}$$

Expanded form:

$$x^3+\sqrt{3}\,x^2-4\sqrt{2}\,x^2+6\sqrt{3}-6\sqrt{2}$$

Roots:

$x = 3\sqrt{2} - 2\sqrt{3}$

$x = 3\sqrt{2} - 2\sqrt{3}$

Input:

$3\sqrt{2} - 2\sqrt{3}$

Decimal approximation:

0.778539071981530559350173489617349501823405118510082963418425255 0

…

0.77853907198…

Alternate form:

$\sqrt{6\left(5 - 2\sqrt{6}\right)}$

Minimal polynomial:

$x^4 - 60x^2 + 36$

$x = \frac{1}{2}\left(\sqrt{2} + \sqrt{3} - \sqrt{5 + 6\sqrt{6}}\right)$

$x = \frac{1}{2}\left(\sqrt{2} + \sqrt{3} + \sqrt{5 + 6\sqrt{6}}\right)$

Polynomial discriminant:

$$\Delta_x = -12\left(409\sqrt{2} - 279\sqrt{3}\right)\left(\sqrt{2} - \sqrt{3}\right)$$

Properties as a real function:

Domain

$\mathbb{R}$ (all real numbers)

Range

$\mathbb{R}$ (all real numbers)

Surjectivity

surjective onto $\mathbb{R}$

$\mathbb{R}$ is the set of real numbers

Derivative:

$$\frac{d}{dx}\Big(x^3 - \left(\sqrt{2} + \sqrt{3}\right)x^2 - \sqrt{6}\,x + 2\sqrt{3}\,x^2 + 2\sqrt{3} \times (-1)\left(\sqrt{2} + \sqrt{3}\right)x + 2\sqrt{3}\left(-\sqrt{6}\right) - \left(3\sqrt{2}\right)x^2 - 3\sqrt{2} \times (-1)\left(\sqrt{2} + \sqrt{3}\right)x - 3\sqrt{2}\left(-\sqrt{6}\right)\Big) = x\left(3x + 2\sqrt{3} - 8\sqrt{2}\right)$$

Indefinite integral:

$$\int\left(-6\sqrt{2}+6\sqrt{3}-\sqrt{6}\,x+3\sqrt{2}\left(\sqrt{2}+\sqrt{3}\right)x-2\sqrt{3}\left(\sqrt{2}+\sqrt{3}\right)x-3\sqrt{2}\,x^2+2\sqrt{3}\,x^2-\left(\sqrt{2}+\sqrt{3}\right)x^2+x^3\right)dx=\frac{x^4}{4}+\frac{x^3}{\sqrt{3}}-\frac{4\sqrt{2}\,x^3}{3}-\sqrt{3}\left(\sqrt{2}+\sqrt{3}\right)x^2+\frac{3\left(\sqrt{2}+\sqrt{3}\right)x^2}{\sqrt{2}}-\sqrt{\frac{3}{2}}\,x^2+6\sqrt{3}\,x-6\sqrt{2}\,x+\text{constant}$$

Local maximum:

$$\max\left\{x^3-\left(\sqrt{2}+\sqrt{3}\right)x^2-\sqrt{6}\,x+2\sqrt{3}\,x^2+2\sqrt{3}\,(-1)\left(\sqrt{2}+\sqrt{3}\right)x+2\sqrt{3}\left(-\sqrt{6}\right)-\left(3\sqrt{2}\right)x^2-3\sqrt{2}\,(-1)\left(\sqrt{2}+\sqrt{3}\right)x-3\sqrt{2}\left(-\sqrt{6}\right)\right\}=6\sqrt{3}-6\sqrt{2}\ \text{ at }\ x=0$$

Local minimum:

$$\min\left\{x^3-\left(\sqrt{2}+\sqrt{3}\right)x^2-\sqrt{6}\,x+2\sqrt{3}\,x^2+2\sqrt{3}\,(-1)\left(\sqrt{2}+\sqrt{3}\right)x+2\sqrt{3}\left(-\sqrt{6}\right)-\left(3\sqrt{2}\right)x^2-3\sqrt{2}\,(-1)\left(\sqrt{2}+\sqrt{3}\right)x-3\sqrt{2}\left(-\sqrt{6}\right)\right\}=\frac{62}{\sqrt{3}}-\frac{818\sqrt{2}}{27}\ \text{ at }\ x=\frac{8\sqrt{2}}{3}-\frac{2}{\sqrt{3}}$$

Definite integrals:

$$\int_{\text{root of }\left\{-3+\#1^2\&,-2+\#2^2\&,6\#1-6\#2+\#1\#3^2-4\#2\#3^2+\#3^3\&\right\}(x)\text{ near }x=0.778539}^{\text{root of }\left\{-3+\#1^2\&,-2+\#2^2\&,6\#1-6\#2+\#1\#3^2-4\#2\#3^2+\#3^3\&\right\}(x)\text{ near }x=-0.645929}\left(-6\sqrt{2}+6\sqrt{3}-\sqrt{6}\,x+3\sqrt{2}\left(\sqrt{2}+\sqrt{3}\right)x-2\sqrt{3}\left(\sqrt{2}+\sqrt{3}\right)x-3\sqrt{2}\,x^2+2\sqrt{3}\,x^2-\left(\sqrt{2}+\sqrt{3}\right)x^2+x^3\right)dx=-\frac{4619}{24}+245\sqrt{\frac{2}{3}}-\frac{1}{2}\sqrt{\frac{3125\sqrt{\frac{2}{3}}}{3}-\frac{103\,175}{144}}\approx 1.79489$$

$$\int_{\text{root of } \{-3+\#1^2\&,-2+\#2^2\&,6\#1-6\#2+\#1\#3^2-4\#2\#3^2+\#3^3\&\}(x) \text{ near } x=0.778539}^{\text{root of } \{-3+\#1^2\&,-2+\#2^2\&,6\#1-6\#2+\#1\#3^2-4\#2\#3^2+\#3^3\&\}(x) \text{ near } x=3.79219} \left(-6\sqrt{2}+6\sqrt{3}-\sqrt{6}\,x+3\sqrt{2}\left(\sqrt{2}+\sqrt{3}\right)x-2\sqrt{3}\left(\sqrt{2}+\sqrt{3}\right)x-3\sqrt{2}\,x^2+2\sqrt{3}\,x^2-\left(\sqrt{2}+\sqrt{3}\right)x^2+x^3\right)dx =$$

$$\frac{1}{24}\left(4619-1960\sqrt{6}-12\sqrt{\frac{3125\sqrt[3]{\frac{2}{3}}}{3}-\frac{103\,175}{144}}\right)\approx$$

$$-13.3718$$

From the first definite integral, we obtain, after some calculations:

((-4619/24 + 245 sqrt(2/3) - 1/2 sqrt(-103175/144 + (3125 sqrt(2/3))/3)))-1/2((((√(10-2√5) -2))/((√5-1))))

where (((((√(10-2√5) -2))/((√5-1)))) = κ

Input:

$$\left(-\frac{4619}{24}+245\sqrt{\frac{2}{3}}-\frac{1}{2}\sqrt{-\frac{103\,175}{144}+\frac{1}{3}\left(3125\sqrt{\frac{2}{3}}\right)}\right)-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

Result:

$$-\frac{4619}{24}+245\sqrt{\frac{2}{3}}-\frac{1}{2}\sqrt{\frac{3125\sqrt[3]{\frac{2}{3}}}{3}-\frac{103\,175}{144}}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}$$

Decimal approximation:

1.652848053052724532812412280376369777416528974167686854790028594 7

…

1.652848053…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

Alternate forms:

$$\frac{1}{24}\left(-3\sqrt{5\left(10-2\sqrt{5}\right)}+6\sqrt{5}+1960\sqrt{6}-3\sqrt{2\left(5-\sqrt{5}\right)}-5\sqrt{2000\sqrt{6}-4127}-4613\right)$$

$$\frac{1}{24}\left(-4619+1960\sqrt{6}-5\sqrt{2000\sqrt{6}-4127}-\frac{12\left(\sqrt{10-2\sqrt{5}}-2\right)}{\sqrt{5}-1}\right)$$

$$\frac{1}{24\left(\sqrt{5}-1\right)}\left(4643-4619\sqrt{5}-1960\sqrt{6}+1960\sqrt{30}-12\sqrt{2\left(5-\sqrt{5}\right)}+5\sqrt{2000\sqrt{6}-4127}-5\sqrt{5\left(2000\sqrt{6}-4127\right)}\right)$$

Expanded form:

$$-\frac{4613}{24}+245\sqrt{\frac{2}{3}}+\frac{\sqrt{5}}{4}-\frac{1}{2}\sqrt{\frac{3125\sqrt{\frac{2}{3}}}{3}-\frac{103\,175}{144}}-\frac{1}{8}\sqrt{10-2\sqrt{5}}-\frac{1}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

We have that from:

$$x=\frac{1}{2}\left(\sqrt{2}+\sqrt{3}-\sqrt{5+6\sqrt{6}}\right)$$

$$x=\frac{1}{2}\left(\sqrt{2}+\sqrt{3}+\sqrt{5+6\sqrt{6}}\right)$$

we obtain:

1/2(sqrt2+sqrt3-sqrt(5+6sqrt6))

Input:

$$\frac{1}{2}\left(\sqrt{2}+\sqrt{3}-\sqrt{5+6\sqrt{6}}\right)$$

Decimal approximation:

$$-0.645929466758133126694072979863364078964926148349458387745157608\ldots$$

-0.645929466758…

Alternate forms:

$$\frac{1}{2}\left(\sqrt{2}+\sqrt{3}\right)-\frac{1}{2}\sqrt{5+6\sqrt{6}}$$

$$-\sqrt{\frac{5}{2}+2\sqrt{6}}-\frac{1}{2}\sqrt{97+40\sqrt{6}}$$

$$-\frac{\sqrt{5-i\sqrt{191}}+\sqrt{2}\left(-\sqrt{2}-\sqrt{3}+\sqrt{\frac{1}{2}i\left(\sqrt{191}+-5i\right)}\right)}{2\sqrt{2}}$$

Minimal polynomial:

$$x^8-10x^6-59x^4-60x^2+36$$

Expanded form:

$$\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}-\frac{1}{2}\sqrt{5+6\sqrt{6}}$$

From which:

((-1/2(sqrt2+sqrt3-sqrt(5+6sqrt6))))^1/32

Input:

$$\sqrt[32]{-\frac{1}{2}\left(\sqrt{2}+\sqrt{3}-\sqrt{5+6\sqrt{6}}\right)}$$

Result:

$$\frac{1}{2}\sqrt[32]{-\sqrt{2}-\sqrt{3}+\sqrt{5+6\sqrt{6}}}\;\;2^{31/32}$$

Decimal approximation:

0.986434570919700182368588374706207485148633247729486460997407113 4

...

0.9864345709197….. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the Omega mesons ($\omega/\omega_3 \mid 5+3 \mid m_{u/d}=255-390 \mid 0.988-1.18$) Regge slope value (0.988) connected to the dilaton scalar field $\mathbf{0.989117352243 = \phi}$

A_1^{**} above the two low-lying pseudo-scalars. (bound states of gluons, or 'glueballs')

A_1^{**}	0.943(39) [2.5]	0.988(38)	0.152(53)
A_4	1.03(10) [2.5]	0.999(32)	0.035(21)

(**Glueball Regge trajectories** - *Harvey Byron Meyer*, Lincoln College -Thesis submitted for the degree of Doctor of Philosophy at the University of Oxford Trinity Term, 2004)

Note that

$$\sqrt[32]{\zeta(2)-1}$$

$$\sqrt[32]{\frac{\pi^2}{6}-1}$$

= 0.98638703135648129 15… (π^2/6 – 1)^1/32

Alternate forms:

$$\sqrt[64]{\frac{5}{2}+2\sqrt{6}}-\frac{1}{2}\sqrt{97+40\sqrt{6}}$$

$$\frac{1}{2^{3/64}\sqrt[32]{\frac{1}{\sqrt{5-i\sqrt{191}}+\sqrt{2}\left(-\sqrt{2}-\sqrt{3}+\sqrt{\frac{1}{2}i\left(\sqrt{191}+-5i\right)}\right)}}}$$

$$-2^{15/32}\left(-\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5+6\sqrt{6}}}\right)^{31/32}-\frac{\sqrt{3}\left(-\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5+6\sqrt{6}}}\right)^{31/32}}{\sqrt[32]{2}}+\frac{\sqrt{5+6\sqrt{6}}\left(-\frac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5+6\sqrt{6}}}\right)^{31/32}}{\sqrt[32]{2}}$$

Minimal polynomial:

$$x^{256}-10x^{192}-59x^{128}-60x^{64}+36$$

Expanded form:

$$-\frac{2^{15/32}}{\left(-\sqrt{2}-\sqrt{3}+\sqrt{5+6\sqrt{6}}\right)^{31/32}}-\frac{\sqrt{3}}{\sqrt[32]{2}\left(-\sqrt{2}-\sqrt{3}+\sqrt{5+6\sqrt{6}}\right)^{31/32}}+\frac{\sqrt{5+6\sqrt{6}}}{\sqrt[32]{2}\left(-\sqrt{2}-\sqrt{3}+\sqrt{5+6\sqrt{6}}\right)^{31/32}}$$

From

$$x=\frac{1}{2}\left(\sqrt{2}+\sqrt{3}+\sqrt{5+6\sqrt{6}}\right)$$

we obtain:

Input:

$$\frac{1}{2}\left(\sqrt{2}+\sqrt{3}+\sqrt{5+6\sqrt{6}}\right)$$

Decimal approximation:

3.792193836700105469023208045578934524477403277536787088977644326 1

…

3.7921938367….

Alternate forms:

$$\frac{1}{2}\left(\sqrt{5+6\sqrt{6}}+\sqrt{2}+\sqrt{3}\right)$$

$$\sqrt{\frac{1}{2}\left(5+4\sqrt{6}+\sqrt{97+40\sqrt{6}}\right)}$$

$$\frac{1}{2}\left(\sqrt{2}+\sqrt{3}\right)+\frac{1}{2}\sqrt{5+6\sqrt{6}}$$

Minimal polynomial:

$$x^8-10x^6-59x^4-60x^2+36$$

Expanded form:

$$\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}+\frac{1}{2}\sqrt{5+6\sqrt{6}}$$

From:

$$\frac{1}{2}\left(\sqrt{5+6\sqrt{6}}+\sqrt{2}+\sqrt{3}\right)$$

we obtain also:

1/((1/2 (sqrt(2) + sqrt(3) + sqrt(5 + 6 sqrt(6))))^1/3

Input:

$$1+\frac{1}{\sqrt[3]{\frac{1}{2}\left(\sqrt{2}+\sqrt{3}+\sqrt{5+6\sqrt{6}}\right)}}$$

Result:

$$1+\sqrt[3]{\frac{2}{\sqrt{2}+\sqrt{3}+\sqrt{5+6\sqrt{6}}}}$$

Decimal approximation:

1.64126345509060444470177789940802711382099619514835925827107853 93

…

$1.64126345509\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms:

$$\frac{1}{2}\left(2 + 2^{2/3}\sqrt[6]{\frac{1}{3}\left(5 + 4\sqrt{6} - \sqrt{97 + 40\sqrt{6}}\right)}\right)$$

$$1 + \frac{\sqrt{2}}{\sqrt[3]{\sqrt{5 - i\sqrt{191}} + \sqrt{2}\left(\sqrt{2} + \sqrt{3} + \sqrt{\frac{1}{2}i\left(\sqrt{191} + -5i\right)}\right)}}$$

Integrating:

(((x^2-(sqrt3+sqrt2)x - sqrt6)(x+(2sqr3-3sqrt2))))dxdy

Indefinite integral:

$$\int\int \left(x^2 - \left(\sqrt{3} + \sqrt{2}\right)x - \sqrt{6}\right)\left(x + \left(2\sqrt{3} - 3\sqrt{2}\right)\right)dx\,dy =$$
$$c_1 x + c_2 + \frac{x^4 y}{4} + \frac{1}{3}\left(-\sqrt{2} - \sqrt{3}\right)x^3 y + \frac{2x^3 y}{\sqrt{3}} - \sqrt{2}\,x^3 y -$$
$$\sqrt{3}\left(\sqrt{2} + \sqrt{3}\right)x^2 y + \frac{3\left(\sqrt{2} + \sqrt{3}\right)x^2 y}{\sqrt{2}} - \sqrt{\frac{3}{2}}\,x^2 y + 6\sqrt{3}\,x y - 6\sqrt{2}\,x y$$

Definite integral area below the axis between the smallest and largest real roots:

$$\int_{\frac{1}{2}\left(\sqrt{2}+\sqrt{3}-\sqrt{4\sqrt{6}+(-\sqrt{2}-\sqrt{3})^2}\right)}^{\frac{1}{2}\left(\sqrt{2}+\sqrt{3}+\sqrt{4\sqrt{6}+(-\sqrt{2}-\sqrt{3})^2}\right)} \left(-3\sqrt{2}+2\sqrt{3}+x\right)\left(-\sqrt{6}-\left(\sqrt{2}+\sqrt{3}\right)x+x^2\right)$$
$$\theta\left(-\left(-3\sqrt{2}+2\sqrt{3}+x\right)\left(-\sqrt{6}-\left(\sqrt{2}+\sqrt{3}\right)x+x^2\right)\right)dx =$$
$$\frac{1}{24}\left(4619-1960\sqrt{6}-12\sqrt{\frac{3125\sqrt{\frac{2}{3}}}{3}-\frac{103\,175}{144}}\right) \approx -13.3718$$

From:

$$\frac{1}{24}\left(4619-1960\sqrt{6}-12\sqrt{-\frac{103\,175}{144}+\frac{1}{3}\left(3125\sqrt{\frac{2}{3}}\right)}\right)$$

we obtain:

1/24 (4619 - 1960 sqrt(6) - 12 sqrt(-103175/144 + (3125 sqrt(2/3))/3))

Input:

$$\frac{1}{24}\left(4619-1960\sqrt{6}-12\sqrt{-\frac{103\,175}{144}+\frac{1}{3}\left(3125\sqrt{\frac{2}{3}}\right)}\right)$$

Result:

$$\frac{1}{24}\left(4619-1960\sqrt{6}-12\sqrt{\frac{3125\sqrt{\frac{2}{3}}}{3}-\frac{103\,175}{144}}\right)$$

Decimal approximation:

$-13.371770412946158691396507338722661159142682155532231997836842 72$

…

-13.3717704….

Alternate forms:

$$\frac{1}{24}\left(-5\sqrt{2000\sqrt{6}-4127}-1960\sqrt{6}+4619\right)$$

$$\frac{4619}{24}-245\sqrt{\frac{2}{3}}-\frac{5}{24}\sqrt{2000\sqrt{6}-4127}$$

$$\frac{1}{24}\left(4619-1960\sqrt{6}\right)-\frac{5}{24}\sqrt{2000\sqrt{6}-4127}$$

Minimal polynomial:

$$144x^4-110856x^3+20529529x^2+334601184x+533790624$$

Expanded form:

$$\frac{4619}{24}-245\sqrt{\frac{2}{3}}-\frac{1}{2}\sqrt{\frac{3125\sqrt{\frac{2}{3}}}{3}-\frac{103175}{144}}$$

From which:

-(2*11)/((1/24 (4619 - 1960 sqrt(6) - 12 sqrt(-103175/144 + (3125 sqrt(2/3))/3))))

Input:

$$-\frac{2\times 11}{\frac{1}{24}\left(4619-1960\sqrt{6}-12\sqrt{-\frac{103\,175}{144}+\frac{1}{3}\left(3125\sqrt{\frac{2}{3}}\right)}\right)}$$

Result:

$$-\frac{528}{4619-1960\sqrt{6}-12\sqrt{\frac{3125\sqrt{\frac{2}{3}}}{3}-\frac{103\,175}{144}}}$$

Decimal approximation:

1.64525708418537015113510287724244487191406381728856757827269365 77

…

$1.6452570841\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms:

$$\frac{528}{5\sqrt{2000\sqrt{6}-4127}+1960\sqrt{6}-4619}$$

$$\frac{76\,679\,438+\frac{190\,871\,395}{\sqrt{6}}-11\,120\,638\sqrt{\frac{41\,778\,885\,688\,019\,425}{742\,011\,537\,162\,264}+\frac{4\,264\,042\,072\,745\,125}{30\,917\,147\,381\,761\sqrt{6}}}}{22\,241\,276}$$

$$\frac{1}{-\frac{4619}{528}+\frac{245}{11\sqrt{6}}+\frac{5}{528}\sqrt{2000\sqrt{6}-4127}}$$

Minimal polynomial:

$$133\,447\,656\,x^4 - 1\,840\,306\,512\,x^3 + 2\,484\,073\,009\,x^2 + 295\,098\,672\,x + 8\,433\,216$$

24*(-13.3717704129 + 1/24*1960 sqrt(6) + 1/24*(12 sqrt(-103175/144 + (3125 sqrt(2/3))/3)))-16^2+3^2

Input interpretation:

$$24\left(-13.3717704129 + \frac{1}{24}\times 1960\sqrt{6} + \frac{1}{24}\left(12\sqrt{-\frac{103\,175}{144} + \frac{1}{3}\left(3125\sqrt{\frac{2}{3}}\right)}\right)\right) - 16^2 + 3^2$$

Result:

4372.000000001…

≈ 4372

Where 4372 is a value indicated in the fundamental Ramanujan paper "**Modular equations and Approximations to π**"

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

And:

27sqrt((24*(-13.3717704129 + 1/24*1960 sqrt(6) + 1/24*(12 sqrt(-103175/144 + (3125 sqrt(2/3))/3)))-521-2))+1

Input interpretation:

$$27\sqrt{\left(24\left(-13.3717704129+\frac{1}{24}\times 1960\sqrt{6}+\frac{1}{24}\left(12\sqrt{-\frac{103\,175}{144}+\frac{1}{3}\left(3125\sqrt{\frac{2}{3}}\right)}\right)\right)-521-2\right)}+1$$

Result:

1729.000000000…

1729

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(((27sqrt((24*(-13.3717704129 + 1/24*1960 sqrt(6) + 1/24*(12 sqrt(-103175/144 + (3125 sqrt(2/3))/3)))-521-2))+1)))^1/15

Input interpretation:

$$\left(27\sqrt{\left(24\left(-13.3717704129+\frac{1}{24}\times 1960\sqrt{6}+\frac{1}{24}\left(12\sqrt{-\frac{103\,175}{144}+\frac{1}{3}\left(3125\sqrt{\frac{2}{3}}\right)}\right)\right)-521-2\right)}+1\right)^{(1/15)}$$

Result:

1.64381522874874…

1.64381522874874…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

(((((27sqrt((24*(-13.3717704129 + 1/24*1960 sqrt(6) + 1/24*(12 sqrt(-103175/144 + (3125 sqrt(2/3))/3)))-521-2))+1)))^1/15-1))^1/32

Input interpretation:

$$\left(\left(27\sqrt{\left(24\left(-13.3717704129+\frac{1}{24}\times 1960\sqrt{6}+\frac{1}{24}\left(12\sqrt{-\frac{103\,175}{144}+\frac{1}{3}\left(3125\sqrt{\frac{2}{3}}\right)}\right)\right)-521-2\right)}+1\right)\text{^}(1/15)-1\right)\text{^}(1/32)$$

Result:

0.9863335115537802…

0.9863335115…. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the Omega mesons ($\omega/\omega_3 \mid 5+3 \mid m_{u/d}=255-390 \mid 0.988-1.18$) Regge slope value (0.988) connected to the dilaton scalar field $\mathbf{0.989117352243} = \boldsymbol{\phi}$

A_1^{**} above the two low-lying pseudo-scalars. (bound states of gluons, or 'glueballs')

A_1^{**}	0.943(39)	[2.5]	0.988(38)	0.152(53)
A_4	1.03(10)	[2.5]	0.999(32)	0.035(21)

(**Glueball Regge trajectories** - *Harvey Byron Meyer*, Lincoln College -Thesis submitted for the degree of Doctor of Philosophy at the University of Oxford Trinity Term, 2004)

Note that

$\sqrt[32]{\zeta(2)-1}$

$\sqrt[32]{\frac{\pi^2}{6}-1}$

= 0.98638703135648129 15… $(\pi^2/6 - 1)^{1/32}$

From:

$\Delta_x = -12\left(409\sqrt{2} - 279\sqrt{3}\right)\left(\sqrt{2} - \sqrt{3}\right)$

-12 (409 sqrt(2) - 279 sqrt(3)) (sqrt(2) - sqrt(3))

Input:

$-12\left(409\sqrt{2} - 279\sqrt{3}\right)\left(\sqrt{2} - \sqrt{3}\right)$

Decimal approximation:

362.98731641791837871677732077183933207086240030146858034030983522 …

362.9873164179….

Alternate form:

$-12\left(1655 - 688\sqrt{6}\right)$

Minimal polynomial:

$x^2 + 39720\,x - 14549616$

Expanded form:

$8256\sqrt{6} - 19860$

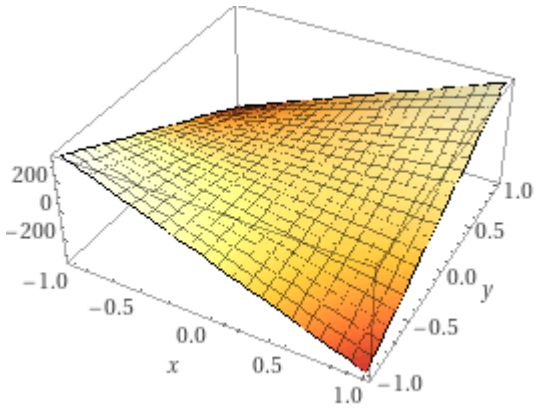
Integrating:

((-12 (409 sqrt(2) - 279 sqrt(3)) (sqrt(2) - sqrt(3))))dxdy

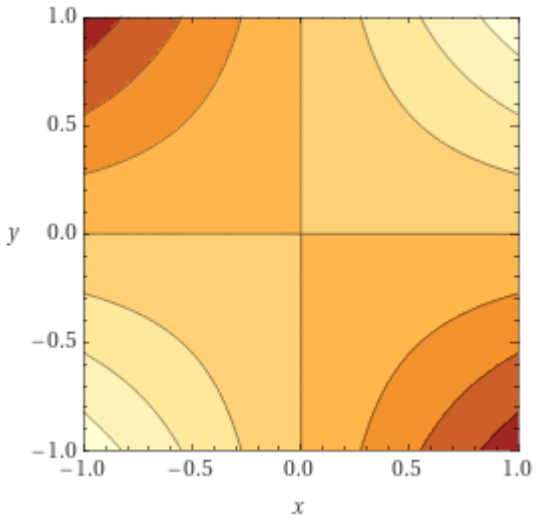
Indefinite integral:

$$\int\int -12\left(409\sqrt{2} - 279\sqrt{3}\right)\left(\sqrt{2} - \sqrt{3}\right)dx\,dy = c_1\,x + c_2 + 8256\sqrt{6}\,x\,y - 19860\,x\,y$$

3D plot: **(figure that can be related to a D-brane)**



Contour plot:



Definite integral over a disk of radius R:

$$\iint_{x^2+y^2<R^2} -12\left(409\sqrt{2}-279\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)dy\,dx =$$
$$-12\left(409\sqrt{2}-279\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)\pi R^2$$

Definite integral over a square of edge length 2 L:

$$\int_{-L}^{L}\int_{-L}^{L} -12\left(409\sqrt{2}-279\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)dx\,dy =$$
$$-48\left(409\sqrt{2}-279\sqrt{3}\right)\left(\sqrt{2}-\sqrt{3}\right)L^2$$

0.3543 + 1 + 8256 sqrt(6) 0.3543*0.3543 - 19860 0.3543*0.3543

Input:

$$0.3543 + 1 + 8256\sqrt{6}\ (0.3543\times0.3543) - 19860\,(0.3543\times0.3543)$$

Result:

46.9195…

46.9195…

From:

String (In)Stability Issues with Broken Supersymmetry
J. Mourad, A. Sagnotti - arXiv:2107.04064v1 [hep-th] 8 Jul 2021

Now, from the following equation :

$$A'' + A'\left(24\,\Omega' - \frac{2}{\phi'}\,e^{2\Omega}\,V_\phi\right) + A\left(m^2 - \frac{7}{4}\,e^{2\Omega}\,V - 14\,e^{2\Omega}\,\Omega'\,\frac{V_\phi}{\phi'}\right) = 0 \qquad (3.3)$$

we obtain:

d(d(A)) + d(A) (((24 d (Ω)-(1/(d(φ))) 2 (e^(2Ω)) Vφ))) + A(((m^2 – 7/4 e^(2Ω) V – 14e^(2Ω) d (Ω) Vφ 1/(d(φ)))))

Input:

$$d\,d(A) + d(A)\left(24\,d\,\Omega - \frac{1}{d(\phi)} \times 2\,e^{2\,\Omega}\,(V\,\phi)\right) + A\left(m^2 - \frac{7}{4}\,e^{2\,\Omega}\,V - \left(14\,e^{2\,\Omega}\right)d\,\Omega\,(V\,\phi) \times \frac{1}{d(\phi)}\right)$$

ϕ is the golden ratio

Alternate forms:

$$-\frac{V\,e^{2\,\Omega}\left(28\,\sqrt{5}\;A\,d\,\Omega + 28\,A\,d\,\Omega + 7\,A\,d(\phi) + 4\,\sqrt{5}\;d(A) + 4\,d(A)\right)}{4\,d(\phi)} + 24\,d\,\Omega\,d(A) + d\,d(A) + A\,m^2$$

$$A\left(V\,e^{2\Omega}\left(-\frac{14\,d\,\Omega\,\phi}{d\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}-\frac{7}{4}\right)+m^2\right)+d(A)\left(d\,(24\,\Omega+1)-\frac{2\,V\,e^{2\Omega}\,\phi}{d\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)}\right)$$

$$\frac{1}{4\,d(\phi)}\left(4\,A\,m^2\,d(\phi)-28\,\sqrt{5}\;A\,d\,V\,e^{2\Omega}\,\Omega-28\,A\,d\,V\,e^{2\Omega}\,\Omega-4\,\sqrt{5}\;V\,e^{2\Omega}\,d(A)-\right.$$
$$\left.4\,V\,e^{2\Omega}\,d(A)-7\,A\,V\,e^{2\Omega}\,d(\phi)+96\,d\,\Omega\,d(A)\,d(\phi)+4\,d\,d(A)\,d(\phi)\right)$$

Expanded form:

$$-\frac{14\,A\,d\,V\,e^{2\Omega}\,\Omega\,\phi}{d(\phi)}-\frac{2\,V\,e^{2\Omega}\,\phi\,d(A)}{d(\phi)}+24\,d\,\Omega\,d(A)+d\,d(A)+A\,m^2-\frac{7}{4}\,A\,V\,e^{2\Omega}$$

Indefinite integral:

$$\int\left(d\,d(A)+d(A)\left(24\,d\,\Omega-\frac{2\,e^{2\Omega}\,(V\,\phi)}{d(\phi)}\right)+A\left(m^2-\frac{7}{4}\,e^{2\Omega}\,V-\frac{\left(14\,e^{2\Omega}\right)d\,\Omega\,(V\,\phi)}{d(\phi)}\right)\right)$$
$$dm=m\,d(A)\left(24\,d\,\Omega-\frac{2\,V\,e^{2\Omega}\,\phi}{d(\phi)}\right)-$$
$$\frac{7\,A\,m\,V\,e^{2\Omega}\,(8\,d\,\Omega\,\phi+d(\phi))}{4\,d(\phi)}+d\,m\,d(A)+\frac{A\,m^3}{3}+\text{constant}$$

From:

d(A) (24 d Ω - (2 V e^(2 Ω) ϕ)/d(ϕ)) - (7 A V e^(2 Ω) (8 d Ω ϕ + d(ϕ)))/(4 d(ϕ)) + d d(A) + (A)/3

Input:

$$d(A)\left(24\,d\,\Omega-\frac{2\,V\,e^{2\Omega}\,\phi}{d(\phi)}\right)-\frac{7\,A\,V\,e^{2\Omega}\,(8\,d\,\Omega\,\phi+d(\phi))}{4\,d(\phi)}+d\,d(A)+\frac{A}{3}$$

Alternate forms:

$$\frac{1}{3}\left(72\, d\, \Omega\, d(A) + 3\, d\, d(A) + A\right) - \frac{V\, e^{2\Omega}\left(56\, A\, d\, \Omega\, \phi + 8\, \phi\, d(A) + 7\, A\, d(\phi)\right)}{4\, d(\phi)}$$

$$A\left(V\, e^{2\Omega}\left(-\frac{14\, d\, \Omega\, \phi}{d(\phi)} - \frac{7}{4}\right) + \frac{1}{3}\right) + d(A)\left(d\,(24\,\Omega + 1) - \frac{2\, V\, e^{2\Omega}\, \phi}{d(\phi)}\right)$$

$$-\frac{1}{12\, d(\phi)}\left(168\, A\, d\, V\, e^{2\Omega}\, \Omega\, \phi + 24\, V\, e^{2\Omega}\, \phi\, d(A) + 21\, A\, V\, e^{2\Omega}\, d(\phi) - 288\, d\, \Omega\, d(A)\, d(\phi) - 4\, A\, d(\phi) - 12\, d\, d(A)\, d(\phi)\right)$$

Expanded form:

$$-\frac{14\, A\, d\, V\, e^{2\Omega}\, \Omega\, \phi}{d(\phi)} - \frac{2\, V\, e^{2\Omega}\, \phi\, d(A)}{d(\phi)} + 24\, d\, \Omega\, d(A) + d\, d(A) - \frac{7}{4}\, A\, V\, e^{2\Omega} + \frac{A}{3}$$

Series expansion at A=0:

$$d(0)\left(-\frac{2\, V\, e^{2\Omega}\, \phi}{d(\phi)} + 24\, d\, \Omega + d\right) + A\left(-\frac{2\, V\, e^{2\Omega}\, \phi\,(d'(0) + 7\, d\, \Omega)}{d(\phi)} + 24\, d\, \Omega\, d'(0) + d\, d'(0) - \frac{7}{4}\, V\, e^{2\Omega} + \frac{1}{3}\right) + \frac{A^2\, d''(0)\left((24\, d\, \Omega + d)\, d(\phi) - 2\, V\, e^{2\Omega}\, \phi\right)}{2\, d(\phi)} + \frac{A^3\, d^{(3)}(0)\left((24\, d\, \Omega + d)\, d(\phi) - 2\, V\, e^{2\Omega}\, \phi\right)}{6\, d(\phi)} + \frac{A^4\, d^{(4)}(0)\left((24\, d\, \Omega + d)\, d(\phi) - 2\, V\, e^{2\Omega}\, \phi\right)}{24\, d(\phi)} + O\left(A^5\right)$$

(Taylor series)

From:

$$\frac{1}{3}(72\,d\,\Omega\,d(A)+3\,d\,d(A)+A)-\frac{V\,e^{2\Omega}(56\,A\,d\,\Omega\,\phi+8\,\phi\,d(A)+7\,A\,d(\phi))}{4\,d(\phi)}$$

we obtain:

((1/3 (72 d (0.595999682594) d(3) + 3 d d(3) + 3) - (V e^(2*0.595999682594) (56*3 d (0.595999682594) ϕ + 8 ϕ d(3) + 7*3 d(ϕ)))/(4 d(ϕ)))) dr

Indefinite integral:

$$\int\left(\frac{1}{3}(72\,d\,0.595999682594\,d(3)+3\,d\,d(3)+3)-\frac{V\,e^{2\times0.595999682594}((56\times3)\,d\,0.595999682594\,\phi+8\,\phi\,d(3)+7\times3\,d(\phi))}{4\,d(\phi)}\right)dr=0.333333333333\,(45.9119771468\,d(3.00000000000)\,d+3.00000000000)\,r-\frac{1}{d(\phi)}0.823414964372\,r\,V\,(100.127946676\,d\,\phi+8.00000000000\,d(3.00000000000)\,\phi+21.0000000000\,d(\phi))+\text{constant}$$

Alternate forms of the integral:

$$r\left(\frac{(-82.446849645\,d-6.5873197150\,d(3.00000000000))\,V\,\phi}{d(\phi)}+15.3039923823\,d\,d(3.00000000000)-17.2917142518\,V+1.00000000000\right)+\text{constant}$$

$$\frac{-82.4468496\,d\,r\,V\,\phi-6.587319715\,d(3.00000000000)\,r\,V\,\phi}{d(\phi)}+15.303992382\,r(1.0000000000\,d\,d(3.00000000000)-1.1298825705\,V+0.06534242667)+\text{constant}$$

$$r\left(d\left(15.3039923823\,d(3.00000000000)-\frac{82.446849645\,V\,\phi}{d(\phi)}\right)+V\left(-\frac{6.5873197150\,d(3.00000000000)\,\phi}{d(\phi)}-17.2917142518\right)+1.00000000000\right)+\text{constant}$$

Expanded form of the integral:

$$-\frac{82.446849645\,d\,r\,V\,\phi}{d(\phi)}-\frac{6.5873197150\,d(3)\,r\,V\,\phi}{d(\phi)}+15.3039923823\,d\,d(3)\,r-17.2917142518\,r\,V+1.00000000000\,r+\text{constant}$$

From the result of the integral, we obtain:

0.33333 (45.9119771468 d(3) d + 3) r - (0.823414964372 r V (100.127946676 d ϕ + 8 d(3) ϕ + 21 d(ϕ)))/d(ϕ)

Input interpretation:

$$0.33333\,(45.9119771468\,d(3)\,d+3)\,r-\frac{0.823414964372\,r\,V\,(100.127946676\,d\,\phi+8\,d(3)\,\phi+21\,d(\phi))}{d(\phi)}$$

Result:

$$0.33333\,(45.9119771468\,d(3)\,d+3)\,r-\frac{0.823414964372\,r\,V\,(100.1279466760\,d\,\phi+8\,d(3)\,\phi+21\,d(\phi))}{d(\phi)}$$

Alternate forms:

$$r\left(\frac{(-82.4468\,d-6.58732\,d(3))\,V\,\phi}{d(\phi)}+15.3038\,d(3)\,d-17.2917\,V+0.99999\right)$$

$$\frac{1}{d(\phi)}15.3038\,r\,(-5.38733\,d\,V\,\phi - 0.430436\,d(3)\,V\,\phi - 1.12989\,V\,d(\phi) + d\,d(3)\,d(\phi) + 0.0653424\,d(\phi))$$

$$\frac{-82.44684964\,d\,r\,V\,\phi - 6.587319715\,d(3)\,r\,V\,\phi}{d(\phi)} + 15.3038\,r\,(d(3)\,d - 1.12989\,V + 0.0653424)$$

Expanded form:

$$-\frac{82.446849645\,d\,r\,V\,\phi}{d(\phi)} - \frac{6.58731971498\,d(3)\,r\,V\,\phi}{d(\phi)} + 15.3038\,d\,d(3)\,r - 17.2917142518\,r\,V + 0.99999\,r$$

Property as a function:

Parity

odd

Indefinite integral:

$$\int\left(0.33333\,(45.9119771468\,d(3)\,d + 3)\,r - \frac{0.823414964372\,r\,V\,(100.127946676\,d\,\phi + 8\,d(3)\,\phi + 21\,d(\phi))}{d(\phi)}\right)dr = 0.166665\,(45.912\,d\,d(3) + 3)\,r^2 - \frac{0.411707\,r^2\,V\,(100.128\,d\,\phi + 8\,d(3)\,\phi + 21\,d(\phi))}{d(\phi)} + \text{constant}$$

And:

0.33333 (45.9119771468 (3) 1+3) - (0.823414964372 (5.21893*10^-6 * e^(2*0.9991104684)) (100.1279466760 * 0.9991104 + 8 (3) 0.9991104 + 21 (0.9991104)))/(0.9991104)

where 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

Input interpretation:

$$0.33333\,(45.9119771468\times 3\times 1+3)-\frac{1}{0.9991104}$$
$$0.823414964372\left(5.21893\times 10^{-6}\,e^{2\times 0.9991104684}\right)$$
$$(100.1279466760\times 0.9991104+8\times 3\times 0.9991104+21\times 0.9991104)$$

Result:

46.9069…

46.9069….

Now, we have:

$x^3 + q = 0$

$$(x^3+q)=\left(x-\sqrt[3]{q}\right)\left(x^2-\sqrt[3]{q}\right)x+\left(-\sqrt[3]{q}\right)$$

If we put $-\sqrt[3]{q}=1, \quad q=1$, we obtain:

$$(x^3 + 1) = (x - 1)(x^2 - x + 1)$$

From the right-hand side, we obtain:

$x_1 = -1$

$x^2 - x + 1 = 0$

from which:

$$\frac{1 \pm \sqrt{1-4}}{2} = \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm \sqrt{3}i}{2}$$

i.e.

$$x_2 = \frac{1+\sqrt{3}i}{2} \; ; \quad x_3 = \frac{1-\sqrt{3}i}{2}$$

We have that:

$$x_3 = \left(\frac{1}{2} - \frac{\sqrt{3}i}{2}\right)^3$$

Input:

$$\left(\frac{1}{2} - \frac{1}{2}\left(\sqrt{3}\ i\right)\right)^3$$

i is the imaginary unit

Result:

-1 -1

-1

Indeed performing the third root:

(((1/2-(√3 i)/2)^3))^(1/3)

Input:

$$\sqrt[3]{\left(\frac{1}{2}-\frac{1}{2}\left(\sqrt{3}\ i\right)\right)^3}$$

i is the imaginary unit

Result:

$$\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)e^{(2i\pi)/3}$$

Decimal approximation:

0.5 +
0.866025403784438646763723170752936183471402626905190314027903489…
i

Polar coordinates:

$r = 1$ (radius), $\theta = \frac{\pi}{3}$ (angle)

1

Polar forms:

$$\cos\left(\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{3}\right)$$

Approximate form

$e^{(i\pi)/3}$

Alternate forms:

$\sqrt[3]{-1}$

$\frac{1}{2} + \frac{i\sqrt{3}}{2}$

$\frac{1}{2}\left(1 + i\sqrt{3}\right)$

Minimal polynomial:

$x^2 - x + 1$

Expanded form:

$\frac{1}{2}\, e^{(2i\pi)/3} - \frac{1}{2}\, i\sqrt{3}\; e^{(2i\pi)/3}$

All 3rd roots of (1/2-(i sqrt(3))/2)³:

$e^{(i\pi)/3} \approx 0.5 + 0.8660\, i$ (principal root)

$e^{i\pi} = -1$ (real root)

$e^{-(i\pi)/3} \approx 0.5 - 0.8660\, i$

From

$$x_3 = \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)^3$$

1/8+3(((1/4)((√3 i)/2)))+3(((1/2) ((√3 i)/2)^2))+(((√3 i)/2))^3

Input:

$$\frac{1}{8} + 3\left(\frac{1}{4}\left(\frac{1}{2}\left(\sqrt{3}\ i\right)\right)\right) + 3\left(\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{3}\ i\right)\right)^2\right) + \left(\frac{1}{2}\left(\sqrt{3}\ i\right)\right)^3$$

i is the imaginary unit

Exact result:

-1

-1

q*[1/8+3(((1/4)((√3 i)/2)))+3(((1/2) ((√3 i)/2)^2))+(((√3 i)/2))^3]

Input:

$$q\left(\frac{1}{8} + 3\left(\frac{1}{4}\left(\frac{1}{2}\left(\sqrt{3}\ i\right)\right)\right) + 3\left(\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{3}\ i\right)\right)^2\right) + \left(\frac{1}{2}\left(\sqrt{3}\ i\right)\right)^3\right)$$

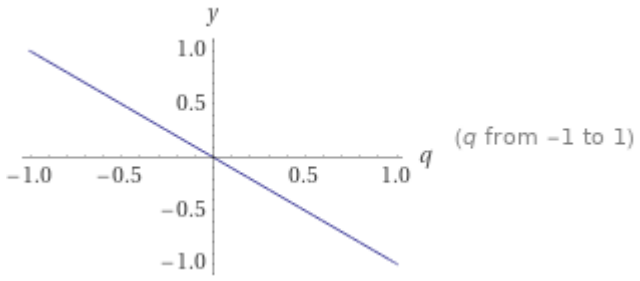
i is the imaginary unit

Result:

$-q$

- q

Plot:



Geometric figure:
line

Property as a function:
Parity
odd

Derivative:

$$\frac{d}{dq}\left(q\left(\frac{1}{8}+\frac{3\left(\sqrt{3}\ i\right)}{4\times 2}+\frac{3}{2}\left(\frac{\sqrt{3}\ i}{2}\right)^2+\left(\frac{\sqrt{3}\ i}{2}\right)^3\right)\right)=-1$$

Indefinite integral:

$$\int q\left(\frac{1}{8}+\frac{3\left(\sqrt{3}\ i\right)}{4\times 2}+\frac{3}{2}\left(\frac{\sqrt{3}\ i}{2}\right)^2+\left(\frac{\sqrt{3}\ i}{2}\right)^3\right)dq=-\frac{q^2}{2}+\text{constant}$$

From which, performing the third root:

(((q*[1/8+3(((1/4)((√3 i)/2)))+3(((1/2) ((√3 i)/2)^2))+(((√3 i)/2))^3])))^1/3

Input:

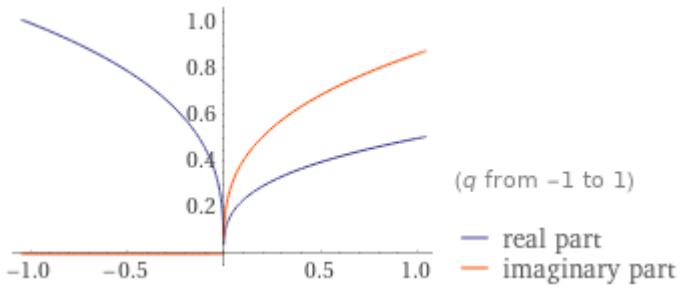
$$\sqrt[3]{q\left(\frac{1}{8}+3\left(\frac{1}{4}\left(\frac{1}{2}\left(\sqrt{3}\ i\right)\right)\right)+3\left(\frac{1}{2}\left(\frac{1}{2}\left(\sqrt{3}\ i\right)\right)^2\right)+\left(\frac{1}{2}\left(\sqrt{3}\ i\right)\right)^3\right)}$$

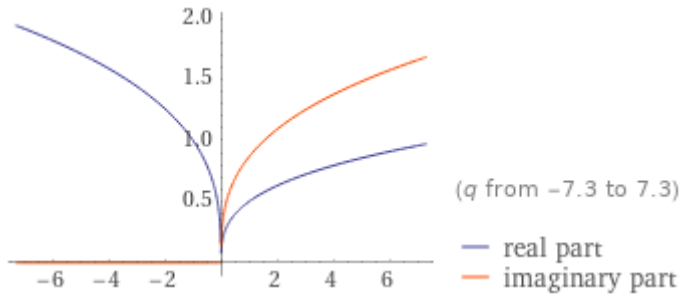
i is the imaginary unit

Result:

$\sqrt[3]{-q}$

Plots: **(figures that can be related to the open strings)**





Alternate forms assuming q>0:

$\sqrt[3]{-1}\ \sqrt[3]{q}$

$e^{(i\pi)/3}\ \sqrt[3]{q}$

Root:

$q = 0$

Series expansion at q=0:

$$\begin{cases} \sqrt[3]{-q} + O\left(q^{12}\right) & \text{Im}(q) \le 0 \\ \sqrt[3]{-q}^{*} + O\left(q^{12}\right) & \text{(otherwise)} \end{cases}$$

Series expansion at q=∞:

$$\frac{\sqrt[3]{-q}\ \sqrt[3]{q}}{\sqrt[3]{q}} + O\left(\left(\frac{1}{q}\right)^{37/3}\right)$$

(Puiseux series)

Derivative:

$$\frac{d}{dq}\left(\sqrt[3]{q\left(\frac{1}{8} + \frac{3\left(\sqrt{3}\ i\right)}{4\times 2} + \frac{3}{2}\left(\frac{\sqrt{3}\ i}{2}\right)^2 + \left(\frac{\sqrt{3}\ i}{2}\right)^3\right)}\right) = -\frac{1}{3\,(-q)^{2/3}}$$

Indefinite integral:

$$\int \sqrt[3]{q\left(\frac{1}{8} + \frac{3\left(\sqrt{3}\ i\right)}{4\times 2} + \frac{3}{2}\left(\frac{\sqrt{3}\ i}{2}\right)^2 + \left(\frac{\sqrt{3}\ i}{2}\right)^3\right)}\, dq = \frac{3}{4}\sqrt[3]{-q}\ q + \text{constant}$$

Global minimum:

$$\min\left\{\sqrt[3]{q\left(\frac{1}{8} + \frac{3\left(\sqrt{3}\ i\right)}{4\times 2} + \frac{3}{2}\left(\frac{\sqrt{3}\ i}{2}\right)^2 + \left(\frac{\sqrt{3}\ i}{2}\right)^3\right)}\right\} = 0 \text{ at } q = 0$$

Series representations:

$$\sqrt[3]{q\left(\frac{1}{8} + \frac{3\left(\sqrt{3}\ i\right)}{4\times 2} + \frac{3}{2}\left(\frac{\sqrt{3}\ i}{2}\right)^2 + \left(\frac{\sqrt{3}\ i}{2}\right)^3\right)} = \frac{1}{2}\sqrt[3]{q\left(1 + i\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}\right)^3}$$

$$\sqrt[3]{q\left(\frac{1}{8}+\frac{3\left(\sqrt{3}\,i\right)}{4\times 2}+\frac{3}{2}\left(\frac{\sqrt{3}\,i}{2}\right)^2+\left(\frac{\sqrt{3}\,i}{2}\right)^3\right)}=\frac{1}{2}\sqrt[3]{q\left(1+i\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^3}$$

$$\sqrt[3]{q\left(\frac{1}{8}+\frac{3\left(\sqrt{3}\,i\right)}{4\times 2}+\frac{3}{2}\left(\frac{\sqrt{3}\,i}{2}\right)^2+\left(\frac{\sqrt{3}\,i}{2}\right)^3\right)}=$$
$$\frac{1}{4}\sqrt[3]{\frac{q\left(2\sqrt{\pi}+i\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^3}{\sqrt{\pi}^{\,3}}}$$

Integral representation:

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

$$x_1=\sqrt[3]{-q}\qquad x_2=\sqrt[3]{-q}\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)\qquad x_3=\sqrt[3]{-q}\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)$$

So the third degree binomial in *x* can be decomposed into factors as follows:

$$\left(x-\sqrt[3]{-q}\right)\left[\left(x-\sqrt[3]{-q}\right)\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)\right]\left[\left(x-\sqrt[3]{-q}\right)\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)\right]$$

$$\left(x^3+q\right)=\left(x-\sqrt[3]{q}\right)\left[\left(x+\sqrt[3]{q}\right)\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)\right]\left[\left(x+\sqrt[3]{-q}\right)\left(\frac{1}{2}-\frac{\sqrt{3}i}{2}\right)\right]$$

(1/2 - (i sqrt(3))/2) (1/2 + (i sqrt(3))/2) (x - q^(1/3))^2 (x + q^(1/3))

Input:

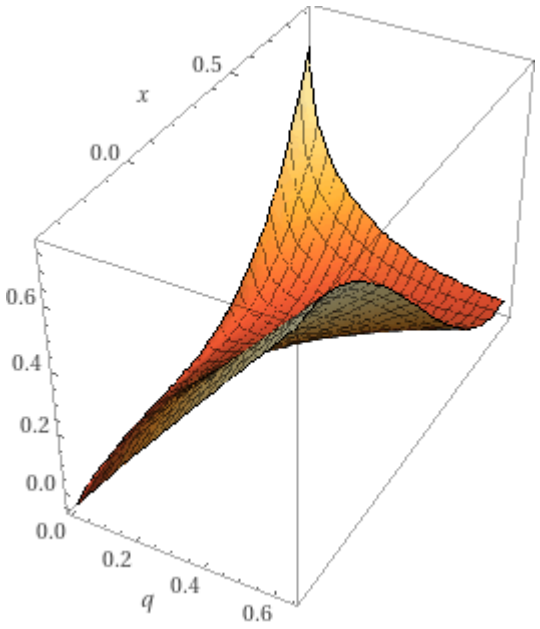
$$\left(\frac{1}{2}-\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(x-\sqrt[3]{q}\right)^2\left(x+\sqrt[3]{q}\right)$$

i is the imaginary unit

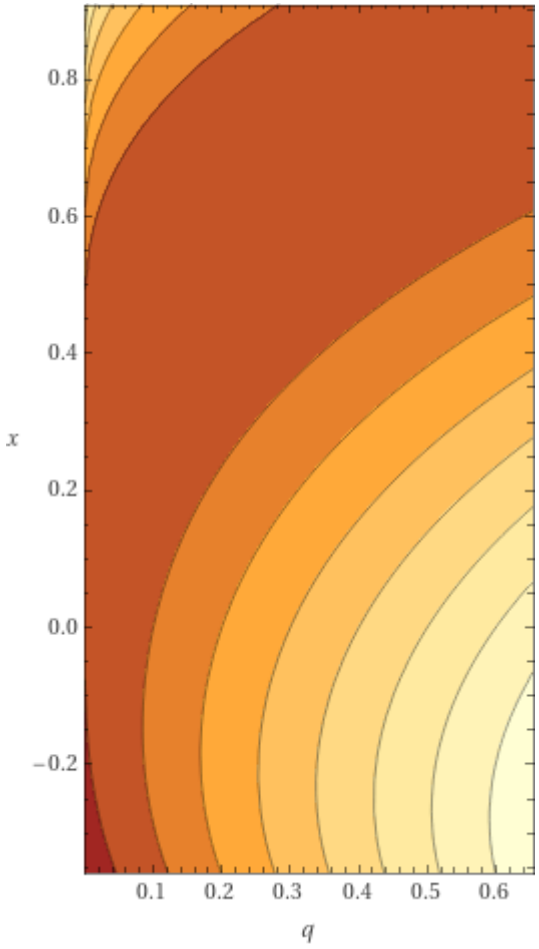
Exact result:

$$\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\sqrt[3]{q}\right)^2\left(\sqrt[3]{q}+x\right)$$

3D plot: **(figure that can be related to a D-brane)**



Contour plot:



Alternate forms:

$$\left(\sqrt[3]{q}-x\right)^2\left(\sqrt[3]{q}+x\right)$$

$$\left(x-\sqrt[3]{q}\right)^2\left(\sqrt[3]{q}+x\right)$$

$$\frac{1}{4}\left(\sqrt{3}+-i\right)\left(\sqrt{3}+i\right)\left(\sqrt[3]{q}-x\right)^2\left(\sqrt[3]{q}+x\right)$$

Alternate form assuming q and x are positive:

$$x\left(x-\sqrt[3]{q}\right)^2+\sqrt[3]{q}\left(x-\sqrt[3]{q}\right)^2$$

Expanded forms:

$$x\left(q^{2/3} - 2\sqrt[3]{q}\ x + x^2\right) + \sqrt[3]{q}\ \left(q^{2/3} - 2\sqrt[3]{q}\ x + x^2\right)$$

$$-q^{2/3}\,x - \sqrt[3]{q}\ x^2 + q + x^3$$

Roots:

$$x = -\sqrt[3]{q}$$

$$x = \sqrt[3]{q}$$

Integer root:

$$q = 0, \quad x = 0$$

Derivative:

$$\frac{\partial}{\partial x}\left(\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\left(x - \sqrt[3]{q}\right)^2\left(x + \sqrt[3]{q}\right)\right) = -q^{2/3} - 2\sqrt[3]{q}\ x + 3x^2$$

Indefinite integral:

$$\int\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\left(x - \sqrt[3]{q}\right)^2\left(x + \sqrt[3]{q}\right)dx =$$
$$\frac{1}{12}\,x\left(-6q^{2/3}\,x - 4\sqrt[3]{q}\ x^2 + 12q + 3x^3\right) + \text{constant}$$

Series representations:

$$\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\sqrt[3]{q}+x\right)^2\left(\sqrt[3]{q}+x\right)=\sum_{n=-\infty}^{\infty}\left(\begin{cases}1 & n=3\\ -\sqrt[3]{q} & n=2\\ -q^{2/3} & n=1\\ q & n=0\end{cases}\right)x^n$$

$$\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\sqrt[3]{q}+x\right)^2\left(\sqrt[3]{q}+x\right)=$$
$$\sum_{n=-\infty}^{\infty}\left(\begin{cases}1 & n=3\\ 3-\sqrt[3]{q} & n=2\\ \left(-1+\sqrt[3]{q}\right)^2\left(1+\sqrt[3]{q}\right) & n=0\\ 3-2\sqrt[3]{q}-q^{2/3} & n=1\end{cases}\right)(-1+x)^n$$

For q = 1, from:

$$\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(x-\sqrt[3]{q}\right)^2\left(\sqrt[3]{q}+x\right)$$

(1/2 - (i sqrt(3))/2) (1/2 + (i sqrt(3))/2) (-1^(1/3) + x)^2 (1^(1/3) + x)

Input:

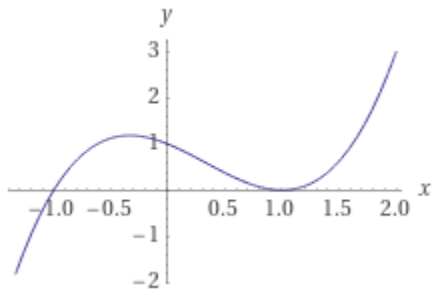
$$\left(\frac{1}{2}-\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(-\sqrt[3]{1}+x\right)^2\left(\sqrt[3]{1}+x\right)$$

i is the imaginary unit

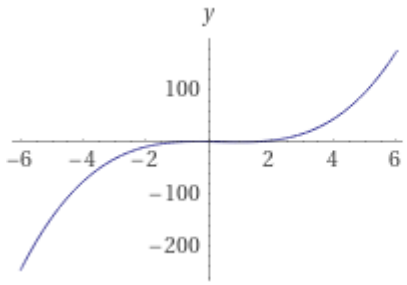
Result:

$$\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(x-1)^2(x+1)$$

Plots: **(figures that can be related to the open strings)**



(x from −1.3 to 2)



(x from −6 to 6)

Alternate forms:

$$(x-1)^2\,(x+1)$$

$$x\,((x-1)\,x-1)+1$$

$$\frac{1}{4}\left(\sqrt{3}+-i\right)\left(\sqrt{3}+i\right)(x-1)^2\,(x+1)$$

Alternate form assuming x>0:

$$x\,(x-1)^2+(x-1)^2$$

Expanded form:

$$x^3-x^2-x+1$$

Roots:

$$x = -1$$

$$x = 1$$

Polynomial discriminant:

$$\Delta = 0$$

Derivative:

$$\frac{d}{dx}\left(\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\left(-\sqrt[3]{1} + x\right)^2\left(\sqrt[3]{1} + x\right)\right) = 3x^2 - 2x - 1$$

Indefinite integral:

$$\int\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)\left(-\sqrt[3]{1} + x\right)^2\left(\sqrt[3]{1} + x\right)dx =$$
$$\frac{x^4}{4} - \frac{x^3}{3} - \frac{x^2}{2} + x + \text{constant}$$

Definite integral area below the axis between the smallest and largest real roots:

$$\int_{-1}^{1}\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)(-1+x)^2(1+x)$$
$$\theta\left(-\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)(-1+x)^2(1+x)\right)dx = 0$$

$\theta(x)$ is the Heaviside step function

Definite integral area above the axis between the smallest and largest real roots:

$$\int_{-1}^{1}\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)^2(1+x)$$
$$\theta\left(\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)(-1+x)^2(1+x)\right)dx=\frac{4}{3}\approx 1.33333$$

For q = 1 and x = (((√(10-2√5) -2))/((√5-1))) = κ

(1/2 - (i sqrt(3))/2) (1/2 + (i sqrt(3))/2) (-1^(1/3) + (((((√(10-2√5) -2))/((√5-1))))))^2 (1^(1/3) + (((((√(10-2√5) -2))/((√5-1))))))

Input:

$$\left(\frac{1}{2}-\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(i\sqrt{3}\right)\right)$$
$$\left(-\sqrt[3]{1}+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)^2\left(\sqrt[3]{1}+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)$$

i is the imaginary unit

Result:

$$\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}-1\right)^2\left(1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)$$

Decimal approximation:

0.658145488414010813898216110868858678642091776563601996187999185 2

…

0.658145488….

Alternate forms:

$$\frac{1}{64}\left(352\sqrt{10-2\sqrt{5}}-384\sqrt{5}+160\sqrt{5\left(10-2\sqrt{5}\right)}-768\right)$$

$$2\left(2+\sqrt{5}\right)\left(\sqrt{5+2\sqrt{5}}-3\right)$$

$$2\left(-6-3\sqrt{5}+\sqrt{85+38\sqrt{5}}\right)$$

Minimal polynomial:

$$x^4+48x^3-176x^2+192x-64$$

Expanded form:

$$1-\frac{68}{\left(\sqrt{5}-1\right)^3}+\frac{12\sqrt{5}}{\left(\sqrt{5}-1\right)^3}+\frac{22\sqrt{10-2\sqrt{5}}}{\left(\sqrt{5}-1\right)^3}-\frac{2\sqrt{5\left(10-2\sqrt{5}\right)}}{\left(\sqrt{5}-1\right)^3}-\frac{14}{\left(\sqrt{5}-1\right)^2}+\frac{2\sqrt{5}}{\left(\sqrt{5}-1\right)^2}+\frac{4\sqrt{10-2\sqrt{5}}}{\left(\sqrt{5}-1\right)^2}+\frac{2}{\sqrt{5}-1}-\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}$$

From which:

1+((((1/2 - (i sqrt(3))/2) (1/2 + (i sqrt(3))/2) (-1^(1/3) + (((((√(10-2√5) -2))/((√5-1))))))^2 (1^(1/3) + (((((√(10-2√5) -2))/((√5-1)))))))))

Input:

$$1+\left(\frac{1}{2}-\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(-\sqrt[3]{1}+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)^2\left(\sqrt[3]{1}+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)$$

i is the imaginary unit

Result:

$$1+\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}-1\right)^2\left(1+\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)$$

Decimal approximation:

1.658145488414010813898216110868858678642091776563601996187999185 2

…

1.658145488…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

Alternate forms:

$$\frac{1}{64}\left(352\sqrt{10-2\sqrt{5}}-384\sqrt{5}+160\sqrt{5\left(10-2\sqrt{5}\right)}-704\right)$$

$$-11-6\sqrt{5}+2\sqrt{85+38\sqrt{5}}$$

$$\frac{4\left(\sqrt{50-10\sqrt{5}}+3\sqrt{10-2\sqrt{5}}+2\left(\sqrt{5}-8\right)\right)}{\left(\sqrt{5}-1\right)^3}$$

Minimal polynomial:

$$x^4+44x^3-314x^2+684x-479$$

Expanded form:

$$2-\frac{68}{(\sqrt{5}-1)^3}+\frac{12\sqrt{5}}{(\sqrt{5}-1)^3}+\frac{22\sqrt{10-2\sqrt{5}}}{(\sqrt{5}-1)^3}-\frac{2\sqrt{5\left(10-2\sqrt{5}\right)}}{(\sqrt{5}-1)^3}-$$
$$\frac{14}{(\sqrt{5}-1)^2}+\frac{2\sqrt{5}}{(\sqrt{5}-1)^2}+\frac{4\sqrt{10-2\sqrt{5}}}{(\sqrt{5}-1)^2}+\frac{2}{\sqrt{5}-1}-\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}$$

For q = 1 and x = 17, subtracting 2*256 = 8*64 = 8^3 = 512 , we obtain:

(1/2 - (i sqrt(3))/2) (1/2 + (i sqrt(3))/2) (-1^(1/3) + 17)^2 (1^(1/3) + 17) - 8^3

Input:

$$\left(\frac{1}{2}-\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(-\sqrt[3]{1}+17\right)^2\left(\sqrt[3]{1}+17\right)-8^3$$

i is the imaginary unit

Result:

4096

4096 = 64^2, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

(1/2 - (i sqrt(3))/2) (1/2 + (i sqrt(3))/2) (-1^(1/3) + 17)^2 (1^(1/3) + 17) - 8^3 + 276

Input:

$$\left(\frac{1}{2}-\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(-\sqrt[3]{1}+17\right)^2\left(\sqrt[3]{1}+17\right)-8^3+276$$

i is the imaginary unit

Result:

4372

4372

Where 4372 is a value indicated in the fundamental Ramanujan paper "**Modular equations and Approximations to π**"

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

27sqrt(((1/2 - (i sqrt(3))/2) (1/2 + (i sqrt(3))/2) (-1^(1/3) + 17)^2 (1^(1/3) + 17) - 8^3))+1

Input:

$$27\sqrt{\left(\frac{1}{2}-\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(-\sqrt[3]{1}+17\right)^2\left(\sqrt[3]{1}+17\right)-8^3}+1$$

i is the imaginary unit

Result:

1729

1729

This result is very near to the mass of candidate glueball **$f_0(1710)$ scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

(((27sqrt(((1/2 - (i sqrt(3))/2) (1/2 + (i sqrt(3))/2) (-1^(1/3) + 17)^2 (1^(1/3) + 17) - 8^3))+1)))^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{27\sqrt{\left(\frac{1}{2}-\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(-\sqrt[3]{1}+17\right)^2\left(\sqrt[3]{1}+17\right)-8^3}+1}+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

i is the imaginary unit

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}+\sqrt[15]{1+27\sqrt{-512+4608\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}}$$

Decimal approximation

1.64493802413026025794173319194636695742024244172581638027130567 82

…

1.64493802413026…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}+\sqrt[15]{1729}$$

$$C_{\text{MRB}}^{\,-1/(4\pi)}\left(\sqrt[15]{1729}\;\sqrt[4\pi]{C_{\text{MRB}}}+C_{\text{MRB}}^{\,1+\pi}\right)$$

(((((27sqrt(((1/2 - (i sqrt(3))/2) (1/2 + (i sqrt(3))/2) (-1^(1/3) + 17)^2 (1^(1/3) + 17) - 8^3))+1)))^1/15-1))^1/32

Input:

$$\sqrt[32]{\sqrt[15]{27\sqrt{\left(\frac{1}{2}-\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(\frac{1}{2}+\frac{1}{2}\left(i\sqrt{3}\right)\right)\left(-\sqrt[3]{1}+17\right)^2\left(\sqrt[3]{1}+17\right)-8^3}+1}-1}$$

i is the imaginary unit

Exact result:

$$\sqrt[32]{-1+\sqrt[15]{1+27\sqrt{-512+4608\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}}}$$

Decimal approximation:

0.986333511553779452661373221744691718129356139495548067920927289 9…

0.98633351155377….. result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the Omega mesons ($\omega/\omega_3 \mid 5+3 \mid m_{u/d}=255-390 \mid 0.988-1.18$) Regge slope value (0.988) connected to the dilaton scalar field $\mathbf{0.989117352243 = \phi}$

A_1^{**} above the two low-lying pseudo-scalars. (bound states of gluons, or 'glueballs')

A_1^{**}	0.943(39) [2.5]	0.988(38)	0.152(53)
A_4	1.03(10) [2.5]	0.999(32)	0.035(21)

(**Glueball Regge trajectories** - *Harvey Byron Meyer*, Lincoln College -Thesis submitted for the degree of Doctor of Philosophy at the University of Oxford Trinity Term, 2004)

Note that

$\sqrt[32]{\zeta(2)-1}$

$\sqrt[32]{\frac{\pi^2}{6}-1}$

= 0.98638703135648129 15… $(\pi^2/6 - 1)^{1/32}$

3. *On the development of a Cubic Equation (M. Nardelli, A. Grablovitz)*

The most general cubic equation (equation of third degree):

$$x^3+rx^2+px+q=0 \quad (1)$$

can be reduced in the following form:

$$x^3+px+q=0 \quad (2)$$

Indeed:

$$(x-u)(x-v)(x-z)=(x^2-vx-ux+uv)(x-z)=$$

$$=[x^2-(u+v)x+uv](x-z)=x^3-(u+v)x^2+uvx-zx^2+z(u+v)x-zuv=$$

$$=x^3-(u+v+z)x^2+[uv+z(u+v)]x-zuv. \quad (2b)$$

Thence:

$$x^3-(u+v+z)x^2+[uv+z(u+v)]x-zuv=0 \quad (3)$$

Putting:

$$\begin{cases} u+v+z=-r \\ z(u+v)+uv=p \\ uvz=-q \end{cases} \quad (4)$$

because the term r become null, is necessary and sufficient that $z=-(u+v)$. Thence:

$$x^3-(u+v+z)x^2+[uv+z(u+v)]x-zuv=0;$$

$$x^3-[u+v-(u+v)]x^2+[uv-(u+v)(u+v)]x-[-(u+v)uv]=0;$$

$$x^3-[uv-(u+v)^2]x-[-(u+v)uv]=0;$$

$$x^3-[uv-(u^2+2uv+v^2)]x+u^2v+uv^2=0;$$

$$x^3-[uv-u^2-2uv-v^2]x+u^2v+uv^2=0;$$

$$x^3-(-uv-u^2-v^2)x+uv(u+v)=0;$$

$$x^3-(u^2+uv+v^2)x+uv(u+v)=0 \quad (5)$$

Putting:

$$\begin{cases} u^2+uv+v^2=-p \\ uv(u+v)=q \end{cases} \quad (6)$$

the $x^3-(u^2+uv+v^2)x+uv(u+v)=0$, become:

$$x^3+px+q=0 \quad (7)$$

From the $u^2+uv+v^2=0$, we obtain two quadratic equations (of second degree):

1) $u^2+uv+v^2=0$;

$$u=\frac{-v\pm\sqrt{v^2-4v^2}}{2}=\frac{-v\pm\sqrt{-3v^2}}{2}=-\frac{1}{2}v\pm v\frac{\sqrt{3i^2}}{2}=-v\left(\frac{1}{2}\pm\frac{\sqrt{3i^2}}{2}\right)=-v\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right);$$

2) $v^2+uv+u^2=0$;

$$v=\frac{-u\pm\sqrt{u^2-4u^2}}{2}=\frac{-u\pm\sqrt{-3u^2}}{2}=-\frac{1}{2}u\pm u\frac{\sqrt{3i^2}}{2}=-u\left(\frac{1}{2}\pm\frac{\sqrt{3i^2}}{2}\right)=-u\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right).$$

Now, we take the eq. (6). We have that:

$$\begin{cases} u^2+uv+v^2=-p \\ u=-v\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right) \\ v=-u\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right) \end{cases} \qquad (7a) \qquad \begin{cases} uv=-uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2 \\ u^2=u\cdot u=-uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2 \\ v^2=v\cdot v=-uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2 \end{cases}$$

thence:

$$-\left[uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2+uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2+uv\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)^2\right]=-(uv+uv+uv)=p;$$

$$3uv=-p; \quad uv=-\frac{p}{3} \quad (8)$$

Indeed, we have, for example:

$$-\left[uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2+uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2+uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\right]=p$$

$$-\left[uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\right]=$$

$$=-\left[uv\left(\frac{1}{4}+\frac{i\sqrt{3}}{2}-\frac{3}{4}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{4}+\frac{i\sqrt{3}}{2}-\frac{3}{4}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(\frac{1}{4}+\frac{i\sqrt{3}}{2}-\frac{3}{4}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\right]=$$

$$=-\left[uv\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)+uv\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\right]=$$

$$=-\left[uv\left(\frac{1}{4}+\frac{3}{4}\right)+uv\left(\frac{1}{4}+\frac{3}{4}\right)+uv\left(\frac{1}{4}+\frac{3}{4}\right)\right]=$$

$$=-(uv+uv+uv)=p;\quad 3uv=-p;\quad uv=-\frac{p}{3}.$$

Now:

$$\begin{cases} u^2v+uv^2=q \\ u=-v\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right) \\ v=-u\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right) \end{cases} \quad (8b)$$

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q;$$

that for the (8b) and for sign + , for example, become:

$$v^2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\cdot v\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)+u^2\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^2\cdot u\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$$v^2\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-v)+u^2\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\left(-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)(-u)=q$$

$$-v^3\left(\frac{1}{4}+\frac{3}{4}\right)-u^3\left(\frac{1}{4}+\frac{3}{4}\right)=q;\quad -v^3-u^3=q;$$

$$u^3+v^3=-q \quad (9)$$

Finally, we have:

$$\begin{cases} u^3+v^3=-q \\ uv=-\dfrac{p}{3} \end{cases} \qquad \text{thence:} \qquad \begin{cases} u^3+v^3=-q \\ u^3v^3=-\dfrac{p^3}{27} \end{cases} \qquad (10)$$

We obtain the same result putting in the (2) $x=u+v$. Indeed, we have:

$$x^3+px+q=0; \qquad x=u+v$$

$$(u+v)^3+p(u+v)+q=0; \quad u^3+3u^2v+3uv^2+v^3+p(u+v)+q=0;$$
$$u^3+v^3+3uv(u+v)+p(u+v)+q=0;$$

from this equation, we obtain:

$$u^3+v^3=-q; \qquad 3uv(u+v)=-p(u+v); \qquad \text{thence, in conclusion:}$$

$$\begin{cases} u^3+v^3=-q \\ 3uv=-p \end{cases} \qquad \begin{cases} u^3+v^3=-q \\ uv=-\dfrac{p}{3} \end{cases} \qquad \begin{cases} u^3+v^3=-q \\ u^3v^3=-\dfrac{p^3}{27} \end{cases} \qquad (11)$$

From the relations (10) and (11), we obtain the sum and the product of u^3 and v^3. We have that u^3 and v^3 must be roots of the following quadratic equation:

$$t^2+qt-\frac{p^3}{27}; \quad (12)$$

This equation is defined the “resolving” of the cubic equation (2). Thence, we have:

$$t^2+qt-\frac{p^3}{27}$$

$$\frac{-q\pm\sqrt{q^2-4(1)\left(-\frac{p^3}{27}\right)}}{2}=\frac{-q\pm\sqrt{q^2+\frac{4p^3}{27}}}{2}=-\frac{q}{2}\pm\sqrt{\frac{1}{4}q^2+\frac{1}{4}\cdot\frac{4p^3}{27}}=$$

$$=-\frac{q}{2}\pm\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}\,.$$

Thence, we have that:

$$\begin{aligned} t_1=u^3&=-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}\\ t_2=v^3&=-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}\end{aligned}\quad;\qquad(13)$$

from this, we obtain:

$$u=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\,;$$

if $uv=-\frac{p}{3}$; $v=-\frac{p}{3u}$, we obtain:

$$v=-\frac{p}{3\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}}\,.$$

From the $x=u+v$, we obtain:

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}-\frac{p}{3\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}}. \quad (14)$$

The expression (14) is the "resolving" formula of the cubic equation (2), from this we have the three roots in correspondence of the three values of the cubic radical. If $p=0$ the eq. (2) become the binomial equation $x^3+q=0$, solved from the formula $x=\sqrt[3]{-q}$.

The expression (14) is written generally in the "Cardano" or "Tartaglia" form:

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}. \quad (15)$$

The solution that Cardano gives of the cubic equation of type $x^3+px=q$, leads to the following formula:

$$x=\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}+\frac{q}{2}}-\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}-\frac{q}{2}}\ ; \quad (15b)$$

while for the cubic equation of type $x^3+px^2=q$, we have the following formula:

$$x=\sqrt[3]{\sqrt{\left(\frac{q}{2}\right)^2-\left(\frac{p}{3}\right)^3}+\frac{q}{2}}-\sqrt[3]{\sqrt{\left(\frac{q}{2}\right)^2-\left(\frac{p}{3}\right)^3}-\frac{q}{2}}\ . \quad (15c)$$

4. *Mathematical connections with some sectors of string theory*

Now we show two examples of solutions of cubic equations and the possible mathematical connections with the "modes" that correspond to the physical vibrations of the bosonic strings by the Ramanujan function.

1) Let

$$x^3 - 8x = 3 \quad (16)$$

Putting

$$q = u - v = 3; \qquad \left(\frac{p}{3}\right)^3 = uv = \left(-\frac{8}{3}\right)^3 = -\frac{512}{27}; \quad \text{we apply the formula:}$$

$$x = \sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} + \frac{q}{2}} - \sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3 + \left(\frac{q}{2}\right)^2} - \frac{q}{2}} =$$

$$= \sqrt[3]{\sqrt{-\frac{512}{27} + \frac{9}{4}} + \frac{3}{2}} - \sqrt[3]{\sqrt{-\frac{512}{27} + \frac{9}{4}} - \frac{3}{2}} = \sqrt[3]{\sqrt{\frac{-2048 + 243}{108}} + \frac{3}{2}} - \sqrt[3]{\sqrt{\frac{-2048 + 243}{108}} - \frac{3}{2}} =$$

$$= \sqrt[3]{\sqrt{-\frac{1805}{108}} + \frac{3}{2}} - \sqrt[3]{\sqrt{-\frac{1805}{108}} - \frac{3}{2}} = \left(\sqrt[3]{\sqrt{\frac{1805}{108} i^2} + \frac{3}{2}}\right)^3 - \left(\sqrt[3]{\sqrt{\frac{1805}{108} i^2} - \frac{3}{2}}\right)^3 =$$

$$= \sqrt{\frac{1805}{108} i^2} + \frac{3}{2} - \sqrt{\frac{1805}{108} i^2} + \frac{3}{2} = \frac{3}{2} + \frac{3}{2} = \frac{6}{2} = 3.$$

Indeed, for $x=3$, the eq. (16) become:

$$x^3-8x=3; \qquad 3^3-8\cdot 3=3; \qquad 27-24=3; \qquad 3=3$$

But, we have also that:

$$8x=x^3-3; \qquad 24=x^3-3; \qquad 24=27-3; \qquad 24=24$$

We know that the number 24 is related to the "modes" that correspond to the physical vibrations of the bosonic strings by the following Ramanujan function:

$$24=\frac{4\left[antilog\frac{\int_0^{\infty}\frac{\cos\pi txw'}{\cosh\pi x}e^{-\pi x^2w'}dx}{e^{-\frac{\pi^2}{4}w'}\phi_{w'}(itw')}\right]\cdot\frac{\sqrt{142}}{t^2w'}}{\log\left[\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)}+\sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right]}, \quad \text{(a)}$$

Thence, we have the following mathematical connection:

$$\frac{4\left[antilog\frac{\int_0^{\infty}\frac{\cos\pi txw'}{\cosh\pi x}e^{-\pi x^2w'}dx}{e^{-\frac{\pi^2}{4}w'}\phi_{w'}(itw')}\right]\cdot\frac{\sqrt{142}}{t^2w'}}{\log\left[\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)}+\sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right]}=x^3-3 \quad (17) \qquad \text{for } x=3$$

A mathematical connection that link the cubic equation $x^3-8x=3$ with the number 24, i.e. with the "modes" that correspond to the physical vibrations of the bosonic strings.

We note also that 3 and 8, i.e. p and q are also Fibonacci's numbers.

2) Let

$$x^3+8x=24. \quad (18)$$

Putting

$$q=u-v=24; \qquad \left(\frac{p}{3}\right)^3=uv=\left(\frac{8}{3}\right)^3=\frac{512}{27}; \quad \text{for the formula:}$$

$$x=\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}+\frac{q}{2}}-\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}-\frac{q}{2}}\ ; \quad \text{we obtain:}$$

$$\sqrt[3]{\sqrt{\frac{512}{27}+\left(\frac{24}{2}\right)^2}+\frac{24}{2}}-\sqrt[3]{\sqrt{\frac{512}{27}+\left(\frac{24}{2}\right)^2}-\frac{24}{2}}=$$

$$=\sqrt[3]{\sqrt{\frac{512}{27}+144}+12}-\sqrt[3]{\sqrt{\frac{512}{27}+144}-12}=\sqrt[3]{\sqrt{162,962}+12}-\sqrt[3]{\sqrt{162,962}+12}=$$

$$=\sqrt[3]{12,765657+12}-\sqrt[3]{12,765657-12}=\sqrt[3]{24,765657}-\sqrt[3]{0,765657}\cong 2,914852-0,914852=2.$$

Thence, we have:

$$x^3+8x=24; \qquad 2^3+8\cdot 2=24; \qquad 8+16=24; \qquad 24=24;$$

$$24=x^3+8x;$$

$$\frac{4\left[antilog\frac{\int_0^{\infty}\frac{\cos\pi t x w}{\cosh\pi x}e^{-\pi x^2 w}dx}{e^{-\frac{\pi t^2}{4}w}\phi_w(itw)}\right]\cdot\frac{\sqrt{142}}{t^2 w}}{\log\left[\sqrt{\left(\frac{10+11\sqrt{2}}{4}\right)}+\sqrt{\left(\frac{10+7\sqrt{2}}{4}\right)}\right]}=x^3+8x;\quad (19)\qquad \text{for } x=2$$

The further connection that link this cubic equation with the “modes” that correspond to the physical vibrations of the bosonic strings.

Creation and solution of cubic equation incomplete, without the first degree, defined “Premonica”.

Let u,v and z three roots of a same equation. Obviously the equation is cubic and the factorization of it is:

$$x^3+px^2+rx+q=0\quad (20)$$

$$(x-u)(x-v)=x^2-ux-vx+uv=x^2-(u+v)x+uv;$$

$$\left[x^2-(u+v)x+uv\right](x-z)=x^3-(u+v)x^2+uvx-zx^2+z(u+v)x-zuv=$$

$$=x^3-(u+v+z)x^2+\left[uv+z(u+v)\right]x-uvz;\quad \text{thence:}$$

$$x^3-(u+v+z)x^2+\left[uv+z(u+v)\right]x-uvz=0;\quad (21)$$

from this, putting

$$\begin{cases}u+v+z=-p\\ z(u+v)+uv=r\\ uvz=-q\end{cases}$$

we obtain the eq. (20).

to cancel the term r, necessary and sufficient condition is that:

$$z(u+v)+uv=0; \qquad z(u+v)=-uv \quad \text{whence}$$

$$z=-\frac{uv}{u+v}. \quad (22)$$

The (22) is defined the "fundamental relation" . Substituting in the eq. (21) the value found of z, we obtain:

$$x^3-(u+v+z)x^2+[uv+z(u+v)]x-uvz=0$$

$$x^3-\left(u+v-\frac{uv}{u+v}\right)x^2+\left[uv-\frac{uv}{u+v}(u+v)\right]x-uv\left(-\frac{uv}{u+v}\right)=0;$$

$$x^3-\frac{(u+v)^2-uv}{u+v}x^2+\frac{u^2v^2}{u+v}=0; \quad \text{that we can rewrite also as follows:}$$

$$(u+v)x^3-\left[(u+v)^2-uv\right]x^2+u^2v^2=0. \quad (23)$$

With this relation we can solve a cubic equation in "Premonica" form, i.e. without the first degree.

$x^3+px^2+q=0$. For example, $3x^3-7x^2+4=0$, we will use the following system:

$$\begin{cases} u+v=3 \\ (u+v)^2-uv=7 \\ u^2v^2=4 \end{cases} \quad (23b) \quad \begin{cases} 9-uv=7 \\ u^2v^2=4 \end{cases} \quad \begin{cases} uv=2 \\ u^2v^2=4 \end{cases} \quad \begin{cases} uv=2 \\ uv=\pm 2 \end{cases}$$

The uv will be equal only if $uv=2$, thence

$$\begin{cases} u+v=3 \\ uv=2 \end{cases} \quad \text{whence} \quad t^2-3t+2=0; \qquad t=\frac{3\pm\sqrt{9-8}}{2}=\frac{3\pm1}{2}=\begin{cases} t_1=2=u=x_1 \\ t_2=1=v=x_2 \end{cases}$$

Because the equation not admit the term of first degree, the third root must be necessarily

$$z=-\frac{uv}{u+v}=-\frac{1\cdot2}{2+1}=-\frac{2}{3}=x_3$$

Now, we have the following cubic equation:

$$x^3-2x+1=0; \quad (24)$$

We want solve this equation by a system of relations between roots and coefficients. This procedure will be valid for each equation in form “Monica” form, i.e. without the second degree, whatever the coefficients.

Let u and v two of the roots of the equation, the third root will be necessarily $u+v$ and thence we have

$$x_1=u; \qquad x_2=v; \qquad x_3=u+v; \quad (25) \qquad \begin{cases} x_1=-(u+v)+u=-v \\ x_2=-(u+v)+v=-u \end{cases} \qquad (25b)$$

Now:

$$(x-u)(x-v)=x^2-(u+v)x+uv;$$

$$[x^2-(u+v)x+uv][x+(u+v)]=x^3-(u+v)x^2+uvx+(u+v)x^2-(u+v)^2x+uv(u+v)=$$

$$=x^3-[(u+v)^2-uv]x+uv(u+v);\qquad \text{from this, we have:}$$

$$x^3-[(u+v)^2-uv]x+uv(u+v)=0. \quad (26)$$

This equation is a cubic equation in the form "Monica", i.e. of type $x^3+px+q=0$, where we put:

$$\begin{cases}(u+v)^2-uv=-p\\ uv(u+v)=q\end{cases}$$

Now, we put $u+v=t$ and $uv=z$, we have:

$$\begin{cases}t^2-z=-p\\ tz=q\end{cases}\qquad\begin{cases}t^2=z-p\\ tz=q\end{cases}\qquad\begin{cases}t=\pm\sqrt{z-p}\\ tz=q\end{cases}\qquad tz=q;\quad \pm z\left(\sqrt{z-p}\right)=q;$$

we raise all to the square: $z^2(z-p)=q^2$; from which developing:

$$z^3-pz^2=q^2;\qquad z^3-pz^2-q^2=0. \quad (27)$$

This last equation is a "Premonica" and we can find u and v applying the Caccioppoli-Grablovitz method (comparison system between roots and coefficients). We resume the eq. (24)

$$x^3-2x+1=0$$

Decompose the equation. We have that:

$$\left(x^2+x-1\right)\left(x-1\right)=x^3-x^2+x^2-x-x+1=x^3-2x+1;\quad \text{thence:}$$

$$\left(x^2+x-1\right)\left(x-1\right)=0;\qquad x-1=0;\qquad x=1$$

$$x^2+x-1=0;\quad \frac{-1\pm\sqrt{1-4(1)(-1)}}{2}=\frac{-1\pm\sqrt{1+4}}{2}=\frac{-1\pm\sqrt{5}}{2}\quad \begin{cases} x_1=\dfrac{-1+\sqrt{5}}{2} \\ x_2=\dfrac{-1-\sqrt{5}}{2} \end{cases}$$

The three roots are: $1;\ \frac{-1+\sqrt{5}}{2};\ \frac{-1-\sqrt{5}}{2}$.

Now, $x^3-2x+1=0$ applying the relation (27), we have:

$$z^3-pz^2-q^2=0;\quad z^3-2z^2-1=0\quad \text{whence, from (23b)}$$

$$\begin{cases} u+v=1 \\ (u+v)^2-uv=2 \\ u^2v^2=1 \end{cases}\qquad \begin{cases} u+v=1 \\ 1-uv=2 \\ uv=\pm 1 \end{cases}\qquad \begin{cases} u+v=1 \\ uv=-1 \\ uv=\pm 1 \end{cases}\qquad \begin{cases} u+v=1 \\ uv=-1 \end{cases}$$

Thence:

$$t^2-t-1=0;\qquad \frac{1\pm\sqrt{1+4}}{2}=\begin{cases} u=\dfrac{1+\sqrt{5}}{2} \\ v=\dfrac{1-\sqrt{5}}{2} \end{cases}$$

For the relations (25b), the third root will be $x_3 = u + v = \frac{1}{2} + \frac{\sqrt{5}}{2} + \frac{1}{2} - \frac{\sqrt{5}}{2} = 1$. The other two roots, will be: $x_1 = -u = \frac{-1-\sqrt{5}}{2}$; and $x_2 = -v = \frac{-1+\sqrt{5}}{2}$.

We note that the $x_2 = \frac{\sqrt{5}-1}{2} = 0{,}61803398...$ is equal to the aurea section.

From:

Compactifications of Heterotic Theory on Non-Kahler Complex Manifolds: I
Katrin Becker, Melanie Becker, Keshav Dasgupta, Paul S. Green - arXiv:hep-th/0301161v2 22 Jan 2003

We have that:

The three-form field strength, that is invariant under the transformation [13] is

$$\mathcal{H} = dB - \alpha' \left[\Omega_3(A) - \Omega_3(\omega - \frac{1}{2}\mathcal{H}) \right], \tag{2.47}$$

where we have defined

$$\Omega_3(A) = A \wedge F - \frac{1}{3} A \wedge A \wedge A, \tag{2.48}$$

From the above analysis it is easy to infer, what the background torsion is. If we concentrate only to the lowest order in α' and linear order in $\mathcal{H}$, the three-form background is given by

$$\mathcal{H} = dB \left(1 - \frac{\alpha'}{2} \mathcal{R}_\omega e^{-2} \right) + \alpha' [\Omega_3(\omega) - \Omega_3(A)] + \mathcal{O}(\alpha'^2). \tag{4.41}$$

The reason of why the above argument failed is, that to the approximation taken in (4.41) all terms besides $\mathcal{H}$ contain either ω or $\mathcal{R}_\omega$ and therefore involve derivatives of t. But we also have terms cubic in $\mathcal{H}$, which originate from the third term in the bracket on the left hand side of (4.42), so that this equation becomes

$$\mathcal{H} + \frac{\alpha'}{12}\tilde{\mathcal{H}} \wedge \tilde{\mathcal{H}} \wedge \tilde{\mathcal{H}} + \ldots = dB + \alpha'[\Omega_3(\omega) - \Omega_3(A)], \tag{4.44}$$

where $\ldots$ represent the ω dependent terms. We can write the above equation as a cubic equation for $\mathcal{H}$

$$\tilde{\mathcal{H}}^3 + a\ \mathcal{H} + b = 0. \tag{4.45}$$

The variables a, b can be determined from (4.44). In general, the equation (4.45) is not a typical cubic equation, because $\mathcal{H}$ defined above are forms on a manifold and not real numbers[13]. Therefore, let us first take a simple situation, which can convert (4.45) into a cubic equation with real variables. For the present purpose, this will suffice to clarify the basic ideas involved here. We will soon generalize this to our six dimensional non-Kähler manifold.

Let us choose a background, that can convert equation (4.45) into a simple cubic equation in terms of functions and not forms. The simplest background will be

$$\mathcal{H}_{ijk} = h\mathcal{C}_{ijk}, \qquad e_{ai} = t^{1/2}e^o_{ai} \qquad \text{and} \qquad e^o \cdot e^o = \eta, \tag{4.46}$$

where $\mathcal{C}_{ijk}$ is an antisymetric in six dimensions, e_{ai} are the vielbeins and $\eta \equiv \eta_{ij}$ is the flat metric. The size of the six manifold is given by t, as defined earlier. We will also assume, that the only non-zero component of $\mathcal{H}$ is $\mathcal{H}_{12\bar{3}} = h$. It is easy to see, that (4.45) becomes a simple cubic equation

$$h^3 + ph + q = 0 \qquad \text{with} \qquad p, q \in \mathbb{R}, \tag{4.47}$$

where p, q are given below. The solution to this equation is well known using Vieta's substitution[14]. Observe, that for simplicity we have ignored all ω dependent terms and $\mathcal{O}(\alpha'^2)$ terms appearing in (4.42). To leading order in α' the presence of ω would induce quadratic and linear terms in $\mathcal{H}$ in the previous equation. This does not change the analysis, that we perform here, because *any* generic cubic equation can always be brought to the form (4.45) with additive contributions to p, q. For the simplest case we can specify p, q as

$$p \sim \pm\frac{t^3}{\alpha'}, \qquad q \sim \mp\frac{t^3 f}{\alpha'}, \qquad \text{and} \qquad \left\langle dB + \alpha'\Omega_3(\omega) - \alpha'\Omega_3(A)\right\rangle_{abc} \equiv f\epsilon_{abc}, \tag{4.48}$$

where the sign ambiguity reflects the sign ambiguity in the torsional-spin connection, $\omega \mp \frac{1}{2}\mathcal{H}$, i.e the choice of either $\tilde{\omega}$ or $\widehat{\omega}$ in terms of earlier notations. The reader might be

[14] Define $\mathcal{H} = w - \frac{p}{3w}$ and substitute it in (4.45). The equation will become a quadratic equation in $w^3 \equiv \lambda$, given by $\lambda^2 + q\lambda - \frac{p^3}{27} = 0$, whose solutions are $w^3 = \frac{1}{2}\left(-q \pm \sqrt{q^2 + \frac{4p^3}{27}}\right)$. Calling these roots as $\delta^3_\pm$, w can be written as $w = \delta_\pm(1, \omega, \omega^2)$, where ω is the cube-root of unity. The solutions are therefore $\delta_+ + \delta_-, \delta_+\omega + \delta_-\omega^2$ and $\delta_+\omega^2 + \delta_-\omega$. This is Cardan's solution for a cubic equation. For a generic cubic equation of the form $h^3 + a_2h^2 + a_1h + a_0 = 0$, we can define $h = y - \frac{a_2}{3}$. In this form the equation looks in the same way as (4.47) with $p = a_1 - \frac{1}{3}a_2^2$ and $q = \frac{1}{3}(3a_0 - a_1a_2) + \frac{2}{27}a_2^3$. Calling the roots as α_i, we get: $\sum \alpha_i = -a_2$, $\sum \alpha_i\alpha_j = a_1$, $\sum \alpha_i\alpha_j\alpha_k = -a_0$. Observe also, that since $\frac{p^3}{27} + \frac{q^2}{4} > 0$, we have one real root and pair of complex conjugate roots.

[14] Define $\mathcal{H} = w - \frac{p}{3w}$ and substitute it in (4.45). The equation will become a quadratic equation in $w^3 \equiv \lambda$, given by $\lambda^2 + q\lambda - \frac{p^3}{27} = 0$, whose solutions are $w^3 = \frac{1}{2}\left(-q \pm \sqrt{q^2 + \frac{4p^3}{27}}\right)$. Calling these roots as $\delta^3_\pm$, w can be written as $w = \delta_\pm(1, \omega, \omega^2)$, where ω is the cube-root of unity. The solutions are therefore $\delta_+ + \delta_-, \delta_+\omega + \delta_-\omega^2$ and $\delta_+\omega^2 + \delta_-\omega$. This is Cardan's solution for a cubic equation.

concerned by the fact, that since (4.44) involves *wedge* products, it will be difficult to get a simple cubic equation. But it is easy to see, that all the other factors due to wedging can be absorbed into the definition of p and q and therefore, we can express (4.44) as a cubic equation in h with p, q *proportional* to (4.48), as we are not very concerned about precise factors. Nevertheless, the exact factors appearing here can be worked out easily. In this section we take a simple example, where the background is just $\mathcal{H}_{12\bar{3}}$. This implies

$$\tilde{\mathcal{H}}_1^{ab} = \mathcal{H}_{1\alpha\beta}e^{a\alpha}e^{b\beta} = 2h\ \mathcal{C}_{12\bar{3}}e^{2[a}e^{b]\bar{3}} \equiv 2t^{-1}\ h\mathcal{C}_{12\bar{3}}e_{\rm o}^{2[a}e_{\rm o}^{b]\bar{3}} = t^{-1}h\alpha_1^{ab}, \tag{4.49}$$

where $\alpha_1^{ab} = -\alpha_1^{ba} = 2\mathcal{C}_{12\bar{3}}\ e_{\rm o}^{2[a}e_{\rm o}^{b]\bar{3}}$ and $e^{ai} = t^{-\frac{1}{2}}e_{\rm o}^{ai}$. We have also defined the anti-symmetrization between a, b as $[...]$, as usual. The above formula can easily be generalized to the case, in which other components of $\mathcal{H}$ are turned on and not all $\mathcal{C}$ are equal. Some aspects of this will be dealt with in the next sub-section. We are also using complex coordinates, but it is easy to infer the corresponding case with real coordinates. Similarly to the above equation we can obtain the other components of $\tilde{\mathcal{H}}_i^{ab}$.

$$\tilde{\mathcal{H}}_2^{cd} = 2h\ \mathcal{C}_{2\bar{3}1}e^{\bar{3}[c}e^{d]1} = t^{-1}h\alpha_2^{cd} \qquad \text{and} \qquad \tilde{\mathcal{H}}_{\bar{3}}^{ef} = t^{-1}h\alpha_{\bar{3}}^{ef}. \tag{4.50}$$

Now we can determine the cubic wedge product between the $\mathcal{H}$'s, using the result of the earlier footnote. We again use M^{ab} as the tensors of the holonomy group of the manifold. The result will be

$$[\tilde{\mathcal{H}} \wedge \tilde{\mathcal{H}} \wedge \tilde{\mathcal{H}}]_{12\bar{3}} = t^{-3}h^3\alpha_1^{ab}\alpha_2^{cd}\alpha_{\bar{3}}^{ef}{\rm Tr}(M^{ab}M^{cd}M^{ef}) = t^{-3}h^3\mathcal{Q}, \tag{4.51}$$

where $\mathcal{Q}$ is an integer determined in terms of α_i and M in an obvious way. In general, $\mathcal{Q}$ is a non-zero integer and for the analysis done in this sub-section and the next one, we shall normalize $\mathcal{Q}$ to 1 by defining variables appropriately. This will not alter any of our results because we only require the form of the moduli and not the precise factors. (Later we will consider the case when $\mathcal{Q}$ vanishes.) From here the set of equations (4.48) can be derived. To determine the solution for a cubic equation, we define, as usual, two variables A and B as

$$A = \left(-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}\right)^{1/3} \qquad \text{and} \qquad B = \left(-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}\right)^{1/3}, \tag{4.52}$$

where p, q have been introduced above. Observe, that if $p \to \infty,\ q \to \infty,\ q/p = $ constant, we recover the situation, where no torsion is switched on. This implies, that expanding

in $1/p$ is a legitimate thing to do (we will give further justification later). In fact, for the limit mentioned above we get $h \sim -q/p$, which is a reasonable estimate in the absence of torsion, because this implies $\mathcal{H} = dB + \mathcal{O}(\alpha')$. Therefore, the solution of the cubic equation for $\mathcal{H}$ (4.45) will be typically

$$A+B, \qquad -\frac{A+B}{2}+i\frac{\sqrt{3}(A-B)}{2}, \qquad \text{and} \qquad -\frac{A+B}{2}-i\frac{\sqrt{3}(A-B)}{2}, \tag{4.53}$$

giving two complex and one real solution.

This by itself is interesting, because it implies, that we can actually have a complex three-form in the heterotic string theory. This means, that the superpotential written in terms of $\int \mathcal{H}\wedge\Omega$ can have a complex part and this is precisely, what we have been looking for, when we mentioned the apparent i-puzzle in an earlier section. To see, whether the present solution is related to the solution obtained by performing T-dualities from a Type IIB background, we have to study the set of solutions (4.53) carefully. Let us define a quantity

$$s \equiv \sqrt{1+z^2}-1 \qquad \text{with} \qquad z=\sqrt{\frac{27q^2}{4p^3}},$$

where z is a small quantity and therefore s can be expanded in powers of z. We can express A and B in terms of these variables as

$$A = a(1+s-z)^{1/3} \qquad \text{and} \qquad B = -a(1+s+z)^{1/3}, \tag{4.54}$$

where $a=\sqrt{p/3}$. Expanding the above relations in powers of $1/p$, we can determine the solutions of (4.53) order by order. The solutions we obtain are

$$\begin{aligned} A+B &= -\frac{2az}{3}+\frac{4asz}{9}-\frac{10az^3}{81}-\frac{10as^2z}{27}+\frac{80as^3z}{243}+\dots, \\ A-B &= 2a+\frac{2as}{3}-\frac{2a(s^2+z^2)}{9}+\frac{10as(s^2+3z^2)}{81}+\dots \end{aligned} \tag{4.55}$$

What we need to know is, how the background three-form field $\mathcal{H}_{12\bar{3}} = h$ depends on the size t of the six-manifold. For the real solution we get

$$h = f - \frac{4\alpha'}{9}\frac{f^3}{t^3}+\dots. \tag{4.56}$$

Comparing with (4.48) we see, that to the lowest order in α' this is exactly, what we expect. The other two complex solutions are

$$h = -\frac{f}{2} \pm i\sqrt{\frac{t^3}{\alpha'}}+\dots. \tag{4.57}$$

From the above analysis we concluded, that the size of the six-manifold can indeed be stabilized by a potential generated by the complex three-form flux. However, the model discussed above is not realistic, since we have chosen very simple fluxes. In this section we will consider the extension of the above analysis to non-Kähler six-dimensional manifolds.

The first difficulty we find, when dealing with the background (2.7) is, that $\mathcal{H}$ has many components. Thus, when we consider the equation (4.44) we have to be careful with the wedge products involved. As a consequence $\tilde{\mathcal{H}}$ will also be more complicated. In the toy example $\tilde{\mathcal{H}}$ was simply proportional to $t^{-1}h$, because only one component $\mathcal{H} = \mathcal{H}_{12\bar{3}}$ was non-vanishing.

From (2.7) we can see that the possible components of $\mathcal{H}$ are

$$\mathcal{H}_{\bar{1}2\bar{3}} \equiv h_1 \, \mathcal{C}_{\bar{1}2\bar{3}} \qquad \text{and} \qquad \mathcal{H}_{1\bar{2}\bar{3}} \equiv h_2 \, \mathcal{C}_{1\bar{2}\bar{3}}, \tag{4.60}$$

and their complex conjugates, which we denote as h_3 and h_4 respectively. In the analysis below we shall ignore the spin connection dependent terms, as they do not change the form of the cubic equation. The one-form $\mathcal{H}_i^{ab}$, for example, can be shown to be proportional to

$$\begin{aligned}\mathcal{H}_{\bar{1}} \equiv \mathcal{H}_{\bar{1}}^{ab} = \mathcal{H}_{\bar{1}jk}\, e^{ja}\, e^{kb} = t^{-1}(\alpha_{\bar{1}}^{ab}\, h_1 + \beta_{\bar{1}}^{ab}\, h_4),\\ \mathcal{H}_2 = t^{-1}(\alpha_2^{ab}\, h_1 + \beta_2^{ab}\, h_4), \quad \mathcal{H}_{\bar{3}} = t^{-1}(\alpha_{\bar{3}}^{ab}\, h_1 + \beta_{\bar{3}}^{ab}\, h_2),\end{aligned} \tag{4.61}$$

where $\alpha_i^{ab}, \beta_i^{ab}$ are constants, whose exact form can be determined from the vielbeins. Due to (4.61) the cubic equation will be far more complicated, than the one discussed before. For simplicity we shall assume all components of $dB + \mathcal{O}(\alpha')$ to be proportional to a function f, as before. We will show later, that this simplification does not substantially affect the results. We will also concentrate mainly on the lowest order in α'. The generic cubic equation for the $\mathcal{H}_{\bar{1}2\bar{3}}$ component of the three-form is

$$h_1^3 + mh_1^2 + nh_1 + s = 0, \tag{4.62}$$

where m, n, s depend on h_4, h_2, t and α'. These are identified as

$$m = (ah_2 + bh_4), \qquad n = \Big(\frac{t^3}{\alpha'} + ch_4h_2 + dh_4^2\Big) \qquad \text{and} \qquad s = \Big(-\frac{ft^3}{\alpha'} + eh_4^2h_2\Big),$$

where $a, b, \ldots$ are integers. These expressions can be easily determined, but for the analysis below we do not need the explicit form for these constants.

The equation above can now be written as (4.47) by using the shift technique for cubic equations. If we identify h_1 with $h - m/3$, then the p, q variables appearing in (4.47) can be given in terms of m, n and s as

$$p = n - \frac{m^2}{3} = \frac{t^3}{\alpha'} + B \qquad \text{and} \qquad q = s - \frac{mn}{3} + \frac{2m^3}{27} = -\frac{ft^3}{\alpha'} + D, \tag{4.63}$$

where B, D are *independent* of α'.

This is very important because it immediately implies, that to lowest order in α' the solution given in (4.55), (4.56) and (4.57) is still satisfied! Thus every component of $\mathcal{H}$ is proportional to the already determined solutions in (4.56) and (4.57). Therefore, we expect a similar potential for the radial modulus in the most generic case as well. This completes the argument[15].

So far we have discussed the case, when the cubic contribution $\tilde{\mathcal{H}}^3$ is non-zero. This is the usual case for some choices of holonomies. However, we could also have situations for which

$$\mathcal{Q} = \alpha_1^{ab}\ \alpha_2^{cd}\ \alpha_3^{ef}\ \mathrm{Tr}(M^{ab}M^{cd}M^{ef})\ =\ 0. \tag{4.64}$$

This is the case when we restrict $\mathcal{C}_{ijk} = \epsilon_{ijk}$. In this case, the cubic equation simplifies very much and to lowest order in α' we have $h = -q/p$. In fact, the relations for the Chern-Simons form will change to a much simpler relation between the spin-connection ω and the background three-form $\mathcal{H}$ as

$$\begin{aligned} \Omega_3(\tilde{\omega}) &= \tilde{\omega} \wedge d\tilde{\omega}, \\ \Omega_3(\widehat{\omega}) &= \widehat{\omega} \wedge d\widehat{\omega}, \end{aligned} \tag{4.65}$$

where now comparing to the earlier expressions of Ω_3 we see that the forms are similar (in the usual case the form of $\Omega_3(\widehat{\omega})$ differs from $\Omega_3(\tilde{\omega})$).

Let us consider now a simpler situation in which we ignore the spin connection ω. In this case, we can show that the three-form equation will eventually be

$$\mathcal{H} - \frac{\alpha'}{4}\tilde{\mathcal{H}} \wedge d\tilde{\mathcal{H}} = f, \tag{4.66}$$

where f is defined as before, but in this case it has no ω dependence. A brief reflection shows us, that this case is more complicated, than the $\mathcal{Q} \neq 0$ case. Of course, to the zeroth order in α' we simply obtain $\mathcal{H} = f$, which is the real root obtained earlier. Having $\mathcal{Q} = 0$ means, that there are no terms of order α' and therefore the next non-trivial equation is of order α'^2. This is simply because (2.47) takes now the form

$$d\mathcal{H} = \alpha' \Big[d\omega \wedge d\omega - d\omega \wedge d\tilde{\mathcal{H}} + \frac{1}{4} d\tilde{\mathcal{H}} \wedge d\tilde{\mathcal{H}} - \mathrm{tr}\ F \wedge F \Big]. \tag{4.67}$$

This makes the situation a little involved and therefore (4.66) can only be solved iteratively. However, in the next sections we will not consider this case. To the zeroth order it is clear, that the only root is the real one and the second Chern class for the gauge bundle satisfies $c_2(F) = \int d\omega \wedge d\omega$. It is also clear, that even for this case the radius would be stabilized,

because from (4.67) we see, that the left hand side scales as t, whereas the right hand side scales as $a\ t^{-1} + b\ t^{-2}$, where a, b can be easily ascertained from (4.67)[16].

(d) Superpotential for the heterotic theory and radius of the six manifold

In the above sections we argued, how a *complex* three-form could arise in the heterotic theory. In this section we would like to compare the superpotential, that we get from this setup to the superpotential obtained by doing U-dualities from the Type IIB theory. In order to do this, we must first incorporate back the spin connection ω into the calculations.

The main equation, which takes into account all the variables has already been spelled out in (4.42). Let us concentrate on the component h_1 for the time being. Using the earlier definitions, we can show, that the cubic equation, that we get at this stage is the same as (4.62) with m, n, s defined as

$$m = a_1 + a_2\omega, \qquad n = \frac{t^3}{\alpha'} - a_3\omega + a_4(d\omega + \omega^2) + a_5 \qquad \text{and} \qquad s = -\frac{ft^3}{\alpha'} + \mathcal{O}(\omega^4), \quad (4.68)$$

where ω is the one-form spin connection ω_μ^{ab} and a_i are constants independent of α'. We can now define the shift β in $h_1 = h - \beta$ as $\beta = \frac{1}{3}(a_1 + a_2\omega)$. This results, as before, in a cubic equation of the form (4.47) with p and q given by

$$\begin{aligned} p &= \frac{t^3}{\alpha'} + \mathcal{A}\ \omega + \mathcal{O}(\omega^2), \\ q &= -\frac{f\ t^3}{\alpha'} + \mathcal{O}(\omega^4), \end{aligned} \qquad (4.69)$$

where the α' dependence is shown explicitly and $\mathcal{A}$ can be determined from (4.68). All

From:

$$\mathcal{H} + \frac{\alpha'}{12}\tilde{\mathcal{H}} \wedge \tilde{\mathcal{H}} \wedge \tilde{\mathcal{H}} + \ldots = dB + \alpha'[\Omega_3(\omega) - \Omega_3(A)], \qquad (4.44)$$

$$\tilde{\mathcal{H}}^3 + a\ \mathcal{H} + b = 0. \qquad (4.45)$$

$$h^3 + ph + q = 0 \qquad \text{with} \qquad p, q \in \mathbb{R}, \qquad (4.47)$$

We note that (4.45) and (4.47) are equals to:

$$x^3 + px + q = 0$$

We have:

$$p \sim \pm\frac{t^3}{\alpha'}, \qquad q \sim \mp\frac{t^3 f}{\alpha'}, \qquad \text{and} \qquad \left\langle dB + \alpha'\Omega_3(\omega) - \alpha'\Omega_3(A)\right\rangle_{abc} \equiv f\epsilon_{abc}, \tag{4.48}$$

$$A = \left(-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}\right)^{1/3} \qquad \text{and} \qquad B = \left(-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}\right)^{1/3}, \tag{4.52}$$

On the possible application of the cubic equation

The most general cubic equation (equation of third degree):

$$x^3 + rx^2 + px + q = 0 \quad (1)$$

can be reduced in the following form:

$$x^3 + px + q = 0 \quad (2)$$

Putting in the (2) $x = u + v$. Indeed, we have:

$$x^3 + px + q = 0; \qquad x = u + v$$

$$(u+v)^3+p(u+v)+q=0;\quad u^3+3u^2v+3uv^2+v^3+p(u+v)+q=0;$$

$$u^3+v^3+3uv(u+v)+p(u+v)+q=0;$$

from this equation, we obtain:

$u^3+v^3=-q;$ $\quad 3uv(u+v)=-p(u+v);$ thence, in conclusion:

$$\begin{cases}u^3+v^3=-q\\3uv=-p\end{cases}\qquad\begin{cases}u^3+v^3=-q\\uv=-\frac{p}{3}\end{cases}\qquad\begin{cases}u^3+v^3=-q\\u^3v^3=-\frac{p^3}{27}\end{cases}\qquad(11)$$

From the relations (10) and (11), we obtain the sum and the product of u^3 and v^3. We have that u^3 and v^3 must be roots of the following quadratic equation:

$$t^2+qt-\frac{p^3}{27};\quad(12)$$

This equation is defined the “resolving” of the cubic equation (2). Thence, we have:

$$t^2+qt-\frac{p^3}{27}$$

$$\frac{-q\pm\sqrt{q^2-4(1)\left(-\frac{p^3}{27}\right)}}{2}=\frac{-q\pm\sqrt{q^2+\frac{4p^3}{27}}}{2}=-\frac{q}{2}\pm\sqrt{\frac{1}{4}q^2+\frac{1}{4}\cdot\frac{4p^3}{27}}=$$

$$=-\frac{q}{2}\pm\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}.$$

Thence, we have that:

$$t_1 = u^3 = -\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \quad ; \qquad (13)$$
$$t_2 = v^3 = -\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}$$

We note that

$$A = \left(-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \right)^{1/3} \quad \text{and} \quad B = \left(-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}} \right)^{1/3},$$

Are equals to the cubic roots of (13). Indeed:

$$A = u = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} ;$$

$$B = v = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

From which:

$$A + B, \quad -\frac{A+B}{2} + i\frac{\sqrt{3}(A-B)}{2}, \quad \text{and} \quad -\frac{A+B}{2} - i\frac{\sqrt{3}(A-B)}{2}, \qquad (4.53)$$

$$s \equiv \sqrt{1+z^2} - 1 \quad \text{with} \quad z = \sqrt{\frac{27q^2}{4p^3}},$$

$$A = a(1+s-z)^{1/3} \quad \text{and} \quad B = -a(1+s+z)^{1/3}, \qquad (4.54)$$

where $a = \sqrt{p/3}$.

From:

$$A=u=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\;;\quad B=v=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$ we obtain:

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\quad+\quad\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

The expression is the “Cardano” or “Tartaglia” formula:

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

we have also:

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}-\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

The solution that Cardano gives of the cubic equation of type $x^3+px=q$, leads to the following formula:

$$x=\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}+\frac{q}{2}}-\sqrt[3]{\sqrt{\left(\frac{p}{3}\right)^3+\left(\frac{q}{2}\right)^2}-\frac{q}{2}}$$

Note that:

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\quad\text{and}\quad x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}-\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

can be connected to

$$A+B=-\frac{2az}{3}+\frac{4asz}{9}-\frac{10az^3}{81}-\frac{10as^2z}{27}+\frac{80as^3z}{243}+\ldots,$$
$$A-B=2a+\frac{2as}{3}-\frac{2a(s^2+z^2)}{9}+\frac{10as(s^2+3z^2)}{81}+\ldots \qquad (4.55)$$

where: (for p = 2 and q = 1)

$$s\equiv\sqrt{1+z^2}-1$$

s = sqrt(1+27/32) – 1 = 1/8 (sqrt(118) - 8) = 0.3578475614

$$z=\sqrt{\frac{27q^2}{4p^3}},$$

z = sqrt(27/32) = 0.918558653

$$a=\sqrt{p/3}.$$

a = sqrt(2/3) = 0.81649658092

For:

s = 0.3578475614

z = 0.918558653

a = 0.81649658092

from:

$$A+B=-\frac{2az}{3}+\frac{4asz}{9}-\frac{10az^3}{81}-\frac{10as^2z}{27}+\frac{80as^3z}{243}+\ldots,$$
$$A-B=2a+\frac{2as}{3}-\frac{2a(s^2+z^2)}{9}+\frac{10as(s^2+3z^2)}{81}+\ldots$$

we obtain:

$$A+B=-\frac{2az}{3}+\frac{4asz}{9}-\frac{10az^3}{81}-\frac{10as^2z}{27}+\frac{80as^3z}{243}+\ldots,$$
$$A-B=2a+\frac{2as}{3}-\frac{2a(s^2+z^2)}{9}+\frac{10as(s^2+3z^2)}{81}+\ldots$$

-2/3*(0.81649658*0.918558653)+4/9(0.81649658*0.3578475614*0.918558653)-10/81(0.81649658*0.918558653^3)-10/27(0.81649658*0.3578475614^2*0.918558653)+80/243*(0.81649658*0.3578475614^3*0.918558653)

Input interpretation:

$$-\frac{2}{3}(0.81649658\times 0.918558653)+\frac{4}{9}(0.81649658\times 0.3578475614\times 0.918558653)-\frac{10}{81}\left(0.81649658\times 0.918558653^3\right)-\frac{10}{27}\left(0.81649658\times 0.3578475614^2\times 0.918558653\right)+\frac{80}{243}\left(0.81649658\times 0.3578475614^3\times 0.918558653\right)$$

Result:

$-0.48309867893156214167565908161423013810962674351934156378600823 0\ldots$

-0.483098678… = A + B

Repeating decimal:

$-0.48309867893156214167565908161423013810962674351934156378600823 0\ldots$

(period 27)

2*(0.81649658092)+2/3(0.81649658092*0.3578475614)-
2/9*0.81649658092(0.3578475614^2+
0.918558653^2)+10/81*0.81649658092*0.3578475614(0.3578475614^2+3*0.91855
8653^2)

Input interpretation:

$$2\times 0.81649658092+\frac{2}{3}(0.81649658092\times 0.3578475614)+\frac{2}{9}\left(0.3578475614^2+0.918558653^2\right)\times(-0.81649658092)+\frac{10}{81}\times 0.81649658092\times 0.3578475614\left(0.3578475614^2+3\times 0.918558653^2\right)$$

Result:

1.747378669631893560046007658305826901703269135802469135802469135
8
…

Repeating decimal:

1.747378669631893560046007658305826901703$\overline{2691358024}$

(period 9)

1.747378669…. = A - B

From:

$$A+B,\qquad -\frac{A+B}{2}+i\frac{\sqrt{3}(A-B)}{2},\qquad \text{and}\qquad -\frac{A+B}{2}-i\frac{\sqrt{3}(A-B)}{2},$$

A + B = -0.483098678

A – B = 1.747378669

$$-\frac{A+B}{2}+i\frac{\sqrt{3}(A-B)}{2},$$

-1/2(-0.483098678)+i*sqrt3/2(1.747378669)

Input interpretation:

$$-\frac{-0.483098678}{2}+i\times\frac{\sqrt{3}}{2}\times 1.747378669$$

i is the imaginary unit

Result:

0.241549339… +
1.513274317… i

Polar coordinates:

$r = 1.53243$ (radius), $\theta = 1.41251$ (angle)
1.53243

Polar forms:

$$1.53243\,(\cos(1.41251)+i\sin(1.41251))$$

$$1.53243\,e^{1.41251\,i}$$

$$-\frac{A+B}{2}-i\frac{\sqrt{3}(A-B)}{2},$$

-1/2(-0.483098678)-i*sqrt3/2(1.747378669)

Input interpretation:

$$-\frac{-0.483098678}{2}+i\times\frac{\sqrt{3}}{2}\times(-1.747378669)$$

i is the imaginary unit

Result:

0.241549339… −
1.513274317… i

Polar coordinates:

$r = 1.53243$ (radius), $\theta = -1.41251$ (angle)

1.53243

Polar forms:

$1.53243\,(\cos(-1.41251) + i\sin(-1.41251))$

$1.53243\,e^{-1.41251\,i}$

From which:

-1/(5+(2*(((√(10-2√5) -2))/((√5-1)))))[((-1/2(-0.483098678)+i*sqrt3/2(1.747378669)))-((-1/2(-0.483098678)-i*sqrt3/2(1.747378669)))]^2

Input interpretation:

$$-\frac{1}{5+2\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}\left(\left(-\frac{-0.483098678}{2}+i\times\frac{\sqrt{3}}{2}\times 1.747378669\right)-\left(-\frac{-0.483098678}{2}+i\times\frac{\sqrt{3}}{2}\times(-1.747378669)\right)\right)^2$$

i is the imaginary unit

Result:

1.64506763…

$1.64506763\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

From

$$x^3 + px^2 + q = 0$$

$$x^3 - 2x + 1 = 0$$

Decompose the equation. We have that:

$$(x^2 + x - 1)(x - 1) = x^3 - x^2 + x^2 - x - x + 1 = x^3 - 2x + 1; \quad \text{thence:}$$

$$(x^2 + x - 1)(x - 1) = 0; \qquad x - 1 = 0; \qquad x = 1$$

$$x^2 + x - 1 = 0; \qquad \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2} = \frac{-1 \pm \sqrt{1+4}}{2} = \frac{-1 \pm \sqrt{5}}{2} \qquad \begin{cases} x_1 = \dfrac{-1+\sqrt{5}}{2} \\ x_2 = \dfrac{-1-\sqrt{5}}{2} \end{cases}$$

The three roots are: $1; \ \frac{-1+\sqrt{5}}{2}; \ \frac{-1-\sqrt{5}}{2}$.

The last two roots are equal to 0.61803398…. and -1.61803398… result that are equals to the value of the golden ratio and golden ratio conjugate

Now, $x^3 - 2x + 1 = 0$ applying the relation (27), we have:

$$z^3 - pz^2 - q^2 = 0; \quad z^3 - 2z^2 - 1 = 0 \quad \text{whence, from (23b)}$$

$$\begin{cases} u + v = 1 \\ (u+v)^2 - uv = 2 \\ u^2 v^2 = 1 \end{cases} \qquad \begin{cases} u + v = 1 \\ 1 - uv = 2 \\ uv = \pm 1 \end{cases} \qquad \begin{cases} u + v = 1 \\ uv = -1 \\ uv = \pm 1 \end{cases} \qquad \begin{cases} u + v = 1 \\ uv = -1 \end{cases}$$

Thence:

$$t^2-t-1=0;\qquad \frac{1\pm\sqrt{1+4}}{2}=\begin{cases}u=\dfrac{1+\sqrt{5}}{2}\\ v=\dfrac{1-\sqrt{5}}{2}\end{cases}$$

For the relations (25b), the third root will be $x_3=u+v=\frac{1}{2}+\frac{\sqrt{5}}{2}+\frac{1}{2}-\frac{\sqrt{5}}{2}=1$. The other two roots, will be: $x_1=-u=\frac{-1-\sqrt{5}}{2}$; and $x_2=-v=\frac{-1+\sqrt{5}}{2}$.

We note that the $x_2=\frac{\sqrt{5}-1}{2}=0{,}61803398...$ is equal to the aurea section.

From:

$$h=f-\frac{4\alpha'}{9}\frac{f^3}{t^3}+.... \qquad (4.56)$$

$$h=-\frac{f}{2}\pm i\sqrt{\frac{t^3}{\alpha'}}+.... \qquad (4.57)$$

And:

$$\mathcal{H}_{\bar{1}}\equiv\mathcal{H}^{ab}_{\bar{1}}=\mathcal{H}_{\bar{1}jk}\,e^{ja}\,e^{kb}=t^{-1}(\alpha^{ab}_{\bar{1}}\,h_1+\beta^{ab}_{\bar{1}}\,h_4),$$
$$\mathcal{H}_2=t^{-1}(\alpha^{ab}_{2}\,h_1+\beta^{ab}_{2}\,h_4),\quad \mathcal{H}_{\bar{3}}=t^{-1}(\alpha^{ab}_{\bar{3}}\,h_1+\beta^{ab}_{\bar{3}}\,h_2), \qquad (4.61)$$

$$h_1^3+mh_1^2+nh_1+s=0, \qquad (4.62)$$

$$m = (ah_2 + bh_4), \qquad n = \left(\frac{t^3}{\alpha'} + ch_4h_2 + dh_4^2\right) \qquad \text{and} \qquad s = \left(-\frac{ft^3}{\alpha'} + eh_4^2h_2\right),$$

$$p = n - \frac{m^2}{3} = \frac{t^3}{\alpha'} + B \qquad \text{and} \qquad q = s - \frac{mn}{3} + \frac{2m^3}{27} = -\frac{ft^3}{\alpha'} + D, \tag{4.63}$$

$$m = a_1 + a_2\omega, \qquad n = \frac{t^3}{\alpha'} - a_3\omega + a_4(d\omega + \omega^2) + a_5 \qquad \text{and} \qquad s = -\frac{ft^3}{\alpha'} + \mathcal{O}(\omega^4), \tag{4.68}$$

$$p = \frac{t^3}{\alpha'} + \mathcal{A}\,\omega + \mathcal{O}(\omega^2),$$
$$q = -\frac{f\ t^3}{\alpha'} + \mathcal{O}(\omega^4), \tag{4.69}$$

For p = 2 and q = 1 , we obtain from (4.63) and (4.69):

$$p = n - \frac{m^2}{3} = \frac{t^3}{\alpha'} + B \qquad q = s - \frac{mn}{3} + \frac{2m^3}{27} = -\frac{ft^3}{\alpha'} + D,$$
$$p = \frac{t^3}{\alpha'} + \mathcal{A}\,\omega + \mathcal{O}(\omega^2), \qquad q = -\frac{f\ t^3}{\alpha'} + \mathcal{O}(\omega^4),$$

p = 2 and q = 1

From:

Properties Of Heterotic Vacua From Superpotentials - *Katrin Becker, Melanie Becker, Keshav Dasgupta, Sergey Prokushkin* - arXiv:hep-th/0304001v4 13 Jun 2003

We have that:

$G_{ijk}e^{aj}e^{bk}$. We see, that G appears on both sides of the above expression and therefore we need to solve iteratively this equation in order to determine G. For the case considered here we can do this order by order in α'. The equation to be solved is

$$G + \frac{\alpha'}{2}\text{tr}\left(\omega_o \wedge \mathcal{R}_{\tilde{G}} + \tilde{G} \wedge \mathcal{R}_{\omega_o} - \frac{1}{2}\tilde{G} \wedge \mathcal{R}_{\tilde{G}}\right) = d\mathcal{B} + \alpha'\left(\Omega_3(\omega_o) - \Omega_3(\mathcal{A})\right), \tag{3.3}$$

where we have introduced the curvature polynomials $\mathcal{R}_{\tilde{G}}$ and $\mathcal{R}_{\omega_o}$ as

$$\mathcal{R}_{\tilde{G}} = d\tilde{G} - \frac{1}{3}\tilde{G} \wedge \tilde{G}, \quad \text{and} \quad \mathcal{R}_{\omega_o} = d\omega_o + \frac{2}{3}\omega_o \wedge \omega_o. \tag{3.4}$$

To the lowest order in α' we can ignore the contributions from $d\tilde{G}$ (3.3), because they are of $O(\alpha'^2)$. We will also ignore the contributions from $d\omega_o$, because they are higher derivatives in the vielbeins. The above formula reduces to the usual heterotic three-form equation in the absence of torsion and the superpotential becomes the superpotential computed in [23], as can be easily seen. Now if we denote the size of the internal manifold as t (we shall take t to be a function of all the spatial coordinates), we obtain from (3.3) a cubic equation, which takes the generic form

$$h^3 + ph + q = 0, \qquad \text{with} \qquad G_{ijk} = h\,\mathcal{C}_{ijk}, \qquad \text{and} \qquad g_{ij} = t\,g^o_{ij}, \tag{3.5}$$

for *every* component of the three-form G. Here $\mathcal{C}$ is a constant antisymmetric tensor in six-dimensions, whose contractions are done with respect to the metric g^o_{ij}. And g^o_{ij} is chosen to be constant locally, so that we can ignore the twist of the fiber[1]. Also, we can relax the condition on $\mathcal{C}$ a little bit. What we actually require is, that $\mathcal{C}$ should be at least

In the following analysis we will show that the complex root of the cubic equation does satisfy all the above three conditions, modulo possible geometric terms (and hence gauge invariant) that could in principle contribute to the imaginary part of the three-form G.[6] We will ignore these contributions for the time being and only mention them later.

Let us now study the solutions of (3.5) carefully. The three roots of the cubic equation (3.5) can be written in terms of p and q. We define two variables A and B, that are functions of p, q, such that the roots of the cubic equations are

$$A + B, \quad -\frac{1}{2}(A + B) \pm i\frac{\sqrt{3}}{2}(A - B). \tag{3.12}$$

The variables A, B are defined in [17] and are real. Therefore, the real root of the cubic equation is $A + B$. This is in fact the heterotic three-form that appears in the lagrangian. The series expansion of this three-form in terms of powers of p, q can be written as

$$A + B = -\frac{q}{p} + \frac{q^3}{p^4} - \frac{3q^5}{p^7} + \frac{12q^7}{p^{10}} - \frac{55q^9}{p^{13}} + \mathcal{O}(q^{11}). \tag{3.13}$$

The above considerations will tell us, that the equation satisfied by the component $G_{\bar{1}2\bar{3}}$ takes the form

$$G_{\bar{1}2\bar{3}} + \frac{\alpha'}{2}\mathrm{tr}\Big[-\frac{1}{3}\omega_o \wedge \tilde{G} \wedge \tilde{G} + \frac{2}{3}\tilde{G} \wedge \omega_o \wedge \omega_o + \frac{1}{6}\tilde{G} \wedge \tilde{G} \wedge \tilde{G}\Big]_{\bar{1}2\bar{3}} = f_{\bar{1}2\bar{3}}, \tag{4.3}$$

where f_{ijk} is defined earlier in (3.7). For the non-Kähler metric studied in [11],[13],[20] and [17], the base manifold was an orbifold. As a result, the only non-trivial factors in the metric are the warp factors. This would mean, that up to powers of the warp factor all components of the spin-connection are the same[11]. Let us therefore take

$$\omega_{oi}{}^{ab} = f_i(\Delta)\omega_o\epsilon^{ab}, \tag{4.4}$$

where f_i needs to be worked out for every components individually and ϵ^{ab} is the anti-symmetric tensor. As we discussed in [17], if we consider the set of equations (4.2) to the leading order in α', we can replace the complex three-forms h_2, h_3 by their real parts. Therefore, up to proportionality constants, we can write the above three one-forms as $h_1+\alpha_1, h_1+\alpha_2, h_1+\alpha_3$, where α_i can be easily calculated from the vielbeins e_o^{ai}. This will transform the cubic equation (4.3) into[12]

$$\frac{h_1 t^3}{\alpha'} - \mathcal{A}_1\omega_o(h_1+\beta_1)(h_1+\beta_2) + \mathcal{A}_2\omega_o^2(h_1+\beta_3) + (h_1+\beta_4)(h_1+\beta_5)(h_1+\beta_6) = \frac{f t^3}{\alpha'}, \tag{4.5}$$

where it is an easy exercise to relate α_i and $f_i(\Delta)$ to $\beta_i, \mathcal{A}_1$ and $\mathcal{A}_2$. We have absorbed the traces of the holonomy matrices discussed in the previous section into the definition of t. One can also check, that similar equations hold for all other components of G. Therefore, the generic cubic equation, that we get is

$$h_1^3 + m h_1^2 + n h_1 + s = 0, \tag{4.6}$$

where m, n and s are integers given in terms of f, t, β_i and $\mathcal{A}_j$ as

$$\begin{aligned} m &= \beta_4 + \beta_5 + \beta_6 - \mathcal{A}_1\omega_o, \\ s &= -\frac{f t^3}{\alpha'} + \beta_4\beta_5\beta_6 - \mathcal{A}_1\omega_o\beta_1\beta_2 + \mathcal{A}_2\omega_o^2\beta_3, \\ n &= \frac{t^3}{\alpha'} + \beta_4\beta_5 + \beta_5\beta_6 + \beta_6\beta_4 - \mathcal{A}_1\omega_o(\beta_1+\beta_2) + \mathcal{A}_2\omega_o^2. \end{aligned} \tag{4.7}$$

There is a quadratic term in this equation, that can be removed by shifting the three-form h_1 by $h_1 = h_1 - \frac{m}{3}$. This will give us precisely the cubic equation (3.5), with p and q defined as

$$p = \frac{t^3}{\alpha'} - \mathcal{A}_3\widehat{\omega}_o + \mathcal{O}(\widehat{\omega}_o^2), \qquad q = -\frac{ft^3}{\alpha'} + \mathcal{A}_4\widehat{\omega}_o + \mathcal{A}_5 + \mathcal{O}(\widehat{\omega}_o^2), \tag{4.8}$$

where $\widehat{\omega}_o$ is a shifted spin-connection, that is introduced to absorb the constant term in p and $\mathcal{A}_i$ are constants determined by (4.7). To order $\widehat{\omega}_o$ the above expression precisely coincides with the simplified discussion presented in the previous section (see eq. (3.14) and identify $\widehat{\omega}_o$ with ω_o there). There is one difference though, that we would like to discuss in some detail. Notice, that in the expression for q given above, there is a constant $\mathcal{A}_5$, that did not appear in our analysis in the previous section. This constant is rather harmless, because adding a constant l into the definition of q in (3.14), will give an additional constants in the real and the imaginary parts of G in (3.17), that have the form

$$-\frac{l\alpha'}{t^3} + \mathcal{O}(\alpha'^2), \qquad \text{and} \qquad 0 + \mathcal{O}(\alpha'^{3/2}), \tag{4.9}$$

respectively. The above shift appearing in the real root doesn't change the final expression for G, because we can tune l to scale smaller than f^3. To summarize, what we have just shown is, that we can trust the analysis done in the previous section, even if some simplifying assumptions were done.

There is one more point, that we would like to discuss, before we derive the torsional constraints from our superpotential in the next section. This has to do with the signs of p and q in the cubic equation (3.5). According to the conventions, that we have chosen, the signs of p and q are fixed. However, is an interesting question to ask, what happens, if we reverse the signs of p and q. Observe, that transforming $q \to -q$ does not change much the real and complex solutions for G, as they still retain their original form, except for an overall sign change in the function (3.13). However, changing $p \to -p$ makes (3.16) pure imaginary and therefore all the roots of the cubic equation become real. One may now wonder, if this is consistent with dual Type IIB picture.

To see, that this is indeed so we need to remind ourselves, that the tensor field H_{RR} of the Type IIB theory turns into the real root $\mathcal{H}$ of the heterotic theory and the tensor field H_{NS} turns into the spin-connection. Let us therefore make the transformation

$$H_{RR} \to iH_{RR}, \qquad \text{and} \qquad H_{NS} \to iH_{NS}, \tag{4.10}$$

which shifts the i in the superpotential to the H_{RR} part, while giving a relative minus sign between the three-forms. Recall, that we are considering the case, when we have a vanishing axion-dilaton, as this is directly related to the superpotential of the heterotic theory. The above transformation (4.10) will, in turn, convert our cubic equation (3.5) into

$$G^3 - p\, G + iq = 0, \tag{4.11}$$

where we have taken $G \to iG$ in (3.5). The roots of the above equation are, in fact, slightly different, because the real root (3.13) will become purely imaginary, while the i in the complex root (3.17) will trade places. A simple way to see this, would be to go back to the series expansions (3.13) and (3.16) and write them as

$$\begin{aligned} A + B &= \sum_{n=0}^{\infty} a_n \frac{(-1)^{n+1}|q|^{2n+1}}{p^{3n+1}}, \\ A - B &= \sqrt{p} + \sum_{n=1}^{\infty} b_n \frac{(-1)^{n+1}|q|^{2n}}{p^{3n-\frac{1}{2}}}, \end{aligned} \tag{4.12}$$

where a_n, b_n can be determined from (3.13) and (3.16). Of course, this expansion is not very meaningful beyond the first few orders of p and q, but we shall still use these series, to illustrate the generic behavior (we believe that putting higher order α' corrections to the system will change the coefficients a_n and b_n, without altering the p, q behavior). If we change $p \to -p$ and $q \to iq$, it is easy to check from (4.12), that both $A \pm B$ become purely imaginary, as mentioned above. Let us now make the transformation $q \to iq$, which will give us

$$G^3 - p\, G - q = 0, \qquad p > 0, \quad q > 0, \tag{4.13}$$

whose roots are all real[13]. This is the case alluded to above. The question now is to trace this back to the Type IIB theory.

We have the following equations:

$$G + \frac{\alpha'}{2}\mathrm{tr}\left(\omega_o \wedge \mathcal{R}_{\tilde{G}} + \tilde{G} \wedge \mathcal{R}_{\omega_o} - \frac{1}{2}\tilde{G} \wedge \mathcal{R}_{\tilde{G}}\right) = d\mathcal{B} + \alpha'\left(\Omega_3(\omega_o) - \Omega_3(\mathcal{A})\right), \tag{3.3}$$

$$h^3 + ph + q = 0, \qquad \text{with} \qquad G_{ijk} = h\, \mathcal{C}_{ijk}, \qquad \text{and} \qquad g_{ij} = t\, g^{o}_{ij}, \tag{3.5}$$

$$A+B, \qquad -\frac{1}{2}(A+B) \pm i\frac{\sqrt{3}}{2}(A-B). \tag{3.12}$$

$$A+B = -\frac{q}{p} + \frac{q^3}{p^4} - \frac{3q^5}{p^7} + \frac{12q^7}{p^{10}} - \frac{55q^9}{p^{13}} + \mathcal{O}(q^{11}). \tag{3.13}$$

For p = 2 and q = 1:

(-1/2+1/16-3/128+12/1024-55/8192+1)

Input:

$$-\frac{1}{2} + \frac{1}{16} - \frac{3}{128} + \frac{12}{1024} - \frac{55}{8192} + 1$$

Exact result:

$$\frac{4457}{8192}$$

Decimal form:

0.5440673828125

0.5440673828125

$$p = \frac{t^3}{\alpha'} - c\ f\ \omega_o, \qquad q = -\frac{ft^3}{\alpha'} + b\ f^2\omega_o, \tag{3.14}$$

For p = 2 and q = 1

$$(A-B)=\frac{2}{\sqrt{3}}\left[\sqrt{p}+\frac{3}{8}\frac{q^2}{p^{5/2}}-\frac{105}{128}\frac{q^4}{p^{11/2}}+\frac{3003}{1024}\frac{q^6}{p^{17/2}}-\frac{415701}{32768}\frac{q^8}{p^{23/2}}\right]+\mathcal{O}(q^{10}). \tag{3.15}$$

(2/sqrt3)[sqrt2+3/8*1/(2^2.5)-105/128*1/(2^5.5)+3003/1024*1/(2^8.5)-415701/32768*1/(2^11.5)]+1

Input:

$$\frac{2}{\sqrt{3}}\left(\sqrt{2}+\frac{3}{8}\times\frac{1}{2^{2.5}}-\frac{105}{128}\times\frac{1}{2^{5.5}}+\frac{3003}{1024}\times\frac{1}{2^{8.5}}-\frac{415\,701}{32\,768}\times\frac{1}{2^{11.5}}\right)+1$$

Result:

2.692904694753610665103185798278083331159849061087539939583057400 2…

2.692904694….

(2/sqrt3)[sqrt2+3/8*1/(2^2.5)-105/128*1/(2^5.5)+3003/(x)*1/(2^8.5)-415701/32768*1/(2^11.5)]+1 = 2.692904694753

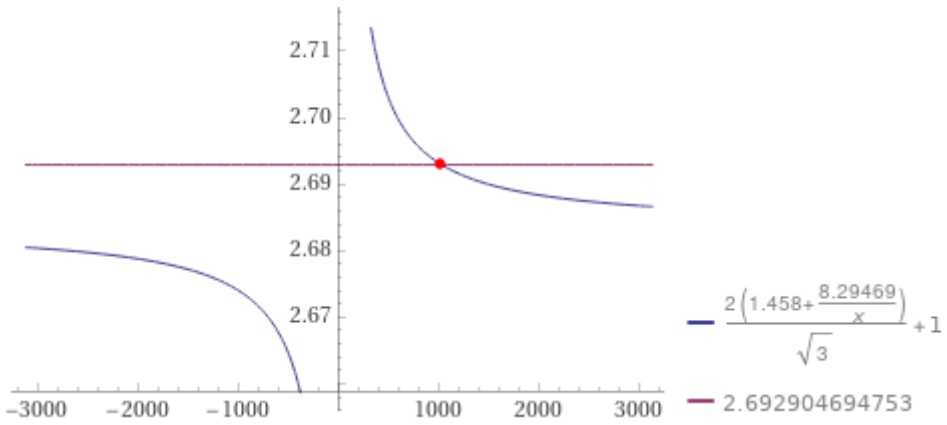
Input interpretation:

$$\frac{2}{\sqrt{3}}\left(\sqrt{2}+\frac{3}{8}\times\frac{1}{2^{2.5}}-\frac{105}{128}\times\frac{1}{2^{5.5}}+\frac{3003}{x}\times\frac{1}{2^{8.5}}-\frac{415\,701}{32\,768}\times\frac{1}{2^{11.5}}\right)+1=2.692904694753$$

Result:

$$\frac{2\left(1.458+\frac{8.29469}{x}\right)}{\sqrt{3}}+1=2.692904694753$$

Plot: **(figures that can be related to the open strings)**



Alternate form assuming x is real:

$$\frac{1024.}{x} = 1$$

Alternate forms:

$$2.68355 + \frac{9.57789}{x} = 2.692904694753$$

$$\frac{0.971999\,(2.76086\,x + 9.85381)}{x} = 2.692904694753$$

Alternate form assuming x is positive:

$x = 1024.$ (for $x \neq 0$)

Expanded form:

$$2.68355 + \frac{9.57789}{x} = 2.692904694753$$

Solution:

$x \approx 1024.$

1024

From:

$$\frac{2\left(1.458 + \frac{8.29469}{x}\right)}{\sqrt{3}} + 1 = 2.692904694753$$

(2 (1.458 + 8.29469/x)) = sqrt3(2.692904694753 - 1)

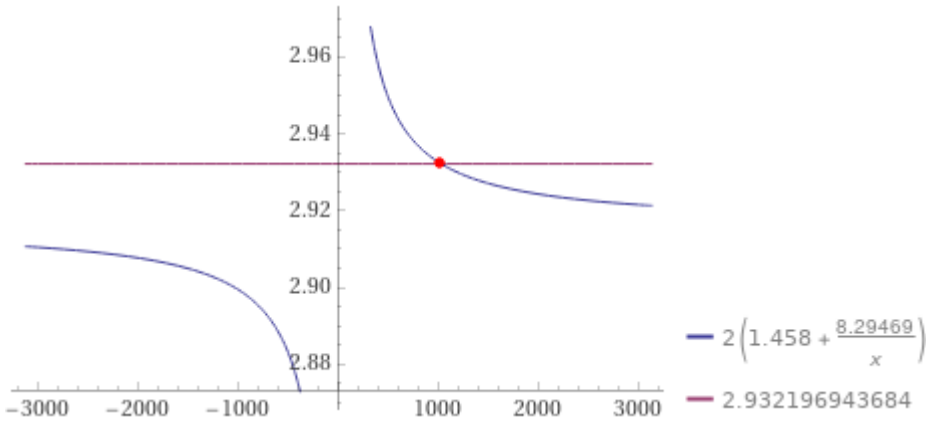
Input interpretation:

$$2\left(1.458 + \frac{8.29469}{x}\right) = \sqrt{3}\ (2.692904694753 - 1)$$

Result:

$$2\left(1.458 + \frac{8.29469}{x}\right) = 2.932196943684$$

Plot: **(figures that can be related to the open strings)**



Alternate form assuming x is real:

$$\frac{1024.23}{x} = 1$$

Alternate form:

$$\frac{2.916\,(x + 5.68909)}{x} = 2.932196943684$$

Alternate form assuming x is positive:

$x = 1024.23$ (for $x \neq 0$)

Expanded forms:

$$2.916 + \frac{16.5894}{x} = 2.932196943684$$

$$\frac{1}{x} = 0.000976344$$

Solution:

$x \approx 1024.23$

1024.23

From:

$$2.916 + \frac{16.5894}{x} = 2.932196943684$$

(2*8.29469)/(2.932196943684-2*1.458)

Input interpretation:

$$\frac{2 \times 8.29469}{2.932196943684 - 2 \times 1.458}$$

Result:

1024.22903503626209187725913192105163091582679852454069939087651 39

…

1024.229035036….

We have also, after the following calculations, we obtain:

1/2((-1/2+1/16-3/128+12/1024-55/8192+1)+[(2/sqrt3)[sqrt2+3/8*1/(2^2.5)-105/128*1/(2^5.5)+3003/1024*1/(2^8.5)-415701/32768*1/(2^11.5)]+1])

Input:

$$\frac{1}{2}\left(\left(-\frac{1}{2}+\frac{1}{16}-\frac{3}{128}+\frac{12}{1024}-\frac{55}{8192}+1\right)+\left(\frac{2}{\sqrt{3}}\left(\sqrt{2}+\frac{3}{8}\times\frac{1}{2^{2.5}}-\frac{105}{128}\times\frac{1}{2^{5.5}}+\frac{3003}{1024}\times\frac{1}{2^{8.5}}-\frac{415\,701}{32\,768}\times\frac{1}{2^{11.5}}\right)+1\right)\right)$$

Result:

1.61848603878305533255159289913904166557992453054376996979152870 01

…

1.61848603878…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Figs.

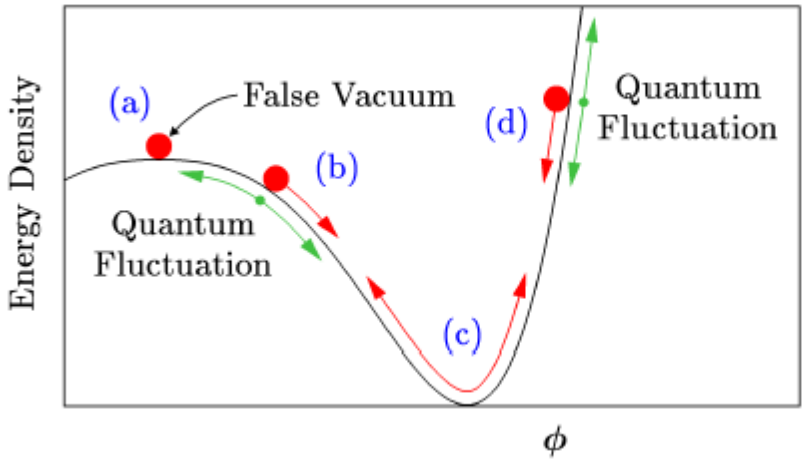


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, ϕ. Red arrows show the classical motion of ϕ. When ϕ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_f$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of ϕ: Even near regions (b) and (d), ϕ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of "slow roll," ρ remains nearly constant. Only after ϕ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.

Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at $y = 0$). The case shown has two critical points. Here the function is $f(x) = (x^3 + 3x^2 - 6x - 8)/4$.

The ratio between M_0 and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{\left(3\sqrt{3}\right) M_s}{2}.$$

i.e. the gravitating mass M_0 and the Wheelerian mass q of the wormhole, is equal to:

$$\frac{\sqrt{3\,(2.17049\times10^{37})^2 - 0.001^2}}{\frac{1}{2}\left((3\sqrt{3})(4.2\times10^6\times1.9891\times10^{30})\right)}$$

1.732050787905194420703947625671018160083566548802082460520...

1.7320507879

$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q of the wormhole

(**Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing?** - *Ramil N. Izmailov and Eduard R. Zhdanov, Amrita Bhattacharya, Alexander A. Potapov, K.K. Nandi* - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

We note that:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)$$

i is the imaginary unit

$i\sqrt{3}$

1.732050807568877293527446341505872366942805253810380628055... i

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

1.73205

This result is very near to the ratio between M_0 and q, that is equal to $1.7320507879 \approx \sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.732050807568877293527446341505872366942805253810380628055\ldots\, i$$

$$r \approx 1.73205 \text{ (radius)}, \quad \theta = 90° \text{ (angle)}$$

can be related with:

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q$$

Considering:

$$(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$$= i\sqrt{3} = 1.732050807568877293527446341505872366942805253810380628055\ldots\, i$$

$$r \approx 1.73205 \text{ (radius)}, \quad \theta = 90° \text{ (angle)}$$

Thence:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q=1.73205\approx\sqrt{3}$$

We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.

From

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

Cubic equations and Cardano's Formulas

From

$$\sqrt[3]{190 + \sqrt{33,903}} + \sqrt[3]{190 - \sqrt{33,903}} + 1.$$

we obtain:

(190+sqrt(33903))^(1/3)+(190-sqrt(33903))^(1/3) + 1

Input

$$\sqrt[3]{190 + \sqrt{33\,903}} + \sqrt[3]{190 - \sqrt{33\,903}} + 1$$

Exact result

$$1 + \sqrt[3]{190 - 3\sqrt{3767}} + \sqrt[3]{190 + 3\sqrt{3767}}$$

Decimal approximation

10.009791228534877323142138669958259126893239265465987494249959287

…

10.0097912285348….

Alternate forms

root of $x^3 - 3x^2 - 36x - 342$ near $x = 10.0098$

$$1+\frac{1}{3}\sqrt[3]{5130-81\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}$$

Minimal polynomial

$x^3-3x^2-36x-342$

The minimal polynomial of α is the irreducible polynomial with rational coefficients of smallest degree with α as a root »

From

$$\sqrt[3]{1520+\sqrt{2{,}169{,}792}}+\sqrt[3]{1520-\sqrt{2{,}169{,}792}}-4$$

(1520+sqrt(2169792))^(1/3)+(1520-sqrt(2169792))^(1/3) - 4

Input

$$\sqrt[3]{1520+\sqrt{2\,169\,792}}+\sqrt[3]{1520-\sqrt{2\,169\,792}}-4$$

Result

$$\sqrt[3]{1520+24\sqrt{3767}}+\sqrt[3]{1520-24\sqrt{3767}}-4$$

Decimal approximation

14.019582457069754646284277339916518253786478530931974988499918575

…

14.019582457….

Alternate forms

2 root of $x^3+6x^2-27x-450$ near $x=7.00979$

$$2\left(-2+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)$$

$$-4+2\sqrt[3]{190-3\sqrt{3767}}+2\sqrt[3]{190+3\sqrt{3767}}$$

Minimal polynomial

$$x^3+12x^2-108x-3600$$

From the sum of the two results

$$1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}$$

and

$$\sqrt[3]{1520+24\sqrt{3767}}+\sqrt[3]{1520-24\sqrt{3767}}-4$$

we obtain:

(1 + (190 - 3 sqrt(3767))^(1/3) + (190 + 3 sqrt(3767))^(1/3))+((1520 + 24 sqrt(3767))^(1/3) + (1520 - 24 sqrt(3767))^(1/3) – 4)

Input

$$\left(1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)+\left(\sqrt[3]{1520+24\sqrt{3767}}+\sqrt[3]{1520-24\sqrt{3767}}-4\right)$$

Result

$$-3+\sqrt[3]{1520-24\sqrt{3767}}+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}+\sqrt[3]{1520+24\sqrt{3767}}$$

Decimal approximation

24.029373685604631969426416009874777380679717796397962482749877862…

24.0293736856…. ≈ 24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles). Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Alternate forms

3 root of $x^3 + 3x^2 - 36x - 418$ near $x = 8.00979$

$$3\left(-1 + \sqrt[3]{190 - 3\sqrt{3767}} + \sqrt[3]{190 + 3\sqrt{3767}}\right)$$

$$-3 + 3\sqrt[3]{190 - 3\sqrt{3767}} + 3\sqrt[3]{190 + 3\sqrt{3767}}$$

Minimal polynomial

$x^3 + 9x^2 - 324x - 11286$

From the alternate form

$$3\left(-1 + \sqrt[3]{190 - 3\sqrt{3767}} + \sqrt[3]{190 + 3\sqrt{3767}}\right)$$

((3 (-1 + (190 - 3 sqrt(3767))^(1/3) + (190 + 3 sqrt(3767))^(1/3))))

after some easy calculations, we obtain:

72((3 (-1 + (190 - 3 sqrt(3767))^(1/3) + (190 + 3 sqrt(3767))^(1/3))))-2^(1/6)

Input

$$72\left(3\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)-\sqrt[6]{2}$$

Result

$$216\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)-\sqrt[6]{2}$$

Decimal approximation

1728.992443315224128817268419661304791892707270230039312004550796 5

…

1728.99244331522….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$216\sqrt[3]{190-3\sqrt{3767}}+216\sqrt[3]{190+3\sqrt{3767}}-\sqrt[6]{2}-216$$

$$216\ \text{root of}\ x^3+3x^2-36x-418\ \text{near}\ x=8.00979-\sqrt[6]{2}$$

$$\begin{aligned}&\text{root of}\ x^{18}+3888x^{17}-3\,779\,136x^{16}-52\,484\,640\,768x^{15}-\\&79\,245\,760\,942\,080x^{14}+189\,079\,249\,327\,423\,488x^{13}+\\&750\,616\,335\,381\,939\,093\,498x^{12}+350\,061\,346\,782\,301\,359\,424\,320x^{11}-\\&2\,268\,214\,834\,118\,432\,481\,015\,144\,192x^{10}-\\&4\,215\,964\,967\,231\,551\,434\,336\,184\,012\,800x^9+\\&721\,387\,678\,983\,322\,964\,871\,315\,877\,576\,704x^8+\\&10\,447\,436\,031\,272\,039\,614\,062\,342\,147\,342\,336\,000x^7+\\&10\,296\,991\,255\,542\,063\,623\,252\,869\,774\,818\,778\,546\,188x^6-\\&6\,299\,462\,642\,271\,869\,981\,214\,358\,119\,146\,843\,289\,928\,512x^5-\\&19\,963\,765\,723\,148\,590\,829\,828\,631\,023\,299\,680\,065\,461\,950\,720x^4-\\&11\,156\,236\,352\,486\,132\,387\,048\,574\,914\,185\,254\,316\,165\,284\,569\,088x^3+\\&8\,167\,622\,184\,177\,255\,332\,395\,556\,006\,287\,361\,741\,384\,332\,952\,322\,048x^2+\\&13\,367\,461\,119\,607\,178\,804\,259\,034\,171\,541\,481\,798\,053\,102\,762\,536\,992\,768\\&\quad x+\\&5\,587\,598\,747\,995\,800\,740\,245\,788\,945\,355\,027\,852\,173\,166\,740\,704\,628\,572\,152\\&\text{near}\ x=1728.99\end{aligned}$$

Minimal polynomial

$$x^{18} + 3888\,x^{17} - 3\,779\,136\,x^{16} - 52\,484\,640\,768\,x^{15} - 79\,245\,760\,942\,080\,x^{14} + 189\,079\,249\,327\,423\,488\,x^{13} + 750\,616\,335\,381\,939\,093\,498\,x^{12} + 350\,061\,346\,782\,301\,359\,424\,320\,x^{11} - 2\,268\,214\,834\,118\,432\,481\,015\,144\,192\,x^{10} - 4\,215\,964\,967\,231\,551\,434\,336\,184\,012\,800\,x^{9} + 721\,387\,678\,983\,322\,964\,871\,315\,877\,576\,704\,x^{8} + 10\,447\,436\,031\,272\,039\,614\,062\,342\,147\,342\,336\,000\,x^{7} + 10\,296\,991\,255\,542\,063\,623\,252\,869\,774\,818\,778\,546\,188\,x^{6} - 6\,299\,462\,642\,271\,869\,981\,214\,358\,119\,146\,843\,289\,928\,512\,x^{5} - 19\,963\,765\,723\,148\,590\,829\,828\,631\,023\,299\,680\,065\,461\,950\,720\,x^{4} - 11\,156\,236\,352\,486\,132\,387\,048\,574\,914\,185\,254\,316\,165\,284\,569\,088\,x^{3} + 8\,167\,622\,184\,177\,255\,332\,395\,556\,006\,287\,361\,741\,384\,332\,952\,322\,048\,x^{2} + 13\,367\,461\,119\,607\,178\,804\,259\,034\,171\,541\,481\,798\,053\,102\,762\,536\,992\,768\,x + 5\,587\,598\,747\,995\,800\,740\,245\,788\,945\,355\,027\,852\,173\,166\,740\,704\,628\,572\,152$$

(1/27((72((3 (-1 + (190 - 3 sqrt(3767))^(1/3) + (190 + 3 sqrt(3767))^(1/3))))-2^(1/6))-1))^2

Input

$$\left(\frac{1}{27}\left(\left(72\left(3\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)-\sqrt[6]{2}\right)-1\right)\right)^2$$

Result

$$\frac{1}{729}\left(-1-\sqrt[6]{2}+216\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)^2$$

Decimal approximation

4095.964175794949347043021900607929265223381407470809942442376850 1

…

4095.9641757949…. ≈ 4096

The number $4096 = 64^2$, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{1}{729}\left(216\sqrt[3]{190-3\sqrt{3767}}+216\sqrt[3]{190+3\sqrt{3767}}-\sqrt[6]{2}-217\right)^2$$

$$\frac{1}{729}\left(-216\ \text{root of}\ x^3+3x^2-36x-418\ \text{near}\ x=8.00979+1+\sqrt[6]{2}\right)^2$$

$$\frac{1}{729}\left(217+\sqrt[6]{2}-216\sqrt[3]{190-3\sqrt{3767}}-216\sqrt[3]{190+3\sqrt{3767}}\right)^2$$

Expanded form

$$\frac{47089}{729} + \frac{434\sqrt[6]{2}}{729} + \frac{\sqrt[3]{2}}{729} - \frac{3472}{27}\sqrt[3]{190 - 3\sqrt{3767}} - \frac{16}{27}\sqrt[6]{2}\sqrt[3]{190 - 3\sqrt{3767}} + 64\left(190 - 3\sqrt{3767}\right)^{2/3} - \frac{3472}{27}\sqrt[3]{190 + 3\sqrt{3767}} - \frac{16}{27}\sqrt[6]{2}\sqrt[3]{190 + 3\sqrt{3767}} + 64\left(190 + 3\sqrt{3767}\right)^{2/3} + 128\sqrt[3]{\left(190 - 3\sqrt{3767}\right)\left(190 + 3\sqrt{3767}\right)}$$

Minimal polynomial

$$
\begin{aligned}
&3\,381\,391\,913\,522\,726\,342\,930\,221\,472\,392\,241\,170\,198\,527\,451\,848\,561\,x^{18} - \\
&\quad 105\,210\,965\,749\,780\,147\,837\,328\,492\,279\,696\,808\,627\,649\,959\,896\,535\,923\,490\,x^{17} + \\
&\quad 1\,681\,001\,167\,345\,330\,917\,019\,886\,748\,342\,655\,348\,621\,798\,209\,454\,905\,635\,604\,729\,x^{16} - \\
&\quad 18\,580\,655\,129\,009\,095\,187\,325\,076\,706\,400\,561\,617\,888\,893\,902\,940\,725\,292\,788\cdot\cdot\cdot \\
&\qquad 037\,788\,x^{15} + \\
&\quad 158\,425\,967\,661\,845\,219\,441\,188\,791\,126\,491\,866\,894\,069\,586\,570\,338\,106\,116\,574\cdot\cdot\cdot \\
&\qquad 300\,463\,120\,x^{14} - \\
&\quad 1\,097\,773\,475\,089\,797\,581\,648\,739\,589\,064\,628\,602\,744\,912\,129\,832\,014\,512\,489\cdot\cdot\cdot \\
&\qquad 150\,035\,036\,607\,284\,x^{13} + \\
&\quad 6\,378\,582\,752\,418\,871\,529\,314\,034\,916\,052\,650\,615\,718\,876\,935\,654\,925\,535\,003\cdot\cdot\cdot \\
&\qquad 595\,317\,857\,869\,359\,820\,x^{12} - \\
&\quad 31\,676\,616\,986\,039\,448\,478\,197\,961\,428\,435\,706\,476\,834\,260\,593\,420\,622\,206\,308\cdot\cdot\cdot \\
&\qquad 197\,812\,027\,380\,505\,457\,052\,x^{11} + \\
&\quad 135\,985\,849\,299\,338\,000\,630\,030\,563\,986\,963\,697\,546\,166\,455\,948\,161\,124\,541\,383\cdot\cdot\cdot \\
&\qquad 126\,602\,963\,641\,776\,519\,269\,426\,x^{10} - \\
&\quad 508\,108\,498\,458\,023\,149\,771\,687\,550\,506\,107\,412\,427\,590\,275\,638\,607\,296\,990\,813\cdot\cdot\cdot \\
&\qquad 781\,740\,334\,237\,921\,268\,122\,244\,872\,x^{9} + \\
&\quad 1\,656\,650\,165\,958\,840\,706\,505\,835\,766\,306\,386\,451\,530\,913\,708\,191\,839\,025\,059\cdot\cdot\cdot \\
&\qquad 645\,752\,933\,043\,164\,956\,579\,552\,559\,762\,926\,x^{8} - \\
&\quad 4\,707\,999\,718\,876\,136\,990\,246\,754\,867\,967\,496\,333\,622\,309\,736\,168\,901\,075\,437\cdot\cdot\cdot \\
&\qquad 274\,725\,700\,419\,092\,463\,033\,258\,413\,633\,694\,452\,x^{7} + \\
&\quad 11\,609\,850\,152\,354\,652\,444\,267\,828\,236\,408\,858\,607\,161\,408\,608\,767\,882\,620\,139\cdot\cdot\cdot \\
&\qquad 657\,273\,734\,678\,279\,263\,375\,278\,771\,658\,338\,685\,080\,x^{6} - \\
&\quad 24\,603\,253\,431\,799\,698\,287\,540\,708\,341\,341\,315\,495\,542\,780\,980\,082\,858\,949\,188\cdot\cdot\cdot \\
&\qquad 365\,227\,931\,004\,573\,949\,112\,692\,694\,274\,476\,677\,980\,652\,x^{5} + \\
&\quad 44\,112\,309\,964\,138\,316\,615\,762\,962\,775\,201\,356\,186\,865\,845\,400\,428\,174\,277\,795\cdot\cdot\cdot \\
&\qquad 572\,704\,712\,036\,552\,550\,161\,309\,450\,661\,069\,479\,168\,818\,732\,x^{4} - \\
&\quad 65\,243\,822\,737\,637\,594\,971\,693\,425\,423\,387\,412\,288\,456\,424\,962\,894\,445\,777\,025\cdot\cdot\cdot \\
&\qquad 730\,281\,929\,325\,722\,600\,499\,387\,700\,162\,764\,848\,554\,218\,059\,604\,x^{3} + \\
&\quad 75\,965\,436\,916\,703\,044\,527\,191\,091\,666\,890\,875\,851\,675\,921\,061\,230\,557\,227\,502\cdot\cdot\cdot \\
&\qquad 926\,661\,714\,575\,345\,847\,560\,142\,447\,526\,708\,916\,771\,017\,857\,798\,901\,x^{2} - \\
&\quad 64\,157\,889\,883\,607\,681\,042\,020\,337\,490\,555\,019\,504\,597\,099\,124\,058\,762\,757\,244\cdot\cdot\cdot \\
&\qquad 192\,499\,643\,533\,198\,746\,001\,092\,694\,254\,783\,299\,536\,979\,360\,682\,924\,598\,x + \\
&\quad 31\,370\,913\,843\,675\,766\,930\,586\,804\,866\,712\,590\,839\,158\,664\,605\,710\,756\,759\,297\cdot\cdot\cdot \\
&\qquad 259\,885\,064\,223\,742\,548\,836\,495\,780\,406\,256\,792\,811\,304\,731\,866\,945\,890\,081
\end{aligned}
$$

(72((3 (-1 + (190 - 3 sqrt(3767))^(1/3) + (190 + 3 sqrt(3767))^(1/3))))-2^(1/6))^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{72\left(3\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)-\sqrt[6]{2}}+C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}+\sqrt[15]{216\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)-\sqrt[6]{2}}$$

Decimal approximation

1.64493754517059768301449181271101019985142521869801064987935443 08

…

1.64493754517…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate form

$$C_{\text{MRB}}{}^{-1/(4\pi)}\left(\sqrt[15]{-216-\sqrt[6]{2}+216\sqrt[3]{190-3\sqrt{3767}}+216\sqrt[3]{190+3\sqrt{3767}}}\right.$$

$$\left.\sqrt[4\pi]{C_{\text{MRB}}}+C_{\text{MRB}}{}^{1+\pi}\right)$$

For V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , we obtain:

from the octahedron volume V = $1/3*\sqrt{2}*a^3$

and, from the sphere volume, V = $(4/3*\pi*(a/2)^3)$

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

And for a = ((3 (-1 + (190 - 3 sqrt(3767))^(1/3) + (190 + 3 sqrt(3767))^(1/3))))

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot(((3\,(-1\,+\,(190\,-\,3\,\mathrm{sqrt}(3767))\text{^}(1/3)\,+\,(190\,+\,3\,\mathrm{sqrt}(3767))\text{^}(1/3)))))^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{(((3\,(-1\,+\,(190\,-\,3\,\mathrm{sqrt}(3767))\text{^}(1/3)\,+\,(190\,+\,3\,\mathrm{sqrt}(3767))\text{^}(1/3)))))}{2}\right)^3}$$

(1/3·√2·(((3 (-1 + (190 - 3 sqrt(3767))^(1/3) + (190 + 3 sqrt(3767))^(1/3)))))^3)/(4/3·π·((((((3 (-1 + (190 - 3 sqrt(3767))^(1/3) + (190 + 3 sqrt(3767))^(1/3))))))/2)^3)

Input

$$\frac{\frac{1}{3}\sqrt{2}\left(3\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\left(3\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)\right)^3}$$

Result

$$\frac{\sqrt{2}\;2}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\left(3\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{3}{2}\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)^3\right)3} = \frac{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\left(3\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{3}{2}\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\sqrt{2}\left(3\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{3}{2}\left(-1+\sqrt[3]{190-3\sqrt{3767}}+\sqrt[3]{190+3\sqrt{3767}}\right)\right)^3\right)3}=\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

and

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

For $a=x=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$

we obtain:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)}{2}\right)^3}$$

(1/3·√2·(∛(-q/2+√(q^2/4+p^3/27)) +∛(-q/2-√(q^2/4+p^3/27)))^3)/(4/3·π·(((∛(-q/2+√(q^2/4+p^3/27)) +∛(-q/2-√(q^2/4+p^3/27))))/2)^3)

Input

$$\frac{\frac{1}{3}\sqrt{2}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)\right)^3}$$

$\sqrt[3]{x}$ is the real-valued 3rd root of x

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)\right)^3\right)3} = \frac{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\sqrt{2}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$$x = \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

and

$$\frac{\frac{1}{3} \cdot \sqrt{2} \cdot a^3}{\frac{4}{3} \cdot \pi \cdot \left(\frac{a}{2}\right)^3}$$

for

$$a = x = \sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}$$

we obtain:

$$\frac{\frac{1}{3} \cdot \sqrt{2} \cdot \left(\sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}\right)^3}{\frac{4}{3} \cdot \pi \cdot \left(\frac{\left(\sqrt[3]{\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}}\right)}{2}\right)^3}$$

(1/3·√2·(∛(q/2+√(q^2/4+p^3/27)) +∛(q/2-√(q^2/4+p^3/27))
)^3)/(4/3·π·(((∛(q/2+√(q^2/4+p^3/27)) +∛(q/2-√(q^2/4+p^3/27))))/2)^3)

Input

$$\frac{\frac{1}{3}\sqrt{2}\left(\sqrt[3]{\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\left(\sqrt[3]{\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)\right)^3}$$

$\sqrt[3]{x}$ is the real-valued 3^{rd} root of x

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\left(\sqrt[3]{\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(\sqrt[3]{\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)\right)^3\right)3} = \frac{2\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\left(\sqrt[3]{\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(\sqrt[3]{\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{\sqrt{2}\left(\sqrt[3]{\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)^3}{\frac{1}{3}\left(4\pi\left(\frac{1}{2}\left(\sqrt[3]{\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we consider V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , we obtain:

from the octahedron volume V = $1/3*\sqrt{2}*a^3$

and, from the sphere volume, V = $(4/3*\pi*(a/2)^3)$

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

For a = $\Lambda_Q = 3.516 * 10^{121}$ (Quantum vacuum value) and $\Lambda = 2.888 * 10^{-122}$ (Cosmological Constant – energy density of the vacuum), from

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot(2.888\times10^{-122})^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{(3.516\times10^{121})}{2}\right)^3}$$

we consider

1/(((1/3·√2·(2.888×10^(-122))^3)/(4/3·π·(((3.516×10^121))/2)^3)))

and obtain:

Input

$$\frac{1}{\frac{\frac{1}{3}\sqrt{2}\left(\frac{2.888}{10^{122}}\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}(3.516\times10^{121})\right)^3}}$$

Result

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation

$2.00429\ldots \times 10^{729}$

$2.00429\ldots*10^{729}$

From which:

ln(2.00429*10^729)+64-4Pi-φ-1/(Pi^2)

Input interpretation

$$\log(2.00429 \times 10^{729}) + 64 - 4\,\pi - \phi - \frac{1}{\pi^2}$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result

1728.99410…

1728.9941….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternative representations

$$\log(2.00429\times 10^{729}) + 64 - 4\,\pi - \phi - \frac{1}{\pi^2} = 64 - \phi - 4\,\pi + \log_e(2.00429\times 10^{729}) - \frac{1}{\pi^2}$$

$$\log(2.00429\times 10^{729}) + 64 - 4\,\pi - \phi - \frac{1}{\pi^2} =$$
$$64 - \phi - 4\,\pi + \log(a)\log_a(2.00429\times 10^{729}) - \frac{1}{\pi^2}$$

$$\log(2.00429\times 10^{729}) + 64 - 4\,\pi - \phi - \frac{1}{\pi^2} =$$
$$64 - \phi - 4\,\pi - \text{Li}_1(1 - 2.00429\times 10^{729}) - \frac{1}{\pi^2}$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\log\left(2.00429\times 10^{729}\right)+64-4\,\pi-\phi-\frac{1}{\pi^2}=64-\phi-\frac{1}{\pi^2}-4\,\pi+\log\left(2.004290000000000\times 10^{729}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,e^{-1679.279822675991199\,k}}{k}$$

$$\log\left(2.00429\times 10^{729}\right)+64-4\,\pi-\phi-\frac{1}{\pi^2}=64-\phi-\frac{1}{\pi^2}-4\,\pi+2\,i\,\pi\left\lfloor\frac{\arg\left(2.004290000000000\times 10^{729}-x\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(2.004290000000000\times 10^{729}-x\right)^k\,x^{-k}}{k}\quad\text{for } x<0$$

$$\log\left(2.00429\times 10^{729}\right)+64-4\,\pi-\phi-\frac{1}{\pi^2}=64-\phi-\frac{1}{\pi^2}-4\,\pi+\left\lfloor\frac{\arg\left(2.004290000000000\times 10^{729}-z_0\right)}{2\,\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg\left(2.004290000000000\times 10^{729}-z_0\right)}{2\,\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(2.004290000000000\times 10^{729}-z_0\right)^k\,z_0^{-k}}{k}$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

i is the imaginary unit

Integral representations

$$\log\left(2.00429\times 10^{729}\right)+64-4\,\pi-\phi-\frac{1}{\pi^2}=64-\phi-\frac{1}{\pi^2}-4\,\pi+\int_1^{2.004290000000000\times 10^{729}}\frac{1}{t}\,dt$$

$$\log(2.00429\times 10^{729}) + 64 - 4\,\pi - \phi - \frac{1}{\pi^2} = 64 - \phi - \frac{1}{\pi^2} - 4\,\pi + \frac{1}{2\,i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{e^{-1679.279822675991199\,s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

(1/27((ln(2.00429*10^729)+64-4Pi-φ-1/(Pi^2))-1))^2

Input interpretation

$$\left(\frac{1}{27}\left(\left(\log(2.00429\times 10^{729}) + 64 - 4\,\pi - \phi - \frac{1}{\pi^2}\right) - 1\right)\right)^2$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

Result

4095.97201…

4095.97201…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality.

The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

((ln(2.00429*10^729)+64-4Pi-φ-1/(Pi^2)))^1/15+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\sqrt[15]{\log(2.00429\times 10^{729}) + 64 - 4\,\pi - \phi - \frac{1}{\pi^2}} + C_{\text{MRB}}{}^{1-1/(4\,\pi)+\pi}$$

$\log(x)$ is the natural logarithm
ϕ is the golden ratio
C_{MRB} is the MRB constant

Result

1.644937650…

1.64493765…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

And:

sqrt(1/(((ln(2.00429*10^729)+64-4Pi-φ-1/(Pi^2)))^1/15+(MRB const)^(1-1/(4π)+π))*(4/3))

Input interpretation

$$\sqrt{\frac{1}{\sqrt[15]{\log(2.00429\times 10^{729}) + 64 - 4\,\pi - \phi - \frac{1}{\pi^2}} + C_{\text{MRB}}{}^{1-1/(4\,\pi)+\pi}} \times \frac{4}{3}}$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

C_{MRB} is the MRB constant

Result

0.90031533559…

$0.9003153\ldots \approx 0.9003163161571\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed form

$$\frac{2\sqrt{2}}{\pi} \approx 0.90031\,63161$$

But also from:

(((1/3·√2·(2.888×10^(-122))^3)/(4/3·π·(((3.516×10^121))/2)^3)))

Input

$$\frac{\frac{1}{3}\sqrt{2}\left(\frac{2.888}{10^{122}}\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\left(3.516\times 10^{121}\right)\right)^3}$$

Result

$4.98931\ldots \times 10^{-730}$

$4.98931\ldots*10^{-730}$

and the previous result

$2.00429\ldots \times 10^{729}$

we obtain, after some calculations:

(0.9568666373+√π/2)*((((2.00429*10^729)/(4.98931 × 10^-730))^1/122)^1/4)-64-π-Φ

Input interpretation

$$\left(0.9568666373 + \frac{\sqrt{\pi}}{2}\right) \sqrt[4]{\sqrt[122]{\frac{2.00429 \times 10^{729}}{\frac{4.98931}{10^{730}}}}} - 64 - \pi - \Phi$$

Φ is the golden ratio conjugate

Result

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation

1728.9994…

1728.9994….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons.

This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(1/27(((0.9568666373+√π/2)*((((2.00429*10^729)/(4.98931 × 10^-730))^1/122)^1/4)-64-π-Φ)-1))^2

Input interpretation

$$\left(\frac{1}{27}\left(\left(\left(0.9568666373+\frac{\sqrt{\pi}}{2}\right)\sqrt[4]{122\sqrt{\frac{2.00429\times 10^{729}}{\frac{4.98931}{10^{730}}}}}-64-\pi-\Phi\right)-1\right)\right)^2$$

Φ is the golden ratio conjugate

Result

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation

4095.9973…

4095.9973…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

((((0.9568666373+√π/2)*((((2.00429*10^729)/(4.98931 × 10^-730))^1/122)^1/4)-64-π-Φ)))^1/15+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\sqrt[15]{\left(0.9568666373+\frac{\sqrt{\pi}}{2}\right)\sqrt[4]{\sqrt[122]{\frac{2.00429\times 10^{729}}{\frac{4.98931}{10^{730}}}}-64-\pi-\Phi}}+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

Φ is the golden ratio conjugate

C_{MRB} is the MRB constant

Result

$\tilde{\infty}$

$\tilde{\infty}$ is complex infinity

Decimal approximation

1.644937988…

1.644937988…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

The Einstein field equations may be written in the form:

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}\,,$$

Where $G_{\mu\nu}$ is the Einstein tensor, $g_{\mu\nu}$ is the metric tensor, $T_{\mu\nu}$ is the stress–energy tensor, Λ is the cosmological constant and κ is the Einstein gravitational constant, that is equal to

$$\kappa = \frac{8\pi G}{c^4} \approx 2.076647442844 \times 10^{-43}\ \mathrm{N}^{-1}$$

We consider $G_{\mu\nu} = (2\sqrt{2})/\pi$; $\Lambda = 2.888 * 10^{-122}$, and obtain:

$$\frac{2\sqrt{2}}{\pi} = \frac{8\pi \cdot G}{c^4}x - 2.888 \cdot 10^{-122}y$$

$$\frac{2\sqrt{2}}{\pi} = 2.076647442844 \cdot 10^{-43}x - 2.888 \cdot 10^{-122}y$$

$(2\sqrt{2})/\pi$=2.076647442844·10^(-43) x-2.888·10^(-122) y

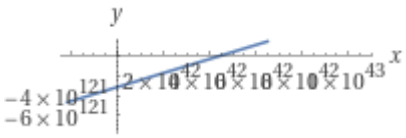
Input interpretation

$$\frac{2\sqrt{2}}{\pi} = \frac{2.076647442844\,x}{10^{43}} - \frac{2.888}{10^{122}}\,y$$

Result

$$\frac{2\sqrt{2}}{\pi} = 2.076647442844\times10^{-43}\,x - 2.888\times10^{-122}\,y$$

Implicit plot



Alternate forms

$$y = 7.19061\times10^{78}\,x - 3.11744\times10^{121}$$

$$-2.076647442844\times10^{-43}\,x + 2.888\times10^{-122}\,y + \frac{2\sqrt{2}}{\pi} = 0$$

Alternate form assuming x and y are real

$$\frac{2\sqrt{2}}{\pi} = 2.076647442844\times10^{-43}\,x - 2.888\times10^{-122}\,y + 0$$

Real solution

$$y \approx 7.19061\times10^{78}\,x - 3.11744\times10^{121}$$

Solution

$$y = 7\,190\,607\,489\,072\,021\,365\,609\,388\,590\,170\,614\,199\,603\,704\,410\,736\,525\,257\,449\,\ddots\,787\,611\,327\,599\,740\,977\,152\,x - 31\,174\,387\,678\,570\,159\,190\,114\,518\,541\,828\,008\,441\,782\,278\,420\,964\,519\,474\,\ddots\,531\,925\,002\,332\,254\,378\,864\,639\,933\,064\,096\,324\,393\,011\,154\,394\,665\,189\,181\,\ddots\,921\,165\,312$$

Solution for the variable y

$$y \approx$$
$$34\,626\,038\,781\,163\,437\,244\,290\,702\,154\,256\,680\,235\,531\,973\,456\,950\,472\,874\,564\,781$$
$$788\,750\,392\,707\,871\,273\,893\,533\,355\,315\,447\,095\,153\,375\,512\,603\,458\,388\,623\,360$$
$$\left(2.07665 \times 10^{-43}\, x - 0.900316\right)$$

Logarithmic form

$$\log_\pi(2) + \log_\pi\left(\sqrt{2}\right) - \log_\pi(\pi) = \log_\pi\left(\frac{2.076647442844\, x}{10^{43}} - \frac{2.888\, y}{10^{122}}\right)$$

$\log_b(x)$ is the base-b logarithm

From

$$\frac{2\sqrt{2}}{\pi} = 2.076647442844 \times 10^{-43}\, x - 2.888 \times 10^{-122}\, y$$

for

$$y \approx 7.19061 \times 10^{78}\, x - 3.11744 \times 10^{121}$$

we obtain:

(2 sqrt(2))/π = 2.076647442844×10^-43 x - 2.888×10^-122 (7.19061×10^78 x - 3.11744×10^121)

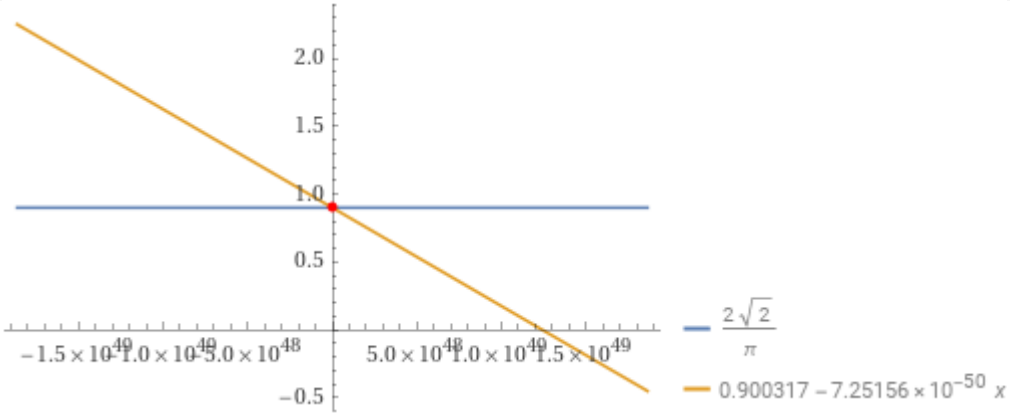
Input interpretation

$$\frac{2\sqrt{2}}{\pi} = 2.076647442844 \times 10^{-43}\, x - \frac{2.888}{10^{122}}\left(7.19061 \times 10^{78}\, x - 3.11744 \times 10^{121}\right)$$

Result

$$\frac{2\sqrt{2}}{\pi} =$$
$$2.076647442844 \times 10^{-43}\, x - 2.888 \times 10^{-122}\left(7.19061 \times 10^{78}\, x - 3.11744 \times 10^{121}\right)$$

Plot



Expanded form

$$\frac{2\sqrt{2}}{\pi} = 0.900317 - 7.25156\times10^{-50}\,x$$

Alternate form

$$7.25156\times10^{-50}\,x - 3.55843\times10^{-7} = 0$$

Solution

$$x = 4\,907\,121\,971\,120\,310\,370\,621\,852\,586\,282\,802\,334\,400\,512$$

Integer solution

$$x = 4\,907\,121\,971\,120\,309\,751\,651\,832\,943\,592\,664\,884\,838\,400$$

Logarithmic form

$$\log_\pi(2) + \log_\pi\left(\sqrt{2}\right) - \log_\pi(\pi) = \log_\pi\left(2.076647442844\times10^{-43}\,x - \frac{2.888\left(7.19061\times10^{78}\,x - 3.11744\times10^{121}\right)}{10^{122}}\right)$$

$\log_b(x)$ is the base- b logarithm

From the result

$$\frac{2\sqrt{2}}{\pi} = 2.076647442844\times10^{-43}\,x - 2.888\times10^{-122}\left(7.19061\times10^{78}\,x - 3.11744\times10^{121}\right)$$

for

$$x = 4\,907\,121\,971\,120\,310\,370\,621\,852\,586\,282\,802\,334\,400\,512$$

$4.907121971120310370621852586282802334400512 \times 10^{42}$

we obtain:

2.076647442844×10^-43 (4.907121971120310370621852586282802334400512 × 10^42) - 2.888×10^-122 (7.19061×10^78 (4.907121971120310370621852586282802334400512 × 10^42) - 3.11744×10^121)

Input interpretation

$$2.076647442844\times10^{-43}\times 4.907121971120310370621852586282802334400512\times10^{42} - \frac{2.888}{10^{122}}\left(7.19061\times10^{78}\times4.907121971120310370621852586282802334400512\times10^{42} - 3.11744\times10^{121}\right)$$

Result

0.900316316157105991028021288133986594150819039546232012 8

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Thence:

$$y \approx 7.19061\times10^{78}\,x - 3.11744\times10^{121}$$

For x = $4.907121971120310370621852586282802334400512 \times 10^{42}$, we obtain:

7.19061×10^78 (4.907121971120310370621852586282802334400512 × 10^42) - 3.11744×10^121

Input interpretation

$7.19061 \times 10^{78} \times 4.907121971120310370621852586282802334400512 \times 10^{42} - 3.11744 \times 10^{121}$

Result

4.1108×10^{120}

$4.1108*10^{120}$

Scientific notation

$4.110800316757414954097199425450981293763665592 32 \times 10^{120}$

Thence, we obtain:

$x = T_{\mu\nu} = 4.907121971120310370621852586282802334400512 \times 10^{42}$ that is the energy-momentum or stress–energy tensor

$y = g_{\mu\nu} = 4.110800316757414954097199425450981293763665592 32 \times 10^{120}$ that is the metric tensor

Now, we consider

$$\frac{\frac{1}{3} \cdot \sqrt{2} \cdot a^3}{\frac{4}{3} \cdot \pi \cdot \left(\frac{a}{2}\right)^3}$$

and

$l_P = \sqrt{\frac{\hbar G}{c^3}}$ (Planck length)

For $a = \sqrt{\frac{\hbar G}{c^3}}$, we obtain:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot\left(\sqrt{\frac{\hbar G}{c^3}}\right)^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{\left(\sqrt{\frac{\hbar G}{c^3}}\right)}{2}\right)^3}$$

Thus:

(1/3·√2·(√(1.054571817e-34*6.6743e-11/299792458^3))^3)/(4/3·π·(((√(1.054571817e-34*6.6743e-11/299792458^3)))/2)^3)

Input interpretation

$$\frac{\frac{1}{3}\sqrt{2}\sqrt{1.054571817\times10^{-34}\times\frac{6.6743\times10^{-11}}{299\,792\,458^3}}^3}{\frac{4}{3}\pi\left(\frac{1}{2}\sqrt{1.054571817\times10^{-34}\times\frac{6.6743\times10^{-11}}{299\,792\,458^3}}\right)^3}$$

Result

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

We consider also:

$$t_{\mathrm{P}}=\frac{l_{\mathrm{P}}}{c}=\sqrt{\frac{\hbar G}{c^5}}$$

From

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

We obtain, for a = t_P :

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot\left(\sqrt{\frac{\hbar G}{c^5}}\right)^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{\left(\sqrt{\frac{\hbar G}{c^5}}\right)}{2}\right)^3}$$

Thus:

(1/3·√2·(√(1.054571817e-34*6.6743e-11/299792458^5))^3)/(4/3·π·(((√(1.054571817e-34*6.6743e-11/299792458^5)))/2)^3)

Input interpretation

$$\frac{\frac{1}{3}\sqrt{2}\sqrt{1.054571817\times10^{-34}\times\frac{6.6743\times10^{-11}}{299\,792\,458^5}}^3}{\frac{4}{3}\pi\left(\frac{1}{2}\sqrt{1.054571817\times10^{-34}\times\frac{6.6743\times10^{-11}}{299\,792\,458^5}}\right)^3}$$

Result

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Now, from the previous formulas, we perform the following calculations:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot\left(\sqrt{\frac{\hbar G}{c^5}}\right)^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{\left(\sqrt{\frac{\hbar G}{c^3}}\right)}{2}\right)^3}$$

(1/3·√2·(√(1.054571817e-34*6.6743e-11/299792458^5))^3)/(4/3·π·(((√(1.054571817e-34*6.6743e-11/299792458^3)))/2)^3)

Input interpretation

$$\frac{\frac{1}{3}\sqrt{2}\left(\sqrt{1.054571817\times10^{-34}\times\frac{6.6743\times10^{-11}}{299\,792\,458^5}}\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\sqrt{1.054571817\times10^{-34}\times\frac{6.6743\times10^{-11}}{299\,792\,458^3}}\right)^3}$$

Result

$3.34143\ldots\times10^{-26}$

3.34143…*10^-26

-ln((1/3·√2·(√(1.054571817e-34*6.6743e-11/299792458^5))^3)/(4/3·π·(((√(1.054571817e-34*6.6743e-11/299792458^3)))/2)^3))

Input interpretation

$$-\log\left(\frac{\frac{1}{3}\sqrt{2}\left(\sqrt{1.054571817\times10^{-34}\times\frac{6.6743\times10^{-11}}{299\,792\,458^5}}\right)^3}{\frac{4}{3}\pi\left(\frac{1}{2}\sqrt{1.054571817\times10^{-34}\times\frac{6.6743\times10^{-11}}{299\,792\,458^3}}\right)^3}\right)$$

$\log(x)$ is the natural logarithm

Result

58.660812074644943523436121694793902275978512471525842160908345466

…

58.660812074….

From which:

27*(58.6608120746449435234-φ

Input interpretation

$27\,(58.660812074644943523436 + 2\,e) - \phi$

Result

1729.0111107634520229940…

1729.011110763452….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons.

This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(1/27((27*(58.660812074644943523436+2e)-φ)-1))^2

Input interpretation

$$\left(\frac{1}{27}\left((27\,(58.660812074644943523436 + 2\,e) - \phi) - 1\right)\right)^2$$

ϕ is the golden ratio

Result

4096.0526734182980191415…

4096.052673418298…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group.

The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

(27*(58.6608120746449435234 36+2e)-φ)^1/15+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\sqrt[15]{27\,(58.660812074644943523436 + 2\,e) - \phi} + C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Result

1.644938728351867909565 20…

1.6449387283518…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

sqrt(1/((27*(58.660812074644943523436+2e)-φ)^1/15+(MRB const)^(1-1/(4π)+π))*(4/3))

Input interpretation

$$\sqrt{\frac{1}{\sqrt[15]{27\,(58.660812074644943523436 + 2\,e) - \phi} + C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}} \times \frac{4}{3}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Result

0.900315040471408514961419…

$0.90031504\ldots \approx 0.9003163161571\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed form

$$\frac{2\sqrt{2}}{\pi} \approx 0.90031\,63161$$

From

Constraining the bispectrum from bouncing cosmologies with Planck - *Paola C. M. Delgado* - Based on Phys.Rev.Lett. 130.191002 and JCAP11(2021)024 - In collaboration with *Ruth Durrer, Nelson Pinto-Neto, Bartjan van Tent*

We have that:

n	γ	q	$f_{\rm nl}$ for $R_B = 1\ l_{Pl}^{-2}$	$f_{\rm nl}$ for $R_B = 10^{-3}\ l_{Pl}^{-2}$
$1/6$	0.6468	-0.7	3326	8518
0.21	0.751	-1.24	959	4372

We analyze the following equation

$$B(k_1, k_2, k_3) = \frac{3}{5}(2\pi^2)^2 f_{\rm nl} \left[\frac{\mathscr{P}_{\mathscr{R}}(k_1)}{k_1^3} \frac{\mathscr{P}_{\mathscr{R}}(k_2)}{k_2^3} + \frac{\mathscr{P}_{\mathscr{R}}(k_1)}{k_1^3} \frac{\mathscr{P}_{\mathscr{R}}(k_3)}{k_3^3} + \frac{\mathscr{P}_{\mathscr{R}}(k_3)}{k_3^3} \frac{\mathscr{P}_{\mathscr{R}}(k_2)}{k_2^3} \right] \times \exp\left(-\gamma \frac{k_1 + k_2 + k_3}{k_b} \right)$$

We consider:

$$\frac{3}{5}(2\pi^2)^2 f_{\rm nl} \left[\frac{\mathscr{P}_{\mathscr{R}}(k_1)}{k_1^3} \frac{\mathscr{P}_{\mathscr{R}}(k_2)}{k_2^3} + \frac{\mathscr{P}_{\mathscr{R}}(k_1)}{k_1^3} \frac{\mathscr{P}_{\mathscr{R}}(k_3)}{k_3^3} + \frac{\mathscr{P}_{\mathscr{R}}(k_3)}{k_3^3} \frac{\mathscr{P}_{\mathscr{R}}(k_2)}{k_2^3} \right] \times \exp\left(-\gamma \frac{k_1 + k_2 + k_3}{k_b} \right)$$

And for $f_{\rm nl}$ = 4372, we obtain:

$$B = \frac{3}{5}(2\pi^2)^2\, 4372 \left[\frac{\mathcal{P}_{\Re(x)}}{x^3} \frac{\mathcal{P}_{\Re(y)}}{y^3} + \frac{\mathcal{P}_{\Re(x)}}{x^3} \frac{\mathcal{P}_{\Re(z)}}{z^3} + \frac{\mathcal{P}_{\Re(z)}}{z^3} \frac{\mathcal{P}_{\Re(y)}}{y^3} \right] \times exp\left(-\gamma \frac{x + y + z}{t} \right)$$

Input

$$B = \frac{3}{5}(2\pi^2)^2 \times 4372 \left(\left(\frac{P(x)}{x^3} \times \frac{P(y)}{y^3} + \frac{P(x)}{x^3} \times \frac{P(z)}{z^3} + \frac{P(z)}{z^3} \times \frac{P(y)}{y^3} \right) \exp\left(-\gamma \times \frac{x + y + z}{t} \right) \right)$$

γ is the Euler-Mascheroni constant

Exact result

$$B = \frac{52\,464}{5} \pi^4 \left(\frac{P(x)\,P(y)}{x^3\,y^3} + \frac{P(x)\,P(z)}{x^3\,z^3} + \frac{P(y)\,P(z)}{y^3\,z^3} \right) e^{-(\gamma\,(x+y+z))/t}$$

Alternate form

$$B = \frac{P(x)\left(\frac{52464\,\pi^4\,P(y)}{5\,y^3} + \frac{52464\,\pi^4\,P(z)}{5\,z^3}\right)e^{(-\gamma\,x-\gamma\,y-\gamma\,z)/t}}{x^3} + \frac{52464\,\pi^4\,P(y)\,P(z)\,e^{(-\gamma\,x-\gamma\,y-\gamma\,z)/t}}{5\,y^3\,z^3}$$

Expanded form

$$B = \frac{52464\,\pi^4\,P(x)\,P(y)\,e^{-(\gamma\,(x+y+z))/t}}{5\,x^3\,y^3} + \frac{52464\,\pi^4\,P(x)\,P(z)\,e^{-(\gamma\,(x+y+z))/t}}{5\,x^3\,z^3} + \frac{52464\,\pi^4\,P(y)\,P(z)\,e^{-(\gamma\,(x+y+z))/t}}{5\,y^3\,z^3}$$

Dividing the exact result

$$B = \frac{52464}{5}\,\pi^4\left(\frac{P(x)\,P(y)}{x^3\,y^3} + \frac{P(x)\,P(z)}{x^3\,z^3} + \frac{P(y)\,P(z)}{y^3\,z^3}\right)e^{-(\gamma\,(x+y+z))/t}$$

by 12 :

1/12*B = 1/12*((52464/5 π^4 ((P(x) P(y))/(x^3 y^3) + (P(x) P(z))/(x^3 z^3) + (P(y) P(z))/(y^3 z^3)) e^(-(euler-mascheroni const (x + y + z))/t)))

Input

$$\frac{1}{12}\,B = \frac{1}{12}\left(\frac{52464}{5}\,\pi^4\left(\frac{P(x)\,P(y)}{x^3\,y^3} + \frac{P(x)\,P(z)}{x^3\,z^3} + \frac{P(y)\,P(z)}{y^3\,z^3}\right)e^{-(\gamma\,(x+y+z))/t}\right)$$

γ is the Euler-Mascheroni constant

Exact result

$$\frac{B}{12} = \frac{4372}{5}\,\pi^4\left(\frac{P(x)\,P(y)}{x^3\,y^3} + \frac{P(x)\,P(z)}{x^3\,z^3} + \frac{P(y)\,P(z)}{y^3\,z^3}\right)e^{-(\gamma\,(x+y+z))/t}$$

Alternate form

$$B = \frac{12\,P(x)\left(\frac{4372\,\pi^4\,P(y)}{5\,y^3} + \frac{4372\,\pi^4\,P(z)}{5\,z^3}\right)e^{(-\gamma\,x-\gamma\,y-\gamma\,z)/t}}{x^3} + \frac{52\,464\,\pi^4\,P(y)\,P(z)\,e^{(-\gamma\,x-\gamma\,y-\gamma\,z)/t}}{5\,y^3\,z^3}$$

Expanded form

$$\frac{B}{12} = \frac{4372\,\pi^4\,P(x)\,P(y)\,e^{-(\gamma\,(x+y+z))/t}}{5\,x^3\,y^3} + \frac{4372\,\pi^4\,P(x)\,P(z)\,e^{-(\gamma\,(x+y+z))/t}}{5\,x^3\,z^3} + \frac{4372\,\pi^4\,P(y)\,P(z)\,e^{-(\gamma\,(x+y+z))/t}}{5\,y^3\,z^3}$$

From the exact result

$$\frac{B}{12} = \frac{4372}{5}\,\pi^4\left(\frac{P(x)\,P(y)}{x^3\,y^3} + \frac{P(x)\,P(z)}{x^3\,z^3} + \frac{P(y)\,P(z)}{y^3\,z^3}\right)e^{-(\gamma\,(x+y+z))/t}$$

for B = 2 and P = 8 :

2/12 = 4372/5 π^4 ((8(x) 8(y))/(x^3 y^3) + (8(x) 8(z))/(x^3 z^3) + (8(y) 8(z))/(y^3 z^3)) e^(-(euler-mascheroni const (x + y + z))/t)

Input

$$\frac{2}{12} = \frac{4372}{5}\,\pi^4\left(\frac{8\,x\,(8\,y)}{x^3\,y^3} + \frac{8\,x\,(8\,z)}{x^3\,z^3} + \frac{8\,y\,(8\,z)}{y^3\,z^3}\right)e^{-(\gamma\,(x+y+z))/t}$$

γ is the Euler-Mascheroni constant

Exact result

$$\frac{1}{6} = \frac{4372}{5}\,\pi^4\left(\frac{64}{x^2\,y^2} + \frac{64}{x^2\,z^2} + \frac{64}{y^2\,z^2}\right)e^{-(\gamma\,(x+y+z))/t}$$

Expanded form

$$\frac{1}{6} = \frac{279\,808\,\pi^4\, e^{-(\gamma\,(x+y+z))/t}}{5\,x^2\,y^2} + \frac{279\,808\,\pi^4\, e^{-(\gamma\,(x+y+z))/t}}{5\,x^2\,z^2} + \frac{279\,808\,\pi^4\, e^{-(\gamma\,(x+y+z))/t}}{5\,y^2\,z^2}$$

Alternate form

$$\frac{1}{6} = \frac{279\,808\,\pi^4\left(x^2+y^2+z^2\right) e^{-(\gamma\,(x+y+z))/t}}{5\,x^2\,y^2\,z^2}$$

Logarithmic form

$$\log(2) - \log(12) = \log(4372) - \log(5) + 4\log(\pi) + \log\left(\frac{8\,x\,(8\,y)}{x^3\,y^3} + \frac{8\,x\,(8\,z)}{x^3\,z^3} + \frac{8\,y\,(8\,z)}{y^3\,z^3}\right) - \frac{(\gamma\,(x+y+z))\log(e)}{t}$$

$\log(x)$ is the natural logarithm

From the exact result

$$\frac{1}{6} = \frac{4372}{5}\,\pi^4\left(\frac{64}{x^2\,y^2} + \frac{64}{x^2\,z^2} + \frac{64}{y^2\,z^2}\right) e^{-(\gamma\,(x+y+z))/t}$$

for x = 16, y = 32, z = 64 and t = 128:

4372/5 e^(-(euler mascheroni const (16 + 32 + 64))/128) π^4 (64/(16^2 32^2) + 64/(16^2 64^2) + 64/(32^2 64^2)) - 1/6

Input

$$\frac{4372}{5}\, e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times 32^2} + \frac{64}{16^2\times 64^2} + \frac{64}{32^2\times 64^2}\right) - \frac{1}{6}$$

γ is the Euler-Mascheroni constant

Exact result

$$\frac{22\,953\,e^{-(7\gamma)/8}\,\pi^4}{81\,920}-\frac{1}{6}$$

Decimal approximation

16.303675551303572336627532663326380143337526314237210098159592361

…

16.3036755513….

Alternate forms

$$\frac{68\,859\,e^{-(7\gamma)/8}\,\pi^4-40\,960}{245\,760}$$

$$-\frac{e^{-(7\gamma)/8}\left(40\,960\,e^{(7\gamma)/8}-68\,859\,\pi^4\right)}{245\,760}$$

Alternative representations

$$\frac{1}{5}\,e^{-1/128\,(\gamma\,(16+32+64))}\,4372\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)-\frac{1}{6}=-\frac{1}{6}+\frac{4372}{5}\,e^{(112\,\psi(1))/128}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)$$

$$\frac{1}{5}\,e^{-1/128\,(\gamma\,(16+32+64))}\,4372\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)-\frac{1}{6}=-\frac{1}{6}+\frac{4372}{5}\,(180\,^\circ)^4\,e^{(112\,\psi(1))/128}\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)$$

$$\frac{1}{5}\,e^{-1/128\,(\gamma\,(16+32+64))}\,4372\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)-\frac{1}{6}=-\frac{1}{6}+\frac{4372}{5}\,(180\,^\circ)^4\,e^{-(112\,\gamma_0)/128}\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)$$

$\psi(x)$ is the digamma function

γ_n is the n^{th} Stieltjes constant

Series representations

$$\frac{1}{5}\,e^{-1/128\,(\gamma\,(16+32+64))}\,4372\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)-\frac{1}{6}=$$
$$-\frac{1}{6}+\frac{22\,953\,\exp\left(-\frac{7}{8}\left(1+\sum_{k=2}^{\infty}\left(\frac{1}{k}+\log\left(\frac{-1+k}{k}\right)\right)\right)\right)\pi^4}{81\,920}$$

$$\frac{1}{5}\,e^{-1/128\,(\gamma\,(16+32+64))}\,4372\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)-\frac{1}{6}=$$
$$-\frac{1}{960}\,e^{-\frac{7}{8}-\frac{7}{8}\sum_{k=2}^{\infty}\left(\frac{1}{k}+\log\left(\frac{-1+k}{k}\right)\right)}\left(160\,e^{\frac{7}{8}+\frac{7}{8}\sum_{k=2}^{\infty}\left(\frac{1}{k}+\log\left(\frac{-1+k}{k}\right)\right)}-68\,859\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k}\right)^4\right)$$

$$\frac{1}{5}\,e^{-1/128\,(\gamma\,(16+32+64))}\,4372\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)-\frac{1}{6}=$$
$$-\frac{\exp\left(-\frac{7}{8}\sum_{k=1}^{\infty}k\sum_{j=2^k}^{-1+2^{1+k}}\frac{(-1)^j}{j}\right)\left(40\,960\exp\left(\frac{7}{8}\sum_{k=1}^{\infty}k\sum_{j=2^k}^{-1+2^{1+k}}\frac{(-1)^j}{j}\right)-68\,859\,\pi^4\right)}{245\,760}$$

$\log(x)$ is the natural logarithm

Integral representations

$$\frac{1}{5}\,e^{-1/128\,(\gamma\,(16+32+64))}\,4372\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)-\frac{1}{6}=$$
$$-\frac{1}{6}+\frac{22\,953\,e^{7/8\int_{-\infty}^{\infty}e^{-e^t+t}\,t\,dt}\,\pi^4}{81\,920}$$

$$\frac{1}{5}\,e^{-1/128\,(\gamma\,(16+32+64))}\,4372\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)-\frac{1}{6}=$$
$$\frac{-2560+68\,859\,e^{7/8\int_{-\infty}^{\infty}e^{-e^t+t}\,t\,dt}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^4}{15\,360}$$

$$\frac{1}{5}\, e^{-1/128\,(\gamma\,(16+32+64))}\, 4372\,\pi^4 \left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)-\frac{1}{6}= \frac{1}{960}\left(-160+68\,859\, e^{7/8\int_{-\infty}^{\infty} e^{-e^t+t}\,t\,dt}\left(\int_0^1 \sqrt{1-t^2}\;dt\right)^4\right)$$

From which, we obtain:

((5(16.3036755513 + 1/6)))/((e^(-(euler mascheroni const (16 + 32 + 64))/128) π^4 (64/(16^2 32^2) + 64/(16^2 64^2) + 64/(32^2 64^2))))

Input interpretation

$$\frac{5\left(16.3036755513+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)}$$

γ is the Euler-Mascheroni constant

Result

4372.00000000…

4372

Where 4372 is a value indicated in the fundamental Ramanujan paper "**Modular equations and Approximations to π**"

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Alternative representations

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}=\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{(112\,\psi(1))/128}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}=\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{(180\,^\circ)^4\,e^{(112\,\psi(1))/128}\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}=\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{(180\,^\circ)^4\,e^{-(112\,\gamma_0)/128}\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}$$

$\psi(x)$ is the digamma function

γ_n is the n^{th} Stieltjes constant

Series representations

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}=\frac{1003.9065732855873\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{7/8\sum_{k=2}^{\infty}\left((-1)^k\,\zeta(k)\right)/k}}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4}$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)}=\frac{1003.9065732855873\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{7/8\,\sum_{k=2}^{\infty}((-1+k)\,(-1+\zeta(k)))/k}}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4}$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)}=\frac{257\,000.0827611103\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{7/8\,\sum_{k=2}^{\infty}\left((-1)^k\,\zeta(k)\right)/k}}{\left(\sum_{k=1}^{\infty}4^{-k}\left(-1+3^k\right)\zeta(1+k)\right)^4}$$

$n!$ is the factorial function

$\zeta(s)$ is the Riemann zeta function

Integral representations

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)}=\frac{16\,062.50517256940\,e^{-7/8\int_0^{\infty}\mathcal{A}^{-t}\log(t)\,dt}}{\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^4}$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)}=\frac{16\,062.50517256940\,e^{-7/8\int_0^{1}\log(\log(1/t))\,dt}}{\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^4}$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}=\frac{16\,062.50517256940\,e^{-7/8\int_0^1\log(-\log(t))\,dt}}{\left(\int_0^\infty\frac{1}{1+t^2}\,dt\right)^4}$$

$\log(x)$ is the natural logarithm

And also:

((5(16.3036755513 + 1/6)))/((e^(-(euler mascheroni const (16 + 32 + 64))/128) π^4 (64/(16^2 32^2) + 64/(16^2 64^2) + 64/(32^2 64^2))))-276

Input interpretation

$$\frac{5\left(16.3036755513+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}-276$$

γ is the Euler-Mascheroni constant

Result

4096.00000000…

4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson.

This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternative representations

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}-276=$$
$$-276+\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{(112\,\psi(1))/128}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}-276=$$
$$-276+\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{(180^\circ)^4\,e^{(112\,\psi(1))/128}\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}-276=$$
$$-276+\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{(180^\circ)^4\,e^{-(112\,\gamma_0)/128}\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}$$

$\psi(x)$ is the digamma function

γ_n is the n^{th} Stieltjes constant

Series representations

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}-276=$$
$$\frac{1}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4}1003.90657328559\left(-0.274925981505138\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4+1.00000000000000\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{7/8\sum_{k=2}^{\infty}\left((-1)^k\zeta(k)\right)/k}\right)$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}-276=$$
$$\frac{1}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4}1003.90657328559\left(-0.274925981505138\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}\right)^4+1.00000000000000\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{7/8\sum_{k=2}^{\infty}((-1+k)(-1+\zeta(k)))/k}\right)$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times32^2}+\frac{64}{16^2\times64^2}+\frac{64}{32^2\times64^2}\right)}-276=$$
$$\left(1003.90657328559\left(-0.274925981505138\left(\sum_{k=1}^{\infty}\tan^{-1}\left(\frac{1}{F_{1+2k}}\right)\right)^4+1.00000000000000\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{7/8\sum_{k=2}^{\infty}\left((-1)^k\zeta(k)\right)/k}\right)\right)\Big/\left(\sum_{k=1}^{\infty}\tan^{-1}\left(\frac{1}{F_{1+2k}}\right)\right)^4$$

$n!$ is the factorial function

$\zeta(s)$ is the Riemann zeta function

F_n is the n^{th} Fibonacci number

$\tan^{-1}(x)$ is the inverse tangent function

Integral representations

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)}-276=\frac{1}{\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^4}$$

$$16\,062.5051725694\left(1.00000000000000\,e^{7/8\int_0^1 (1/(1-t)+1/\log(t))\,dt}-0.0171828738440711\left(\int_0^\infty \frac{\sin(t)}{t}\,dt\right)^4\right)$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)}-276=$$

$$-\frac{1}{\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^4}276.00000000000\,e^{-7/8\int_0^\infty \mathcal{A}^{-t}\log(t)\,dt}\left(-58.1974825093094+1.00000000000000\,e^{7/8\int_0^\infty \mathcal{A}^{-t}\log(t)\,dt}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^4\right)$$

$$\frac{5\left(16.30367555130000+\frac{1}{6}\right)}{e^{-1/128\,(\gamma\,(16+32+64))}\,\pi^4\left(\frac{64}{16^2\times 32^2}+\frac{64}{16^2\times 64^2}+\frac{64}{32^2\times 64^2}\right)}-276=$$

$$-\frac{1}{\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^4}276.00000000000\,e^{-7/8\int_0^1 \log(\log(1/t))\,dt}\left(-58.1974825093094+1.00000000000000\,e^{7/8\int_0^1 \log(\log(1/t))\,dt}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^4\right)$$

$\log(x)$ is the natural logarithm

A general cubic equation is:

$ax^3+bx^2+cx+d = 0$

for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), where r = (a/2), considering (1/3*√2*a^3) and (4/3*π*(a/2)^3) , we obtain the following ratio:

$$\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$$

From the previous general formula, for a = b = c = d = $\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}$, we obtain the following equation:

$$\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)^3 x^3+\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)^2 x^2+\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)x+\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)=0$$

((1/3·√2·a^3)/(4/3·π·(a/2)^3))^3 x^3+((1/3·√2·a^3)/(4/3·π·(a/2)^3))^2 x^2+((1/3·√2·a^3)/(4/3·π·(a/2)^3))x+((1/3·√2·a^3)/(4/3·π·(a/2)^3))=0

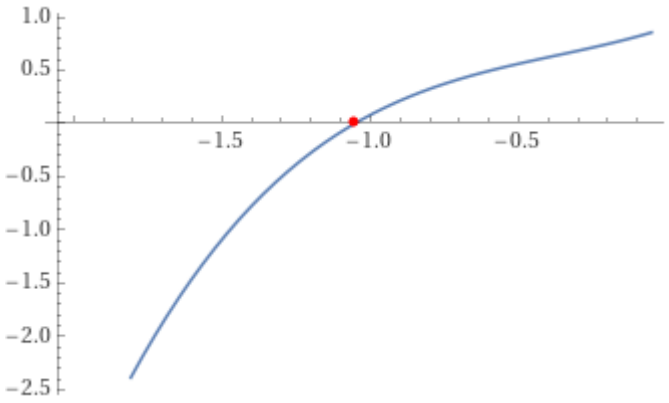
Input

$$\left(\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}\right)^3 x^3+\left(\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}\right)^2 x^2+\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}x+\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}=0$$

Result

$$\frac{16\sqrt{2}\,x^3}{\pi^3}+\frac{8x^2}{\pi^2}+\frac{2\sqrt{2}\,x}{\pi}+\frac{2\sqrt{2}}{\pi}=0$$

Root plot (figure that can be related to an open string)



Alternate forms

$$\frac{2\left(8\sqrt{2}\,x^3+4\pi x^2+\sqrt{2}\,\pi^2(x+1)\right)}{\pi^3}=0$$

$$8\sqrt{2}\,x^3+4\pi x^2+\sqrt{2}\,\pi^2 x=-\sqrt{2}\,\pi^2$$

$$\frac{16\sqrt{2}\left(x+\frac{\pi}{6\sqrt{2}}\right)^3}{\pi^3}+\frac{4\sqrt{2}\left(x+\frac{\pi}{6\sqrt{2}}\right)}{3\pi}+\frac{2\sqrt{2}}{\pi}-\frac{7}{27}=0$$

Real solution

$x\approx -1.05238$

Complex solutions

$x \approx -1.0524$

$x \approx -0.02917 + 1.08233\,i$

$x \approx -0.02917 - 1.08233\,i$

Sum of roots

$$-\frac{\pi}{2\sqrt{2}}$$

We note that this value is the inverse of the DN Constant with minus sign

Product of roots

$$-\frac{\pi^2}{8}$$

Or:

$$\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)^1 x^3+\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)^1 x^2+\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)x+\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)=0$$

((1/3·√2·a^3)/(4/3·π·(a/2)^3)) x^3+((1/3·√2·a^3)/(4/3·π·(a/2)^3))
x^2+((1/3·√2·a^3)/(4/3·π·(a/2)^3))x+((1/3·√2·a^3)/(4/3·π·(a/2)^3))=0

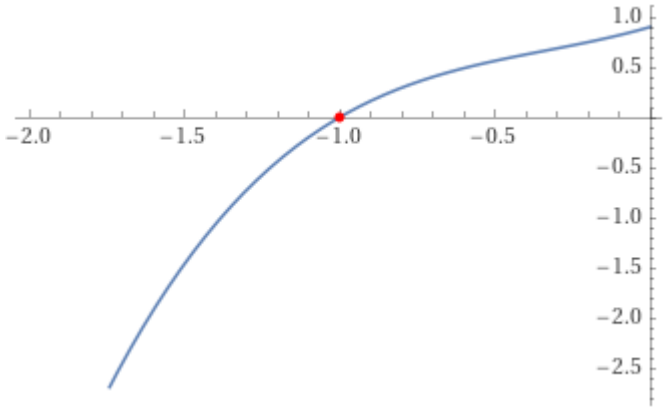
Input

$$\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}x^3+\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}x^2+\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}x+\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}=0$$

Result

$$\frac{2\sqrt{2}\,x^3}{\pi}+\frac{2\sqrt{2}\,x^2}{\pi}+\frac{2\sqrt{2}\,x}{\pi}+\frac{2\sqrt{2}}{\pi}=0$$

Root plot (figure that can be related to an open string)



Alternate forms

$$\frac{2\sqrt{2}\,(x+1)\left(x^2+1\right)}{\pi}=0$$

$$\frac{2\sqrt{2}\left(x^3+x^2+x+1\right)}{\pi}=0$$

$$\frac{2\sqrt{2}\left(x+\frac{1}{3}\right)^3}{\pi}+\frac{4\sqrt{2}\left(x+\frac{1}{3}\right)}{3\pi}+\frac{40\sqrt{2}}{27\pi}=0$$

Real solution

$x=-1$

Complex solutions

$x = -i$

$x = i$

Sum of roots

-1

Product of roots

-1

We note that the sum and the product of the roots is equal to -1 that is:

(1/2+(√3 i)/2)^3

Input

$$\left(\frac{1}{2} + \frac{1}{2}\left(\sqrt{3}\ i\right)\right)^3$$

i is the imaginary unit

Result

-1

-1 practically equal to the square of the imaginary unit

Alternate complex forms

$-1 + 0\,i$

$\cos(\pi) + i\sin(\pi)$

$e^{i\pi}$

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

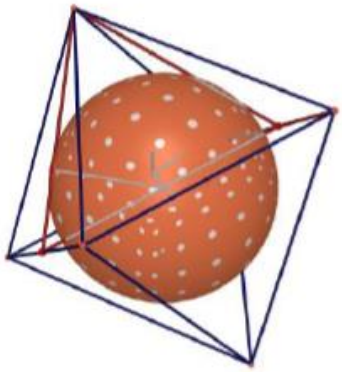


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$
$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x\in\mathbb{R}\ \text{and}\ x<0)$

$$\frac{\left(6\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

Indeed:

From:

https://www.academia.edu/75787512/The_Theory_of_String_A_Candidate_for_a_Generalized_Unification_Model

The Einstein's field equation and the String Theory.

The Einstein's field equation which includes the cosmological constant is:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -8\pi G T_{\mu\nu} \quad (8)$$

where $R_{\mu\nu}$ is the Ricci tensor, R its trace, λ the cosmological constant, $g_{\mu\nu}$ the metric tensor of the space geometry, G the Newton's gravitational constant and $T_{\mu\nu}$ the tensor representing the properties of energy, matter and momentum.

The left hand-side of (8) represents the gravitational field and, consequently, the warped space-time, while the right hand-side represents the matter, i.e. the sources of the gravitational field.

In string theory the gravity is related to the gravitons which are bosons, whereas the matter is related to fermions. It follows that the left and right hand of (8) may be respectively related to the action of bosonic and of superstrings.

The actions of bosonic string and superstring (also containing fermions) are connected by the Palumbo-Nardelli relation (Palumbo et al. 2005):

$$-\int d^{26}x\sqrt{g}\left[-\frac{R}{16\pi G}-\frac{1}{8}g^{\mu\rho}g^{\nu\sigma}Tr\left(G_{\mu\nu}G_{\rho\sigma}\right)f(\varphi)-\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right]=$$

$$=\int_0^\infty \frac{1}{2\kappa_{10}^2}\int d^{10}x\sqrt{-G}e^{-2\Phi}\left[R+4\partial_\mu\Phi\partial^\mu\Phi-\frac{1}{2}|\widetilde{H}_3|^2-\frac{\kappa_{10}^2}{g_{10}^2}Tr_\nu\left(|F_2|^2\right)\right]$$

The sign minus in the above equation comes from the inversion of any relationship.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^{\,8}\right)^{-k}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{-1+256\sqrt{2}^{\,8}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^{\,8}\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^{\,8}-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^{0} \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

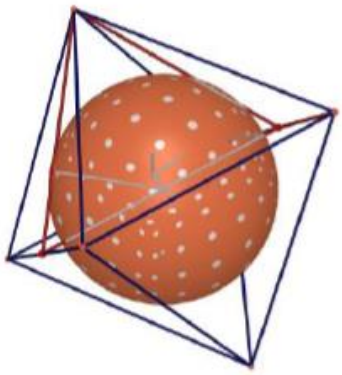
$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard to a sphere inscribed in an octahedron



we have the following formulas:

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\,2}{\pi} \quad (\text{for } l \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor\arg(2-z_0)/(2\,\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor\arg(2-z_0)/(2\,\pi)\rfloor)}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Now, we have the following convex regular polyhedrons volumes:

$$\frac{\sqrt{2}}{12}d^3$$ Tetrahedron Volume

$$\frac{\sqrt{2}}{3}d^3$$ Octahedron Volume

$$\frac{5}{12}(3+\sqrt{5})d^3$$ Icosahedron Volume

Regarding the Sphere the volume is

$$V = \frac{4}{3}\pi r^3$$

With regard the sphere inscribed in a Tetrahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_T}{V_S} = \frac{\frac{\sqrt{2}}{12}d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(((√2 d^3)/12))*1/(4/3*π*(d/2)^3)

Input

$$\left(\frac{1}{12}\left(\sqrt{2}\ d^3\right)\right) \times \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.225079079039276517388799797751685145666143537488805156392859367 8

…

0.225079079….

Property

$\frac{1}{\sqrt{2}\ \pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{2\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Octahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_O}{V_S} = \frac{\frac{\sqrt{2}}{3}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

((1/3*√2*d^3)/(4/3*π*(d/2)^3))

Input

$$\frac{\frac{1}{3}\sqrt{2}\, d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{\sqrt{2}\ 2}{\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.9003163161571…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,d^3}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

With regard the sphere inscribed in a Icosahedron, we have, for $r = \frac{d}{2}$, the following ratio:

$$\frac{V_I}{V_S} = \frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Thus:

(5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3)

Input

$$\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}$$

Result

$$\frac{5\left(3+\sqrt{5}\right)}{2\pi} \quad (\text{for } d \neq 0)$$

Decimal approximation

4.166730504921372682725163141597223093105782245731637719697502271 9

…

4.166730504921….

Property

$\frac{5\left(3+\sqrt{5}\right)}{2\pi}$ is a transcendental number

Expanded form

$$\frac{15}{2\pi} + \frac{5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

Alternate forms

$$\frac{15+5\sqrt{5}}{2\pi} \quad (\text{for } d \neq 0)$$

$$\frac{\frac{15}{2}+\frac{5\sqrt{5}}{2}}{\pi} \quad (\text{for } d \neq 0)$$

Series representations

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sqrt{4}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{2\pi}$$

$$\frac{5\left(\left(3+\sqrt{5}\right)d^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{d}{2}\right)^3\right)12} = \frac{15}{2\pi} + \frac{5\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{4\pi\sqrt{\pi}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\ a^3\right)\left(\sqrt{2}\ d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-\lfloor \arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)d^3 \times \left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

Indeed, we obtain:

$$\frac{\sqrt[2\pi]{\frac{5}{12}\left(3+\sqrt{5}\right)\left(d^3\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2\right)}}{\sqrt[2\pi]{\frac{4}{3}\left(\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\right)\sqrt{2}\left(d^3 \times \frac{\sqrt{2}}{12}\right)d^3}}$$

$= 1.6180085459001 07\ldots$

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5)))

$$\frac{4 \times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13
…

3.1415926535…. = π (Ramanujan Recurring Number)

In conclusion, from the following further formula concerning the tetrahedron, the cube, the octahedron, the dodecahedron and the icosahedron volumes and their ratios (and/or the inverses) with the sphere volume, raised the expression

$$(2\log(2)log^{2/3}(3^2))/(3e^{7/3})$$

$$\left(\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\frac{\frac{1}{4}\left(\left(15+7\sqrt{5}\right)d^3\right)}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\right)^{(2\log(2)\log^{2/3}(3^2))/(3e^{7/3})}$$

i.e.

$$\left(\frac{3}{2}\right)^{(4\log(2)\log^{2/3}(9))/(3e^{7/3})}\left(\frac{5\left(3+\sqrt{5}\right)\left(15+7\sqrt{5}\right)}{\pi}\right)^{(2\log(2)\log^{2/3}(9))/(3e^{7/3})}$$

we obtain:

1.61803817115191398574568170847431346739030216149893707129177220 71
…

1.6180381711…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Observations

We note that, from the number 8, we obtain as follows:

8^2

64

$8^2 \times 2 \times 8$

1024

$8^4 = 8^2 \times 2^6$

True

$8^4 = 4096$

$8^2 \times 2^6 = 4096$

$2^{13} = 2 \times 8^4$

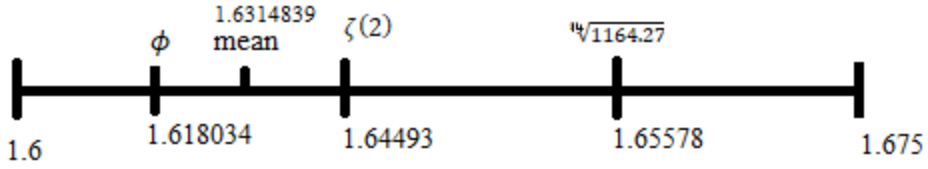
True

$2^{13} = 8192$

$2 \times 8^4 = 8192$

We notice how from the numbers 8 and 2 we get 64, 1024, 4096 and 8192, and that 8 is the fundamental number. In fact $8^2 = 64$, $8^3 = 512$, $8^4 = 4096$. We define it "fundamental number", since 8 is a Fibonacci number, which by rule, divided by the previous one, which is 5, gives 1.6 , a value that tends to the golden ratio, as for all numbers in the Fibonacci sequence

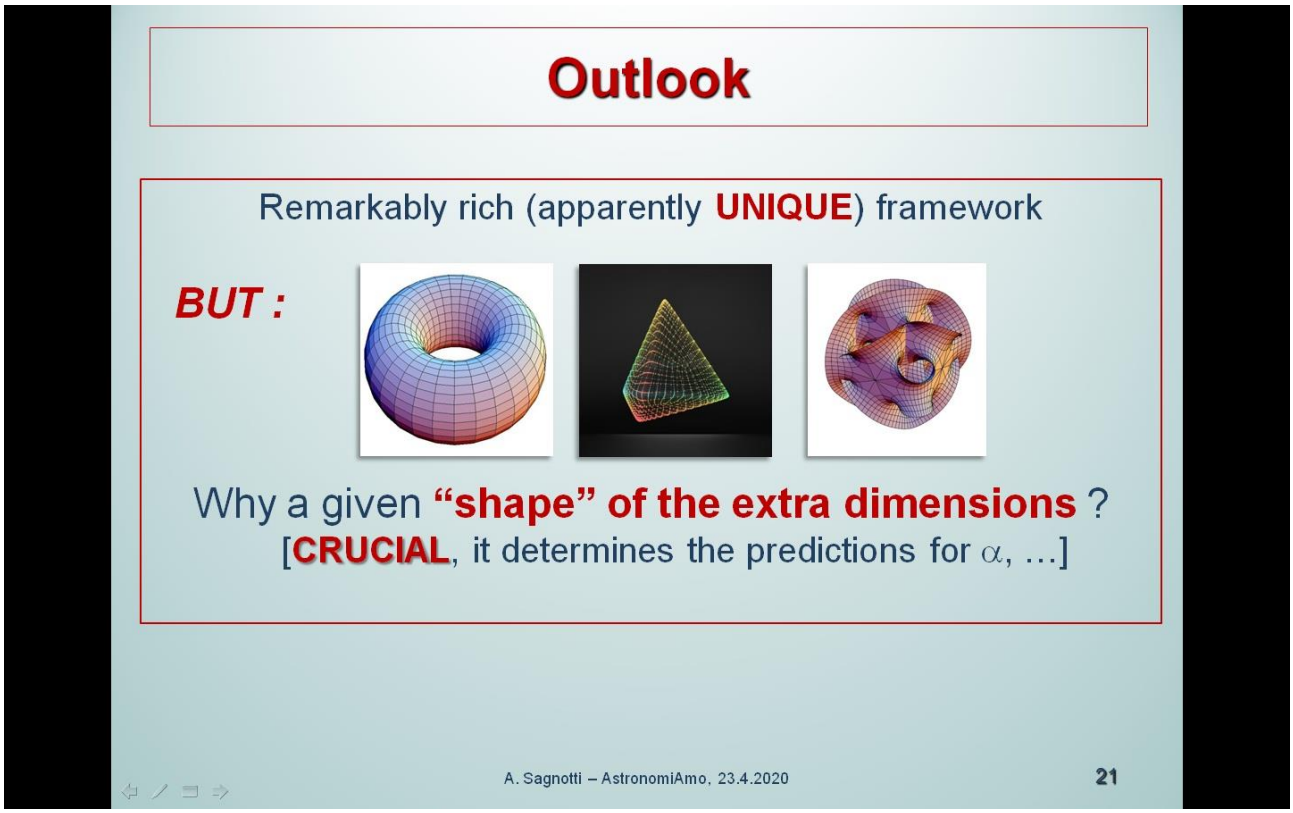
“Golden” Range



Finally we note how $8^2 = 64$, multiplied by 27, to which we add 1, is equal to 1729, the so-called "Hardy-Ramanujan number". Then taking the 15th root of 1729, we obtain a value close to $\zeta(2)$ that 1.6438 ..., which, in turn, is included in the range of what we call "golden numbers"

Furthermore for all the results very near to 1728 or 1729, adding $64 = 8^2$, one obtain values about equal to 1792 or 1793. These are values almost equal to the Planck multipole spectrum frequency 1792.35 and to the hypothetical Gluino mass

Appendix


Outlook
Remarkably rich (apparently UNIQUE) framework
BUT :
Why a given "shape" of the extra dimensions ?
[CRUCIAL, it determines the predictions for α, …]
A. Sagnotti – AstronomiAmo, 23.4.2020
21

From: *A. Sagnotti – AstronomiAmo, 23.04.2020*

In the above figure, it is said that: "why a given shape of the extra dimensions? Crucial, it determines the predictions for α".

We propose that whatever shape the compactified dimensions are, their geometry must be based on the values of the golden ratio and ζ(2), (the latter connected to 1728 or 1729, whose fifteenth root provides an excellent approximation to the above mentioned value) which are recurrent as solutions of the equations that we are going to develop. It is important to specify that the initial conditions are **always** values belonging to a fundamental chapter of the work of S. Ramanujan "Modular equations and Approximations to Pi" (see references). These values are some multiples of 8 (64 and 4096), 276, which added to 4096, is equal to 4372, and finally $e^{\pi\sqrt{22}}$

We have, in certain cases, the following connections:

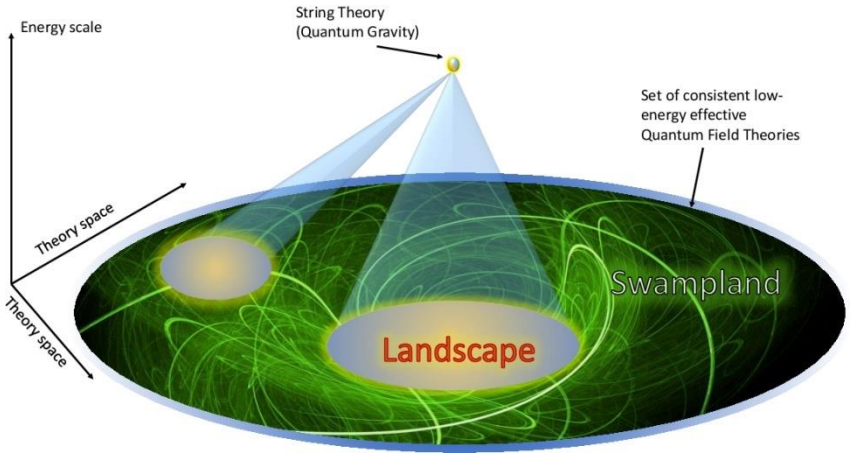


Fig. 1

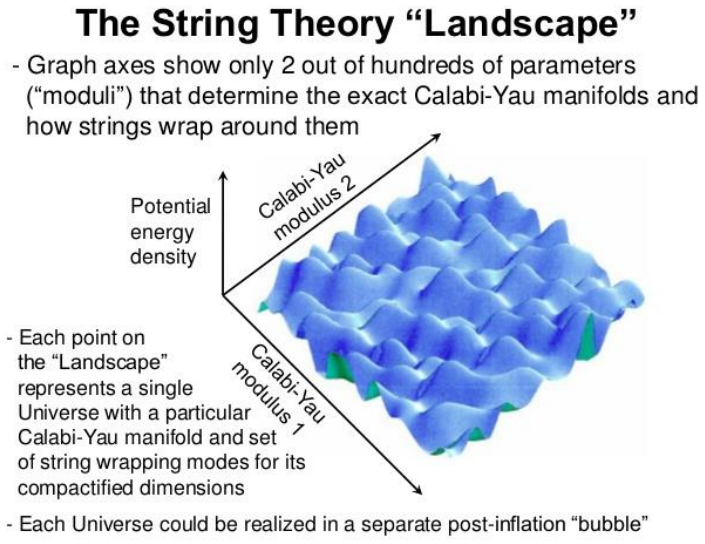

The String Theory "Landscape"
- Graph axes show only 2 out of hundreds of parameters ("moduli") that determine the exact Calabi-Yau manifolds and how strings wrap around them
Potential energy density
Calabi-Yau modulus 2
Calabi-Yau modulus 1
- Each point on the "Landscape" represents a single Universe with a particular Calabi-Yau manifold and set of string wrapping modes for its compactified dimensions
- Each Universe could be realized in a separate post-inflation "bubble"

Fig. 2

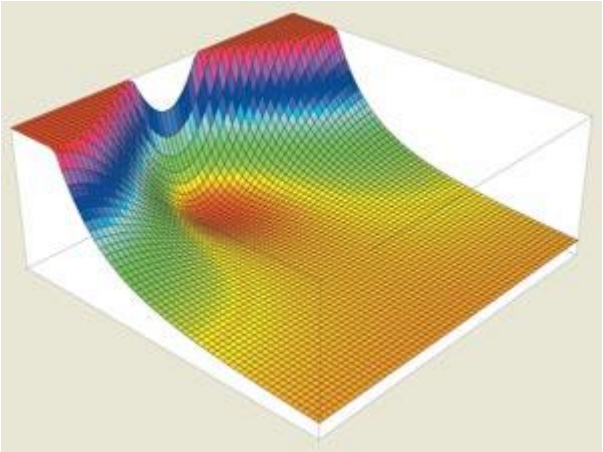


Fig. 3

Stringscape - a small part of the string-theory landscape showing the new de Sitter solution as a local minimum of the energy (vertical axis). The global minimum occurs at the infinite size of the extra dimensions on the extreme right of the figure.

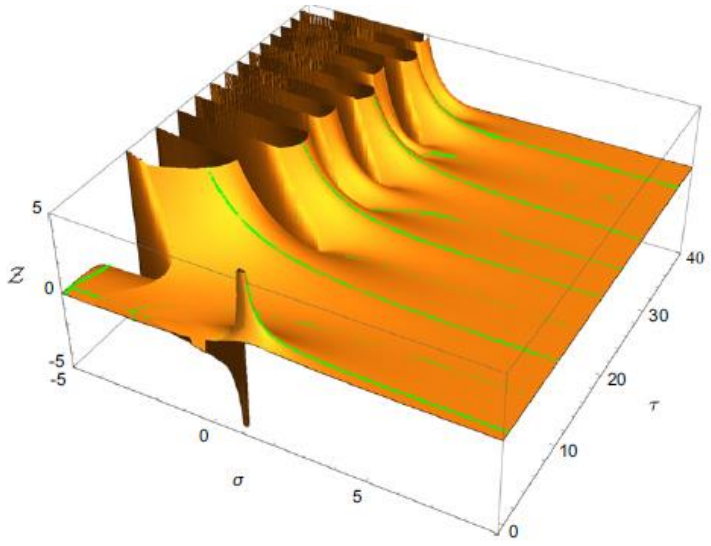


Figure 2. Lines in the complex plane where the Riemann zeta function ζ is real (green) depicted on a relief representing the positive absolute value of ζ for arguments $s \equiv \sigma + i\tau$ where the real part of ζ is positive, and the negative absolute value of ζ where the real part of ζ is negative. This representation brings out most clearly that the lines of constant phase corresponding to phases of integer multiples of 2π run down the hills on the left-hand side, turn around on the right and terminate in the non-trivial zeros. This pattern repeats itself infinitely many times. The points of arrival and departure on the right-hand side of the picture are equally spaced and given by equation (11).

Fig. 4

From: https://www.mdpi.com/2227-7390/6/12/285/htm

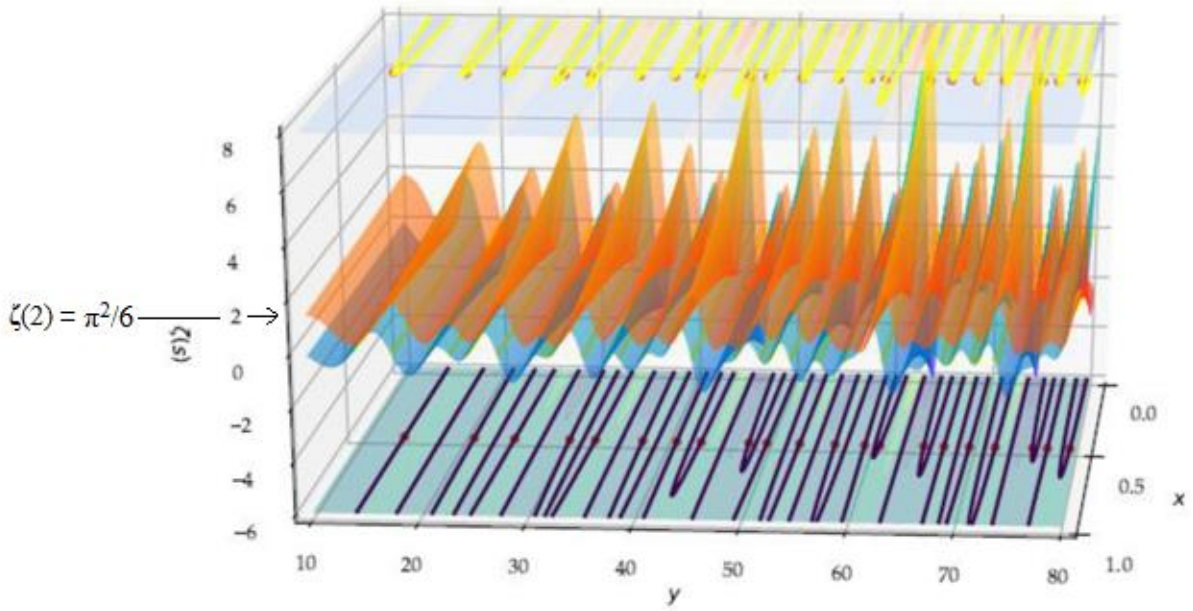


Figure 1. $C(x,y)$ and $S(x,y)$ surfaces of the Riemann $\zeta(x,y) = C - i\,S$ function, in the critical strip $\boldsymbol{s}$: $0 \leqslant x \leqslant 1; 10 \leqslant y \leqslant 80$. On the top and bottom planes, the C and S common zeros are the red points.

Fig. 5

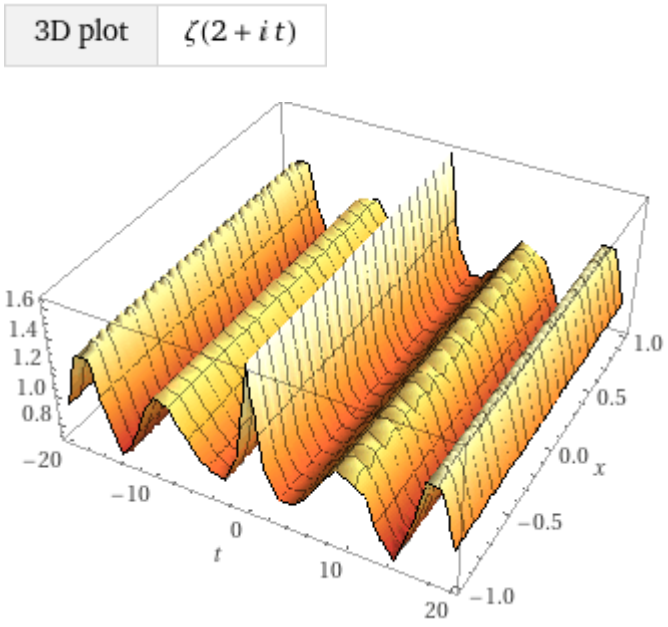


Fig. 6

Where ζ(2+it) :

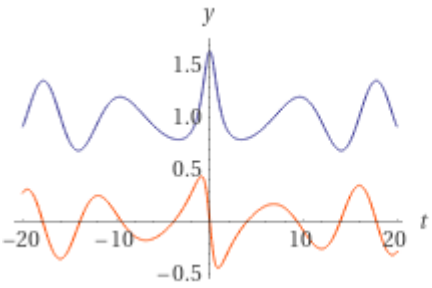
Input

$$\zeta(2 + i\,t)$$

$\zeta(s)$ is the Riemann zeta function

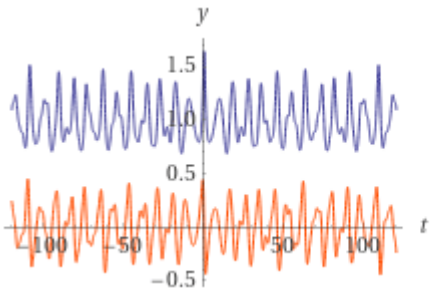
i is the imaginary unit

Plots



(t from −20 to 20)

real part
imaginary part



(t from −120 to 120)

real part
imaginary part

Roots

$$t = 2\,i\,(n+1), \quad n \in \mathbb{Z}, \quad n \geq 1$$

$$t = -i\,(\rho_n - 2), \quad n \neq 0, \quad n \in \mathbb{Z}$$

$\mathbb{Z}$ is the set of integers

ρ_n is the nontrivial n^{th} zero of the Riemann zeta function

Series expansion at t=0

$$\frac{\pi^2}{6} + i\,t\,\zeta'(2) - \frac{1}{2}\,t^2\,\zeta''(2) - \frac{1}{6}\,i\,\zeta^{(3)}(2)\,t^3 + \frac{1}{24}\,\zeta^{(4)}(2)\,t^4 + O\left(t^5\right)$$

(Taylor series)

Alternative representations

$$\zeta(2 + i\,t) = \zeta(2 + i\,t,\, 1)$$

$$\zeta(2 + i\,t) = S_{1+i\,t,1}(1)$$

$$\zeta(2 + i\,t) = \frac{\zeta\left(2 + i\,t,\, \frac{1}{2}\right)}{-1 + 2^{2+i\,t}}$$

$\zeta(s,\, a)$ is the generalized Riemann zeta function

$S_{n,p}(x)$ is the Nielsen generalized polylogarithm function

Series representations

$$\zeta(2 + i\,t) = \sum_{k=1}^{\infty} k^{-2-i\,t} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = \frac{\sum_{k=0}^{\infty} (1 + 2\,k)^{-2-i\,t}}{1 - 2^{-2-i\,t}} \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2 + i\,t) = e^{\sum_{k=1}^{\infty} P(k\,(2+i\,t))/k} \quad \text{for } \mathrm{Im}(t) < 1$$

$\mathrm{Im}(z)$ is the imaginary part of z

$P(z)$ gives the prime zeta function

Integral representations

$$\zeta(2+i\,t) = \frac{1}{\Gamma(2+i\,t)} \int_0^\infty \frac{\tau^{1+i\,t}}{-1+e^\tau}\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(3+i\,t)} \int_0^\infty \tau^{2+i\,t}\,\mathrm{csch}^2(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$$\zeta(2+i\,t) = \frac{2^{1+i\,t}}{\Gamma(2+i\,t)} \int_0^\infty e^{-\tau}\,\tau^{1+i\,t}\,\mathrm{csch}(\tau)\,d\tau \quad \text{for } \mathrm{Im}(t) < 1$$

$\Gamma(x)$ is the gamma function

$\mathrm{csch}(x)$ is the hyperbolic cosecant function

Functional equations

$$\zeta(2+i\,t) = -i\,2^{2+i\,t}\,\pi^{1+i\,t}\,\Gamma(-1-i\,t)\sinh\left(\frac{\pi\,t}{2}\right)\zeta(-1-i\,t)$$

$$\zeta(2+i\,t) = \frac{\pi^{3/2+i\,t}\,\Gamma\left(-\frac{1}{2}-\frac{i\,t}{2}\right)\zeta(-1-i\,t)}{\Gamma\left(1+\frac{i\,t}{2}\right)}$$

$$\zeta(2+i\,t) = -\frac{i\sum_{k=0}^\infty \frac{\Gamma\left(k-\frac{i\,t}{2}\right)\sum_{j=0}^k (-1)^j\,(1+2\,j)\binom{k}{j}\zeta(2+2\,j)}{k!}}{(-i+t)\,\Gamma\left(-\frac{i\,t}{2}\right)}$$

With regard the Fig. 4 the points of arrival and departure on the right-hand side of the picture are equally spaced and given by the following equation:

$$\tau'_k \equiv k\,\frac{\pi}{\ln 2},$$

with $k = \ldots, -2, -1, 0, 1, 2, \ldots$.

we obtain:

2Pi/(ln(2))

Input:

$$2 \times \frac{\pi}{\log(2)}$$

Exact result:

$$\frac{2\pi}{\log(2)}$$

Decimal approximation:

9.064720283654387619255365891433333620343722935447591168372033095 8…

9.06472028365….

Alternative representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log_e(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(a)\log_a(2)}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2\coth^{-1}(3)}$$

Series representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{2i\pi\left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k}}$$

Integral representations:

$$\frac{2\pi}{\log(2)} = \frac{2\pi}{\int_1^2 \frac{1}{t}\, dt}$$

$$\frac{2\pi}{\log(2)} = \frac{4i\pi^2}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds} \quad \text{for } -1 < \gamma < 0$$

From which:

(2Pi/(ln(2)))*(1/12 π log(2))

Input:

$$\left(2\times\frac{\pi}{\log(2)}\right)\left(\frac{1}{12}\pi\log(2)\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\pi^2}{6}$$

Decimal approximation:

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.6449340668…. = ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi =$$
$$\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

1.64381856858…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\right/\right.$$
$$\left.\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\text{^}(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right/\right.$$
$$\left.\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10\text{^-}6$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10\text{^-}6$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

–5.010882647757…

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right)=0.054323\,(0.0814845\,b+1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M}=-\frac{\frac{226\,802\,245}{18\,480\,874}+b}{M}$$

$$\frac{\partial M(b,V)}{\partial V}=\frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b}=-\frac{18\,480\,874\,M}{226\,802\,245+18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M}=\frac{2\,(226\,802\,245+18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b}=\frac{36\,961\,748\,(226\,802\,245+18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}(0.0814845\,b+1)^2\,M^2\right\}=0 \text{ at } (b,M)=(-16,0)$$

Global minima:

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $b=-\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

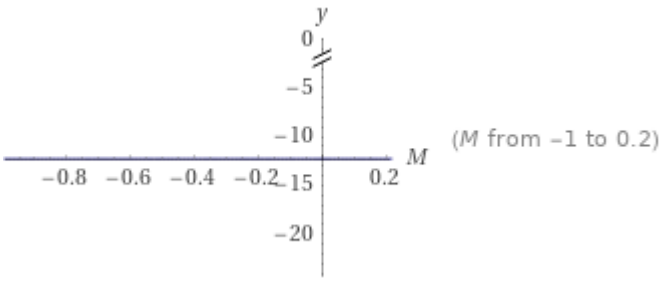
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

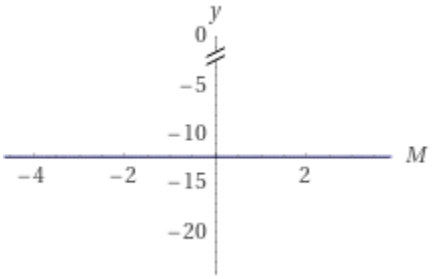
Result:

$$\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774 \times 10^{-7} \sqrt{M^4}}{M^2} - 12.2723\right) + O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545 \times 10^{-10} \sqrt{M^4} - 0.054323\, M^2\right)}{M^2}\right) = \frac{3.55271 \times 10^{-15}}{M}$$

Indefinite integral:

$$\int \frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM = \frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\} = -\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2} = -12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^\infty\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM = 0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

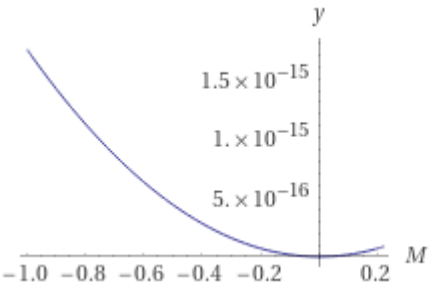
Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

Result:

0

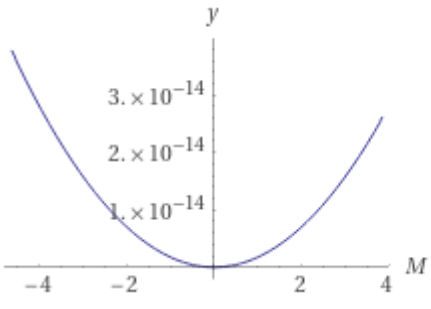
Plots: **(possible mathematical connection with an open string)**



(*M* from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O\left(M^{62194}\right)$

(Taylor series)

Series expansion at M = ∞:

$$1.75541\times10^{-15}\,M^2+O\left(\left(\frac{1}{M}\right)^{62\,194}\right)$$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}\,M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.38851344947464545348970783378088020833333333333333333333\ldots \times 10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 0.2^2 + 6.58545 \times 10^{-10}\sqrt{0.2^4}\right)}{0.2^2} + 1\right)^2 \times 0.2^2$$

Result:

$$7.02162151915943272558353253404940833333333333333333333333\ldots \times 10^{-17}$$

7.021621519159*10^-17

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875\times10^{-14}$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times 2^2+6.58545\times 10^{-10}\sqrt{2^4}\right)}{2^2}+1\right)^2\times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots \times 10^{-8}$

$5.9776991059*10^{-8}$ result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^{2} = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

We note that:

1/55*(([[((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left(\left(1/\left(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}\right)\right)\text{ ^ }(1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

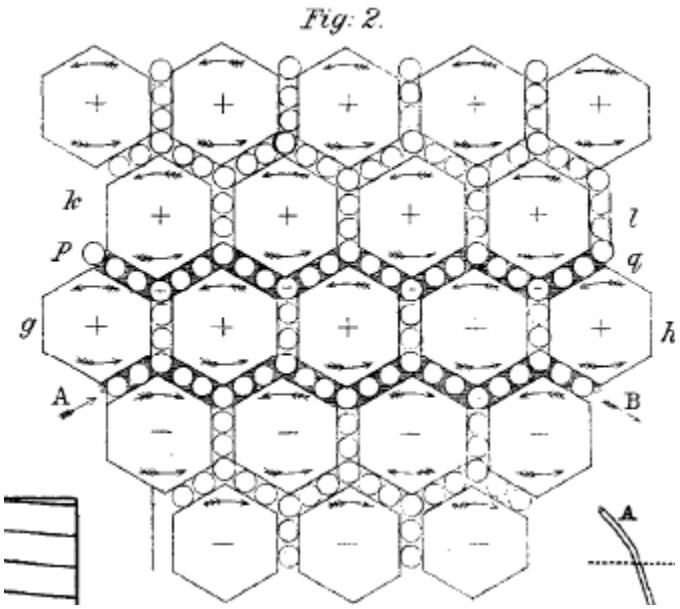
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

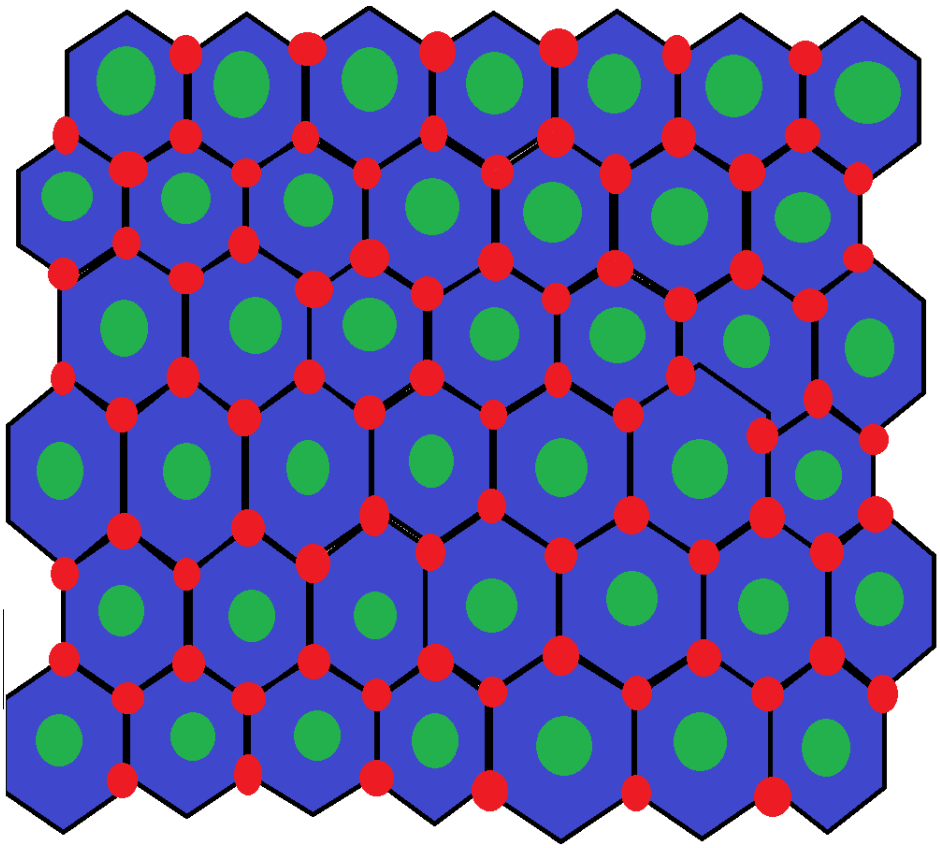
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Fig. 3

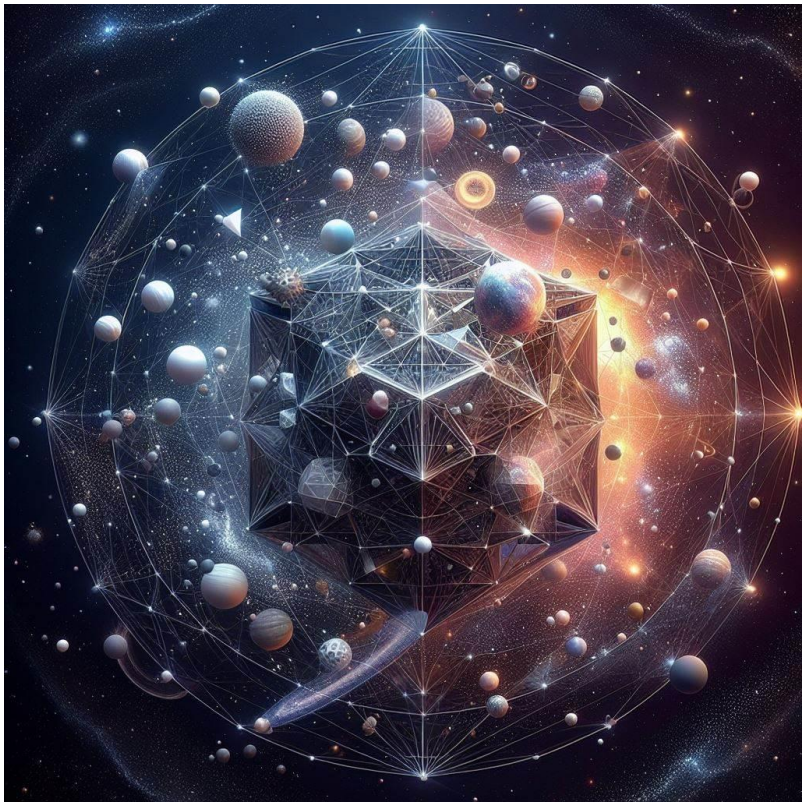


Image representing the solutions of the cubic equations in relation to the boundary conditions of the Plateau problem useful for understanding the formation of the bubbles of a universe with eternal inflation and of an octahedron as a mathematical model of the primordial universe before the Big Bang, therefore highly symmetrical and with extremely low entropy. As for the cubic equations, they represent the geometric characteristics of the quantum vacuum, while the spherical surface would represent the geometry of the bubble of the universe.

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability. I would like to thank also my son **Francesco Nardelli**, student of Educational Sciences, majoring in History and Philosophy, for his useful suggestions regarding the intriguing image at the end of this work

References

String (In)Stability Issues with Broken Supersymmetry
J. Mourad, A. Sagnotti - arXiv:2107.04064v1 [hep-th] 8 Jul 2021

Compactifications of Heterotic Theory on Non-Kahler Complex Manifolds: I
Katrin Becker, Melanie Becker, Keshav Dasgupta, Paul S. Green - arXiv:hep-th/0301161v2 22 Jan 2003

Properties Of Heterotic Vacua From Superpotentials - *Katrin Becker, Melanie Becker, Keshav Dasgupta, Sergey Prokushkin* - arXiv:hep-th/0304001v4 13 Jun 2003

ARS MAGNA *or The Rules of Algebra* - GIROLAMO CARDANO - *Translated and Edited by* - T. RICHARD WITMER - *With a Foreword by Oystein Ore* - DOVER PUBLICATIONS, INC. *New York* Copyright © 1968 by The Massachusetts Institute of Technology.

See also:

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Constraining the bispectrum from bouncing cosmologies with Planck - *Paola C. M. Delgado* - Based on Phys.Rev.Lett. 130.191002 and JCAP11(2021)024 - In collaboration with *Ruth Durrer, Nelson Pinto-Neto, Bartjan van Tent*

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *Emilian Dudas, Sergio Ferrara, Alex Kehagias and Augusto Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

What is the DN constant (Del Gaudio-Nardelli constant)? New possible mathematical connections with the Ramanujan Recurring Numbers and some parameters of Number Theory, Theoretical Cosmology and String Theory

Michele Nardelli[1], Antonio Nardelli

Abstract

In this paper (part II), we analyze the DN Constant (Del Gaudio-Nardelli Constant). We obtain new possible mathematical connections with the Ramanujan Recurring Numbers and some parameters of Number Theory, Theoretical Cosmology and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan

(1887 - 1920)

Introduction

In this paper, an octahedron could serve as a mathematical or conceptual model of the universe in the quantic phase, while the spherical surface could be used to describe the geometry of the bubble-universe.

The values $(2\sqrt{2})/\pi$, the golden ratio φ, $\zeta(2)$ and π, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$ the so called DN Constant:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and bubbles universe in eternal inflation.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and bubbles universe.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of bubbles universe and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

The above values $(2\sqrt{2})/\pi$, the golden ratio φ and π, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

The various mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe.

Regarding the fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the quantum universe and bubbles universe.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

Proposal:

The initial octahedron: Let's imagine a regular octahedron, with perfectly symmetrical faces. Each face represents an ideal symmetry.

The emerging sphere: Inside the octahedron, there is an inscribed sphere. This sphere represents the bubble of the universe that emerges from the perturbations of the quantum vacuum during eternal inflation.

Expansion and transitions: As time passes, the universe expands. The faces of the octahedron begin to break, symbolizing "symmetry breaks." The sphere continues to grow, representing the expanding universe.

Constants and numbers: We integrate the mathematical results you obtained. For example, the golden ratio (φ) could be represented by a proportion between the dimensions of the octahedron and the sphere.

Entropy and complexity: Entropy increases as the universe evolves. We can represent this with a disordered growth of structures within the emerging sphere. Imagine this scene as an abstract work of art, where geometric shapes and cosmological concepts merge

In Fig.1 and Fig.2 let's imagine a regular octahedron representing the universe in its phase of high symmetry and very low entropy. Inside the octahedron we have an inscribed sphere that emerges from perturbations of the quantum vacuum during eternal inflation. As time passes, the universe expands, the faces of the octahedron break (symmetry breaks), and entropy increases. Spheres emerge from the octahedra, symbolizing the transition phases from a regime of very low entropy to a universe in which, with the passage of time, entropy increases, increasing the complexity of the universe itself.

Fig. 1

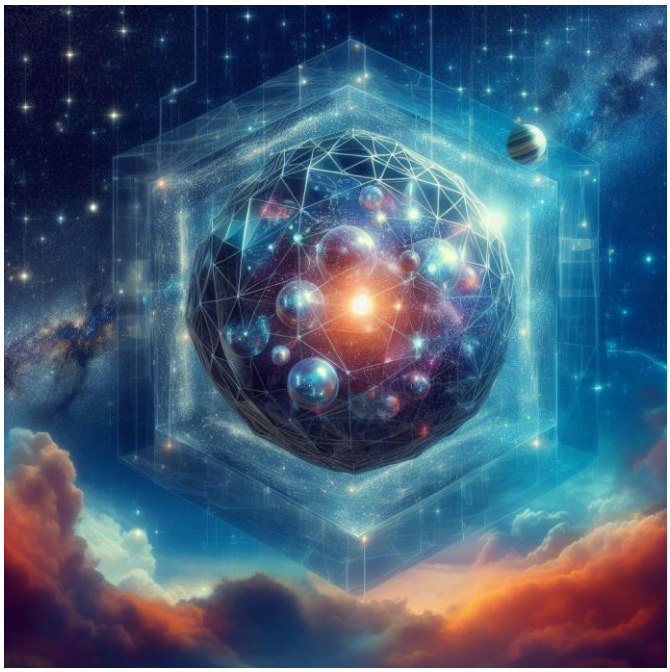
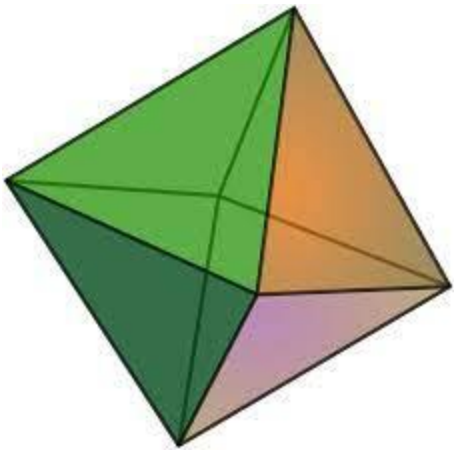
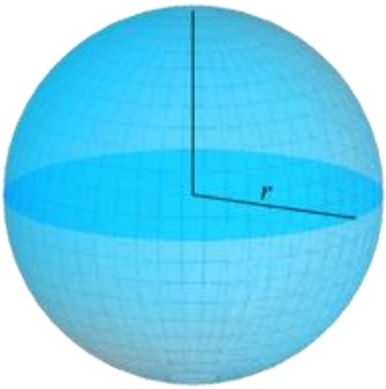


Fig. 2

Now, we have that:



Octahedron



Sphere

From the octahedron volume V = 1/3*√2 l^3 and, from the sphere volume,

V = (4/3*π*r^3) , we consider the following relationship, for r = x:

4/3*π*x^3 = 1/3*√2*l^3

Input

$$\frac{4}{3}\pi x^3 = \frac{1}{3}\sqrt{2}\, l^3$$

Exact result

$$\frac{4\pi x^3}{3} = \frac{\sqrt{2}\, l^3}{3}$$

Alternate forms

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$\frac{4\,\pi\,x^3}{3} - \frac{\sqrt{2}\,l^3}{3} = 0$$

Real solution

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Solutions

$$x = -\frac{\sqrt[3]{-\frac{1}{\pi}}\,l}{\sqrt{2}}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Integer solution

$$l = 0\,, \quad x = 0$$

Implicit derivatives

$$\frac{\partial x(l)}{\partial l} = \frac{l^2}{2\sqrt{2}\,\pi\,x^2}$$

$$\frac{\partial l(x)}{\partial x} = \frac{2\sqrt{2}\,\pi\,x^2}{l^2}$$

From the alternate form

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

for l = 8, we have that:

8/(sqrt(2) π^(1/3)) = 8/(2sqrt2 * Pi)^1/3

Input

$$\frac{8}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{8}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Result

True

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Thence:

l/(sqrt(2) π^(1/3)) = l/(2sqrt2 * Pi)^1/3

Input

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Now, we have that:

l/(2 sqrt(2) π)^(1/3) = (2sqrt2)/Pi

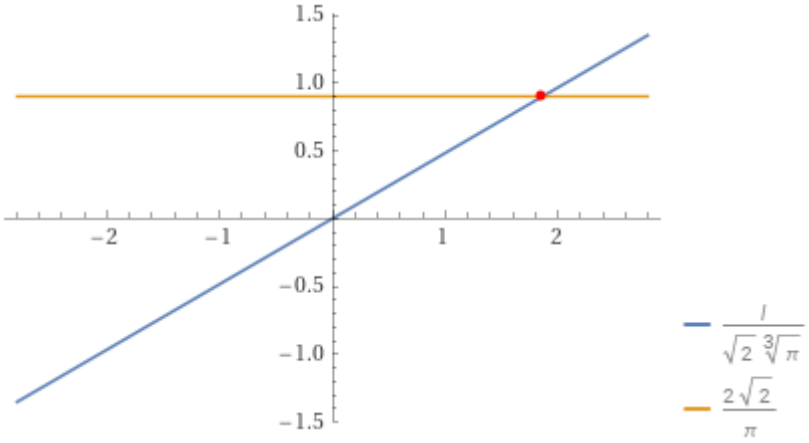
Input

$$\frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}} = \frac{2\sqrt{2}}{\pi}$$

Exact result

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{2\sqrt{2}}{\pi}$$

Plot



Solution

$$l = \frac{4}{\pi^{2/3}}$$

$$l = \frac{4}{\sqrt[3]{\pi^2}}$$

The DN Constant (Del Gaudio-Nardelli Constant)

The DN constant (Del Gaudio-Nardelli constant) is a mathematical concept derived from the ratio of the volume of an octahedron to the volume of a sphere. This constant has intriguing implications in various fields such as geometry, number theory, and cosmology.

Michele Nardelli and his collaborators have proposed that the DN constant could be fundamental to understanding the early universe, particularly in the context of eternal inflation. The octahedron represents a highly symmetric, low-entropy phase of the universe, while the sphere represents the universe emerging from the quantum vacuum, increasing in entropy over time.

The DN constant also connects to other significant mathematical constants such as pi, the golden ratio, and some values from number theory, suggesting a deeper unification of mathematical concepts and potentially offering insights into quantum gravity.

The implications of the DN constant in cosmology are quite fascinating. It plays a role in understanding the structure and evolution of the universe, particularly in the context of eternal inflation.

Eternal inflation

Eternal inflation is a theory that suggests that the universe is continually expanding exponentially. In this context, the DN constant helps describe the transition from a highly symmetric, low-entropy state (represented by the octahedron) to a more complex, high-entropy state (represented by the sphere). This transition is key to understanding how the universe evolves from a quantum vacuum to its current state.

Quantum gravity

The DN constant also has potential implications for quantum gravity, a field that seeks to unify general relativity and quantum mechanics. By connecting geometric shapes such as the octahedron and the sphere, the DN constant could offer insights into the structure of spacetime at the smallest scales.

Mathematical unification

Furthermore, the relationship of the DN constant to other mathematical constants such as pi and the golden ratio suggests a deeper unification of mathematical principles. This unification could provide new ways to address problems in cosmology, such as the nature of dark matter and dark energy.

Early Universe

In the early universe, the DN constant could help explain the initial conditions that led to the Big Bang. By understanding the geometric and entropic properties of the initial state of the universe, cosmologists can better model its subsequent expansion and structure formation.

The DN constant (Del Gaudio-Nardelli constant) plays a significant role in the theory of eternal inflation, which is a model of the exponential expansion of the universe. Here are some key points about its role:

1. **Geometric representation**

The DN constant is derived from the ratio of the volume of an octahedron to the volume of a sphere. In the context of eternal inflation, this geometric relationship helps describe the transition from a highly symmetric, low-entropy state (octahedron) to a more complex, high-entropy state (sphere). This transition is critical to understanding how the universe evolves from a quantum vacuum to its current state1.

2. **Entropy and symmetry**

Eternal inflation implies that the universe is continually expanding and creating new regions of space. The DN constant helps quantify the change in entropy and symmetry during this process. The octahedron represents a low-entropy phase, while the sphere represents a high-entropy phase, illustrating the progression of the universe toward greater complexity.

3. **Quantum fluctuations**

In eternal inflation, quantum fluctuations play a critical role in driving the exponential expansion of the universe. The DN constant can be used to model these fluctuations, providing insights into how different regions of the universe evolve over time. This helps explain why some regions stop inflating and form "pocket universes", while others continue to expand.

4. **Cosmological implications**

The relationship of the DN constant to other mathematical constants suggests a deeper unification of mathematical principles, which could offer new ways to approach problems in cosmology. This includes understanding the nature of dark matter and dark energy, as well as the initial conditions that led to the Big Bang.

5. **Fractal structure**

Eternal inflation leads to a fractal-like structure of the universe, with countless "pocket universes" forming within the inflating space. The DN constant helps describe the geometric and entropic properties of these pocket universes, providing a framework for understanding their formation and evolution.

The DN constant (Del Gaudio-Nardelli constant) has interesting connections with quantum fluctuations, which are temporary changes in energy at a point in space due to the uncertainty principle.

Quantum fluctuations

Quantum fluctuations are tiny, random changes in the values of fields representing elementary particles, such as electric and magnetic fields. These fluctuations are a fundamental aspect of quantum mechanics and play a crucial role in the behavior of the universe at the smallest scales.

Relation to the DN constant

Geometric interpretation: The DN constant, derived from the ratio of the volume of an octahedron to the volume of a sphere, can be used to model the geometric properties of space during quantum fluctuations. This helps to understand how different regions of space evolve over time during the process of eternal inflation.

Entropy and symmetry: Quantum fluctuations contribute to entropy and symmetry changes in the universe. The DN constant helps quantify these changes, providing a framework for understanding, as we discussed earlier, the transition from a low-entropy, highly symmetric state (octahedron) to a high-entropy, less symmetric state (sphere).

Energy distribution: During eternal inflation, quantum fluctuations cause variations in the distribution of energy in different regions of space. The DN constant can be used to describe these variations, providing insights into how energy is distributed and how it affects the formation of structures in the universe.

Fractal structure: The fractal structure of the universe, resulting from eternal inflation, is influenced by quantum fluctuations. The DN constant helps describe the geometric and entropic properties of these fractal structures, helping to understand their formation and evolution.

By connecting geometric shapes and quantum fluctuations, the DN constant provides a deeper understanding of the behavior of the universe at both the macroscopic and microscopic scales.

DN Constant and Ramanujan Recurring Numbers

The DN constant (Del Gaudio-Nardelli constant) and Ramanujan recurring numbers are linked through their profound mathematical properties and implications in number theory and geometry.

DN Constant

The DN constant is derived from the ratio of the volume of an octahedron to the volume of a sphere. This constant has significant implications in various fields, including cosmology and quantum mechanics, as it helps describe transitions between different states of the universe.

Ramanujan's recurring numbers

Ramanujan's recurring numbers are a set of numbers that appear frequently in his work and have special properties. Some of these numbers include:

1729: Known as the Hardy-Ramanujan number, it is the smallest number that can be expressed as the sum of two cubes in two different ways.

4096: A power of 2, which appears frequently in Ramanujan's work.

π (pi): The ratio of the circumference of a circle to its diameter. 1.644934: The value of the Riemann zeta function of 2, also known as ζ(2).

φ (the golden ratio): Approximately 1.6180339887, it appears in various geometric and natural phenomena.

Connections

Geometric and number theory connections: The constant DN and Ramanujan's recurring numbers are related through their geometric and number theory properties. For example, the relationship of the constant DN to the volumes of geometric shapes

can be linked to the properties of numbers such as π and φ, which are fundamental to geometry.

String theory and quantum mechanics: Both the constant DN and Ramanujan recurring numbers have applications in advanced fields such as string theory and quantum mechanics. These connections help us understand the fundamental nature of the universe, including the behavior of particles and the structure of spacetime.

Mathematical unification: The study of these constants and numbers often leads to a deeper unification of mathematical principles. For example, the relationship of the constant DN to geometric shapes and Ramanujan numbers can provide new insights into the solutions of complex equations in number theory and physics.

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}{\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2}}$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}{}^{3\times3} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}{}^{3\times3} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))) = 3.1415926535

Indeed:

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.1415926535884871624221932509858715278931025189245576170078100613
…

3.1415926535…. = π (Ramanujan Recurring Number)

From the above expression, we obtain the following formulas concerning the icosahedron, octahedron and tetrahedron volumes together to the sphere volume.

$$\sqrt[2\pi]{\frac{5}{12}(3+\sqrt{5})d^3\times\left(\frac{4}{3}\pi\left(\frac{d}{2}\right)^3\right)^2}$$

((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

and

$$\sqrt[2\pi]{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3 \times \frac{1}{3}\sqrt{2}d^3 \times \frac{\sqrt{2}}{12}d^3}$$

((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi))

Multiplying each equation by ((5/12 (3+√5) d^3)×((4/3 π(d/2)^3)^2))^(1/(2Pi))

and ((4/3 π(d/2)^3)×(1/3 √2 d^3)×(√2/12 d^3))^(1/(2Pi)) , we obtain, as described in the calculations, the golden ratio and π

Indeed, we obtain:

((5/12 (3+√5) d^3×(4/3 π(d/2)^3)^2)^(1/(2Pi)))/((4/3 π(d/2)^3×1/3 √2 d^3×√2/12 d^3)^(1/(2Pi)))

= 1.6180085459001O7…

From the following extended DN Constant (“Unitary Formula”)

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}} \times \sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}}$$

with regard

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

for q = 1729 and p = 4096, we obtain by changing the sign in the algebraic sum of the aforementioned Cardano's Formula and after some calculations:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

multiplied by

$$\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}$$

and performing the ninth root of the entire expression:

√((2^(-1/π)(5(3+√(5))π)^(1/(2π)))((∛(-1729/2+√(1729^2/4+4096^3/27)) -∛(-1729/2-√(1729^2/4+4096^3/27)))^1/9))

we obtain:

$$\sqrt{\left(\left(2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right)\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}\right)}$$

$$2^{-1/(2\pi)}\sqrt[18]{\sqrt[3]{\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}-\frac{1729}{2}}+\sqrt[3]{\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}}\sqrt[4\pi]{5\left(3+\sqrt{5}\right)\pi}$$

i.e.

2^(-1/(2π)) ((-1729/2+√(274958621851/3)/6)^(1/3)+(1729/2 + √(274958621851/3)/6)^(1/3))^(1/18) (5(3+√(5)) π)^(1/(4π))

= 1.61549140391…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

and:

Exact Result

$$2^{-1/(2\pi)}\sqrt[18]{\sqrt[3]{-\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}+\sqrt[3]{\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}}\sqrt[12]{5\left(3+\sqrt{5}\right)\pi}$$

i.e.

2^(-1/(2π)) ((-1729/2+√(274958621851/3)/6)^(1/3)+(1729/2 + √(274958621851/3)/6)^(1/3))^(1/18) (5(3+√(5)) π)^(1/12)

= 1.64247057108….≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

The general "unitary" formula, which derives from DN Constant, is the following:

$$\sqrt{2 \times \frac{2 \cdot \sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi \cdot 0.9991104684}(C \times \mathrm{R} \times 2.33 \cdot 10^{-13})}} \cong 1.61803398\ldots = \frac{\sqrt{5}+1}{2}$$

Where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$$

C is any constant or solution to an equation, R is the radius of the Universe and $2.33*10^{-13}$ is the temperature of the universe expressed in GeV.

From the previous result, for $C = 9.9128$, inserting a radius of the Universe, which we have calculated, equal to $R = 2.06274*10^{12}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}$$

Result

1.618035912348264235440174408834709854254527350840173356306481810 7

…

1.618035912348…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

And, always from:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(1.616255\cdot 10^{-35}\times R\times 2.33\cdot 10^{-13})}}$$

for C = $1.616255*10^{-35}$ (Planck Length), R = $1.265120782997423\times 10^{48}$ that is equal to the radius of the Multiverse and $2.33*10^{-13}$ is the mean temperature of the CMB is equivalent around to *2.73 Kelvin* , we obtain:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(1.616255\cdot 10^{-35}\times 1.265120782997423*10^{48}\times 2.33\cdot 10^{-13})}}=$$

√(2×(2·(((2√2)/π)^(1/16)))/(1/(π·0.9991104684)(1.616255·10^(-35)×1.265120782997423*10^48×2.33·10^(-13))))

Where $1.616255*10^{-35}$ = Planck Length ; $1.265120782997423*10^{48}$ = radius of Multiverse ; $2.33*10^{-13}$ = mean temperature of the CMB

$$=\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{(4.764285633177547923154 5)}{\pi\cdot 0.9991104684}}}=\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{1.517869417}}=\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{9\sqrt{\frac{2}{5}}0.4142293^{\frac{3}{2}}}}=$$

1.618033719519….

Indeed:

√(2×(2·((2√2)/π)^1/16)/(9√(2/5)(log2 log3)/√(3e)))

Input

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{9\sqrt{\frac{2}{5}}\times\frac{\log(2)\log(3)}{\sqrt{3e}}}}$$

log(x) is the natural logarithm

Exact result

$$\frac{2^{51/64}\sqrt[4]{5e}}{3^{3/4}\sqrt[32]{\pi}\sqrt{\log(2)\log(3)}}$$

Decimal approximation

1.6180337195191494694592859877088514992174074748460976388126912571
…

1.618033719519…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We obtain also

(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))dxdydz

Indefinite integral assuming all variables are real

$0.809018\, x^2\, y\, z$ + constant

Definite integral over a cube of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} 1.61804\, dx\, dy\, dz = 12.9443\, L^3$$

Definite integral over a sphere of radius R

$$\iiint_{x^2+y^2+z^2<R^2} 1.61804\, dz\, dy\, dx = 6.77761\, R^3$$

From which, for L = R = 1 , dividing the two definite integral results by the original expression, we obtain:

12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))

Input interpretation

$$\frac{12.9443}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

8.00001…

8.00001…. ≈ 8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

From which:

((12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))^4)

Input interpretation

$$\left(\frac{12.9443}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}\right)^4$$

Result

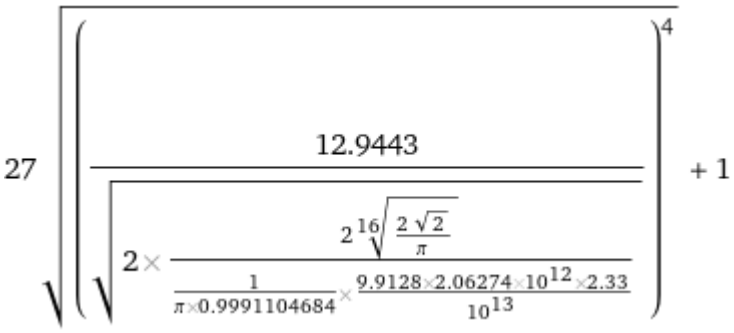
4096.02…

4096.02…. ≈ 4096

The number $4096 = 64^2$, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

27*sqrt(((12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684)
(9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))^4))+1

Input interpretation



Result

1729.00…

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons.

This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(27*sqrt(((12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684)
(9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))^4))+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\sqrt[15]{27\sqrt{\left(\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}\right)^4}+\frac{12.9443}{}}+1+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Result

1.64493823906537945152785663931290225811839380861027509962627119 80…

1.644938239…. ≈ $\zeta(2) = \pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

3*((12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))))

Input interpretation

$$3\times\frac{12.9443}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

24.0000…

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles). Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

And

3*(6.77761/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))

Input interpretation

$$3\times\frac{6.77761}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

12.5664…

12.5664…≈ 4π ≈ S_{BH} - Bekenstein-Hawking Black Hole Entropy

With regard the Planck length and the Planck time, we have that squaring the Planck length:

$$l_p^2 = \left(\sqrt{\frac{\hbar G}{c^3}}\right)^2$$

we obtain:

(√(ħG/c^3))^2

Input interpretation

$$\sqrt{\left(G\,\hbar/c^3\right.}$$ (Newtonian gravitational constant reduced Planck constant per speed of light cubed)$$\left.\right)^2$$

Result

$2.612 \times 10^{-70}\ m^2$ (square meters)

$2.612*10^{-70}\ m^2$

Now, we perform the following calculation

(-ln(2.612×10^-70)-4^2)

(String Theory, Superstring Theory and Beyond – Vol. II – J. Polchinski)

Table B.3. *Supergravities with 32 supercharges. The group G is a symmetry of the low energy supergravity theory, and the moduli space is locally G/H.*

d	scalars	vectors	G	H
10A	1	1	$SO(1,1,\mathbf{R})$	-
10B	2	0	$SL(2,\mathbf{R})$	$SO(2,\mathbf{R})$
9	3	3	$SL(2,\mathbf{R}) \times SO(1,1,\mathbf{R})$	$SO(2,\mathbf{R})$
8	7	6	$SL(2,\mathbf{R}) \times SL(3,\mathbf{R})$	$SO(2,\mathbf{R}) \times SO(3,\mathbf{R})$
7	14	10	$SL(5,\mathbf{R})$	$SO(5,\mathbf{R})$
6	25	16	$SO(5,5,\mathbf{R})$	$SO(5,\mathbf{R}) \times SO(5,\mathbf{R})$
5	42	27	$E_{6(6)}(\mathbf{R})$	$US_p(8)$
4	70	28	$E_{7(7)}(\mathbf{R})$	$SU(8)$
3	128	-	$E_{8(8)}(\mathbf{R})$	$SO(16,\mathbf{R})$

Where 16 = 4^2 are vectors (see above Table) and 233 is a Fibonacci's number and the difference between 248 and 15.

There is a group, the exceptional group $E8$, that has an $SO(16)$ subgroup under which the E adjoint **248** transforms as **120** + **128**. The total matter central charge is $c^{\mathrm{m}} = \tilde{c}^{\,m}$ = 15. Thence, we obtain:

233/(-ln(2.612×10^-70)-4^2)

Input interpretation

$$\frac{233}{-\log(2.612\times10^{-70})-4^2}$$

log(x) is the natural logarithm

Result

1.6155779…

1.6155779…. a result very near to the value of the Golden Ratio.

Furthermore, with regard the Planck time

$$t_p = \left(\sqrt{\frac{\hbar G}{c^5}}\right)$$

we obtain:

(√(ħG/c^5))

Input interpretation

$\sqrt{(G\hbar/c^5}$ (Newtonian gravitational constant reduced Planck constant per speed of light to the fifth))

Result

5.391×10^{-44} seconds

5.391*10^{-44} s

And performing the following calculations:

27*1/2(-ln(5.391×10^-44)+24+2Φ+Pi)+1

Where 27 are vectors (see above table) and 24 is the number of vibration modes of the bosonic strings

we obtain:

Input interpretation

$$27 \times \frac{1}{2}\left(-\log\left(5.391 \times 10^{-44}\right) + 24 + 2\,\Phi + \pi\right) + 1$$

log(x) is the natural logarithm

Φ is the golden ratio conjugate

Result

1729.0901…

1729.0901….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(1/2(-ln(5.391×10^-44)+24+2Φ+Pi))^2-2/5

Input interpretation

$$\left(\frac{1}{2}\left(-\log\left(5.391\times 10^{-44}\right)+24+2\,\Phi+\pi\right)\right)^2-\frac{2}{5}$$

log(x) is the natural logarithm

Φ is the golden ratio conjugate

Result

4096.0271…

4096.0271…. ≈ 4096

The number $4096 = 64^2$, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

And:

(27*1/2(-ln(5.391×10^-44)+24+2Φ+Pi)+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\sqrt[15]{27\times\frac{1}{2}\left(-\log\left(5.391\times10^{-44}\right)+24+2\,\Phi+\pi\right)+1}+C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm

Φ is the golden ratio conjugate

C_{MRB} is the MRB constant

Result

1.64494373…

1.64494373…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

With regard the Planck mass:

$$m_P = \sqrt{\frac{\hbar c}{G}}$$

we obtain:

√(ħc/G)

Input interpretation

$\sqrt{\hbar c/G}$ (reduced Planck constant speed of light per Newtonian gravitational constant)

Result

2.1764×10^{-8} kg (kilograms)

2.1764*10^{-8} kg

From which, after some calculations:

29/(-ln(2.1764×10^-8)+3^2/5^2)

Input interpretation

$$\frac{29}{-\log(2.1764\times10^{-8})+\frac{3^2}{5^2}}$$

log(x) is the natural logarithm

Result

1.610841867097335855559505400203061959486453489238566568482733370

…

1.610841867…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Michele Nardelli's hypothesis involving the DN Constant (Del Gaudio-Nardelli Constant) explores the geometric and mathematical relationships in the universe. According to his work, the DN Constant is the ratio of the volume of an octahedron to the volume of a sphere[12].

In the initial stages of the universe, the octahedron represents a fundamental geometric shape. Over time, this shape transitions into a sphere, symbolizing the evolution and expansion of the universe. This transformation from an octahedron to a sphere reflects the dynamic changes in the universe's structure and geometry[12].

Nardelli's hypothesis connects these geometric shapes with deeper mathematical principles, including Ramanujan's Recurring Numbers and aspects of Number Theory and String Theory[12]. This approach aims to unify various mathematical concepts to better understand the universe's underlying structure.

The choice of an octahedron and a sphere in Michele Nardelli's hypothesis is rooted in their unique geometric and mathematical properties:

1. **Octahedron**: This shape is one of the simplest and most symmetrical polyhedra, consisting of eight equilateral triangles. Its symmetry and simplicity make it a fundamental building block in geometric theories. In the context of the universe's initial state, the octahedron represents a highly ordered and symmetrical structure.
2. **Sphere**: As the universe evolves, the sphere becomes a symbol of expansion and uniformity. Spheres are perfectly symmetrical in all directions, representing a state of equilibrium and balance. This transition from an octahedron to a sphere can be seen as a metaphor for the universe's progression from a highly ordered state to a more expansive and balanced one.

These shapes are not chosen arbitrarily but are deeply connected to the mathematical principles that Nardelli explores, such as the DN Constant and its relationship to the volumes of these shapes. The transformation from an octahedron to a sphere encapsulates the dynamic changes in the universe's structure and geometry over time.

The DN Constant is an intriguing concept in cosmology. It suggests a fundamental constant that influences the dynamics of the universe, particularly in the context of eternal inflation and cyclic models. The idea is that this constant could play a role in the transition between different phases of the universe, such as from a supermassive black hole to a white hole.

In the framework you're mentioning, the DN Constant might be theorized to govern the conditions under which a supermassive black hole at the end of one cosmic cycle could become a white hole, initiating a new cycle. This transformation would involve complex physical processes, potentially linked to quantum gravity and other advanced theories.

While this idea is highly speculative and not part of mainstream cosmology, it offers a fascinating perspective on how the universe might renew itself through cycles of expansion and contraction.

On Srinivasa Ramanujan's Mathematics and a Possible Unitary Mathematical Model

In practice, the intuition is this: all geometric shapes in the microcosm and macrocosm have a "trace", a "signature". It is mainly identified with π and φ (the golden section). The constant DN $(2\sqrt{2})/\pi$ unifies the various mathematical constants, certainly the fundamental ones and is also connected to the Planck length and the Planck mass. This is why Ramanujan studied π so much, this is why I consider his work "Modular equations and approximations to π" of fundamental importance. From π and φ and the connected constant DN, a multiverse emerges that is born from a white hole, through that cosmological process that goes under the name of eternal inflation. "Nothing is created, nothing is destroyed, everything is transformed". The Universe "is" has always existed, exists and will always exist. It only goes through "phases", "cycles" and the information cycle after cycle is not lost, but only reworked.

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

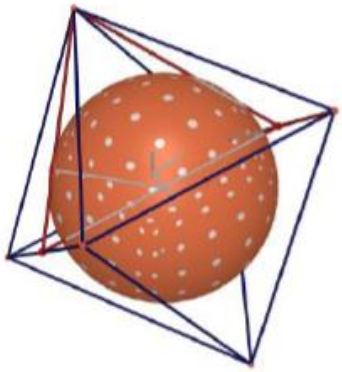


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.14193957152658430892433073219616263267751338681165904468254173 93

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

4096 = 64^2 , (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$).

The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^8\right)^{-k}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^8\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times3}}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\,z_0^{\,-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\right.\right.\right.$$
$$\left.\left.\left.z_0^{\,1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)\Big/$$
$$\left.\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))) = 3.1415926535

Indeed:

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.1415926535884871624221932509858715278931025189245576170078100613
…

3.1415926535…. = π (Ramanujan Recurring Number)

From the following extended DN Constant (“Unitary Formula”)

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}\times\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

with regard

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

for q = 1729 and p = 4096, we obtain by changing the sign in the algebraic sum of the aforementioned Cardano’s Formula and after some calculations:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

multiplied by

$$\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}$$

and performing the ninth root of the entire expression:

√((2^(-1/π)(5(3+√(5))π)^(1/(2π)))((∛(-1729/2+√(1729^2/4+4096^3/27)) -∛(-1729/2-√(1729^2/4+4096^3/27)))^1/9))

we obtain:

$$\sqrt{\left(\left(2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right)\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}\right)}$$

$$2^{-1/(2\pi)}\sqrt[18]{\sqrt[3]{\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}-\frac{1729}{2}}+\sqrt[3]{\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}}\Big/\sqrt[4\pi]{5\left(3+\sqrt{5}\right)\pi}$$

i.e.

2^(-1/(2π)) ((-1729/2+√(274958621851/3)/6)^(1/3)+(1729/2 + √(274958621851/3)/6)^(1/3))^(1/18) (5(3+√(5)) π)^(1/(4π))

= 1.61549140391….

The general "unitary" formula, which derives from DN Constant, is the following:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

Where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

C is any constant or solution to an equation, R is the radius of the Universe and $2.33*10^{-13}$ is the temperature of the universe expressed in GeV.

For example, $C = 9.9128$, inserting a radius of the Universe, which we have calculated, equal to $R = 2.06274*10^{12}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}$$

1.6180359123482642354401744088347098542545273508401733563064818107 …

1.618035912348…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We obtain also:

(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))dxdydz

Indefinite integral assuming all variables are real

$$0.809018\, x^2\, y\, z + \text{constant}$$

Definite integral over a cube of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} 1.61804\, dx\, dy\, dz = 12.9443\, L^3$$

Definite integral over a sphere of radius R

$$\iiint_{x^2+y^2+z^2<R^2} 1.61804\, dz\, dy\, dx = 6.77761\, R^3$$

From which, for L = R = 1 , dividing the two definite integral results by the original expression, we obtain:

12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))

Input interpretation

$$\frac{12.9443}{\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

8.00001…

8.00001…. ≈ 8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

And

3*(6.77761/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))

Input interpretation

$$3\times\frac{6.77761}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

12.5664…

12.5664…. ≈ 4π = Bekenstein-Hawking (S_{BH}) black hole entropy

New fundamental formula deriving from DN Constant

The DN Constant (Del Gaudio-Nardelli Constant) equals $(2\sqrt{2})/\pi$ is defined as the ratio of the volume of an octahedron to the volume of a sphere and is an intriguing mathematical concept. Michele Nardelli hypothesized that the regular octahedron represents a phase in which the universe is highly symmetrical and with very low entropy. On the other hand, the sphere (which is inscribed in the octahedron, i.e. is "inside" it) represents the universe emerging from the quantum vacuum, which over time increases entropy and undergoes various symmetry breakings. This occurs in a regime of eternal inflation.

From the following expression

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×C×R)))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\,C\,R\right)}}$$

which comes from the DN Constant, with $1.616255*10^{-35}$ which is equal to the Planck length, $1.1056*10^{-52}$ which is equal to the Cosmological Constant, C = 1729 which corresponds to the Hardy-Ramanujan number and R = $4.6018401361 \times 10^{-24}$, which represents the radius of the Universe, we obtain:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×4.6018401361 × 10^-24))) = 1.6180329973...

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 4.6018401361\times 10^{-24}\right)}}$$

1.6180329973075324915067570166297467464225772608929671407919395903

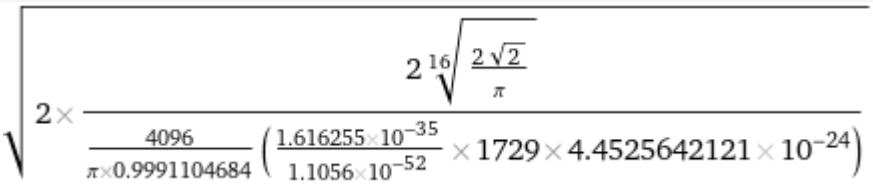
…

1.6180329973075…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We have also the following formula:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(4.4525642121 × 10^-24))))

Input interpretation



Result

1.6449323521020921304838989837041511688766218416551668779141660338

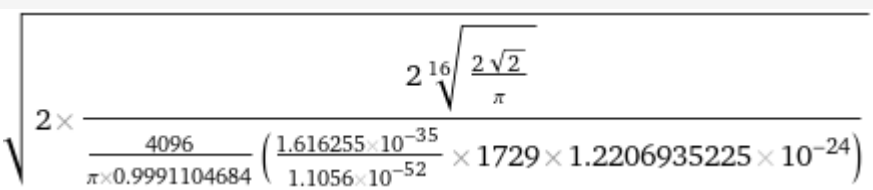
…

1.64493235210209213…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

And again:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(1.2206935225 × 10^-24))))

Input interpretation



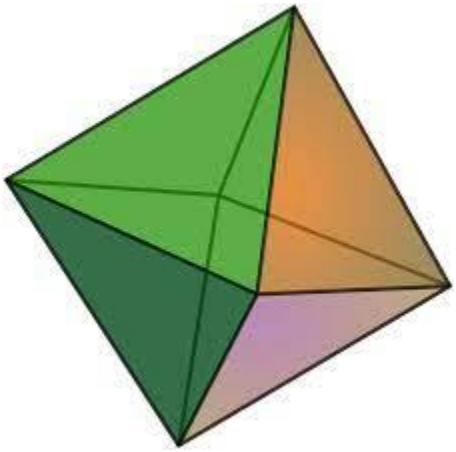
Result

3.14159225731469939939950039231605796469731171781644423955790024797

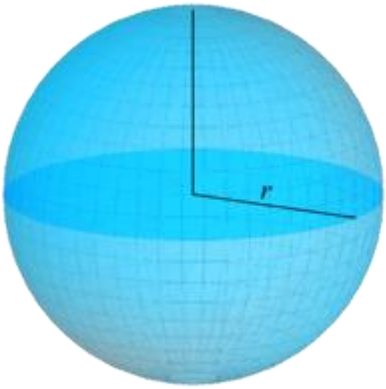
…

3.14159225731469…. ≈ π (Ramanujan Recurring Number)

Now, we have that:



Octahedron



Sphere

Given the value of a volume, independently of the solid, following the Poincaré Conjecture, we compare any solid "without holes" and a sphere. If we compare an octahedron with a sphere, we have:

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

If we consider the radius of the sphere as an unknown, we must find the value of the side of the octahedron which allows us to equalize the two volumes and which will give us the DN Constant as a result (which will therefore be equal to the radius of the sphere).

From

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

To find r we perform the following calculation:

$$r^3 = \frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi} = \frac{1}{3}\sqrt{2}a^3 \cdot \frac{3}{4\pi} = \frac{\sqrt{2}a^3}{4\pi} = \frac{\sqrt{2}\cdot\sqrt{2}a^3}{\sqrt{2}\cdot 4\pi} = \frac{2a^3}{\sqrt{2}\cdot 4\pi} = \frac{a^3}{2\sqrt{2}\pi}$$

$$r^3 = \frac{a^3}{2\sqrt{2}\pi}; \qquad r = \sqrt[3]{\frac{a^3}{2\sqrt{2}\pi}} = \frac{a}{\sqrt[3]{2\sqrt{2}\pi}}$$

To find *a*, we have, for

$$r = \frac{2\sqrt{2}}{\pi}; \qquad \frac{a}{\sqrt[3]{2\sqrt{2}\pi}} = \frac{2\sqrt{2}}{\pi};$$

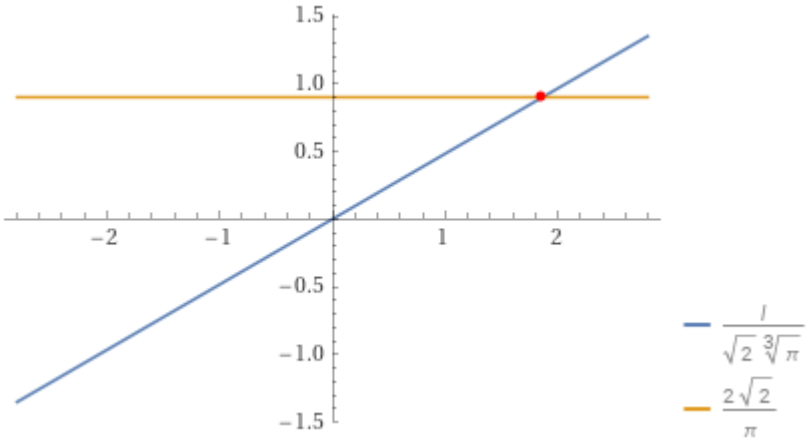
Thus, multiplying both the sides by $\sqrt[3]{2\sqrt{2}\pi}$, we obtain:

$$a = \frac{2\sqrt{2}}{\pi} \cdot \sqrt[3]{2\sqrt{2}\pi} = \sqrt[3]{\frac{(2\sqrt{2})^3 2\sqrt{2}\pi}{\pi^3}} =$$

$$= \sqrt[3]{\frac{8\cdot\sqrt{2^3}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{8\cdot\sqrt{2\cdot 2^2}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{16\cdot\sqrt{2}\cdot 2\sqrt{2}}{\pi^2}}$$

$$= \sqrt[3]{\frac{32\sqrt{2}\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{32\cdot 2}{\pi^2}} = \frac{\sqrt[3]{64}}{\sqrt[3]{\pi^2}} = \frac{4}{\sqrt[3]{\pi^2}}; \; a = \frac{4}{\sqrt[3]{\pi^2}}$$

Plot



Solution

$a = \frac{4}{\sqrt[3]{\pi^2}}$

for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), we obtain:

from the octahedron volume, we have: V = 1/3*√2*a^3 = (1/3*√2*($\frac{4}{\sqrt[3]{\pi^2}}$)^3)

(1/3*√2*(4/∛(π^2))^3)

Input

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3$$

$\sqrt[3]{x}$ is the real-valued 3rd root of x

Exact result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{3\pi^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And, from the sphere volume V = (4/3*π*r^3) = (4/3*π*((2√2)/π)^3)

(4/3*π*((2√2)/π)^3)

Input

$$\frac{4}{3}\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3$$

Result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\sqrt{z_0}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\exp^3\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,3}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3}{3\pi^2}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\left(\frac{1}{z_0}\right)^{3/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{3/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi =$$
$$\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

$1.64381856858\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\text{^}\,(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\right/\right.$$
$$\left.\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\text{^}(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right/\right.$$
$$\left.\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10^{-6}$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

$-5.010882647757...$

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

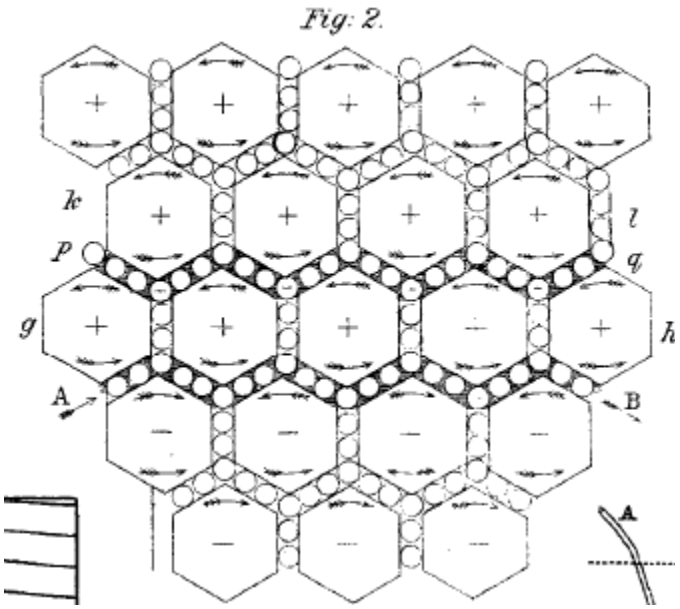
Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

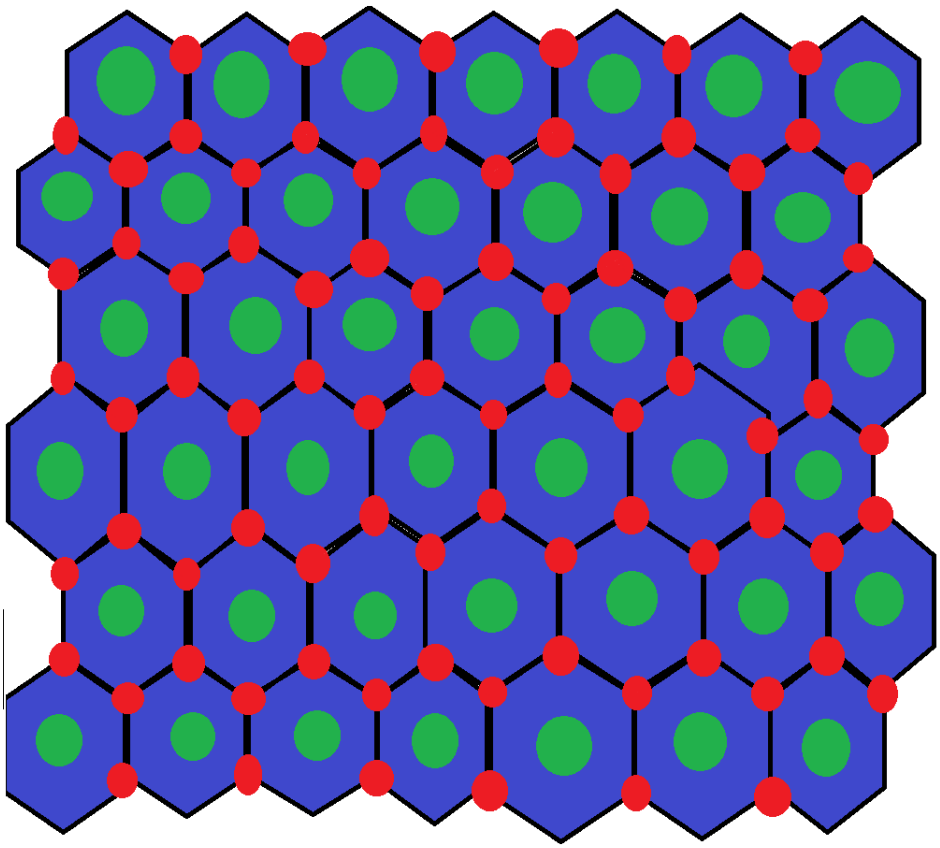
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability.

References

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the study of various equations concerning the Atomic Physics. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory, Theoretical Cosmology and String Theory

Michele Nardelli[1], Antonio Nardelli

Abstract

In this paper, we analyze various equations concerning the Atomic Physics. We obtain new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory, Theoretical Cosmology and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan

(1887 - 1920)

Introduction

In this paper, an octahedron could serve as a mathematical or conceptual model of the universe in the quantic phase, while the spherical surface could be used to describe the geometry of the bubble-universe.

The values $(2\sqrt{2})/\pi$, the golden ratio φ, $\zeta(2)$ and π, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$ the so called DN Constant:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and bubbles universe in eternal inflation.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and bubbles universe.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of bubbles universe and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

The above values $(2\sqrt{2})/\pi$, the golden ratio φ and π, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

The various mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe.

Regarding the fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the quantum universe and bubbles universe.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

Proposal:

The initial octahedron: Let's imagine a regular octahedron, with perfectly symmetrical faces. Each face represents an ideal symmetry.

The emerging sphere: Inside the octahedron, there is an inscribed sphere. This sphere represents the bubble of the universe that emerges from the perturbations of the quantum vacuum during eternal inflation.

Expansion and transitions: As time passes, the universe expands. The faces of the octahedron begin to break, symbolizing "symmetry breaks." The sphere continues to grow, representing the expanding universe.

Constants and numbers: We integrate the mathematical results you obtained. For example, the golden ratio (φ) could be represented by a proportion between the dimensions of the octahedron and the sphere.

Entropy and complexity: Entropy increases as the universe evolves. We can represent this with a disordered growth of structures within the emerging sphere. Imagine this scene as an abstract work of art, where geometric shapes and cosmological concepts merge

In Fig.1 and Fig.2 let's imagine a regular octahedron representing the universe in its phase of high symmetry and very low entropy. Inside the octahedron we have an inscribed sphere that emerges from perturbations of the quantum vacuum during eternal inflation. As time passes, the universe expands, the faces of the octahedron break (symmetry breaks), and entropy increases. Spheres emerge from the octahedra, symbolizing the transition phases from a regime of very low entropy to a universe in which, with the passage of time, entropy increases, increasing the complexity of the universe itself.

Fig. 1

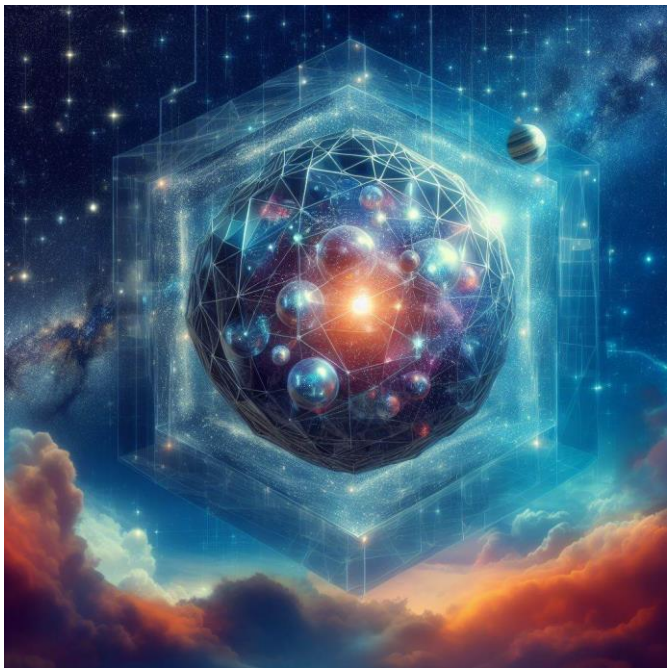
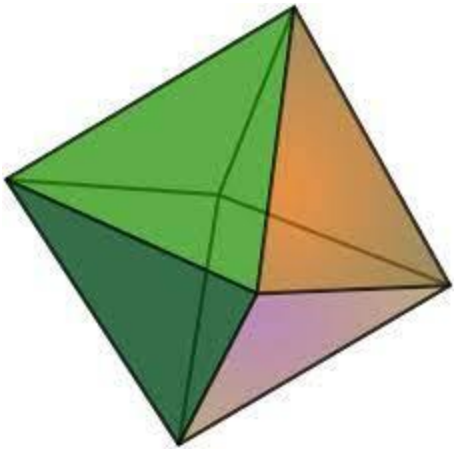
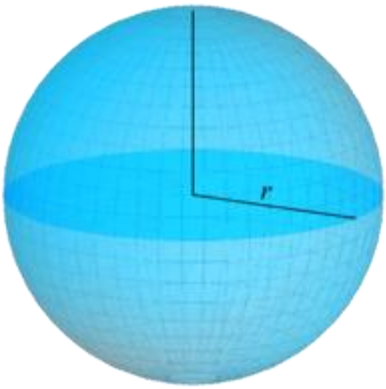


Fig. 2

Now, we have that:



Octahedron



Sphere

From the octahedron volume V = 1/3*√2 l^3 and, from the sphere volume,

V = (4/3*π*r^3) , we consider the following relationship, for r = x:

4/3*π*x^3 = 1/3*√2*l^3

Input

$$\frac{4}{3}\pi x^3 = \frac{1}{3}\sqrt{2}\, l^3$$

Exact result

$$\frac{4\pi x^3}{3} = \frac{\sqrt{2}\, l^3}{3}$$

Alternate forms

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$\frac{4\,\pi\,x^3}{3} - \frac{\sqrt{2}\,l^3}{3} = 0$$

Real solution

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Solutions

$$x = -\frac{\sqrt[3]{-\frac{1}{\pi}}\,l}{\sqrt{2}}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Integer solution

$$l = 0\,, \quad x = 0$$

Implicit derivatives

$$\frac{\partial x(l)}{\partial l} = \frac{l^2}{2\sqrt{2}\,\pi\,x^2}$$

$$\frac{\partial l(x)}{\partial x} = \frac{2\sqrt{2}\,\pi\,x^2}{l^2}$$

From the alternate form

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

for l = 8, we have that:

8/(sqrt(2) π^(1/3)) = 8/(2sqrt2 * Pi)^1/3

Input

$$\frac{8}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{8}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Result

True

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Thence:

l/(sqrt(2) π^(1/3)) = l/(2sqrt2 * Pi)^1/3

Input

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Logarithmic form

$$\log_{\sqrt{2}\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\sqrt[3]{\pi}}\left(\sqrt{2}\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Now, we have that:

l/(2 sqrt(2) π)^(1/3) = (2sqrt2)/Pi

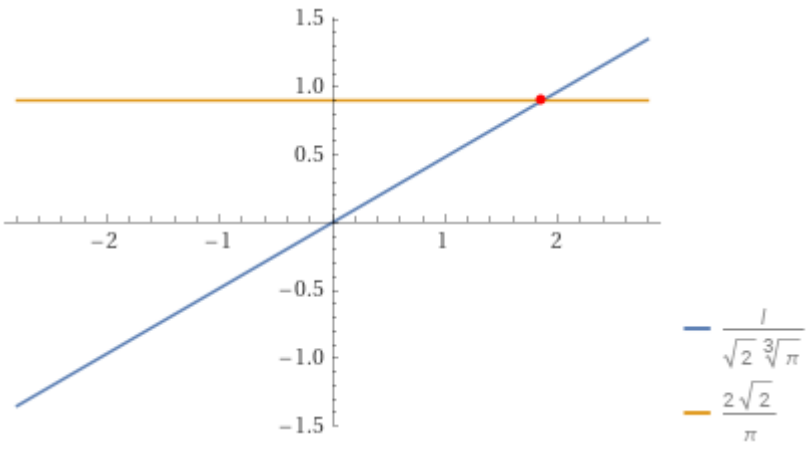
Input

$$\frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}} = \frac{2\sqrt{2}}{\pi}$$

Exact result

$$\frac{l}{\sqrt{2}\sqrt[3]{\pi}} = \frac{2\sqrt{2}}{\pi}$$

Plot



Solution

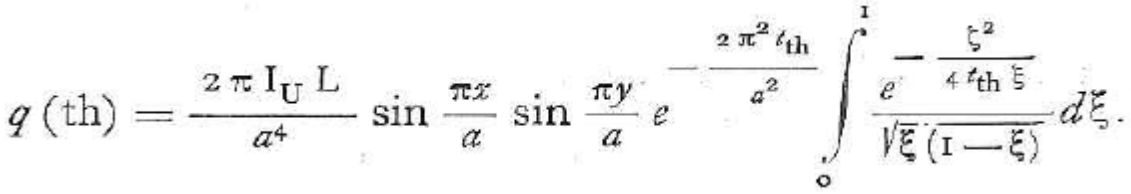
$$l = \frac{4}{\pi^{2/3}}$$

$$l = \frac{4}{\sqrt[3]{\pi^2}}$$

From

Production of Neutrons by Uranium – *H. L. Anderson and E. Fermi* - 1941

We analyze the following equation: (pag.86 Fermi Omnia 2)



$$q\,(\mathrm{th}) = \frac{2\,\pi\,\mathrm{I_U}\,\mathrm{L}}{a^4}\,\sin\frac{\pi x}{a}\,\sin\frac{\pi y}{a}\,e^{-\frac{2\,\pi^2\,t_{\mathrm{th}}}{a^2}}\int_0^1 \frac{e^{-\frac{\zeta^2}{4\,t_{\mathrm{th}}\,\xi}}}{\sqrt{\xi\,(1-\xi)}}\,d\xi.$$

We consider:

integrate((e^((-ζ^2)/(4*t*ξ)))/(sqrt(ξ*(1-ξ))))dξ, ξ=0..1

Input

$$\int_0^1 \frac{e^{-\zeta(u;g_2,g_3)^2/(4\,t\,\xi)}}{\sqrt{\xi\,(1-\xi)}}\,d\xi$$

$\zeta(u; g_2, g_3)$ is the Weierstrass zeta function

Alternative representations

$$\int_0^1 \frac{e^{-\zeta(u;g_2,g_3)^2/(4\,t\,\xi)}}{\sqrt{\xi\,(1-\xi)}}\,d\xi = \int_0^1 \frac{w^a}{\sqrt{\xi\,(1-\xi)}}\,d\xi \quad \text{for } 4\,a + \frac{\zeta(u;\,g_2,\,g_3)^2}{t\,\xi\log(w)} = 0$$

$$\int_0^1 \frac{e^{-\zeta(u;g_2,g_3)^2/(4\,t\,\xi)}}{\sqrt{\xi\,(1-\xi)}}\,d\xi = \int_0^1 \frac{z^{-\zeta(u;g_2,g_3)^2/(4\,t\,\xi)}}{\sqrt{\xi\,(1-\xi)}}\,d\xi \quad \text{for } z = e$$

$$\int_0^1 \frac{e^{-\zeta(u;g_2,g_3)^2/(4\,t\,\xi)}}{\sqrt{\xi\,(1-\xi)}}\,d\xi =$$

$$\left(\int_0^1 \frac{e^{-\left(\frac{\partial \log(\sigma(u;g_2,g_3))}{\partial u}\right)^2/(4\,t\,\xi)}}{\sqrt{(1-\xi)\,\xi}}\,d\xi = \int_0^1 \frac{e^{-\left(\frac{\partial \log(\sigma(u;g_2,g_3))}{\partial u}\right)^2/(4\,t\,\xi)}}{\sqrt{(1-\xi)\,\xi}}\,d\xi\right)$$

$\log(x)$ is the natural logarithm

$\sigma(u; g_2, g_3)$ is the Weierstrass sigma function

From the alternative representation

$$\int_0^1 \frac{e^{-\zeta(u;g_2,g_3)^2/(4\,t\,\xi)}}{\sqrt{\xi\,(1-\xi)}}\,d\xi = \int_0^1 \frac{w^a}{\sqrt{\xi\,(1-\xi)}}\,d\xi \text{ for } 4\,a + \frac{\zeta(u;\,g_2,\,g_3)^2}{t\,\xi\log(w)} = 0$$

We consider:

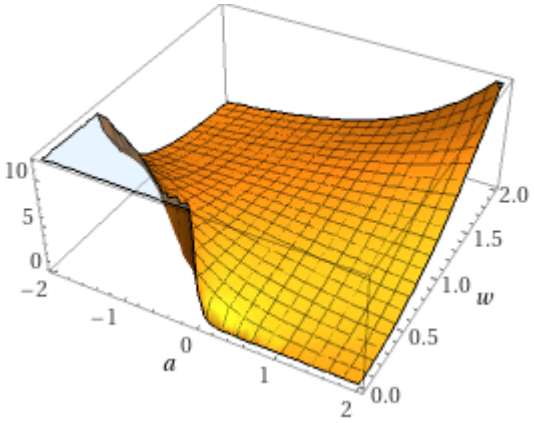
Integrate(w^a/sqrt(ξ (1 - ξ)))dξ, ξ=0..1

Definite integral

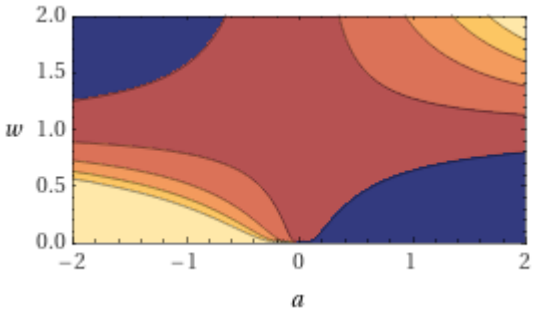
$$\int_0^1 \frac{w^a}{\sqrt{\xi\,(1-\xi)}}\,d\xi = \pi\,w^a$$

3D plot **(figure that can be related to a D-brane/Instanton)**

The key observation from the below plots and is that at $w = 1$, which is taken as the energy density of the universe at the Big Bang, with $a = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.



Contour plot



Indefinite integral

$$\int \frac{w^a}{\sqrt{\xi\,(1-\xi)}}\,d\xi = -2\,w^a \sin^{-1}\left(\sqrt{1-\xi}\right) + \text{constant}$$

$\sin^{-1}(x)$ is the inverse sine function

From

$$q\,(\mathrm{th}) = \frac{2\,\pi\,\mathrm{I_U}\,\mathrm{L}}{a^4}\,\sin\frac{\pi x}{a}\,\sin\frac{\pi y}{a}\,e^{-\frac{2\,\pi^2\,t_{\mathrm{th}}}{a^2}}\int_0^1 \frac{e^{-\frac{\zeta^2}{4\,t_{\mathrm{th}}\,\xi}}}{\sqrt{\xi\,(1-\xi)}}\,d\xi.$$

for I = A:

(2π*A*L)/(a^4)*sin((π*x)/a)*sin((π*y)/a)*e^((-2π^2*t)/(a^2))* π w^a

Input

$$\frac{2\pi\,A\,L}{a^4}\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)e^{(-2\pi^2\,t)/a^2}\,\pi\,w^a$$

Exact result

$$\frac{2\pi^2\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^4}$$

Alternate form

$$-\frac{\pi^2\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\left(e^{-(i\,\pi\,x)/a}-e^{(i\,\pi\,x)/a}\right)\left(e^{-(i\,\pi\,y)/a}-e^{(i\,\pi\,y)/a}\right)}{2\,a^4}$$

Roots

$w \neq 0, \quad a \neq 0, \quad x = a\,n, \quad n \in \mathbb{Z}$

$w \neq 0, \quad a \neq 0, \quad y = a\,n, \quad n \in \mathbb{Z}$

$\mathrm{Re}(a) > 0, \quad w = 0$

$a \neq 0, \quad w \neq 0, \quad A = 0$

$a \neq 0, \quad w \neq 0, \quad L = 0$

$\mathbb{Z}$ is the set of integers

Re(z) is the real part of z

Periodicity

periodic in t with period $-\frac{i\,a^2}{\pi}$
periodic in x with period $-2\,a$
periodic in y with period $-2\,a$

Series expansion at x=0

$$\frac{2\pi^3\,A\,L\,x\,e^{-(2\pi^2\,t)/a^2}\,w^a\,\sin\left(\frac{\pi\,y}{a}\right)}{a^5} - \frac{x^3\left(\pi^5\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\,\sin\left(\frac{\pi\,y}{a}\right)\right)}{3\,a^7} + \frac{\pi^7\,A\,L\,x^5\,e^{-(2\pi^2\,t)/a^2}\,w^a\,\sin\left(\frac{\pi\,y}{a}\right)}{60\,a^9} + O\left(x^6\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial x}\left(\frac{(2\pi\,A\,L)\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)e^{-(2\pi^2\,t)/a^2}\,\pi\,w^a}{a^4}\right) = \frac{2\pi^3\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\cos\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^5}$$

Indefinite integral

$$\int \frac{2\,A\,e^{-(2\pi^2\,t)/a^2}\,L\,\pi^2\,w^a\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^4}\,dx = -\frac{2\pi\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\cos\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^3} + \text{constant}$$

Limit

$$\lim_{t\to\infty} \frac{2\,A\,e^{-(2\pi^2\,t)/a^2}\,L\,\pi^2\,w^a\,\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^4} = 0$$

$$\text{for } \left(A,\,L,\,w^a,\,\sin\left(\frac{\pi\,x}{a}\right),\,\sin\left(\frac{\pi\,y}{a}\right)\right) \in \mathbb{R}^5 \wedge a \neq 0$$

$e_1 \wedge e_2 \wedge \ldots$ is the logical AND function

$\mathbb{R}$ is the set of real numbers

From the indefinite integral result

$$\int \frac{2\,A\,e^{-(2\pi^2\,t)/a^2}\,L\,\pi^2\,w^a\,\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^4}\,dx =$$

$$-\frac{2\,\pi\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\,\cos\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^3} + \text{constant}$$

we obtain:

-(2 A e^(-(2 π^2 t)/a^2) L π w^a cos((π x)/a) sin((π y)/a))/a^3

Input

$$-\frac{2\,A\,e^{-(2\pi^2\,t)/a^2}\,L\,\pi\,w^a\,\cos\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^3}$$

Alternate form

$$-\frac{i\,\pi\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\left(e^{-(i\,\pi\,x)/a} + e^{(i\,\pi\,x)/a}\right)\left(e^{-(i\,\pi\,y)/a} - e^{(i\,\pi\,y)/a}\right)}{2\,a^3}$$

Roots

$$w \neq 0, \quad a \neq 0, \quad x = a\,n - \frac{a}{2}, \quad n \in \mathbb{Z}$$

$w \neq 0, \quad a \neq 0, \quad y = a\,n, \quad n \in \mathbb{Z}$

$\mathrm{Re}(a) > 0, \quad w = 0$

$a \neq 0, \quad w \neq 0, \quad A = 0$

$a \neq 0, \quad w \neq 0, \quad L = 0$

$\mathbb{Z}$ is the set of integers

$\mathrm{Re}(z)$ is the real part of z

Periodicity

periodic in t with period $-\frac{i\,a^2}{\pi}$
periodic in x with period $-2\,a$
periodic in y with period $-2\,a$

Series expansion at x=0

$$-\frac{2\left(\pi\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\,\sin\left(\frac{\pi\,y}{a}\right)\right)}{a^3}+\frac{\pi^3\,A\,L\,x^2\,e^{-(2\pi^2\,t)/a^2}\,w^a\,\sin\left(\frac{\pi\,y}{a}\right)}{a^5}-\frac{x^4\left(\pi^5\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\,\sin\left(\frac{\pi\,y}{a}\right)\right)}{12\,a^7}+O\left(x^6\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial x}\left(-\frac{2\,A\,e^{-(2\pi^2\,t)/a^2}\,L\,\pi\,w^a\cos\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^3}\right)=$$
$$\frac{2\pi^2\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^4}$$

Indefinite integral

$$\int -\frac{2\,A\,e^{-(2\pi^2\,t)/a^2}\,L\,\pi\,w^a\cos\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^3}\,dx =$$
$$-\frac{2\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^2}+\text{constant}$$

Limit

$$\lim_{t\to\infty} -\frac{2\,A\,e^{-(2\pi^2\,t)/a^2}\,L\,\pi\,w^a\cos\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^3}=0$$
$$\text{for}\ \left(A,\,L,\,w^a,\,\cos\left(\frac{\pi\,x}{a}\right),\,\sin\left(\frac{\pi\,y}{a}\right)\right)\in\mathbb{R}^5\wedge a\neq 0$$

$e_1 \wedge e_2 \wedge \ldots$ is the logical AND function

$\mathbb{R}$ is the set of real numbers

From the indefinite integral result

$$\int -\frac{2\,A\,e^{-(2\pi^2\,t)/a^2}\,L\,\pi\,w^a\cos\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^3}\,dx =$$
$$-\frac{2\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^2}+\text{constant}$$

we obtain:

-(2 A e^(-(2 π^2 t)/a^2) L w^a sin((π x)/a) sin((π y)/a))/a^2

Input

$$-\frac{2\,A\,e^{-(2\pi^2\,t)/a^2}\,L\,w^a\,\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^2}$$

Alternate form

$$\frac{A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\left(e^{-(i\,\pi\,x)/a}-e^{(i\,\pi\,x)/a}\right)\left(e^{-(i\,\pi\,y)/a}-e^{(i\,\pi\,y)/a}\right)}{2\,a^2}$$

Roots

$w \neq 0, \quad a \neq 0, \quad x = a\,n, \quad n \in \mathbb{Z}$

$w \neq 0, \quad a \neq 0, \quad y = a\,n, \quad n \in \mathbb{Z}$

$\mathrm{Re}(a) > 0, \quad w = 0$

$a \neq 0, \quad w \neq 0, \quad A = 0$

$a \neq 0, \quad w \neq 0, \quad L = 0$

$\mathbb{Z}$ is the set of integers

$\mathrm{Re}(z)$ is the real part of z

Periodicity

periodic in t with period $-\frac{i\,a^2}{\pi}$
periodic in x with period $-2\,a$
periodic in y with period $-2\,a$

Series expansion at x=0

$$-\frac{2x\left(\pi A L e^{-(2\pi^2 t)/a^2} w^a \sin\left(\frac{\pi y}{a}\right)\right)}{a^3}+\frac{\pi^3 A L x^3 e^{-(2\pi^2 t)/a^2} w^a \sin\left(\frac{\pi y}{a}\right)}{3a^5}-\frac{x^5\left(\pi^5 A L e^{-(2\pi^2 t)/a^2} w^a \sin\left(\frac{\pi y}{a}\right)\right)}{60a^7}+O\left(x^6\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial x}\left(-\frac{2A e^{-(2\pi^2 t)/a^2} L w^a \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{a}\right)}{a^2}\right)=-\frac{2\pi A L e^{-(2\pi^2 t)/a^2} w^a \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{a}\right)}{a^3}$$

Indefinite integral

$$\int -\frac{2A e^{-(2\pi^2 t)/a^2} L w^a \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{a}\right)}{a^2}\,dx=\frac{2A L e^{-(2\pi^2 t)/a^2} w^a \cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{a}\right)}{\pi a}+\text{constant}$$

Limit

$$\lim_{t\to\infty} -\frac{2A e^{-(2\pi^2 t)/a^2} L w^a \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{a}\right)}{a^2}=0 \quad \text{for } \left(A, L, w^a, \sin\left(\frac{\pi x}{a}\right), \sin\left(\frac{\pi y}{a}\right)\right)\in\mathbb{R}^5 \wedge a\neq 0$$

$e_1 \wedge e_2 \wedge \ldots$ is the logical AND function

$\mathbb{R}$ is the set of real numbers

From the indefinite integral result

$$\int -\frac{2\,A\,e^{-(2\pi^2\,t)/a^2}\,L\,w^a\,\sin\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{a^2}\,dx = \frac{2\,A\,L\,e^{-(2\pi^2\,t)/a^2}\,w^a\,\cos\left(\frac{\pi\,x}{a}\right)\sin\left(\frac{\pi\,y}{a}\right)}{\pi\,a} + \text{constant}$$

(2 A e^(-(2 π^2 t)/a^2) L w^a cos((π x)/a) sin((π y)/a))/(a π)

for A = 2, a = 4, L = 8, t = 16, w = 32, x = 1/64 and y = 1/128:

(2*2 e^(-(2 π^2 16)/4^2) 8 32^4 cos((π*(1/64))/4) sin((π*(1/128))/4))/(4 π)

Input

$$\frac{(2\times 2)\,e^{-(2\pi^2\times 16)/4^2}\times 8\times 32^4\cos\left(\frac{1}{4}\left(\pi\times\frac{1}{64}\right)\right)\sin\left(\frac{1}{4}\left(\pi\times\frac{1}{128}\right)\right)}{4\,\pi}$$

Exact result

$$\frac{8\,388\,608\,e^{-2\pi^2}\,\sin\left(\frac{\pi}{512}\right)\cos\left(\frac{\pi}{256}\right)}{\pi}$$

Decimal approximation

0.000043828342962120293400495366751404913667456545699756102570220 6…

0.00004382834….

Alternate forms

$$\frac{2\,097\,152\,i\,e^{-2\pi^2}\left(e^{-(i\,\pi)/512}-e^{(i\,\pi)/512}\right)\left(e^{-(i\,\pi)/256}+e^{(i\,\pi)/256}\right)}{\pi}$$

$$\frac{1}{\pi}2097152\sqrt{\left(\left(2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}\right)\left(2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}\right)\right)}\,e^{-2\pi^2}$$

Alternative representations

$$\frac{e^{-(2\pi^2\,16)/4^2}\left(2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)}{4\,\pi}=\frac{32\cosh\left(\frac{i\,\pi}{4\times64}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{4\times128}\right)32^4\,e^{-(32\pi^2)/4^2}}{4\,\pi}$$

$$\frac{e^{-(2\pi^2\,16)/4^2}\left(2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)}{4\,\pi}=-\frac{32\cosh\left(-\frac{i\,\pi}{4\times64}\right)\cos\left(\frac{\pi}{2}+\frac{\pi}{4\times128}\right)32^4\,e^{-(32\pi^2)/4^2}}{4\,\pi}$$

$$\frac{e^{-(2\pi^2\,16)/4^2}\left(2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)}{4\,\pi}=\frac{32\cosh\left(-\frac{i\,\pi}{4\times64}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{4\times128}\right)32^4\,e^{-(32\pi^2)/4^2}}{4\,\pi}$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Series representations

$$\frac{e^{-(2\pi^2\ 16)/4^2}\left(2\times 2\times 8\left(32^4\cos\left(\frac{\pi}{64\times 4}\right)\sin\left(\frac{\pi}{128\times 4}\right)\right)\right)}{4\pi} = \frac{8\,388\,608\,e^{-2\pi^2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+3k_2}\,2^{-16k_1-18k_2}\times 255^{2k_2}\,\pi^{2k_1+2k_2}}{(2k_1)!\,(2k_2)!}}{\pi}$$

$$\frac{e^{-(2\pi^2\ 16)/4^2}\left(2\times 2\times 8\left(32^4\cos\left(\frac{\pi}{64\times 4}\right)\sin\left(\frac{\pi}{128\times 4}\right)\right)\right)}{4\pi} = \frac{8\,388\,608\,e^{-2\pi^2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1+k_2}\,2^{-9-16k_1-18k_2}\,\pi^{1+2k_1+2k_2}}{(2k_1)!\,(1+2k_2)!}}{\pi}$$

$$\frac{e^{-(2\pi^2\ 16)/4^2}\left(2\times 2\times 8\left(32^4\cos\left(\frac{\pi}{64\times 4}\right)\sin\left(\frac{\pi}{128\times 4}\right)\right)\right)}{4\pi} = \frac{8\,388\,608\,e^{-2\pi^2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_2}\,2^{-17-16k_1-18k_2}\times 127^{1+2k_1}\,e^{3i\pi k_1}\,\pi^{2+2k_1+2k_2}}{(1+2k_1)!\,(1+2k_2)!}}{\pi}$$

$n!$ is the factorial function

Integral representations

$$\frac{e^{-(2\pi^2\ 16)/4^2}\left(2\times 2\times 8\left(32^4\cos\left(\frac{\pi}{64\times 4}\right)\sin\left(\frac{\pi}{128\times 4}\right)\right)\right)}{4\pi} = -\frac{8192\,i\,e^{-2\pi^2}\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^2/(262\,144\,s)+s}}{\sqrt{s}}\,ds\right)\int_0^1\cos\left(\frac{\pi t}{512}\right)dt}{\sqrt{\pi}} \quad \text{for } \gamma > 0$$

$$\frac{e^{-(2\pi^2\ 16)/4^2}\left(2\times 2\times 8\left(32^4\cos\left(\frac{\pi}{64\times 4}\right)\sin\left(\frac{\pi}{128\times 4}\right)\right)\right)}{4\pi} = -\frac{2048\,e^{-2\pi^2}\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^2/(1\,048\,576\,s)+s}}{s^{3/2}}\,ds\right)\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^2/(262\,144\,s)+s}}{\sqrt{s}}\,ds}{\pi} \quad \text{for } \gamma > 0$$

$$\frac{e^{-(2\pi^2\,16)/4^2}\left(2\times 2\times 8\left(32^4\cos\left(\frac{\pi}{64\times 4}\right)\sin\left(\frac{\pi}{128\times 4}\right)\right)\right)}{4\pi} = \frac{4096\,i\,e^{-2\pi^2}\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(1048\,576\,s)+s}}{s^{3/2}}\,ds\right)\int_{\frac{\pi}{2}}^{\frac{\pi}{256}}\sin(t)\,dt}{\sqrt{\pi}} \quad \text{for } \gamma > 0$$

From which:

1/5(1/(((2*2 e^(-(2 π^2 16)/4^2) 8 32^4 cos((π*(1/64))/4) sin((π*(1/128))/4))/(4 π))))-521+47+7-1/4

Input

$$\frac{1}{5}\times\frac{1}{\frac{(2\times 2)\,e^{-(2\pi^2\times 16)/4^2}\times 8\times 32^4\cos\left(\frac{1}{4}\left(\pi\times\frac{1}{64}\right)\right)\sin\left(\frac{1}{4}\left(\pi\times\frac{1}{128}\right)\right)}{4\pi}} - 521 + 47 + 7 - \frac{1}{4}$$

Exact result

$$\frac{e^{2\pi^2}\,\pi\csc\left(\frac{\pi}{512}\right)\sec\left(\frac{\pi}{256}\right)}{41\,943\,040} - \frac{1869}{4}$$

csc(x) is the cosecant function

sec(x) is the secant function

Decimal approximation

4096.007163814174825257906588547322976816580996076572079268145182 3…

4096.007163814…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192.

The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{e^{2\pi^2}\,\pi\csc\left(\frac{\pi}{512}\right)\sec\left(\frac{\pi}{256}\right)-19\,597\,885\,440}{41\,943\,040}$$

$$\frac{\csc\left(\frac{\pi}{512}\right)\sec\left(\frac{\pi}{256}\right)\left(e^{2\pi^2}\,\pi-19\,597\,885\,440\sin\left(\frac{\pi}{512}\right)\cos\left(\frac{\pi}{256}\right)\right)}{41\,943\,040}$$

$$-\frac{1869}{4}-\frac{e^{2\pi^2}\,\pi\sin\left(\frac{\pi}{512}\right)\cos\left(\frac{\pi}{256}\right)}{10\,485\,760\left(\cos\left(\frac{\pi}{256}\right)-1\right)\left(1+\cos\left(\frac{\pi}{128}\right)\right)}$$

$\csc(x)$ is the cosecant function

$\sec(x)$ is the secant function

Alternative representations

$$\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\,2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}=$$
$$-467-\frac{1}{4}+\frac{1}{\frac{5\left(32\cosh\left(\frac{i\,\pi}{4\times64}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{4\times128}\right)32^4\,e^{-(32\pi^2)/4^2}\right)}{4\pi}}$$

$$\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\,2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}=$$
$$-467-\frac{1}{4}+\frac{1}{\frac{5\left(-32\cosh\left(-\frac{i\,\pi}{4\times64}\right)\cos\left(\frac{\pi}{2}+\frac{\pi}{4\times128}\right)32^4\,e^{-(32\pi^2)/4^2}\right)}{4\pi}}$$

$$\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\,2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}=$$
$$-467-\frac{1}{4}+\frac{1}{\frac{5\left(32\cosh\left(-\frac{i\,\pi}{4\times64}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{4\times128}\right)32^4\,e^{-(32\pi^2)/4^2}\right)}{4\pi}}$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Series representations

$$\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\,2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}=$$
$$-\frac{1869}{4}-\frac{e^{2\pi^2}\,\pi\csc\left(\frac{\pi}{512}\right)\sum_{k=1}^{\infty}(-1)^k\,q^{-1+2k}}{20\,971\,520}\quad\text{for } q=\sqrt[256]{-1}$$

$$\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\,2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}=$$

$$-\frac{1869}{4}-\frac{i\,e^{2\pi^2}\,\pi\sec\left(\frac{\pi}{256}\right)\sum_{k=1}^{\infty}q^{-1+2k}}{20\,971\,520}\quad\text{for } q=\sqrt[512]{-1}$$

$$\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\,2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}=$$

$$-\frac{1869}{4}+\frac{e^{2\pi^2}\,\pi\sec\left(\frac{\pi}{256}\right)\sum_{k=-\infty}^{\infty}\frac{e^{i\,k\,\pi}}{\pi-262\,144\,k^2\,\pi}}{81\,920}$$

Integral representation

$$\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\,2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}=$$

$$\frac{-9\,798\,942\,720\,\pi+e^{2\pi^2}\left(\int_0^{\infty}\frac{\sqrt[128]{t}}{1+t^2}\,dt\right)\int_0^{\infty}\frac{\sqrt[512]{t}}{t+t^2}\,dt}{20\,971\,520\,\pi}$$

27*sqrt((1/5(1/(((2*2 e^(-(2 π^2 16)/4^2) 8 32^4 cos((π*(1/64))/4) sin((π*(1/128))/4))/(4 π))))-521+47+7-1/4))+1

Input

$$27\sqrt{\frac{1}{5}\times\frac{1}{\frac{(2\times2)\,e^{-(2\pi^2\times16)/4^2}\times8\times32^4\cos\left(\frac{1}{4}\left(\pi\times\frac{1}{64}\right)\right)\sin\left(\frac{1}{4}\left(\pi\times\frac{1}{128}\right)\right)}{4\pi}}-521+47+7-\frac{1}{4}}+1$$

Exact result

$$1 + 27\sqrt{\frac{e^{2\pi^2}\,\pi \csc\left(\frac{\pi}{512}\right)\sec\left(\frac{\pi}{256}\right)}{41\,943\,040} - \frac{1869}{4}}$$

csc(*x*) is the cosecant function

sec(*x*) is the secant function

Decimal approximation

1729.00151111639177513412106551454475184224557716759926921220251 63

…

1729.00151111….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons.

This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$\frac{20\,480 + 27\sqrt{10\left(e^{2\pi^2}\,\pi\csc\left(\frac{\pi}{512}\right)\sec\left(\frac{\pi}{256}\right) - 19\,597\,885\,440\right)}}{20\,480}$$

$$1 + \frac{27\sqrt{\frac{1}{10}\csc\left(\frac{\pi}{512}\right)\sec\left(\frac{\pi}{256}\right)\left(e^{2\pi^2}\,\pi - 19\,597\,885\,440\sin\left(\frac{\pi}{512}\right)\cos\left(\frac{\pi}{256}\right)\right)}}{2048}$$

$$1 + 27\sqrt{-\frac{1869}{4} - \frac{e^{2\pi^2}\,\pi\sin\left(\frac{\pi}{512}\right)\cos\left(\frac{\pi}{256}\right)}{10\,485\,760\left(\cos\left(\frac{\pi}{256}\right) - 1\right)\left(1 + \cos\left(\frac{\pi}{128}\right)\right)}}$$

csc(x) is the cosecant function

sec(x) is the secant function

Alternative representations

$$27\sqrt{\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\;2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}}+1=$$

$$1+27\sqrt{-467-\frac{1}{4}+\frac{1}{\frac{5\left(32\cosh\left(\frac{i\pi}{4\times64}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{4\times128}\right)32^4\,e^{-(32\pi^2)/4^2}\right)}{4\pi}}}$$

$$27\sqrt{\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\;2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}}+1=$$

$$1+27\sqrt{-467-\frac{1}{4}+\frac{1}{\frac{5\left(-32\cosh\left(-\frac{i\pi}{4\times64}\right)\cos\left(\frac{\pi}{2}+\frac{\pi}{4\times128}\right)32^4\,e^{-(32\pi^2)/4^2}\right)}{4\pi}}}$$

$$27\sqrt{\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\;2\times2\times8\left(32^4\cos\left(\frac{\pi}{64\times4}\right)\sin\left(\frac{\pi}{128\times4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}}+1=$$

$$1+27\sqrt{-467-\frac{1}{4}+\frac{1}{\frac{5\left(32\cosh\left(-\frac{i\pi}{4\times64}\right)\cos\left(\frac{\pi}{2}-\frac{\pi}{4\times128}\right)32^4\,e^{-(32\pi^2)/4^2}\right)}{4\pi}}}$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Series representations

$$27\sqrt{\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\;2\cdot 2\cdot 8\left(32^4\cos\left(\frac{\pi}{64\cdot 4}\right)\sin\left(\frac{\pi}{128\cdot 4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}}+1=$$
$$1+27\sqrt{-\frac{1869}{4}+\frac{e^{2\pi^2}\sec\left(\frac{\pi}{256}\right)\left(131\,072+\sum_{k=1}^{\infty}\frac{(-1)^k}{\frac{1}{262\,144}-k^2}\right)}{10\,737\,418\,240}}$$

$$27\sqrt{\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\;2\cdot 2\cdot 8\left(32^4\cos\left(\frac{\pi}{64\cdot 4}\right)\sin\left(\frac{\pi}{128\cdot 4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}}+1=$$
$$1+27\sqrt{-\frac{1869}{4}-\frac{i\,e^{2\pi^2}\,\pi\sec\left(\frac{\pi}{256}\right)\sum_{k=1}^{\infty}q^{-1+2k}}{20\,971\,520}}\quad\text{for } q=\sqrt[512]{-1}$$

$$27\sqrt{\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\;2\cdot 2\cdot 8\left(32^4\cos\left(\frac{\pi}{64\cdot 4}\right)\sin\left(\frac{\pi}{128\cdot 4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}}+1=$$
$$1+27\sqrt{-\frac{1869}{4}+\frac{e^{2\pi^2}\,\pi\sec\left(\frac{\pi}{256}\right)\sum_{k=-\infty}^{\infty}\frac{e^{i\,k\,\pi}}{\pi-262\,144\,k^2\,\pi}}{81\,920}}$$

Integral representation

$$27\sqrt{\frac{1}{\frac{\left(e^{-(2\pi^2\,16)/4^2}\;2\cdot 2\cdot 8\left(32^4\cos\left(\frac{\pi}{64\cdot 4}\right)\sin\left(\frac{\pi}{128\cdot 4}\right)\right)\right)5}{4\pi}}-521+47+7-\frac{1}{4}}+1=$$
$$\frac{10\,240+27\sqrt{\frac{5}{\pi}}\sqrt{-9\,798\,942\,720\,\pi+e^{2\pi^2}\left(\int_0^{\infty}\frac{\sqrt[128]{t}}{1+t^2}\,dt\right)\int_0^{\infty}\frac{\sqrt[512]{t}}{t+t^2}\,dt}}{10\,240}$$

((27*sqrt((1/5(1/(((2*2 e^(-(2 π^2 16)/4^2) 8 32^4 cos((π*(1/64))/4) sin((π*(1/128))/4))/(4 π))))-521+47+7-1/4))+1))^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{27\sqrt{\frac{1}{5}\times\frac{1}{\frac{(2\times2)\,e^{-(2\pi^2\times16)/4^2}\times8\times32^4\cos\left(\frac{1}{4}\left(\pi\times\frac{1}{64}\right)\right)\sin\left(\frac{1}{4}\left(\pi\times\frac{1}{128}\right)\right)}{4\pi}}-521+47+7-\frac{1}{4}}+1}+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}+\sqrt[15]{1+27\sqrt{\frac{e^{2\pi^2}\,\pi\csc\left(\frac{\pi}{512}\right)\sec\left(\frac{\pi}{256}\right)}{41\,943\,040}-\frac{1869}{4}}}$$

$\csc(x)$ is the cosecant function

$\sec(x)$ is the secant function

Decimal approximation

1.644938119907978557266643990272904218269708943491705199070960291 5…

1.6449381199…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}+\sqrt[15]{1+27\sqrt{-\frac{1869}{4}-\frac{e^{2\pi^2}\,\pi\sin\left(\frac{\pi}{512}\right)\cos\left(\frac{\pi}{256}\right)}{10\,485\,760\left(\cos\left(\frac{\pi}{256}\right)-1\right)\left(1+\cos\left(\frac{\pi}{128}\right)\right)}}}$$

$$\frac{1}{10} C_{\text{MRB}}^{-1/(4\pi)} \left(10 C_{\text{MRB}}^{1+\pi} + \sqrt[5]{2}\, 5^{14/15} \sqrt[15]{20480 + 27 \sqrt{10 \left(e^{2\pi^2}\, \pi \csc\left(\frac{\pi}{512}\right) \sec\left(\frac{\pi}{256}\right) - 19597885440 \right)}} \sqrt[4\pi]{C_{\text{MRB}}} \right)$$

$$\frac{C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{2048\sqrt{10} + 27 \sqrt{\csc\left(\frac{\pi}{512}\right) \sec\left(\frac{\pi}{256}\right) \left(e^{2\pi^2}\, \pi - 19597885440 \sin\left(\frac{\pi}{512}\right) \cos\left(\frac{\pi}{256}\right) \right)}}}{2^{23/30} \sqrt[30]{5}}$$

Now, we analyze the following equation:

$$\mathrm{F}\left(\frac{\zeta_o^2}{4 t_{\text{th}}}\right) = \int_0^1 \frac{e^{-\frac{\zeta_o^2}{4 t \chi}}}{\sqrt{\chi (1-\chi)}}\, d\chi .$$

We consider:

(e^(-(ζ)^2)/(4*t*χ))/(sqrt(χ*(1-χ)))

Input

$$\frac{\frac{e^{-\zeta(u;g_2,g_3)^2}}{4 t \chi}}{\sqrt{\chi (1-\chi)}}$$

$\zeta(u; g_2, g_3)$ is the Weierstrass zeta function

Exact result

$$\frac{e^{-\zeta(u;g_2,g_3)^2}}{4\,t\,\chi\,\sqrt{(1-\chi)\,\chi}}$$

Alternative representations

$$\frac{e^{-\zeta(u;g_2,g_3)^2}}{\sqrt{\chi\,(1-\chi)}\;(4\,t\,\chi)} = \frac{w^a}{\sqrt{\chi\,(1-\chi)}\;(4\,t\,\chi)} \quad \text{for } a + \frac{\zeta(u;\,g_2,\,g_3)^2}{\log(w)} = 0$$

$$\frac{e^{-\zeta(u;g_2,g_3)^2}}{\sqrt{\chi\,(1-\chi)}\;(4\,t\,\chi)} = \frac{z^{-\zeta(u;g_2,g_3)^2}}{\sqrt{\chi\,(1-\chi)}\;(4\,t\,\chi)} \quad \text{for } z = e$$

$$\frac{e^{-\zeta(u;g_2,g_3)^2}}{\sqrt{\chi\,(1-\chi)}\;(4\,t\,\chi)} = \frac{e^{-\left(\frac{\partial \log(\sigma(u;g_2,g_3))}{\partial u}\right)^2}}{(4\,t\,\chi)\,\sqrt{(1-\chi)\,\chi}}$$

$\log(x)$ is the natural logarithm

$\sigma(u;\,g_2,\,g_3)$ is the Weierstrass sigma function

From the alternative representation:

$$\frac{e^{-\zeta(u;g_2,g_3)^2}}{\sqrt{\chi\,(1-\chi)}\;(4\,t\,\chi)} = \frac{w^a}{\sqrt{\chi\,(1-\chi)}\;(4\,t\,\chi)} \quad \text{for } a + \frac{\zeta(u;\,g_2,\,g_3)^2}{\log(w)} = 0$$

we obtain:

w^a/(sqrt(χ*(1 - χ)) (4 t χ))

Input

$$\frac{w^a}{\sqrt{\chi\,(1-\chi)}\;(4\,t\,\chi)}$$

Result

$$\frac{w^a}{4\,t\,\chi\,\sqrt{(1-\chi)\,\chi}}$$

Expanded forms

$$\frac{\sqrt{(1-\chi)\,\chi}\;w^a}{4\,t\,(1-\chi)\,\chi^2}$$

$$\frac{w^a}{4\,t\,\chi\,\sqrt{\chi-\chi^2}}$$

Alternate forms

$$\frac{w^a}{4\,t\,\chi\,\sqrt{-(\chi-1)\,\chi}}$$

$$\frac{w^a}{4\,t\,\sqrt{1-\chi}\;\chi^{3/2}}$$

Root

$$\mathrm{Re}(a) > 0\,,\quad w = 0$$

Re(z) is the real part of z

Derivative

$$\frac{\partial}{\partial w}\left(\frac{w^a}{\sqrt{\chi\,(1-\chi)}\;(4\,t\,\chi)}\right)=\frac{a\,w^{a-1}}{4\,t\,\chi\,\sqrt{-(\chi-1)\,\chi}}$$

Indefinite integral

$$\int\frac{w^a}{4\,t\,\chi\,\sqrt{(1-\chi)\,\chi}}\,dw=\frac{w^{a+1}}{\sqrt{1-\chi}\;\chi^{3/2}\,(4\,a\,t+4\,t)}+\text{constant}$$

Limit

$$\lim_{t\to\pm\infty}\frac{w^a}{4\,t\,\chi\,\sqrt{(1-\chi)\,\chi}}=0$$

Series representations

$$\frac{w^a}{\sqrt{\chi\,(1-\chi)}\;4\,(t\,\chi)}=\frac{w^a}{4\,t\,\chi\,\sqrt{-1+\chi-\chi^2}\;\sum_{k=0}^{\infty}\left(-1+\chi-\chi^2\right)^{-k}\binom{\frac{1}{2}}{k}}$$

for $\left|1-\chi+\chi^2\right|>1$

$$\frac{w^a}{\sqrt{\chi\,(1-\chi)}\;4\,(t\,\chi)}=\frac{w^a}{4\,t\,\chi\,\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\chi-\chi^2\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\quad\text{for}\ \left|1-\chi+\chi^2\right|<1$$

$$\frac{w^a}{\sqrt{\chi\,(1-\chi)}\;4\,(t\,\chi)}=\frac{w^a}{4\,t\,\chi\,\sqrt{-1+\chi-\chi^2}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\chi-\chi^2\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for $\left|1-\chi+\chi^2\right|>1$

$\binom{n}{m}$ is the binomial coefficient

$|z|$ is the absolute value of z

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

From the indefinite integral result

$$\int \frac{w^a}{4\,t\,\chi\,\sqrt{(1-\chi)\,\chi}}\,dw = \frac{w^{a+1}}{\sqrt{1-\chi}\;\chi^{3/2}\,(4\,a\,t+4\,t)} + \text{constant}$$

we obtain:

w^(1 + a)/((4 t + 4 a t) sqrt(1 - χ) χ^(3/2))

Input

$$\frac{w^{1+a}}{(4\,t+4\,a\,t)\,\sqrt{1-\chi}\;\chi^{3/2}}$$

Root

$$\text{Re}(a) > -1\,, \quad w = 0$$

Re(z) is the real part of z

Derivative

$$\frac{\partial}{\partial w}\left(\frac{w^{1+a}}{(4\,t+4\,a\,t)\,\sqrt{1-\chi}\;\chi^{3/2}}\right) = \frac{w^a}{4\,t\,\chi^{3/2}\,\sqrt{1-\chi}}$$

Indefinite integral

$$\int \frac{w^{1+a}}{(4\,t+4\,a\,t)\,\sqrt{1-\chi}\;\chi^{3/2}}\,dw = \frac{w^{a+2}}{(a+2)\,\sqrt{1-\chi}\;\chi^{3/2}\,(4\,a\,t+4\,t)} + \text{constant}$$

Limit

$$\lim_{t\to\pm\infty} \frac{w^{1+a}}{(4\,t+4\,a\,t)\,\sqrt{1-\chi}\;\chi^{3/2}} = 0$$

Series representations

$$\frac{w^{1+a}}{(4\,t+4\,a\,t)\,\sqrt{1-\chi}\;\chi^{3/2}} = \frac{w^{1+a}}{(4\,t+4\,a\,t)\,\chi^{3/2}\,\sqrt{-\chi}\;\sum_{k=0}^{\infty}(-\chi)^{-k}\binom{\frac{1}{2}}{k}} \quad \text{for } |\chi| > 1$$

$$\frac{w^{1+a}}{(4\,t+4\,a\,t)\,\sqrt{1-\chi}\;\chi^{3/2}} = \frac{w^{1+a}}{(4\,t+4\,a\,t)\,\chi^{3/2}\sum_{k=0}^{\infty}\frac{(-1)^k(-\chi)^k\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } |\chi| < 1$$

$$\frac{w^{1+a}}{(4\,t+4\,a\,t)\,\sqrt{1-\chi}\;\chi^{3/2}} = \frac{w^{1+a}}{(4\,t+4\,a\,t)\,\chi^{3/2}\,\sqrt{-\chi}\;\sum_{k=0}^{\infty}\frac{(-1)^k(-\chi)^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for $|\chi| > 1$

$\binom{n}{m}$ is the binomial coefficient

$|z|$ is the absolute value of z

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

And again, from the indefinite integral result

$$\int \frac{w^{1+a}}{(4\,t+4\,a\,t)\,\sqrt{1-\chi}\;\chi^{3/2}}\,dw = \frac{w^{a+2}}{(a+2)\,\sqrt{1-\chi}\;\chi^{3/2}\,(4\,a\,t+4\,t)} + \text{constant}$$

w^(2 + a)/((2 + a) (4 t + 4 a t) sqrt(1 - χ) χ^(3/2))

for a = 2, χ = 4, t = 8 and w = 16:

16^(2 + 2)/((2 + 2) (4*8 + 4*2*8) sqrt(1 - 4) 4^(3/2))

Input

$$\frac{16^{2+2}}{(2+2)\,(4\times 8+4\times 2\times 8)\,\sqrt{1-4}\;\times 4^{3/2}}$$

Result

$$-\frac{64\,i\,\sqrt{3}}{9}$$

Decimal approximation

– 12.316805742712016309528507317375092387148837360429373355063516 2… i

-12.316805742712…i

Alternate complex forms

$$\frac{64\left(\cos\left(-\frac{\pi}{2}\right)+i\sin\left(-\frac{\pi}{2}\right)\right)}{3\,\sqrt{3}}$$

$$\frac{64\,e^{-(i\,\pi)/2}}{3\,\sqrt{3}}$$

Polar coordinates

$r = \frac{64}{3\sqrt{3}}$ (radius), $\theta = -\frac{\pi}{2}$ (angle)

From which, after some calculations, we obtain:

-(1/6(1/4(-(16^(2 + 2)/((2 + 2) (4*8 + 4*2*8) sqrt(1 - 4) 4^(3/2)))))^2-Pi/83)

Input

$$-\left(\frac{1}{6}\left(\frac{1}{4}\left(-\frac{16^{2+2}}{(2+2)\,(4\times 8+4\times 2\times 8)\,\sqrt{1-4}\,\times 4^{3/2}}\right)\right)^2-\frac{\pi}{83}\right)$$

Result

$$\frac{128}{81}+\frac{\pi}{83}$$

Decimal approximation

1.618097427478919121272567918198072249534429677428139754796812511 0…

1.618097427489…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Property

$\frac{128}{81}+\frac{\pi}{83}$ is a transcendental number

Alternate form

$$\frac{10624+81\,\pi}{6723}$$

Series representations

$$-\left(\frac{1}{6}\left(-\frac{16^{2+2}}{4\left((2+2)\left(4\times 8+4\times 2\times 8\right)\sqrt{1-4}\;4^{3/2}\right)}\right)^2-\frac{\pi}{83}\right)=\frac{128}{81}+\frac{4}{83}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k}$$

$$-\left(\frac{1}{6}\left(-\frac{16^{2+2}}{4\left((2+2)\left(4\times 8+4\times 2\times 8\right)\sqrt{1-4}\;4^{3/2}\right)}\right)^2-\frac{\pi}{83}\right)=$$
$$\frac{128}{81}+\sum_{k=0}^{\infty}-\frac{4\,(-1)^k\;1195^{-1-2\,k}\left(5^{1+2\,k}-4\times 239^{1+2\,k}\right)}{83\,(1+2\,k)}$$

$$-\left(\frac{1}{6}\left(-\frac{16^{2+2}}{4\left((2+2)\left(4\times 8+4\times 2\times 8\right)\sqrt{1-4}\;4^{3/2}\right)}\right)^2-\frac{\pi}{83}\right)=$$
$$\frac{128}{81}+\frac{1}{83}\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2\,k}+\frac{2}{1+4\,k}+\frac{1}{3+4\,k}\right)$$

Integral representations

$$-\left(\frac{1}{6}\left(-\frac{16^{2+2}}{4\left((2+2)\left(4\times 8+4\times 2\times 8\right)\sqrt{1-4}\;4^{3/2}\right)}\right)^2-\frac{\pi}{83}\right)=$$
$$\frac{128}{81}+\frac{4}{83}\int_0^1\sqrt{1-t^2}\;dt$$

$$-\left(\frac{1}{6}\left(-\frac{16^{2+2}}{4\left((2+2)\left(4\times 8+4\times 2\times 8\right)\sqrt{1-4}\;4^{3/2}\right)}\right)^2-\frac{\pi}{83}\right)=$$
$$\frac{128}{81}+\frac{2}{83}\int_0^1\frac{1}{\sqrt{1-t^2}}\;dt$$

$$-\left(\frac{1}{6}\left(-\frac{16^{2+2}}{4\left((2+2)\left(4\times 8+4\times 2\times 8\right)\sqrt{1-4}\;4^{3/2}\right)}\right)^2-\frac{\pi}{83}\right)=\frac{128}{81}+\frac{2}{83}\int_0^{\infty}\frac{1}{1+t^2}\;dt$$

Now, we have that:

$$(9) \qquad n = \sum_{j,k=1}^{\infty} C_{jk} \frac{b_{jk}}{2} \sum_{i=1,2,3} f_i \, e^{\frac{3 r_i^2}{4 \lambda^2 N}} \left\{ \left[1 - \Theta\left(\frac{z}{r_i} + \frac{r_i}{2 b_{jk}} \right) \right] e^{\frac{z}{b_{jk}}} + \left[1 + \Theta\left(\frac{z}{r_i} - \frac{r_i}{2 b_{jk}} \right) \right] e^{-\frac{z}{b_{jk}}} \right\}$$

where

$$C_{jk} = \frac{6}{\lambda v a^2} \sin \frac{\pi j u}{a} \sin \frac{\pi k v}{a} \sin \frac{\pi j x}{a} \sin \frac{\pi k y}{a}$$

$$\frac{1}{b_{jk}^2} = \frac{3}{\lambda^2 N} + \frac{\pi^2}{a^2} (j^2 + k^2)$$

$$\Theta(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy.$$

We consider:

6/(λ*v*a^2) sin(π*j*u)/a * sin(π*k*v)/a * sin(π*j*x)/a * sin(π*k*y)/a

Input

$$\frac{6}{\lambda v a^2} \sin\left(\frac{\pi j u}{a}\right) \sin\left(\frac{\pi k v}{a}\right) \sin\left(\frac{\pi j x}{a}\right) \sin\left(\frac{\pi k y}{a}\right)$$

Exact result

$$\frac{6 \sin\left(\frac{\pi j u}{a}\right) \sin\left(\frac{\pi j x}{a}\right) \sin\left(\frac{\pi k v}{a}\right) \sin\left(\frac{\pi k y}{a}\right)}{a^2 \lambda v}$$

Reduced trigonometric form

$$\frac{1}{a^2 \nu \lambda} 3\left(\frac{1}{2}\left(\cos\left(\frac{j\pi u}{a}-\frac{k\pi v}{a}+\frac{j\pi x}{a}-\frac{k\pi y}{a}\right)+\cos\left(\frac{j\pi u}{a}-\frac{k\pi v}{a}-\frac{j\pi x}{a}+\frac{k\pi y}{a}\right)\right)+ -\frac{1}{2}\left(\cos\left(\frac{j\pi u}{a}+\frac{k\pi v}{a}+\frac{j\pi x}{a}-\frac{k\pi y}{a}\right)+\cos\left(\frac{j\pi u}{a}+\frac{k\pi v}{a}-\frac{j\pi x}{a}+\frac{k\pi y}{a}\right)\right)+ -\frac{1}{2}\left(\cos\left(\frac{j\pi u}{a}-\frac{k\pi v}{a}-\frac{j\pi x}{a}-\frac{k\pi y}{a}\right)+\cos\left(\frac{j\pi u}{a}-\frac{k\pi v}{a}+\frac{j\pi x}{a}+\frac{k\pi y}{a}\right)\right)+ \frac{1}{2}\left(\cos\left(\frac{j\pi u}{a}+\frac{k\pi v}{a}-\frac{j\pi x}{a}-\frac{k\pi y}{a}\right)+\cos\left(\frac{j\pi u}{a}+\frac{k\pi v}{a}+\frac{j\pi x}{a}+\frac{k\pi y}{a}\right)\right)\right)\frac{1}{2}$$

Alternate form

$$\frac{1}{8a^2\lambda\nu}3\left(e^{-(i\pi ju)/a}-e^{(i\pi ju)/a}\right)\left(e^{-(i\pi jx)/a}-e^{(i\pi jx)/a}\right)\left(e^{-(i\pi kv)/a}-e^{(i\pi kv)/a}\right) \left(e^{-(i\pi ky)/a}-e^{(i\pi ky)/a}\right)$$

Roots

$a\lambda\nu \neq 0, \quad j = 0$

$a\lambda\nu \neq 0, \quad k = 0$

$a\lambda\nu \neq 0, \quad j \neq 0, \quad u = \frac{2an}{j}, \quad n \in \mathbb{Z}$

$a\lambda\nu \neq 0, \quad j \neq 0, \quad u = \frac{a(2n+1)}{j}, \quad n \in \mathbb{Z}$

$a\lambda\nu \neq 0, \quad j \neq 0, \quad x = \frac{2an}{j}, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

Properties as a function

Periodicity

periodic in u with period $-\frac{2a}{j}$

periodic in x with period $-\frac{2a}{j}$

periodic in y with period $-\frac{2a}{k}$

Parity

odd

Root for the variable u

$$u = \frac{a\,(2\,\pi\,c_1 + \pi)}{\pi\,j}$$

Series expansion at x=0

$$\frac{6\,\pi\,j\,x\,\sin\left(\frac{\pi\,j\,u}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{a^3\,\lambda\,v} - \frac{\pi^3\,j^3\,x^3\,\sin\left(\frac{\pi\,j\,u}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{a^5\,\lambda\,v} + \frac{\pi^5\,j^5\,x^5\,\sin\left(\frac{\pi\,j\,u}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{20\,a^7\,\lambda\,v} + O\left(x^6\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial x}\left(\frac{6\sin\left(\frac{\pi\,j\,u}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,j\,x}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{\lambda\,v\,a^2}\right) = \frac{6\,\pi\,j\sin\left(\frac{\pi\,j\,u}{a}\right)\cos\left(\frac{\pi\,j\,x}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{a^3\,\lambda\,v}$$

Indefinite integral

$$\int \frac{6 \sin\left(\frac{j\pi u}{a}\right) \sin\left(\frac{k\pi v}{a}\right) \sin\left(\frac{j\pi x}{a}\right) \sin\left(\frac{k\pi y}{a}\right)}{a^2 v \lambda}\, dx = -\frac{6 \sin\left(\frac{\pi j u}{a}\right) \cos\left(\frac{\pi j x}{a}\right) \sin\left(\frac{\pi k v}{a}\right) \sin\left(\frac{\pi k y}{a}\right)}{\pi a j \lambda v} + \text{constant}$$

Limit

$$\lim_{v \to \pm\infty} \frac{6 \sin\left(\frac{j\pi u}{a}\right) \sin\left(\frac{k\pi v}{a}\right) \sin\left(\frac{j\pi x}{a}\right) \sin\left(\frac{k\pi y}{a}\right)}{a^2 v \lambda} = 0 \text{ for } \frac{k}{a} \in \mathbb{R}$$

$\mathbb{R}$ is the set of real numbers

From

$$\frac{1}{b_{jk}^2} = \frac{3}{\lambda^2 N} + \frac{\pi^2}{a^2}(j^2 + k^2)$$

we obtain:

3/(λ^2*N) + Pi^2/a^2 * (j^2+k^2)

Input

$$\frac{3}{\lambda^2 N} + \frac{\pi^2}{a^2}\left(j^2 + k^2\right)$$

Result

$$\frac{\pi^2\left(j^2 + k^2\right)}{a^2} + \frac{3}{\lambda^2 N}$$

Alternate forms

$$\frac{3\,a^2+\pi^2\,j^2\,\lambda^2\,N+\pi^2\,k^2\,\lambda^2\,N}{a^2\,\lambda^2\,N}$$

$$\frac{\pi^2\,j^2+\pi^2\,k^2}{a^2}+\frac{3}{\lambda^2\,N}$$

$$\frac{3\,a^2+\pi^2\,\lambda^2\,N\left(j^2+k^2\right)}{a^2\,\lambda^2\,N}$$

Expanded form

$$\frac{\pi^2\,j^2}{a^2}+\frac{\pi^2\,k^2}{a^2}+\frac{3}{\lambda^2\,N}$$

Roots

$$N\left(j^2+k^2\right)\neq 0\,,\quad \lambda=-\frac{\sqrt{3}\;a}{\pi\sqrt{N\left(-\left(j^2+k^2\right)\right)}}\,,\quad a\neq 0$$

$$N\left(j^2+k^2\right)\neq 0\,,\quad \lambda=\frac{\sqrt{3}\;a}{\pi\sqrt{N\left(-\left(j^2+k^2\right)\right)}}\,,\quad a\neq 0$$

Property as a function

Parity

even

Roots for the variable λ

$$\lambda = -\frac{\sqrt{3}\ a}{\pi\sqrt{j^2(-N) - k^2 N}}$$

$$\lambda = \frac{\sqrt{3}\ a}{\pi\sqrt{j^2(-N) - k^2 N}}$$

Derivative

$$\frac{\partial}{\partial a}\left(\frac{3}{\lambda^2 N} + \frac{\pi^2\left(j^2+k^2\right)}{a^2}\right) = -\frac{2\pi^2\left(j^2+k^2\right)}{a^3}$$

Indefinite integral

$$\int\left(\frac{\left(j^2+k^2\right)\pi^2}{a^2} + \frac{3}{N\lambda^2}\right)da = \frac{3a}{\lambda^2 N} - \frac{\pi^2\left(j^2+k^2\right)}{a} + \text{constant}$$

Limit

$$\lim_{\lambda\to\pm\infty}\left(\frac{\left(j^2+k^2\right)\pi^2}{a^2} + \frac{3}{N\lambda^2}\right) = \frac{\pi^2 j^2 N + \pi^2 k^2 N}{a^2 N} \approx \frac{9.8696\, j^2 N + 9.8696\, k^2 N}{a^2 N}$$

Alternative representations

$$\frac{3}{\lambda^2 N} + \frac{\left(j^2+k^2\right)\pi^2}{a^2} = \frac{(180^\circ)^2\left(j^2+k^2\right)}{a^2} + \frac{3}{N\lambda^2}$$

$$\frac{3}{\lambda^2 N} + \frac{\left(j^2+k^2\right)\pi^2}{a^2} = \frac{\left(j^2+k^2\right)(-i\log(-1))^2}{a^2} + \frac{3}{N\lambda^2}$$

$$\frac{3}{\lambda^2 N} + \frac{\left(j^2+k^2\right)\pi^2}{a^2} = \frac{3}{N\lambda^2} + \frac{6\left(j^2+k^2\right)\zeta(2)}{a^2}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

$\zeta(s)$ is the Riemann zeta function

Series representations

$$\frac{3}{\lambda^2 N} + \frac{\left(j^2+k^2\right)\pi^2}{a^2} = \frac{3}{N\lambda^2} - \frac{8\left(j^2+k^2\right)\left(-9+8\sum_{j=0}^{\infty}\frac{j(3+j)}{(3+8j+4j^2)^2}\right)}{7a^2}$$

$$\frac{3}{\lambda^2 N} + \frac{\left(j^2+k^2\right)\pi^2}{a^2} = \frac{3}{N\lambda^2} + \frac{16\left(j^2+k^2\right)\left(\sum_{j=0}^{\infty}\frac{(-1)^j}{1+2j}\right)^2}{a^2}$$

$$\frac{3}{\lambda^2 N} + \frac{\left(j^2+k^2\right)\pi^2}{a^2} = \frac{3\left(a^2+2j^2N\lambda^2\sum_{j=1}^{\infty}\frac{1}{j^2}+2k^2N\lambda^2\sum_{j=1}^{\infty}\frac{1}{j^2}\right)}{a^2N\lambda^2}$$

Integral representations

$$\frac{3}{\lambda^2 N} + \frac{\left(j^2+k^2\right)\pi^2}{a^2} = \frac{3}{N\lambda^2} + \frac{16\left(j^2+k^2\right)\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2}{a^2}$$

$$\frac{3}{\lambda^2 N} + \frac{\left(j^2+k^2\right)\pi^2}{a^2} = \frac{3}{N\lambda^2} + \frac{4\left(j^2+k^2\right)\left(\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt\right)^2}{a^2}$$

$$\frac{3}{\lambda^2 N}+\frac{\left(j^2+k^2\right)\pi^2}{a^2}=\frac{3}{N\lambda^2}+\frac{64\left(j^2+k^2\right)\left(\int_0^{\infty}\frac{\sin^3(t)}{t^3}\,dt\right)^2}{9a^2}$$

From

$$\Theta(x)=\frac{2}{\sqrt{\pi}}\int_0^x e^{-y^2}\,dy.$$

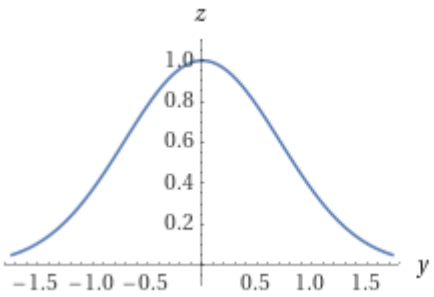
we obtain:

(e^(-y^2))

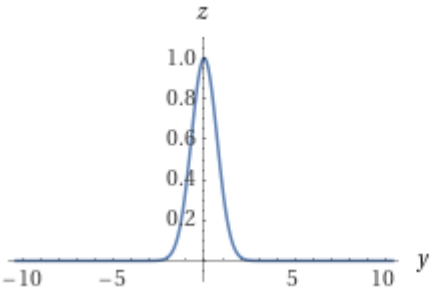
Input

e^{-y^2}

Plots **(figures that can be related to the open strings)**



(y from −1.7 to 1.7)



(y from −10.4 to 10.4)

Roots

(no roots exist)

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\{z \in \mathbb{R} : 0 < z \leq 1\}$

Parity

even

$\mathbb{R}$ is the set of real numbers

Series expansion at y=0

$$1 - y^2 + \frac{y^4}{2} + O(y^5)$$

(Taylor series)

Derivative

$$\frac{d}{dy}\left(e^{-y^2}\right) = -2\,e^{-y^2}\,y$$

Indefinite integral

$$\int e^{-y^2}\,dy = \frac{1}{2}\sqrt{\pi}\,\operatorname{erf}(y) + \text{constant}$$

erf(x) is the error function

Global maximum

$$\max\left\{e^{-y^2}\right\} = 1 \text{ at } y = 0$$

Limit

$$\lim_{y \to \pm\infty} e^{-y^2} = 0$$

Alternative representations

$$e^{-y^2} = z^{-y^2} \text{ for } z = e$$

$$e^{-y^2} = w^a \text{ for } a + \frac{y^2}{\log(w)} = 0$$

$$e^{-y^2} = 1 + \frac{2}{-1 + \coth\left(-\frac{y^2}{2}\right)}$$

log(x) is the natural logarithm

coth(x) is the hyperbolic cotangent function

Series representations

$$e^{-y^2} = \sum_{k=0}^{\infty} \frac{(-y^2)^k}{k!}$$

$$e^{-y^2} = \sum_{k=-\infty}^{\infty} I_k(-y^2)$$

$$e^{-y^2} = \frac{\sum_{k=0}^{\infty} \frac{(-y^2)^{2k}(-2k+y^2)}{(2k)!}}{y^2}$$

$n!$ is the factorial function

$I_n(z)$ is the modified Bessel function of the first kind

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$\arg(z)$ is the complex argument

$|z|$ is the absolute value of z

i is the imaginary unit

Definite integral

$$\int_0^{\infty} e^{-y^2}\,dy = \frac{\sqrt{\pi}}{2} \approx 0.886227$$

From the indefinite integral result

$$\int e^{-y^2}\,dy = \frac{1}{2}\sqrt{\pi}\ \text{erf}(y) + \text{constant}$$

we obtain:

$2/\sqrt{\pi}$ * (1/2 sqrt(π) erf(y))

Input

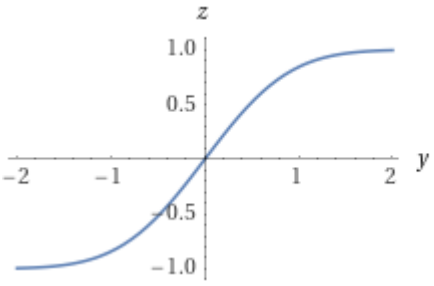
$$\frac{2}{\sqrt{\pi}}\left(\frac{1}{2}\sqrt{\pi}\ \text{erf}(y)\right)$$

erf(*x*) is the error function

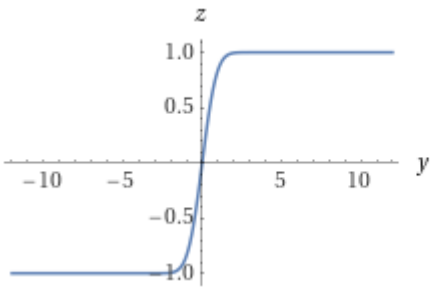
Result

erf(y)

Plots **(figures that can be related to the open strings)**



(*y* from −2 to 2)



(*y* from −12 to 12)

Integer root

$y = 0$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$\{z \in \mathbb{R} : -1 < z < 1\}$

Parity

odd

$\mathbb{R}$ is the set of real numbers

Series expansion at y=0

$$\frac{2y}{\sqrt{\pi}} - \frac{2y^3}{3\sqrt{\pi}} + \frac{y^5}{5\sqrt{\pi}} + O(y^6)$$

(Taylor series)

Series expansion at y=∞

$$e^{-y^2}\left(-\frac{1}{\sqrt{\pi}\ y} + \frac{1}{2\sqrt{\pi}\ y^3} - \frac{3}{4\sqrt{\pi}\ y^5} + O\left(\left(\frac{1}{y}\right)^6\right)\right) + 1$$

Derivative

$$\frac{d}{dy}\left(\frac{2\left(\sqrt{\pi}\ \operatorname{erf}(y)\right)}{\sqrt{\pi}\ 2}\right) = \frac{2e^{-y^2}}{\sqrt{\pi}}$$

Indefinite integral

$$\int \operatorname{erf}(y)\, dy = y \operatorname{erf}(y) + \frac{e^{-y^2}}{\sqrt{\pi}} + \text{constant}$$

Limit

$$\lim_{y \to -\infty} \operatorname{erf}(y) = -1$$

$$\lim_{y \to \infty} \operatorname{erf}(y) = 1$$

Alternative representations

$$\frac{\left(\sqrt{\pi}\, \operatorname{erf}(y)\right) 2}{2\sqrt{\pi}} = \frac{(1 - \operatorname{erfc}(y))\sqrt{\pi}}{\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\pi}\, \operatorname{erf}(y)\right) 2}{2\sqrt{\pi}} = \frac{\operatorname{erf}(0, y)\sqrt{\pi}}{\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\pi}\, \operatorname{erf}(y)\right) 2}{2\sqrt{\pi}} = -\frac{i\left(\operatorname{erfi}(i\, y)\sqrt{\pi}\right)}{\sqrt{\pi}}$$

$\operatorname{erfc}(x)$ is the complementary error function

$\operatorname{erf}(x_0, x_1)$ is the generalized error function

$\operatorname{erfi}(x)$ is the imaginary error function

i is the imaginary unit

Series representations

$$\frac{\left(\sqrt{\pi}\ \mathrm{erf}(y)\right) 2}{2\sqrt{\pi}} = \frac{2\sum_{k=0}^{\infty} \frac{(-1)^k\, y^{1+2k}}{(1+2k)\,k!}}{\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\pi}\ \mathrm{erf}(y)\right) 2}{2\sqrt{\pi}} = \frac{\sum_{k=0}^{\infty} \frac{(-1)^k\, 2^{-1/2-3k}\, H_{1+2k}(y)}{(1+2k)\,k!}}{\sqrt{\pi}}$$

$$\frac{\left(\sqrt{\pi}\ \mathrm{erf}(y)\right) 2}{2\sqrt{\pi}} = \sum_{k=0}^{\infty} \frac{2^k\ {}_2\tilde{F}_2\left(\frac{1}{2}, 1; 1-\frac{k}{2}, \frac{3-k}{2}; -z_0^2\right) (y-z_0)^k\ z_0^{1-k}}{k!}$$

$n!$ is the factorial function

$H_n(x)$ is the n^{th} Hermite polynomial in x

${}_p\tilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ is the regularized generalized hypergeometric function

Integral representations

$$\frac{\left(\sqrt{\pi}\ \mathrm{erf}(y)\right) 2}{2\sqrt{\pi}} = \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2}\, dt$$

$$\frac{\left(\sqrt{\pi}\ \mathrm{erf}(y)\right) 2}{2\sqrt{\pi}} = \frac{2}{\pi} \int_0^{\infty} \frac{e^{-t^2} \sin(2\,t\,y)}{t}\, dt \quad \text{for } y \in \mathbb{R}$$

$$\frac{\left(\sqrt{\pi}\ \mathrm{erf}(y)\right) 2}{2\sqrt{\pi}} = -\frac{i}{2\pi^{3/2}} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{y^{-2s}\, \Gamma(-s)\, \Gamma\left(\frac{1}{2}+s\right)}{\Gamma(1-s)}\, ds$$
$$\text{for } \left(-\frac{1}{2} < \gamma < 0 \text{ and } 2\,|\arg(y)| < \pi\right)$$

$\Gamma(x)$ is the gamma function

arg(z) is the complex argument

$|z|$ is the absolute value of z

And again, from the indefinite integral result

$$\int \operatorname{erf}(y)\,dy = y\operatorname{erf}(y) + \frac{e^{-y^2}}{\sqrt{\pi}} + \text{constant}$$

we obtain:

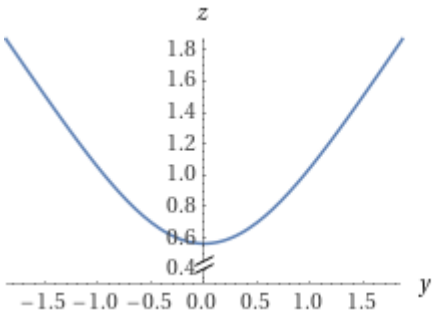
e^(-y^2)/sqrt(π) + y erf(y)

Input

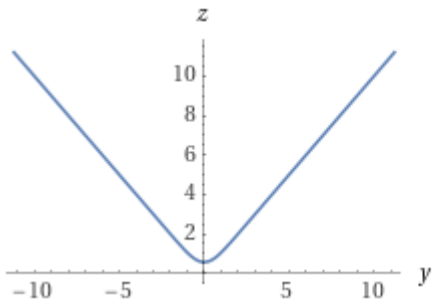
$$\frac{e^{-y^2}}{\sqrt{\pi}} + y\operatorname{erf}(y)$$

erf(x) is the error function

Plots **(figures that can be related to the open strings)**



(y from −1.9 to 1.9)



(y from −11.2 to 11.2)

Alternate form

$$\frac{\sqrt{\pi}\ y\ \mathrm{erf}(y) + e^{-y^2}}{\sqrt{\pi}}$$

Numerical roots

$y \approx -1.96916 \times 10^{-16} - 0.924139\,i\ldots$

$y \approx -1.96916 \times 10^{-16} + 0.924139\,i\ldots$

Properties as a real function

Domain

$\mathbb{R}$ (all real numbers)

Range

$$\{z \in \mathbb{R} : z \geq \frac{1}{\sqrt{\pi}}\}$$

Parity

even

$\mathbb{R}$ is the set of real numbers

Series expansion at y=0

$$\frac{1}{\sqrt{\pi}} + \frac{y^2}{\sqrt{\pi}} - \frac{y^4}{6\sqrt{\pi}} + O(y^5)$$

(Taylor series)

Series expansion at y=∞

$$\frac{e^{-y^2}\left(e^{y^2}\left(\sqrt{\pi}\ y+O\left(\left(\frac{1}{y}\right)^9\right)\right)+\left(\frac{1}{2y^2}-\frac{3}{4y^4}+\frac{15}{8y^6}-\frac{105}{16y^8}+O\left(\left(\frac{1}{y}\right)^9\right)\right)\right)}{\sqrt{\pi}}$$

Derivative

$$\frac{d}{dy}\left(\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(y)\right)=\mathrm{erf}(y)$$

Indefinite integral

$$\int\left(\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(y)\right)dy=\frac{1}{4}\left(2y^2\,\mathrm{erf}(y)+\mathrm{erf}(y)+\frac{2e^{-y^2}y}{\sqrt{\pi}}\right)+\text{constant}$$

Global minimum

$$\min\left\{\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(y)\right\}=\frac{1}{\sqrt{\pi}}\ \text{at}\ y=0$$

Alternative representations

$$\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(y)=y\,\mathrm{erf}(0,y)+\frac{e^{-y^2}}{\sqrt{\pi}}$$

$$\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(y)=-i\,(y\,\mathrm{erfi}(i\,y))+\frac{e^{-y^2}}{\sqrt{\pi}}$$

$$\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(y)=\frac{z^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(0,y)\ \text{for}\ z=e$$

$\operatorname{erf}(x_0, x_1)$ is the generalized error function

$\operatorname{erfi}(x)$ is the imaginary error function

i is the imaginary unit

Series representations

$$\frac{e^{-y^2}}{\sqrt{\pi}} + y \operatorname{erf}(y) = \sum_{k=0}^{\infty} \frac{2(-1)^k y^{2+2k} + (1+2k)\left(-y^2\right)^k}{(1+2k)\sqrt{\pi}\, k!}$$

$$\frac{e^{-y^2}}{\sqrt{\pi}} + y \operatorname{erf}(y) = \sum_{k=0}^{\infty} \frac{2(-1)^k y^{2+2k} + e^{z_0}(1+2k)\left(-y^2 - z_0\right)^k}{(1+2k)\sqrt{\pi}\, k!}$$

$$\frac{e^{-y^2}}{\sqrt{\pi}} + y \operatorname{erf}(y) = \sum_{k=0}^{\infty} \frac{\frac{2(-1)^k y^{4+2k}}{(1+2k)\,k!} + \frac{\left(-y^2\right)^{2k}\left(-2k+y^2\right)}{(2k)!}}{\sqrt{\pi}\, y^2}$$

$n!$ is the factorial function

Integral representations

$$\frac{e^{-y^2}}{\sqrt{\pi}} + y \operatorname{erf}(y) = \frac{e^{-y^2}}{\sqrt{\pi}} + \frac{2y}{\sqrt{\pi}} \int_0^y e^{-t^2}\, dt$$

$$\frac{e^{-y^2}}{\sqrt{\pi}} + y \operatorname{erf}(y) = \frac{e^{-y^2}}{\sqrt{\pi}} + \frac{2y}{\pi} \int_0^{\infty} \frac{e^{-t^2} \sin(2ty)}{t}\, dt \quad \text{for } y \in \mathbb{R}$$

$$\frac{e^{-y^2}}{\sqrt{\pi}} + y\,\mathrm{erf}(y) = \frac{e^{-y^2}}{\sqrt{\pi}} - \frac{i\,y}{2\,\pi^{3/2}} \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{y^{-2\,s}\,\Gamma(-s)\,\Gamma\left(\frac{1}{2}+s\right)}{\Gamma(1-s)}\,ds$$

$$\text{for } \left(-\frac{1}{2} < \gamma < 0 \text{ and } 2\,|\arg(y)| < \pi\right)$$

$\Gamma(x)$ is the gamma function

$\arg(z)$ is the complex argument

$|z|$ is the absolute value of z

Definite integral after subtraction of diverging parts

$$\int_0^\infty \left(\frac{e^{-y^2}}{\sqrt{\pi}} + y\,\mathrm{erf}(y)\right) - y\,dy = \frac{1}{4} = 0.25$$

From which:

(1/(integrate(e^(-y^2)/sqrt(π) + y erf(y)) - y dy, y=0..infinity))^6

Input

$$\left(\frac{1}{\int_0^\infty \left(\left(\frac{e^{-y^2}}{\sqrt{\pi}} + y\,\mathrm{erf}(y)\right) - y\right) dy}\right)^6$$

erf(x) is the error function

Computation result

$$\left(\frac{1}{\int_0^\infty \left(\left(\frac{e^{-y^2}}{\sqrt{\pi}} + y\,\mathrm{erf}(y)\right) - y\right) dy}\right)^6 = 4096$$

4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

27*sqrt((1/(integrate(e^(-y^2)/sqrt(π) + y erf(y)) - y dy, y=0..infinity))^6)+1

Input

$$27\sqrt{\left(\frac{1}{\int_0^\infty\left(\left(\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(y)\right)-y\right)dy}\right)^6}+1$$

erf(x) is the error function

Computation result

$$27\sqrt{\left(\frac{1}{\int_0^\infty\left(\left(\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(y)\right)-y\right)dy}\right)^6}+1=1729$$

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry.

Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(27*sqrt((1/(integrate(e^(-y^2)/sqrt(π) + y erf(y)) - y dy, y=0..infinity))^6)+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{27\sqrt{\left(\frac{1}{\int_0^\infty\left(\left(\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(y)\right)-y\right)dy}\right)^6}+1}+C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi}$$

erf(x) is the error function

C_{MRB} is the MRB constant

Computation result

$$\sqrt[15]{27\sqrt{\left(\frac{1}{\int_0^\infty\left(\left(\frac{e^{-y^2}}{\sqrt{\pi}}+y\,\mathrm{erf}(y)\right)-y\right)dy}\right)^6}+1}+C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi}=$$
$$C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi}+\sqrt[15]{1729}\approx 1.64494$$

Alternate form

$$C_{\mathrm{MRB}}{}^{-1/(4\pi)}\left(\sqrt[15]{1729}\ \sqrt[4\pi]{C_{\mathrm{MRB}}}+C_{\mathrm{MRB}}{}^{1+\pi}\right)$$

From the above alternate form, we obtain:

(MRB const)^(-1/(4 π)) (1729^(1/15) (MRB const)^(1/(4 π)) + (MRB const)^(1 + π))

Input

$$C_{\text{MRB}}^{-1/(4\pi)}\left(\sqrt[15]{1729}\ \sqrt[4\pi]{C_{\text{MRB}}}+C_{\text{MRB}}^{1+\pi}\right)$$

C_{MRB} is the MRB constant

Decimal approximation

1.644938024130260257941733191946366957420242441725816380271305678 2

…

1.64493802413…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Expanded form

$$C_{\text{MRB}}^{1-1/(4\pi)+\pi}+\sqrt[15]{1729}$$

Furthermore, we obtain also:

(((6 sin((π j u)/a) sin((π j x)/a) sin((π k v)/a) sin((π k y)/a))/(a^2 λ v)))+(((π^2 (j^2 + k^2))/a^2 + 3/(λ^2 N)))+((2/√π * (1/2 sqrt(π) erf(y))))

Input

$$\frac{6\sin\left(\frac{\pi j u}{a}\right)\sin\left(\frac{\pi j x}{a}\right)\sin\left(\frac{\pi k v}{a}\right)\sin\left(\frac{\pi k y}{a}\right)}{a^2\,\lambda\,v}+\left(\frac{\pi^2\left(j^2+k^2\right)}{a^2}+\frac{3}{\lambda^2\,N}\right)+\frac{2}{\sqrt{\pi}}\left(\frac{1}{2}\sqrt{\pi}\ \text{erf}(y)\right)$$

erf(x) is the error function

Exact result

$$\frac{\pi^2\left(j^2+k^2\right)}{a^2}+\frac{6\sin\left(\frac{\pi j u}{a}\right)\sin\left(\frac{\pi j x}{a}\right)\sin\left(\frac{\pi k v}{a}\right)\sin\left(\frac{\pi k y}{a}\right)}{a^2\lambda v}+\operatorname{erf}(y)+\frac{3}{\lambda^2 N}$$

Reduced trigonometric form

$$\begin{aligned}&\frac{1}{4a^2 N v\lambda^2}\Bigg(4j^2 N\pi^2 v\lambda^2+4k^2 N\pi^2 v\lambda^2+12a^2 v+4a^2 N v\lambda^2\operatorname{erf}(y)+\\&\quad 3N\lambda\left(\cos\left(\frac{j\pi u}{a}-\frac{k\pi v}{a}+\frac{j\pi x}{a}-\frac{k\pi y}{a}\right)+\cos\left(\frac{j\pi u}{a}-\frac{k\pi v}{a}-\frac{j\pi x}{a}+\frac{k\pi y}{a}\right)\right)+\\&\quad 3N\lambda\left(-\cos\left(\frac{j\pi u}{a}+\frac{k\pi v}{a}+\frac{j\pi x}{a}-\frac{k\pi y}{a}\right)-\right.\\&\qquad\left.\cos\left(\frac{j\pi u}{a}+\frac{k\pi v}{a}-\frac{j\pi x}{a}+\frac{k\pi y}{a}\right)\right)+3N\lambda\\&\quad\left(-\cos\left(\frac{j\pi u}{a}-\frac{k\pi v}{a}-\frac{j\pi x}{a}-\frac{k\pi y}{a}\right)-\cos\left(\frac{j\pi u}{a}-\frac{k\pi v}{a}+\frac{j\pi x}{a}+\frac{k\pi y}{a}\right)\right)+\\&\quad 3N\lambda\left(\cos\left(\frac{j\pi u}{a}+\frac{k\pi v}{a}-\frac{j\pi x}{a}-\frac{k\pi y}{a}\right)+\cos\left(\frac{j\pi u}{a}+\frac{k\pi v}{a}+\frac{j\pi x}{a}+\frac{k\pi y}{a}\right)\right)\Bigg)\end{aligned}$$

Expanded sine form

$$\frac{j^2\pi^2}{a^2}+\frac{k^2\pi^2}{a^2}+\frac{3}{N\lambda^2}+\operatorname{erf}(y)+\frac{6\sin\left(\frac{j\pi u}{a}\right)\sin\left(\frac{k\pi v}{a}\right)\sin\left(\frac{j\pi x}{a}\right)\sin\left(\frac{k\pi y}{a}\right)}{a^2 v\lambda}$$

Alternate forms

$$\frac{1}{a^2\lambda^2 N v}\left(v\left(a^2\left(\lambda^2 N\operatorname{erf}(y)+3\right)+\pi^2\lambda^2 N\left(j^2+k^2\right)\right)+6\lambda N\sin\left(\frac{\pi j u}{a}\right)\sin\left(\frac{\pi j x}{a}\right)\sin\left(\frac{\pi k v}{a}\right)\sin\left(\frac{\pi k y}{a}\right)\right)$$

$$\frac{1}{a^2\lambda^2 N v}\left(a^2\lambda^2 N v\operatorname{erf}(y)+3a^2 v+6\lambda N\sin\left(\frac{\pi j u}{a}\right)\sin\left(\frac{\pi j x}{a}\right)\sin\left(\frac{\pi k v}{a}\right)\sin\left(\frac{\pi k y}{a}\right)+\pi^2 j^2\lambda^2 N v+\pi^2 k^2\lambda^2 N v\right)$$

$$\frac{\pi^2\left(j^2+k^2\right)}{a^2}+\frac{1}{8\,a^2\,\lambda\,\nu}3\left(e^{-(i\,\pi\,j\,u)/a}-e^{(i\,\pi\,j\,u)/a}\right)\left(e^{-(i\,\pi\,j\,x)/a}-e^{(i\,\pi\,j\,x)/a}\right)$$
$$\left(e^{-(i\,\pi\,k\,v)/a}-e^{(i\,\pi\,k\,v)/a}\right)\left(e^{-(i\,\pi\,k\,y)/a}-e^{(i\,\pi\,k\,y)/a}\right)+\operatorname{erf}(y)+\frac{3}{\lambda^2\,N}$$

Series expansion at x=0

$$\left(\frac{\pi^2\left(j^2+k^2\right)}{a^2}+\operatorname{erf}(y)+\frac{3}{\lambda^2\,N}\right)+\frac{6\,\pi\,j\,x\sin\left(\frac{\pi\,j\,u}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{a^3\,\lambda\,\nu}-$$
$$\frac{\pi^3\,j^3\,x^3\sin\left(\frac{\pi\,j\,u}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{a^5\,\lambda\,\nu}+\frac{\pi^5\,j^5\,x^5\sin\left(\frac{\pi\,j\,u}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{20\,a^7\,\lambda\,\nu}+O\left(x^6\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial x}\left(\frac{6\sin\left(\frac{\pi\,j\,u}{a}\right)\sin\left(\frac{\pi\,j\,x}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{a^2\,\lambda\,\nu}+\left(\frac{\pi^2\left(j^2+k^2\right)}{a^2}+\frac{3}{\lambda^2\,N}\right)+\right.$$
$$\left.\frac{2\left(\sqrt{\pi}\,\operatorname{erf}(y)\right)}{\sqrt{\pi}\times 2}\right)=\frac{6\,\pi\,j\sin\left(\frac{\pi\,j\,u}{a}\right)\cos\left(\frac{\pi\,j\,x}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{a^3\,\lambda\,\nu}$$

Indefinite integral

$$\int\left(\frac{\left(j^2+k^2\right)\pi^2}{a^2}+\frac{3}{N\,\lambda^2}+\operatorname{erf}(y)+\frac{6\sin\left(\frac{j\,\pi\,u}{a}\right)\sin\left(\frac{k\,\pi\,v}{a}\right)\sin\left(\frac{j\,\pi\,x}{a}\right)\sin\left(\frac{k\,\pi\,y}{a}\right)}{a^2\,\nu\,\lambda}\right)dx=$$
$$\frac{\pi^2\,j^2\,x}{a^2}+\frac{\pi^2\,k^2\,x}{a^2}-\frac{6\sin\left(\frac{\pi\,j\,u}{a}\right)\cos\left(\frac{\pi\,j\,x}{a}\right)\sin\left(\frac{\pi\,k\,v}{a}\right)\sin\left(\frac{\pi\,k\,y}{a}\right)}{\pi\,a\,j\,\lambda\,\nu}+x\operatorname{erf}(y)+\frac{3\,x}{\lambda^2\,N}+$$

constant

From the indefinite integral result

$$\int\left(\frac{(j^2+k^2)\pi^2}{a^2}+\frac{3}{N\lambda^2}+\text{erf}(y)+\frac{6\sin\left(\frac{j\pi u}{a}\right)\sin\left(\frac{k\pi v}{a}\right)\sin\left(\frac{j\pi x}{a}\right)\sin\left(\frac{k\pi y}{a}\right)}{a^2 v\lambda}\right)dx =$$

$$\frac{\pi^2 j^2 x}{a^2}+\frac{\pi^2 k^2 x}{a^2}-\frac{6\sin\left(\frac{\pi j u}{a}\right)\cos\left(\frac{\pi j x}{a}\right)\sin\left(\frac{\pi k v}{a}\right)\sin\left(\frac{\pi k y}{a}\right)}{\pi a j\lambda v}+x\,\text{erf}(y)+\frac{3x}{\lambda^2 N}+$$

constant

(j^2 π^2 x)/a^2 + (k^2 π^2 x)/a^2 + (3 x)/(N λ^2) + x erf(y) - (6 cos((j π x)/a) sin((j π u)/a) sin((k π v)/a) sin((k π y)/a))/(a j π v λ)

for a = 2, j = 4, k = 8, λ = 16, N = 32, x = 64, y = 128, u = 256 and v = 512:

(4^2 π^2 64)/2^2 + (8^2 π^2 64)/2^2 + (3*64)/(32 16^2) + 64 erf(128) - (6 cos((4 π 64)/2) sin((4 π 256)/2) sin((8 π 512)/2) sin((8 π 128)/2))/(2 4 π 512 16)

Input

$$\frac{4^2\pi^2\times 64}{2^2}+\frac{8^2\pi^2\times 64}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\text{erf}(128)-\frac{6\cos\left(\frac{1}{2}(4\pi\times 64)\right)\sin\left(\frac{1}{2}(4\pi\times 256)\right)\sin\left(\frac{1}{2}(8\pi\times 512)\right)\sin\left(\frac{1}{2}(8\pi\times 128)\right)}{2\times 4\pi\times 512\times 16}$$

erf(x) is the error function

Exact result

$$64\,\text{erf}(128)+\frac{3}{128}+1280\pi^2$$

Decimal approximation

12697.117070894379032108148479841473453201535241268212001809087201

…

12697.117070894379….

Alternate forms

$$\frac{1}{128}\left(8192\,\mathrm{erf}(128)+3+163\,840\,\pi^2\right)$$

$$64\,\mathrm{erf}(128)+\frac{1}{128}\left(3+163\,840\,\pi^2\right)$$

Alternative representations

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\mathrm{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=64\,\mathrm{erf}(0,\,128)+\frac{64\times 4^2\,\pi^2}{4}+\frac{64\times 8^2\,\pi^2}{4}-\frac{6\cosh(-128\,i\,\pi)\cos\left(-\frac{4095\,\pi}{2}\right)\cos^2\left(-\frac{1023\,\pi}{2}\right)}{65\,536\,\pi}+\frac{192}{32\times 16^2}$$

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\mathrm{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=64\,\mathrm{erf}(0,\,128)+\frac{64\times 4^2\,\pi^2}{4}+\frac{64\times 8^2\,\pi^2}{4}+\frac{3\cos^2\left(\frac{1025\,\pi}{2}\right)\cos\left(\frac{4097\,\pi}{2}\right)\left(e^{-128\,i\,\pi}+e^{128\,i\,\pi}\right)}{65\,536\,\pi}+\frac{192}{32\times 16^2}$$

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\mathrm{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=64\,\mathrm{erf}(0,\,128)+\frac{64\times 4^2\,\pi^2}{4}+\frac{64\times 8^2\,\pi^2}{4}-\frac{3\cos\left(-\frac{4095\,\pi}{2}\right)\cos^2\left(-\frac{1023\,\pi}{2}\right)\left(e^{-128\,i\,\pi}+e^{128\,i\,\pi}\right)}{65\,536\,\pi}+\frac{192}{32\times 16^2}$$

$\mathrm{erf}(x_0,\,x_1)$ is the generalized error function

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Series representations

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\text{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=\frac{3}{128}+1280\,\pi^2+\frac{128\sum_{k=0}^{\infty}\frac{(-1)^k\,128^{1+2\,k}}{(1+2\,k)\,k!}}{\sqrt{\pi}}$$

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\text{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=\frac{3}{128}+1280\,\pi^2+\frac{64\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1/2-3\,k}\,H_{1+2\,k}(128)}{(1+2\,k)\,k!}}{\sqrt{\pi}}$$

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\text{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=\frac{3}{128}+1280\,\pi^2+64\sum_{k=0}^{\infty}\frac{{}_2\tilde{F}_2\left(\frac{1}{2},\,1;\,1-\frac{k}{2},\,\frac{3-k}{2};\,-z_0^2\right)(256-2\,z_0)^k\,z_0^{1-k}}{k!}$$

$n!$ is the factorial function

$H_n(x)$ is the n^{th} Hermite polynomial in x

${}_p\tilde{F}_q(a_1,\ldots,a_p;b_1,\ldots,b_q;z)$ is the regularized generalized hypergeometric function

Integral representations

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\text{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=\frac{3}{128}+1280\,\pi^2+\frac{128}{\sqrt{\pi}}\int_0^{128}e^{-t^2}\,dt$$

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\text{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=\frac{3}{128}+1280\,\pi^2+\frac{128}{\pi}\int_0^{\infty}\frac{e^{-t^2}\,\sin(256\,t)}{t}\,dt$$

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\text{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=\frac{3}{128}+1280\,\pi^2-\frac{32\,i}{\pi^{3/2}}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{128^{-2\,s}\,\Gamma(-s)\,\Gamma\left(\frac{1}{2}+s\right)}{\Gamma(1-s)}\,ds\quad\text{for}\quad-\frac{1}{2}<\gamma<0$$

$\Gamma(x)$ is the gamma function

Half-argument formula

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\mathrm{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=\frac{3}{128}+1280\,\pi^2+64\,\mathrm{erf}(128)-\frac{1}{32\,768\,\pi}3\,(-1)^{\lfloor(\pi+\mathrm{Re}(256\,\pi))/(2\,\pi)\rfloor+2\,\lfloor\mathrm{Re}(1024\,\pi)/(2\,\pi)\rfloor+\lfloor\mathrm{Re}(4096\,\pi)/(2\,\pi)\rfloor}\sqrt{\frac{1}{2}\left(1+\cos(256\,\pi)\right)}\sqrt{\frac{1}{2}\left(1-\cos(1024\,\pi)\right)}^{\,2}\sqrt{\frac{1}{2}\left(1-\cos(4096\,\pi)\right)}\left(1-\left(1+(-1)^{\lfloor-(\pi+\mathrm{Re}(256\,\pi))/(2\,\pi)\rfloor+\lfloor(\pi+\mathrm{Re}(256\,\pi))/(2\,\pi)\rfloor}\right)\theta(-\mathrm{Im}(256\,\pi))\right)\left(1-\left(1+(-1)^{\lfloor-\mathrm{Re}(1024\,\pi)/(2\,\pi)\rfloor+\lfloor\mathrm{Re}(1024\,\pi)/(2\,\pi)\rfloor}\right)\theta(-\mathrm{Im}(1024\,\pi))\right)^2\left(1-\left(1+(-1)^{\lfloor-\mathrm{Re}(4096\,\pi)/(2\,\pi)\rfloor+\lfloor\mathrm{Re}(4096\,\pi)/(2\,\pi)\rfloor}\right)\theta(-\mathrm{Im}(4096\,\pi))\right)$$

$\mathrm{Re}(z)$ is the real part of z

$\lfloor x\rfloor$ is the floor function

$\mathrm{Im}(z)$ is the imaginary part of z

$\theta(x)$ is the Heaviside step function

Multiple-argument formulas

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\mathrm{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=\frac{3}{128}+1280\,\pi^2+64\,\mathrm{erf}(128)+\frac{3\cos^2(256\,\pi)\cos(1024\,\pi)\left(-1+2\sin^2(64\,\pi)\right)\sin^2(256\,\pi)\sin(1024\,\pi)}{4096\,\pi}$$

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\mathrm{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=\frac{3}{128}+1280\,\pi^2+64\,\mathrm{erf}(128)-\frac{3\left(-1+2\cos^2(64\,\pi)\right)\cos^2(256\,\pi)\cos(1024\,\pi)\sin^2(256\,\pi)\sin(1024\,\pi)}{4096\,\pi}$$

$$\frac{4^2\left(\pi^2\,64\right)}{2^2}+\frac{8^2\left(\pi^2\,64\right)}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\mathrm{erf}(128)-\frac{6\cos\left(\frac{4\pi\,64}{2}\right)\sin\left(\frac{4\pi\,256}{2}\right)\sin\left(\frac{8\pi\,512}{2}\right)\sin\left(\frac{8\pi\,128}{2}\right)}{2\times 4\,\pi\,512\times 16}=\frac{3}{128}+1280\,\pi^2+64\,\mathrm{erf}(128)-\frac{3\,U_{511}(\cos(\pi))^2\,U_{2047}(\cos(\pi))\left(-1+2\cos^2(64\,\pi)\right)\sin^3(\pi)}{32\,768\,\pi}$$

$U_n(x)$ is the Chebyshev polynomial of the second kind

From which:

1/Pi*((4^2 π^2 64)/2^2 + (8^2 π^2 64)/2^2 + (3*64)/(32 16^2) + 64 erf(128) - (6 cos((4 π 64)/2) sin((4 π 256)/2) sin((8 π 512)/2) sin((8 π 128)/2))/(2 4 π 512 16))+55-Φ

Input

$$\frac{1}{\pi}\left(\frac{4^2\,\pi^2\times 64}{2^2}+\frac{8^2\,\pi^2\times 64}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\mathrm{erf}(128)-\frac{6\cos\left(\frac{1}{2}\,(4\,\pi\times 64)\right)\sin\left(\frac{1}{2}\,(4\,\pi\times 256)\right)\sin\left(\frac{1}{2}\,(8\,\pi\times 512)\right)\sin\left(\frac{1}{2}\,(8\,\pi\times 128)\right)}{2\times 4\,\pi\times 512\times 16}\right)+55-\Phi$$

erf(x) is the error function

Φ is the golden ratio conjugate

Exact result

$$\frac{64\,\mathrm{erf}(128)+\frac{3}{128}+1280\,\pi^2}{\pi}-\Phi+55$$

Exact form

$$\frac{64\,\mathrm{erf}(128)+\frac{3}{128}+1280\,\pi^2}{\pi}-\phi+56$$

ϕ is the golden ratio

Decimal approximation

4095.999855709905485956260231901487978605787721971002197321530096 4

…

4095.9998557…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{64\,\mathrm{erf}(128)+\frac{3}{128}}{\pi}-\Phi+55+1280\,\pi$$

$$-\frac{-8192\,\mathrm{erf}(128)+128\,\pi\,\Phi-3-7040\,\pi-163\,840\,\pi^2}{128\,\pi}$$

$$\frac{64\,\mathrm{erf}(128)}{\pi}+\frac{-128\,\pi\,\Phi+3+7040\,\pi+163\,840\,\pi^2}{128\,\pi}$$

Expanded form

$$\frac{64\,\mathrm{erf}(128)}{\pi}-\Phi+55+\frac{3}{128\,\pi}+1280\,\pi$$

27*sqrt(1/Pi*((4^2 π^2 64)/2^2 + (8^2 π^2 64)/2^2 + (3*64)/(32 16^2) + 64 erf(128) - (6 cos((4 π 64)/2) sin((4 π 256)/2) sin((8 π 512)/2) sin((8 π 128)/2))/(2 4 π 512 16))+55-Φ)+1

Input

$$27\sqrt{\left(\frac{1}{\pi}\left(\frac{4^2\,\pi^2\times 64}{2^2}+\frac{8^2\,\pi^2\times 64}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\mathrm{erf}(128)-\left(6\cos\left(\frac{1}{2}\,(4\,\pi\times 64)\right)\sin\left(\frac{1}{2}\,(4\,\pi\times 256)\right)\sin\left(\frac{1}{2}\,(8\,\pi\times 512)\right)\sin\left(\frac{1}{2}\,(8\,\pi\times 128)\right)\right)\Big/(2\times 4\,\pi\times 512\times 16)\right)+55-\Phi\right)}+1$$

erf(x) is the error function

Φ is the golden ratio conjugate

Exact result

$$27\sqrt{\frac{64\,\mathrm{erf}(128)+\frac{3}{128}+1280\,\pi^2}{\pi}-\Phi+55}+1$$

Exact form

$$27\sqrt{\frac{64\,\mathrm{erf}(128)+\frac{3}{128}+1280\,\pi^2}{\pi}}-\phi+56\ +1$$

ϕ is the golden ratio

Decimal approximation

1728.999969563807920399399711433411748846031063378705904940747779 2

…

1728.9999695638….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$27\sqrt{\frac{64\,\mathrm{erf}(128)+\frac{3}{128}}{\pi}-\Phi+55+1280\,\pi}+1$$

$$\frac{27}{8}\sqrt{\frac{8192\,\mathrm{erf}(128)-128\,\pi\,\Phi+3+7040\,\pi+163\,840\,\pi^2}{2\,\pi}}+1$$

$$\frac{1}{16}\left(27\sqrt{\frac{2\left(8192\,\mathrm{erf}(128)-128\,\pi\,\Phi+3+7040\,\pi+163\,840\,\pi^2\right)}{\pi}}+16\right)$$

(27√(1/Pi((4^2 π^2 64)/2^2+(8^2 π^2 64)/2^2+(3*64)/(32 16^2)+64 erf(128) - (6 cos((4 π 64)/2) sin((4π 256)/2) sin((8π 512)/2) sin((8π 128)/2))/(8π 512 16))+55-Φ)+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\left(27\sqrt{\left(\frac{1}{\pi}\left(\frac{4^2\pi^2\times 64}{2^2}+\frac{8^2\pi^2\times 64}{2^2}+\frac{3\times 64}{32\times 16^2}+64\,\mathrm{erf}(128)-\left(6\cos\left(\frac{1}{2}(4\pi\times 64)\right)\sin\left(\frac{1}{2}(4\pi\times 256)\right)\sin\left(\frac{1}{2}(8\pi\times 512)\right)\sin\left(\frac{1}{2}(8\pi\times 128)\right)\right)\Big/((8\pi)\times 512\times 16)\right)+55-\Phi\right)+1}\right)\,\hat{}\,(1/15)+C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi}$$

erf(x) is the error function

Φ is the golden ratio conjugate

C_{MRB} is the MRB constant

Exact result

$$\sqrt[15]{27\sqrt{\frac{64\,\mathrm{erf}(128)+\frac{3}{128}+1280\pi^2}{\pi}-\Phi+55}+1}+C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi}$$

Decimal approximation

1.64493802220114992609009264376430849837211731328332239094177236 93…

1.6449380222…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$\sqrt[15]{27\sqrt{\frac{64\,\mathrm{erf}(128)+\frac{3}{128}}{\pi}-\Phi+55+1280\,\pi}+1}+C_{\mathrm{MRB}}^{\,1-1/(4\pi)+\pi}$$

$$\sqrt[15]{\frac{27}{8}\sqrt{\frac{8192\,\mathrm{erf}(128)-128\,\pi\,\Phi+3+7040\,\pi+163\,840\,\pi^2}{2\,\pi}}+1}+C_{\mathrm{MRB}}^{\,1-1/(4\pi)+\pi}$$

$$\frac{\sqrt[15]{27\sqrt{\frac{2\left(8192\,\mathrm{erf}(128)-128\,\pi\,\Phi+3+7040\,\pi+163\,840\,\pi^2\right)}{\pi}}+16}}{2^{4/15}}+C_{\mathrm{MRB}}^{\,1-1/(4\pi)+\pi}$$

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

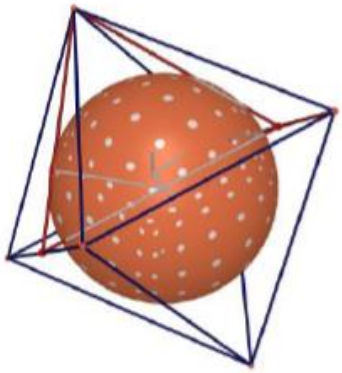


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^{0}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not $(z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0)$)

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

4096 = 64^2 , (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$).

The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^8\right)^{-k}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^8\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^{0} \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459001070581002623979536005212943435960226956084921288971

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left((\sqrt{2}\,a^3)(\sqrt{2}\,d^3)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\frac{\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)}{3\times 3}}}} = \sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}} = \sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}} = \sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))) = 3.1415926535

Indeed:

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13

…

3.1415926535…. = π (Ramanujan Recurring Number)

From the following extended DN Constant (“Unitary Formula”)

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}\times\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

with regard

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

for q = 1729 and p = 4096, we obtain by changing the sign in the algebraic sum of the aforementioned Cardano’s Formula and after some calculations:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

multiplied by

$$\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}$$

and performing the ninth root of the entire expression:

√((2^(-1/π)(5(3+√(5))π)^(1/(2π)))((∛(-1729/2+√(1729^2/4+4096^3/27)) -∛(-1729/2-√(1729^2/4+4096^3/27)))^1/9))

we obtain:

$$\sqrt{\left(\left(2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right)\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}\right)}$$

$$2^{-1/(2\pi)}\sqrt[18]{\sqrt[3]{\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}-\frac{1729}{2}}+\sqrt[3]{\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}}\Big/\sqrt[4\pi]{5\left(3+\sqrt{5}\right)\pi}$$

i.e.

2^(-1/(2π)) ((-1729/2+√(274958621851/3)/6)^(1/3)+(1729/2 + √(274958621851/3)/6)^(1/3))^(1/18) (5(3+√(5)) π)^(1/(4π))

= 1.61549140391….

The general "unitary" formula, which derives from DN Constant, is the following:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

Where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

C is any constant or solution to an equation, R is the radius of the Universe and $2.33*10^{-13}$ is the temperature of the universe expressed in GeV.

For example, $C = 9.9128$, inserting a radius of the Universe, which we have calculated, equal to $R = 2.06274*10^{12}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}$$

1.618035912348264235440174408834709854254527350840173356306481810 7

…

1.618035912348…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We obtain also:

(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))dxdydz

Indefinite integral assuming all variables are real

$$0.809018\, x^2\, y\, z + \text{constant}$$

Definite integral over a cube of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} 1.61804\, dx\, dy\, dz = 12.9443\, L^3$$

Definite integral over a sphere of radius R

$$\iiint_{x^2+y^2+z^2<R^2} 1.61804\, dz\, dy\, dx = 6.77761\, R^3$$

From which, for L = R = 1 , dividing the two definite integral results by the original expression, we obtain:

12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))

Input interpretation

$$\frac{12.9443}{\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

8.00001…

8.00001…. ≈ 8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

And

3*(6.77761/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))

Input interpretation

$$3\times\frac{6.77761}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

12.5664…

12.5664…. ≈ 4π = Bekenstein-Hawking (S_{BH}) black hole entropy

New fundamental formula deriving from DN Constant

The DN Constant (Del Gaudio-Nardelli Constant) equals $(2\sqrt{2})/\pi$ is defined as the ratio of the volume of an octahedron to the volume of a sphere and is an intriguing mathematical concept. Michele Nardelli hypothesized that the regular octahedron represents a phase in which the universe is highly symmetrical and with very low entropy. On the other hand, the sphere (which is inscribed in the octahedron, i.e. is "inside" it) represents the universe emerging from the quantum vacuum, which over time increases entropy and undergoes various symmetry breakings. This occurs in a regime of eternal inflation.

From the following expression

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×C×R)))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\,C\,R\right)}}$$

which comes from the DN Constant, with $1.616255*10^{-35}$ which is equal to the Planck length, $1.1056*10^{-52}$ which is equal to the Cosmological Constant, C = 1729 which corresponds to the Hardy-Ramanujan number and $R = 4.6018401361 \times 10^{-24}$, which represents the radius of the Universe, we obtain:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×4.6018401361 × 10^-24))) = 1.6180329973...

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 4.6018401361\times 10^{-24}\right)}}$$

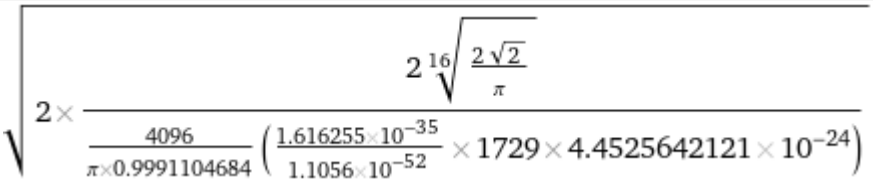
1.618032997307532491506757016629746746422577260892967140791939590 3…

1.6180329973075… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We have also the following formula:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(4.4525642121 × 10^-24))))

Input interpretation



Result

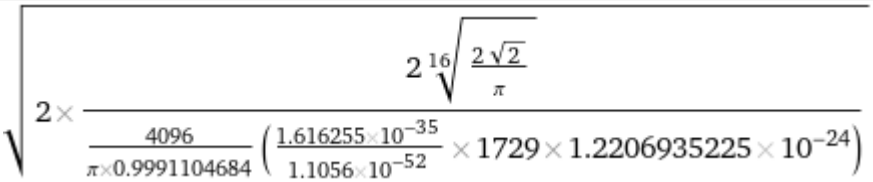
1.644932352102092130483898983704151168876621841655166877914166033 8…

1.64493235210209213…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

And again:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(1.2206935225 × 10^-24))))

Input interpretation



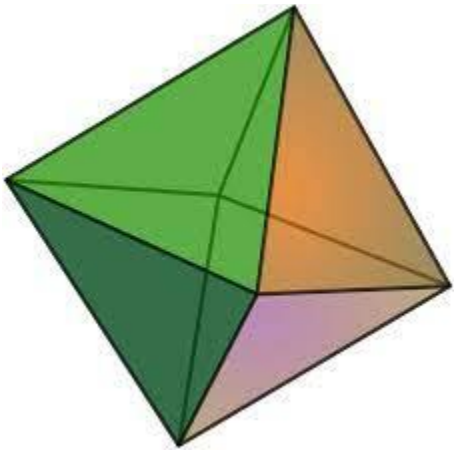
Result

3.14159225731469939939950039231605796469731171781644423955790024797

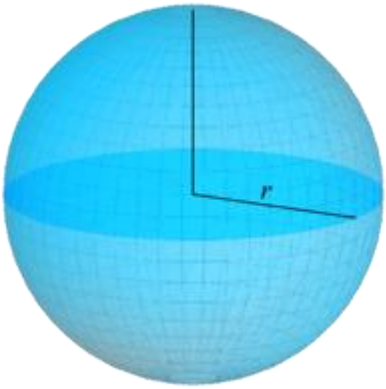
…

3.14159225731469…≈ π (Ramanujan Recurring Number)

Now, we have that:



Octahedron



Sphere

Given the value of a volume, independently of the solid, following the Poincaré Conjecture, we compare any solid "without holes" and a sphere. If we compare an octahedron with a sphere, we have:

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

If we consider the radius of the sphere as an unknown, we must find the value of the side of the octahedron which allows us to equalize the two volumes and which will give us the DN Constant as a result (which will therefore be equal to the radius of the sphere).

From

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

To find r we perform the following calculation:

$$r^3 = \frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi} = \frac{1}{3}\sqrt{2}a^3 \cdot \frac{3}{4\pi} = \frac{\sqrt{2}a^3}{4\pi} = \frac{\sqrt{2}\cdot\sqrt{2}a^3}{\sqrt{2}\cdot 4\pi} = \frac{2a^3}{\sqrt{2}\cdot 4\pi} = \frac{a^3}{2\sqrt{2}\pi}$$

$$r^3 = \frac{a^3}{2\sqrt{2}\pi}; \qquad r = \sqrt[3]{\frac{a^3}{2\sqrt{2}\pi}} = \frac{a}{\sqrt[3]{2\sqrt{2}\pi}}$$

To find *a*, we have, for

$$r = \frac{2\sqrt{2}}{\pi}; \qquad \frac{a}{\sqrt[3]{2\sqrt{2}\pi}} = \frac{2\sqrt{2}}{\pi};$$

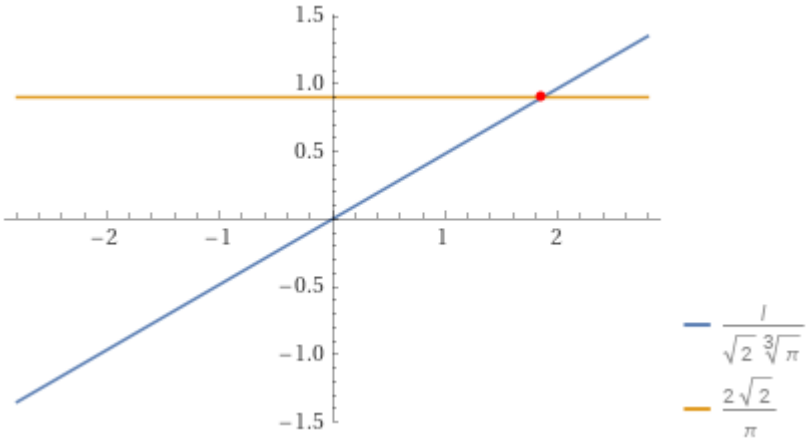
Thus, multiplying both the sides by $\sqrt[3]{2\sqrt{2}\pi}$, we obtain:

$$a = \frac{2\sqrt{2}}{\pi} \cdot \sqrt[3]{2\sqrt{2}\pi} = \sqrt[3]{\frac{(2\sqrt{2})^3 2\sqrt{2}\pi}{\pi^3}} =$$

$$= \sqrt[3]{\frac{8\cdot\sqrt{2^3}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{8\cdot\sqrt{2\cdot 2^2}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{16\cdot\sqrt{2}\cdot 2\sqrt{2}}{\pi^2}}$$

$$= \sqrt[3]{\frac{32\sqrt{2}\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{32\cdot 2}{\pi^2}} = \frac{\sqrt[3]{64}}{\sqrt[3]{\pi^2}} = \frac{4}{\sqrt[3]{\pi^2}};\ a = \frac{4}{\sqrt[3]{\pi^2}}$$

Plot



Solution

$a = \frac{4}{\sqrt[3]{\pi^2}}$

for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), we obtain:

from the octahedron volume, we have: V = 1/3*√2*a^3 = (1/3*√2*($\frac{4}{\sqrt[3]{\pi^2}}$)^3)

(1/3*√2*(4/∛(π^2))^3)

Input

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3$$

$\sqrt[3]{x}$ is the real-valued 3rd root of x

Exact result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{3\pi^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And, from the sphere volume V = (4/3*π*r^3) = (4/3*π*((2√2)/π)^3)

(4/3*π*((2√2)/π)^3)

Input

$$\frac{4}{3}\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3$$

Result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\sqrt{z_0}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\exp^3\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,3}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3}{3\pi^2}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\left(\frac{1}{z_0}\right)^{3/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{3/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

1.64381856858…. $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

$1.6272016\ldots * 10^{-6}$

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10^{-6}$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

$-5.010882647757...$

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t} \, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757 \ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\;\phi\;e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\;\phi\;e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874} + b}{M}$$

$$\frac{\partial M(b,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245 + 18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M} = \frac{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b} = \frac{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right\} = 0 \text{ at } (b, M) = (-16, 0)$$

Global minima:

$$\min\left\{\frac{1}{3}\,M^2\left(1 - \frac{(b\,2)\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\,\sqrt{6}}\right)^2\right\} = 0$$

$$\text{for } b = -\frac{226\,802\,245}{18\,480\,874}$$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

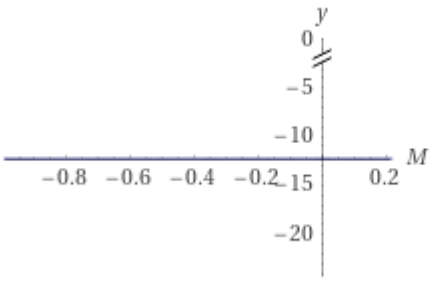
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

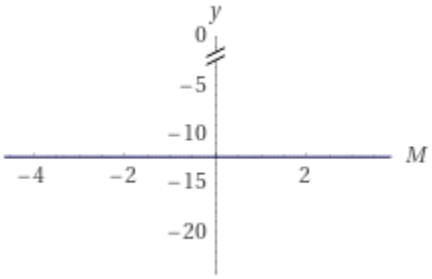
Result:

$$\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723\right)+O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

-12.2723

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times10^{-15}}{M}$$

Indefinite integral:

$$\int\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=$$
$$-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3}(0.0814845\,b + 1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

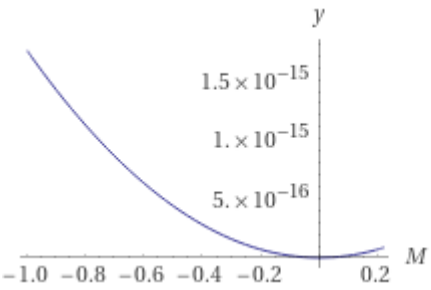
Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

Result:

0

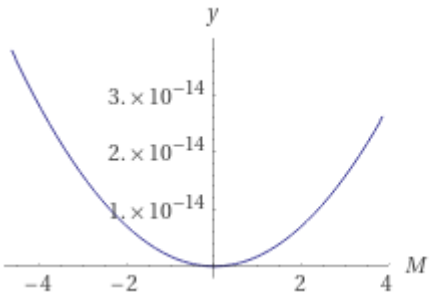
Plots: **(possible mathematical connection with an open string)**



(M from −1 to 0.2)

M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O(M^{62194})$

(Taylor series)

Series expansion at M = ∞:

$1.75541 \times 10^{-15}\, M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.388513449474645453489707833780880208333333333333333333\ldots\times10^{-16}$$

-4.38851344947*10^{-16}

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^2+6.58545\times10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times0.2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-17}$

7.021621519159*10^{-17}

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875 \times 10^{-14}$

1.57986484181*10^-14

For M = 2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\, M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 2^2 + 6.58545 \times 10^{-10}\sqrt{2^4}\right)}{2^2} + 1\right)^2 \times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots \times 10^{-15}$

7.021621519*10^-15

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots\times 10^{-8}$

5.9776991059*10^-8 result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E}=\mathbf{E}_{\mathrm{P}}l_{\mathrm{P}}^{2}=\phi_{\mathrm{P}}l_{\mathrm{P}}=\sqrt{\frac{\hbar c}{\varepsilon_{0}}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left((1/(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}))\,\hat{}\,(1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

$5.729475 * 10^{-35}$ Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

$1.820306 * 10^{61}$ V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

$5.975498*10^{-8}$ V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

$1.042940*10^{27}$ V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

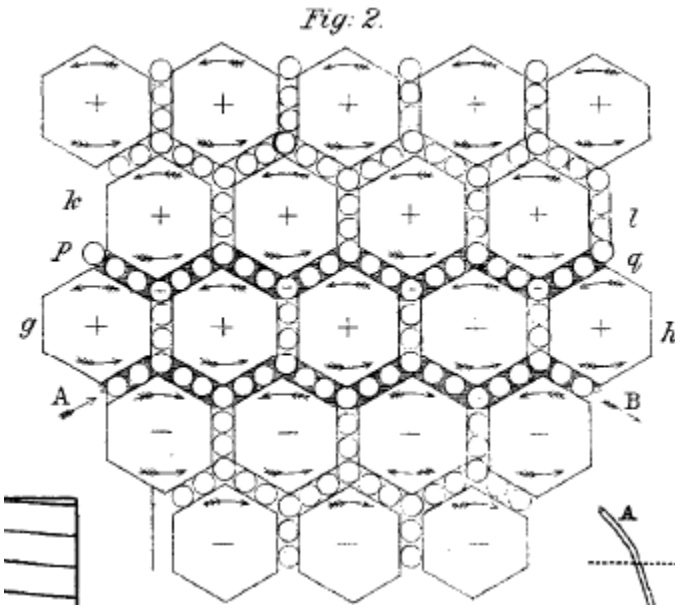
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

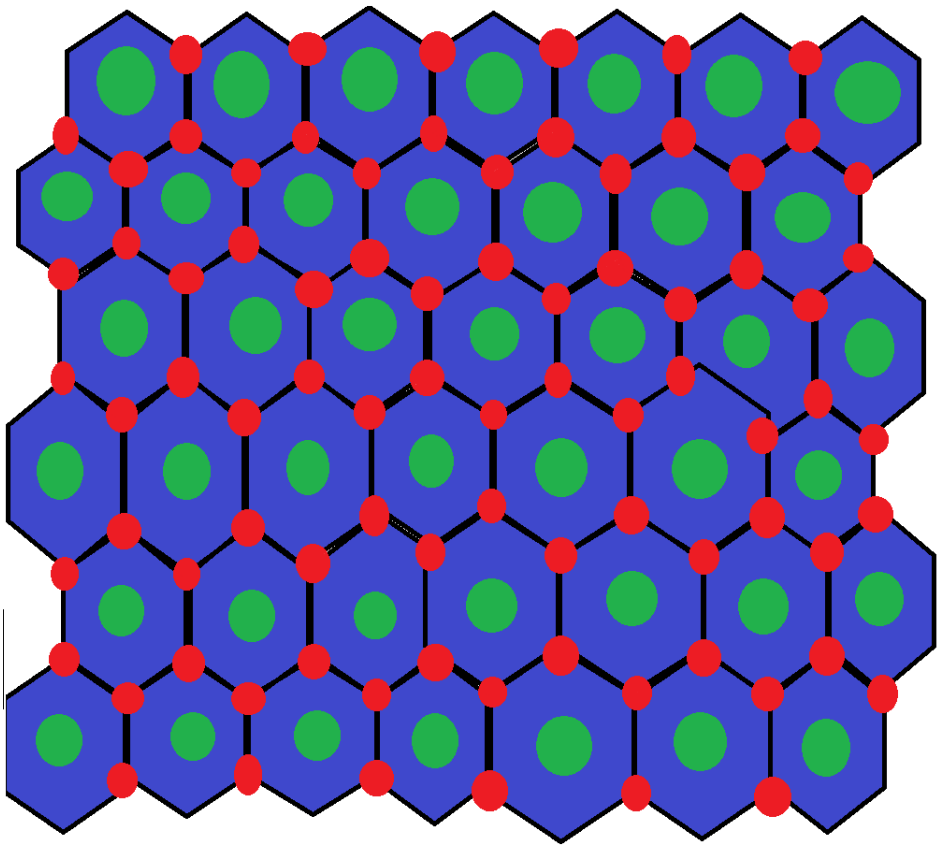
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability.

References

Production of Neutrons by Uranium – *H. L. Anderson and E. Fermi* - 1941

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On the study of various Caccioppoli's equations. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory, Theoretical Cosmology and String Theory II

Michele Nardelli[1], Antonio Nardelli

Abstract

In this paper (part II), we analyze various Caccioppoli's equations. We obtain new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory, Theoretical Cosmology and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

A. Nardelli studied at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**

Srinivasa Ramanujan

(1887 - 1920)

Introduction

In this paper, an octahedron could serve as a mathematical or conceptual model of the universe in the quantic phase, while the spherical surface could be used to describe the geometry of the bubble-universe.

The values $(2\sqrt{2})/\pi$, the golden ratio φ, $\zeta(2)$ and π, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$ the so called DN Constant:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and bubbles universe in eternal inflation.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and bubbles universe.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of bubbles universe and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

The above values $(2\sqrt{2})/\pi$, the golden ratio φ and π, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

The various mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe.

Regarding the fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the quantum universe and bubbles universe.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

Proposal:

The initial octahedron: Let's imagine a regular octahedron, with perfectly symmetrical faces. Each face represents an ideal symmetry.

The emerging sphere: Inside the octahedron, there is an inscribed sphere. This sphere represents the bubble of the universe that emerges from the perturbations of the quantum vacuum during eternal inflation.

Expansion and transitions: As time passes, the universe expands. The faces of the octahedron begin to break, symbolizing "symmetry breaks." The sphere continues to grow, representing the expanding universe.

Constants and numbers: We integrate the mathematical results you obtained. For example, the golden ratio (φ) could be represented by a proportion between the dimensions of the octahedron and the sphere.

Entropy and complexity: Entropy increases as the universe evolves. We can represent this with a disordered growth of structures within the emerging sphere. Imagine this scene as an abstract work of art, where geometric shapes and cosmological concepts merge

In Fig.1 and Fig.2 let's imagine a regular octahedron representing the universe in its phase of high symmetry and very low entropy. Inside the octahedron we have an inscribed sphere that emerges from perturbations of the quantum vacuum during eternal inflation. As time passes, the universe expands, the faces of the octahedron break (symmetry breaks), and entropy increases. Spheres emerge from the octahedra, symbolizing the transition phases from a regime of very low entropy to a universe in which, with the passage of time, entropy increases, increasing the complexity of the universe itself.

Fig. 1

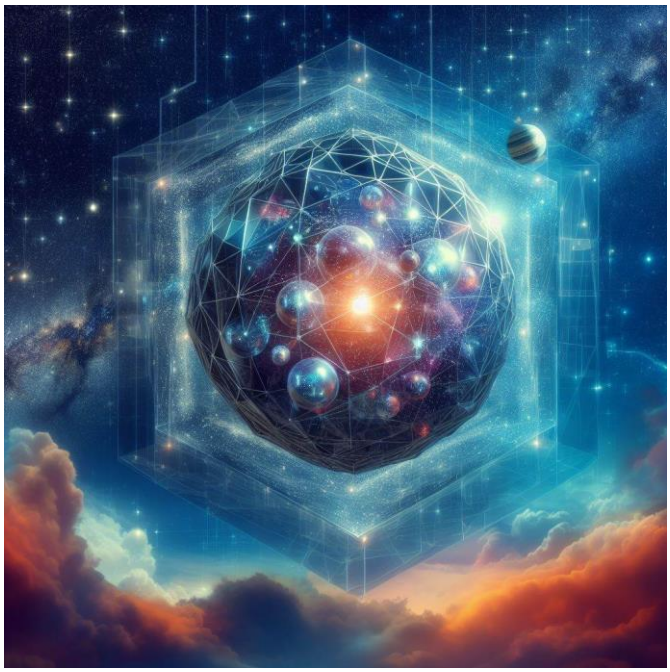
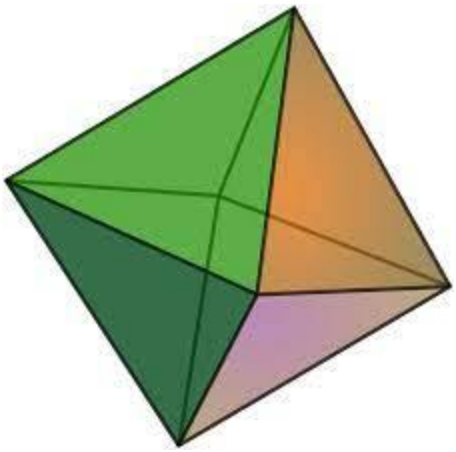
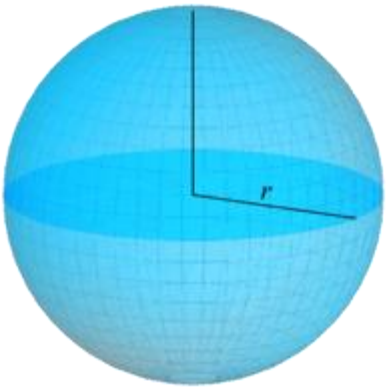


Fig. 2

Now, we have that:



Octahedron



Sphere

From the octahedron volume V = 1/3*√2 l^3 and, from the sphere volume,

V = (4/3*π*r^3) , we consider the following relationship, for r = x:

4/3*π*x^3 = 1/3*√2*l^3

Input

$$\frac{4}{3}\pi x^3 = \frac{1}{3}\sqrt{2}\, l^3$$

Exact result

$$\frac{4\pi x^3}{3} = \frac{\sqrt{2}\, l^3}{3}$$

Alternate forms

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$\frac{4\,\pi\,x^3}{3}-\frac{\sqrt{2}\,l^3}{3}=0$$

Real solution

$$x=\frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Solutions

$$x=-\frac{\sqrt[3]{-\frac{1}{\pi}}\,l}{\sqrt{2}}$$

$$x=\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

$$x=\frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Integer solution

$$l=0\,,\quad x=0$$

Implicit derivatives

$$\frac{\partial x(l)}{\partial l}=\frac{l^2}{2\,\sqrt{2}\,\pi\,x^2}$$

$$\frac{\partial l(x)}{\partial x}=\frac{2\,\sqrt{2}\,\pi\,x^2}{l^2}$$

From the alternate form

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

for l = 8, we have that:

8/(sqrt(2) π^(1/3)) = 8/(2sqrt2 * Pi)^1/3

Input

$$\frac{8}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{8}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Result

True

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Thence:

l/(sqrt(2) π^(1/3)) = l/(2sqrt2 * Pi)^1/3

Input

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Now, we have that:

l/(2 sqrt(2) π)^(1/3) = (2sqrt2)/Pi

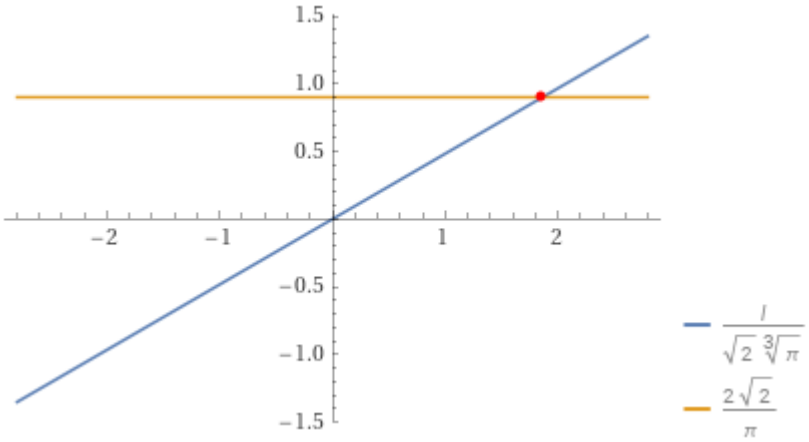
Input

$$\frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}} = \frac{2\sqrt{2}}{\pi}$$

Exact result

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{2\sqrt{2}}{\pi}$$

Plot



Solution

$$l = \frac{4}{\pi^{2/3}}$$

$$l = \frac{4}{\sqrt[3]{\pi^2}}$$

From

Fondamenti per una Teoria Generale delle Funzioni Pseudo-Analitiche di una Variabile Complessa – *Prof. Renato Caccioppoli* – 1952

We have a limitation for the oscillation $w(C_0)$ of $w(z)$ in the circle $C_0 = C(r_0)$. Setting, for example, $R = \sqrt{(Rr_0)}$, remembering that K is the coefficient of the isoperimetric inequality we find:

$$\omega(C_0) < \frac{2\pi K}{\mu} \frac{1}{\log R/r_0}$$

Input

$$\frac{2\,\pi\,K}{\mu} \times \frac{1}{\log\left(\frac{R}{r_0}\right)}$$

log(*x*) is the natural logarithm

Exact result

$$\frac{2\,\pi\,K}{\mu \log\left(\frac{R}{r_0}\right)}$$

Alternate form assuming K, R, μ , and r_0 are positive

$$\frac{2\,\pi\,K}{\mu\,(\log(R) - \log(r_0))}$$

Expanded logarithmic form

$$\frac{2\,\pi\,K}{\mu \log(R) + \mu\,(-1) \log(r_0)}$$

Reduced logarithmic form

$$\frac{2\pi K}{\log\left(\left(\frac{R}{r_0}\right)^{\mu}\right)}$$

Root

$$r_0 (R - r_0) \neq 0, \quad K = 0$$

Integer roots

$$K = 0, \quad R = n_1 + 1, \quad r_0 = n_1 + n_2 + 2, \quad n_1 \in \mathbb{Z}, \quad n_1 \geq 0, \quad n_2 \in \mathbb{Z}, \quad n_2 \geq 0$$

$$K = 0, \quad R = n_1 + n_2 + 2, \quad r_0 = n_2 + 1, \quad n_1 \in \mathbb{Z}, \quad n_1 \geq 0, \quad n_2 \in \mathbb{Z}, \quad n_2 \geq 0$$

$$K = 0, \quad R = -n_1 - n_2 - 2, \quad r_0 = -n_2 - 1, \quad n_1 \in \mathbb{Z}, \quad n_1 \geq 0, \quad n_2 \in \mathbb{Z}, \quad n_2 \geq 0$$

$$K = 0, \quad R = -n_1 - 1, \quad r_0 = -n_1 - n_2 - 2, \quad n_1 \in \mathbb{Z}, \quad n_1 \geq 0, \quad n_2 \in \mathbb{Z}, \quad n_2 \geq 0$$

$\mathbb{Z}$ is the set of integers

Property as a function

Parity

even

Series expansion at $r_0=0$

$$\frac{2\pi K}{\mu \log\left(\frac{R}{r_0}\right)} + O\left(r_0^2\right)$$

(generalized Puiseux series)

Series expansion at $r_0=\infty$

$$\frac{2\pi K}{\mu \log\left(\frac{R}{r_0}\right)} + O\left(\left(\frac{1}{r_0}\right)^2\right)$$

(generalized Puiseux series)

Derivative

$$\frac{\partial}{\partial r_0}\left(\frac{2\pi K}{\mu \log\left(\frac{R}{r_0}\right)}\right) = \frac{2\pi K}{\mu r_0 \log^2\left(\frac{R}{r_0}\right)}$$

Indefinite integral

$$\int \frac{2K\pi}{\mu \log\left(\frac{R}{r_0}\right)}\, dr_0 = -\frac{2\pi K R\, \mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)}{\mu} + \text{constant}$$

(assuming a complex-valued logarithm)

Ei(x) is the exponential integral Ei

Limit

$$\lim_{R\to\pm\infty} \frac{2K\pi}{\mu \log\left(\frac{R}{r_0}\right)} = 0$$

$$\lim_{\mu\to\pm\infty} \frac{2K\pi}{\mu \log\left(\frac{R}{r_0}\right)} = 0$$

$$\lim_{r_0\to\pm\infty} \frac{2K\pi}{\mu \log\left(\frac{R}{r_0}\right)} = 0$$

Alternative representations

$$\frac{2\pi K}{\log\left(\frac{R}{r_0}\right)\mu} = \frac{2K\pi}{\mu \log_e\left(\frac{R}{r_0}\right)}$$

$$\frac{2\pi K}{\log\left(\frac{R}{r_0}\right)\mu} = \frac{2K\pi}{\mu\left(\log(a)\log_a\left(\frac{R}{r_0}\right)\right)}$$

$$\frac{2\pi K}{\log\left(\frac{R}{r_0}\right)\mu} = \frac{2K\pi}{\mu\left(-\mathrm{Li}_1\left(1-\frac{R}{r_0}\right)\right)}$$

$\log_b(x)$ is the base- b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{2\pi K}{\log\left(\frac{R}{r_0}\right)\mu} = -\frac{2K\pi}{\mu\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{R}{r_0}\right)^k}{k}} \quad \text{for} \quad \left|-1+\frac{R}{r_0}\right| < 1$$

$$\frac{2\pi K}{\log\left(\frac{R}{r_0}\right)\mu} = \frac{2K\pi}{\mu\left(\log\left(-1+\frac{R}{r_0}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{R}{r_0}\right)^{-k}}{k}\right)} \quad \text{for} \quad \left|-1+\frac{R}{r_0}\right| > 1$$

$$\frac{2\pi K}{\log\left(\frac{R}{r_0}\right)\mu} = \frac{2K\pi}{\mu\left(2i\pi\left\lfloor\frac{\arg\left(-x+\frac{R}{r_0}\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k x^{-k}\left(-x+\frac{R}{r_0}\right)^k}{k}\right)} \quad \text{for} \quad x < 0$$

$|z|$ is the absolute value of z

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{2\pi K}{\log\left(\frac{R}{r_0}\right)\mu} = \frac{2K\pi}{\mu \int_1^{\frac{R}{r_0}} \frac{1}{t}\, dt}$$

$$\frac{2\pi K}{\log\left(\frac{R}{r_0}\right)\mu} = \frac{4iK\pi^2}{\mu \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{R}{r_0}\right)^{-s}}{\Gamma(1-s)}\, ds} \quad \text{for } \left(-1 < \gamma < 0 \text{ and } \left|\arg\left(-1+\frac{R}{r_0}\right)\right| < \pi\right)$$

$\Gamma(x)$ is the gamma function

Within a class C_μ, functions for which the same conditional isoperimetric inequality is verified in the relative "**riemannian fields**", constitute a normal (i.e. compact) family.

From the indefinite integral result

$$\int \frac{2K\pi}{\mu \log\left(\frac{R}{r_0}\right)}\, dr_0 = -\frac{2\pi K R\, \mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)}{\mu} + \text{constant}$$

(assuming a complex-valued logarithm)

we obtain:

-(2 π K R Ei(-log(R/r_0)))/μ

Input

$$-\frac{2\pi K R\, \mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)}{\mu}$$

$\log(x)$ is the natural logarithm

$\mathrm{Ei}(x)$ is the exponential integral Ei

Expanded logarithmic form

$$-\frac{2\pi K R \operatorname{Ei}(-\log(R)+\log(r_0))}{\mu}$$

Alternate form assuming K, R, μ, and r_0 are positive

$$-\frac{2\pi K R \operatorname{Ei}(\log(r_0)-\log(R))}{\mu}$$

Alternate form assuming K, R, μ, and r_0 are positive

$$-\frac{2\pi K R \operatorname{li}\left(\frac{r_0}{R}\right)}{\mu}$$

$\operatorname{li}(x)$ is the logarithmic integral

Series expansion at $r_0=0$

$$\frac{1}{\mu}\left(\pi K R \log\left(\left((\log(R)-\log(r_0))+O\left(r_0^2\right)\right)+2i\pi\left\lfloor -\frac{\arg(R)}{2\pi}+\frac{\arg(r_0)}{2\pi}+\frac{1}{2}\right\rfloor\right)-\right.$$
$$\pi K R \log\left(\left((\log(r_0)-\log(R))+O\left(r_0^2\right)\right)-2i\pi\left\lfloor -\frac{\arg(R)}{2\pi}+\frac{\arg(r_0)}{2\pi}+\frac{1}{2}\right\rfloor\right)+$$
$$\left(iK\pi^2 R+\frac{2K\pi\left(\log\left(\frac{R}{r_0}\right)-1\right)r_0}{\log^2\left(\frac{R}{r_0}\right)}+O\left(r_0^2\right)\right)+$$
$$2i\pi^2 K R\left\lfloor\frac{\arg\left(2i\pi\left\lfloor -\frac{\arg(R)}{2\pi}+\frac{\arg(r_0)}{2\pi}+\frac{1}{2}\right\rfloor+\log(R)-\log(r_0)\right)}{2\pi}\right\rfloor-$$
$$\left.2i\pi^2 K R\left\lfloor\frac{\arg\left(2i\pi\left\lfloor -\frac{\arg(R)}{2\pi}+\frac{\arg(r_0)}{2\pi}+\frac{1}{2}\right\rfloor+\log(R)-\log(r_0)\right)}{2\pi}+\frac{1}{2}\right\rfloor\right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Derivative

$$\frac{\partial}{\partial r_0}\left(-\frac{2\pi K R\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)}{\mu}\right)=\frac{2\pi K}{\mu\log\left(\frac{R}{r_0}\right)}$$

Indefinite integral

$$\int -\frac{2\pi K R\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)}{\mu}\,dr_0=\frac{2\pi K R\left(R\,\mathrm{Ei}\left(-2\log\left(\frac{R}{r_0}\right)\right)-r_0\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)\right)}{\mu}+\text{constant}$$

(assuming a complex-valued logarithm)

From the indefinite integral result

$$\int -\frac{2\pi K R\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)}{\mu}\,dr_0=\frac{2\pi K R\left(R\,\mathrm{Ei}\left(-2\log\left(\frac{R}{r_0}\right)\right)-r_0\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)\right)}{\mu}+\text{constant}$$

(assuming a complex-valued logarithm)

we obtain:

(2 π K R (R Ei(-2 log(R/r_0)) - r_0 Ei(-log(R/r_0))))/μ

Input

$$\frac{2\pi K R\left(R\,\mathrm{Ei}\left(-2\log\left(\frac{R}{r_0}\right)\right)-r_0\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)\right)}{\mu}$$

log(x) is the natural logarithm

Ei(x) is the exponential integral Ei

Expanded form

$$\frac{2\pi K R^2\,\mathrm{Ei}\left(-2\log\left(\frac{R}{r_0}\right)\right)}{\mu}-\frac{2\pi K r_0 R\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)}{\mu}$$

Alternate form assuming K, R, μ, and r_0 are positive

$$\frac{2\pi K R\,(R\,\mathrm{Ei}(2\log(r_0)-2\log(R))-r_0\,\mathrm{Ei}(\log(r_0)-\log(R)))}{\mu}$$

Expanded logarithmic form

$$\frac{2K\pi R^2\,\mathrm{Ei}(-2\log(R)+2\log(r_0))}{\mu}+-\frac{2K\pi R\,\mathrm{Ei}(-\log(R)+\log(r_0))\,r_0}{\mu}$$

Reduced logarithmic form

$$\frac{2K\pi R\left(R\,\mathrm{Ei}\left(\log\left(\frac{r_0^2}{R^2}\right)\right)-\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)r_0\right)}{\mu}$$

Alternate form

$$-\frac{2\pi K R\left(r_0\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)-R\,\mathrm{Ei}\left(-2\log\left(\frac{R}{r_0}\right)\right)\right)}{\mu}$$

Derivative

$$\frac{\partial}{\partial r_0}\left(\frac{2\pi K R\left(R\,\mathrm{Ei}\left(-2\log\left(\frac{R}{r_0}\right)\right)-r_0\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)\right)}{\mu}\right)=-\frac{2\pi K R\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)}{\mu}$$

Indefinite integral

$$\int\frac{2\pi K R\left(R\,\mathrm{Ei}\left(-2\log\left(\frac{R}{r_0}\right)\right)-r_0\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)\right)}{\mu}\,dr_0=$$
$$-\frac{\pi K R\left(R^2\,\mathrm{Ei}\left(-3\log\left(\frac{R}{r_0}\right)\right)-2r_0\,R\,\mathrm{Ei}\left(-2\log\left(\frac{R}{r_0}\right)\right)+r_0^2\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)\right)}{\mu}+\text{constant}$$

(assuming a complex-valued logarithm)

From the indefinite integral result

$$\int \frac{2\pi K R\left(R\,\mathrm{Ei}\left(-2\log\left(\frac{R}{r_0}\right)\right)-r_0\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)\right)}{\mu}\,dr_0 =$$
$$-\frac{\pi K R\left(R^2\,\mathrm{Ei}\left(-3\log\left(\frac{R}{r_0}\right)\right)-2r_0\,R\,\mathrm{Ei}\left(-2\log\left(\frac{R}{r_0}\right)\right)+r_0^2\,\mathrm{Ei}\left(-\log\left(\frac{R}{r_0}\right)\right)\right)}{\mu} + \text{constant}$$

(assuming a complex-valued logarithm)

-(π K R (R^2 Ei(-3 log(R/r_0)) - 2 r_0 R Ei(-2 log(R/r_0)) + r_0^2 Ei(-log(R/r_0))))/μ

for K = 2, μ = 4, r_0 = 8 and R = 16:

-(π 32 (16^2 Ei(-3 log(16/8)) – 2*8*16 Ei(-2 log(16/8)) + 8^2 Ei(-log(16/8))))/4

Input

$$-\frac{1}{4}\left(\pi\times 32\left(16^2\,\mathrm{Ei}\left(-3\log\left(\frac{16}{8}\right)\right)-(2\times 8\times 16)\,\mathrm{Ei}\left(-2\log\left(\frac{16}{8}\right)\right)+8^2\,\mathrm{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right)$$

log(x) is the natural logarithm

Ei(x) is the exponential integral Ei

Exact result

$$-8\pi\,(256\,\mathrm{Ei}(-3\log(2))-256\,\mathrm{Ei}(-2\log(2))+64\,\mathrm{Ei}(-\log(2)))$$

Decimal approximation

127.6325608716889618649765144240455301730339477624081874732899060 7…

127.632560871….

Alternate forms

$$-512\pi\,(4\,\mathrm{Ei}(-3\log(2))-4\,\mathrm{Ei}(-2\log(2))+\mathrm{Ei}(-\log(2)))$$

$$-512\,\pi\,(\mathrm{Ei}(-\log(2)) - 4\,\mathrm{Ei}(-\log(4)) + 4\,\mathrm{Ei}(-\log(8)))$$

$$\pi\,(-2048\,\mathrm{Ei}(-3\log(2)) + 2048\,\mathrm{Ei}(-2\log(2)) - 512\,\mathrm{Ei}(-\log(2)))$$

Expanded logarithmic form

$$-8\,\pi\,256\,\mathrm{Ei}(-3\log(2)) - 8\,\pi\,(-256)\,\mathrm{Ei}(-2\log(2)) - 8\,\pi\,64\,\mathrm{Ei}(-\log(2))$$

Reduced logarithmic form

$$-8\,\pi\,(64\,\mathrm{Ei}(-\log(2)) - 256\,\mathrm{Ei}(-\log(4)) + 256\,\mathrm{Ei}(-\log(8)))$$

Alternative representations

$$-\frac{1}{4}\left(\pi\,32\left(16^2\,\mathrm{Ei}\left(-3\log\left(\frac{16}{8}\right)\right) - (2\times 8\times 16)\,\mathrm{Ei}\left(-2\log\left(\frac{16}{8}\right)\right) + 8^2\,\mathrm{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right) =$$
$$-\frac{32}{4}\,\pi\left(-256\,\mathrm{Ei}\left(-2\log(a)\log_a\left(\frac{16}{8}\right)\right) + \mathrm{Ei}\left(-\log(a)\log_a\left(\frac{16}{8}\right)\right)8^2 + \mathrm{Ei}\left(-3\log(a)\log_a\left(\frac{16}{8}\right)\right)16^2\right)$$

$$-\frac{1}{4}\left(\pi\,32\left(16^2\,\mathrm{Ei}\left(-3\log\left(\frac{16}{8}\right)\right) - (2\times 8\times 16)\,\mathrm{Ei}\left(-2\log\left(\frac{16}{8}\right)\right) + 8^2\,\mathrm{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right) =$$
$$-\frac{32}{4}\,\pi\left(-256\,\mathrm{Ei}\left(-2\log_e\left(\frac{16}{8}\right)\right) + \mathrm{Ei}\left(-\log_e\left(\frac{16}{8}\right)\right)8^2 + \mathrm{Ei}\left(-3\log_e\left(\frac{16}{8}\right)\right)16^2\right)$$

$$-\frac{1}{4}\left(\pi\,32\left(16^2\,\mathrm{Ei}\left(-3\log\left(\frac{16}{8}\right)\right) - (2\times 8\times 16)\,\mathrm{Ei}\left(-2\log\left(\frac{16}{8}\right)\right) + 8^2\,\mathrm{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right) =$$
$$-\frac{32}{4}\,\pi\left(-256\,\mathrm{li}\left(e^{-2\log(16/8)}\right) + \mathrm{li}\left(e^{-\log(16/8)}\right)8^2 + \mathrm{li}\left(e^{-3\log(16/8)}\right)16^2\right)$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{li}(x)$ is the logarithmic integral

Series representations

$$-\frac{1}{4}\left(\pi\, 32\left(16^2\,\text{Ei}\left(-3\log\left(\frac{16}{8}\right)\right)-(2\times 8\times 16)\,\text{Ei}\left(-2\log\left(\frac{16}{8}\right)\right)+8^2\,\text{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right)=$$
$$\sum_{k=0}^{\infty}\frac{256\,\pi\,(L_k(\log(2))-2\,L_k(\log(4))+L_k(\log(8)))}{1+k}$$

$$-\frac{1}{4}\left(\pi\, 32\left(16^2\,\text{Ei}\left(-3\log\left(\frac{16}{8}\right)\right)-(2\times 8\times 16)\,\text{Ei}\left(-2\log\left(\frac{16}{8}\right)\right)+8^2\,\text{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right)=$$
$$-512\,\gamma\,\pi-512\,\pi\log(\log(2))+2048\,\pi\log(\log(4))-$$
$$2048\,\pi\log(\log(8))+\sum_{k=1}^{\infty}-\frac{512\,e^{i\,k\,\pi}\,\pi\left(\log^k(2)-4\log^k(4)+4\log^k(8)\right)}{k\,k!}$$

$$-\frac{1}{4}\left(\pi\, 32\left(16^2\,\text{Ei}\left(-3\log\left(\frac{16}{8}\right)\right)-(2\times 8\times 16)\,\text{Ei}\left(-2\log\left(\frac{16}{8}\right)\right)+8^2\,\text{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right)=$$
$$-256\,\pi\left(2\,\gamma+2\log(\log(2))-8\log(\log(4))+8\log(\log(8))-\right.$$
$$\left.\sum_{k=1}^{\infty}\sum_{j=1}^{k}\frac{(-1)^{2k}\log^k(2)}{j\,k!}+2\sum_{k=1}^{\infty}\sum_{j=1}^{k}\frac{(-1)^{2k}\log^k(4)}{j\,k!}-\sum_{k=1}^{\infty}\sum_{j=1}^{k}\frac{(-1)^{2k}\log^k(8)}{j\,k!}\right)$$

$L_n(x)$ is the Laguerre polynomial L

$n!$ is the factorial function

γ is the Euler-Mascheroni constant

Integral representations

$$-\frac{1}{4}\left(\pi\, 32\left(16^2\,\text{Ei}\left(-3\log\left(\frac{16}{8}\right)\right)-(2\times 8\times 16)\,\text{Ei}\left(-2\log\left(\frac{16}{8}\right)\right)+8^2\,\text{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right)=$$
$$-512\,\gamma\,\pi+\int_0^{-\log(2)}\frac{512\left(1+4^{1+t/\log(2)}-e^t-4\,e^{3\,t}\right)\pi}{t}\,dt-$$
$$512\,\pi\log(\log(2))+2048\,\pi\log(\log(4))-2048\,\pi\log(\log(8))$$

$$-\frac{1}{4}\left(\pi\,32\left(16^2\,\mathrm{Ei}\left(-3\log\left(\frac{16}{8}\right)\right)-(2\times 8\times 16)\,\mathrm{Ei}\left(-2\log\left(\frac{16}{8}\right)\right)+8^2\,\mathrm{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right)=$$
$$\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}-\frac{1}{\Gamma(1+s)}\,256\,i\,\Gamma(s)^2\,(\log(2)\log(4)\log(8))^{-s}$$
$$\left(4\,(\log(2)\log(4))^s-4\,(\log(2)\log(8))^s+(\log(4)\log(8))^s\right)ds\quad\text{for } 0<\gamma$$

$$-\frac{1}{4}\left(\pi\,32\left(16^2\,\mathrm{Ei}\left(-3\log\left(\frac{16}{8}\right)\right)-(2\times 8\times 16)\,\mathrm{Ei}\left(-2\log\left(\frac{16}{8}\right)\right)+8^2\,\mathrm{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right)=$$
$$-512\,\pi\left(\mathcal{P}\int_{-\infty}^{-\log(2)}\frac{e^t}{t}\,dt-4\,\mathcal{P}\int_{-\infty}^{-\log(4)}\frac{e^t}{t}\,dt+4\,\mathcal{P}\int_{-\infty}^{-\log(8)}\frac{e^t}{t}\,dt\right)$$

$\Gamma(x)$ is the gamma function

$\mathcal{P}\int f\,dx$ is the Cauchy principal value integral

From which:

(1/2(-(π 32 (16^2 Ei(-3 log(16/8)) – 2*8*16 Ei(-2 log(16/8)) + 8^2 Ei(-log(16/8))))/4 + 2 MRB const))^2-e/5

Input

$$\left(\frac{1}{2}\left(-\frac{1}{4}\left(\pi\times 32\left(16^2\,\mathrm{Ei}\left(-3\log\left(\frac{16}{8}\right)\right)-(2\times 8\times 16)\,\mathrm{Ei}\left(-2\log\left(\frac{16}{8}\right)\right)+8^2\,\mathrm{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right)+2\,C_{\mathrm{MRB}}\right)\right)^2-\frac{e}{5}$$

$\log(x)$ is the natural logarithm

$\mathrm{Ei}(x)$ is the exponential integral Ei

C_{MRB} is the MRB constant

Exact result

$$\frac{1}{4}\left(2\,C_{\mathrm{MRB}}-8\,\pi\left(256\,\mathrm{Ei}(-3\log(2))-256\,\mathrm{Ei}(-2\log(2))+64\,\mathrm{Ei}(-\log(2))\right)\right)^2-\frac{e}{5}$$

Decimal approximation

4095.986290797794726053027289637373521363182705020959112979134285 2

…

4095.9862907977…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$(C_{\text{MRB}} - 256\,\pi\,(4\,\text{Ei}(-3\log(2)) - 4\,\text{Ei}(-2\log(2)) + \text{Ei}(-\log(2))))^2 - \frac{e}{5}$$

$$(C_{\text{MRB}} - 256\,\pi\,(\text{Ei}(-\log(2)) - 4\,\text{Ei}(-\log(4)) + 4\,\text{Ei}(-\log(8))))^2 - \frac{e}{5}$$

$$\frac{1}{5}\left(5\left(C_{\text{MRB}}-256\,\pi\left(4\,\text{Ei}(-3\log(2))-4\,\text{Ei}(-2\log(2))+\text{Ei}(-\log(2))\right)\right)^2-e\right)$$

Expanded logarithmic form

$$\begin{aligned}&C_{\text{MRB}}{}^2-2048\,C_{\text{MRB}}\,\pi\,\text{Ei}(-3\log(2))+\\&\quad 1\,048\,576\,\pi^2\,\text{Ei}(-3\log(2))^2+2\times 1024\,C_{\text{MRB}}\,\pi\,\text{Ei}(-2\log(2))+\\&\quad 2\,(-1\,048\,576)\,\pi^2\,\text{Ei}(-3\log(2))\,\text{Ei}(-2\log(2))+2\,(-256)\,C_{\text{MRB}}\,\pi\,\text{Ei}(-\log(2))+\\&\quad 2\times 262\,144\,\pi^2\,\text{Ei}(-3\log(2))\,\text{Ei}(-\log(2))+1\,048\,576\,\pi^2\,\text{Ei}(-2\log(2))^2-\\&\quad 524\,288\,\pi^2\,\text{Ei}(-2\log(2))\,\text{Ei}(-\log(2))+65\,536\,\pi^2\,\text{Ei}(-\log(2))^2-\frac{e}{5}\end{aligned}$$

Reduced logarithmic form

$$\frac{1}{5}\left(-e+5\left(C_{\text{MRB}}-256\,\pi\left(\text{Ei}(-\log(2))-4\,\text{Ei}(-\log(4))+4\,\text{Ei}(-\log(8))\right)\right)^2\right)$$

27*sqrt((1/2(-(π 32 (16^2 Ei(-3 log(16/8)) – 2*8*16 Ei(-2 log(16/8)) + 8^2 Ei(-log(16/8))))/4 + 2 MRB const))^2-e/5)+1

Input

$$27\sqrt{\left(\left(\frac{1}{2}\left(-\frac{1}{4}\left(\pi\times 32\left(16^2\,\text{Ei}\left(-3\log\left(\frac{16}{8}\right)\right)-(2\times 8\times 16)\,\text{Ei}\left(-2\log\left(\frac{16}{8}\right)\right)+8^2\,\text{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)+2\,C_{\text{MRB}}\right)\right)^2-\frac{e}{5}\right)+1}$$

$\log(x)$ is the natural logarithm

$\text{Ei}(x)$ is the exponential integral Ei

C_{MRB} is the MRB constant

Exact result

$$27\sqrt{\frac{1}{4}\left(2\,C_{\text{MRB}}-8\,\pi\left(256\,\text{Ei}(-3\log(2))-256\,\text{Ei}(-2\log(2))+64\,\text{Ei}(-\log(2))\right)\right)^2-\frac{e}{5}}+1$$

Decimal approximation

1728.997108212740139391309249586259031986384662756895132037810948 9…

1728.99710821274….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$27\sqrt{(C_{\text{MRB}} - 256\,\pi\,(\text{Ei}(-\log(2)) - 4\,\text{Ei}(-\log(4)) + 4\,\text{Ei}(-\log(8))))^2 - \frac{e}{5}} + 1$$

$$\frac{27}{\sqrt{\frac{5}{5\left(C_{\text{MRB}} - 256\,\pi\,(4\,\text{Ei}(-3\log(2)) - 4\,\text{Ei}(-2\log(2)) + \text{Ei}(-\log(2)))\right)^2 - e}}} + 1$$

$$\frac{1}{5}\Big(27\sqrt{\big(5\big(-10\,240\,\pi\,\text{Ei}(-3\log(2))\,C_{\text{MRB}} + 10\,240\,\pi\,\text{Ei}(-2\log(2))\,C_{\text{MRB}} - 2560\,\pi\,\text{Ei}(-\log(2))\,C_{\text{MRB}} + 5\,242\,880\,\pi^2\,\text{Ei}(-3\log(2))^2 - 10\,485\,760\,\pi^2\,\text{Ei}(-3\log(2))\,\text{Ei}(-2\log(2)) + 5\,242\,880\,\pi^2\,\text{Ei}(-2\log(2))^2 + 2\,621\,440\,\pi^2\,\text{Ei}(-3\log(2))\,\text{Ei}(-\log(2)) - 2\,621\,440\,\pi^2\,\text{Ei}(-2\log(2))\,\text{Ei}(-\log(2)) + 327\,680\,\pi^2\,\text{Ei}(-\log(2))^2 + 5\,C_{\text{MRB}}{}^2 - e\big)\big) + 5\Big)$$

Expanded logarithmic form

$$27\sqrt{\left(C_{\text{MRB}}{}^2 - 2048\,C_{\text{MRB}}\,\pi\,\text{Ei}(-3\log(2)) + 1\,048\,576\,\pi^2\,\text{Ei}(-3\log(2))^2 + 2\times 1024\,C_{\text{MRB}}\,\pi\,\text{Ei}(-2\log(2)) + 2\,(-1\,048\,576)\,\pi^2 \text{Ei}(-3\log(2))\,\text{Ei}(-2\log(2)) + 2\,(-256)\,C_{\text{MRB}}\,\pi\,\text{Ei}(-\log(2)) + 2\times 262\,144\,\pi^2\,\text{Ei}(-3\log(2))\,\text{Ei}(-\log(2)) + 1\,048\,576\,\pi^2\,\text{Ei}(-2\log(2))^2 - 524\,288\,\pi^2\,\text{Ei}(-2\log(2))\,\text{Ei}(-\log(2)) + 65\,536\,\pi^2\,\text{Ei}(-\log(2))^2 - \frac{e}{5}\right)} + 1$$

Reduced logarithmic form

$$\frac{1}{5}\left(5 + 27\sqrt{5\left(-e + 5\,(C_{\text{MRB}} - 256\,\pi\,(\text{Ei}(-\log(2)) - 4\,\text{Ei}(-\log(4)) + 4\,\text{Ei}(-\log(8))))^2\right)}\right)$$

(27*sqrt((1/2(-(π 32 (16^2 Ei(-3 log(16/8)) – 2*8*16 Ei(-2 log(16/8)) + 8^2 Ei(-log(16/8))))/4 + 2 MRB const))^2-e/5)+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\left(27\sqrt{\left(\left(\frac{1}{2}\left(-\frac{1}{4}\left(\pi\times 32\left(16^2\,\text{Ei}\left(-3\log\left(\frac{16}{8}\right)\right) - (2\times 8\times 16)\,\text{Ei}\left(-2\log\left(\frac{16}{8}\right)\right) + 8^2\,\text{Ei}\left(-\log\left(\frac{16}{8}\right)\right)\right)\right) + 2\,C_{\text{MRB}}\right)\right)^2 - \frac{e}{5}\right) + 1\right)\,\hat{}\,(1/15) + C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}$$

log(x) is the natural logarithm

Ei(x) is the exponential integral Ei

C_{MRB} is the MRB constant

Exact result

$$\left(27\sqrt{\frac{1}{4}\left(2\,C_{\text{MRB}}-8\,\pi\left(256\,\text{Ei}(-3\log(2))-256\,\text{Ei}(-2\log(2))+64\,\text{Ei}(-\log(2))\right)\right)^2-\frac{e}{5}}+1\right)\,\hat{}\,(1/15)+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

Decimal approximation

1.64493784084251604102742128511597604047369858924957173144600713 80

…

1.64493784…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$\sqrt[15]{27\sqrt{\left(C_{\text{MRB}}-256\,\pi\left(\text{Ei}(-\log(2))-4\,\text{Ei}(-\log(4))+4\,\text{Ei}(-\log(8))\right)\right)^2-\frac{e}{5}}+1}+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

$$\sqrt[15]{\frac{27}{\sqrt{\frac{5}{5\left(C_{\text{MRB}}-256\,\pi\left(4\,\text{Ei}(-3\log(2))-4\,\text{Ei}(-2\log(2))+\text{Ei}(-\log(2))\right)\right)^2-e}}}+1}+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

$$1\Big/\Big(\Big(5\Big/\Big(27\sqrt{\big(5\big(-10\,240\,\pi\,\text{Ei}(-3\log(2))\,C_{\text{MRB}}+10\,240\,\pi\,\text{Ei}(-2\log(2))\,C_{\text{MRB}}-2560\,\pi\,\text{Ei}(-\log(2))\,C_{\text{MRB}}+5\,242\,880\,\pi^2\,\text{Ei}(-3\log(2))^2-10\,485\,760\,\pi^2\,\text{Ei}(-3\log(2))\,\text{Ei}(-2\log(2))+5\,242\,880\,\pi^2\,\text{Ei}(-2\log(2))^2+2\,621\,440\,\pi^2\,\text{Ei}(-3\log(2))\,\text{Ei}(-\log(2))-2\,621\,440\,\pi^2\,\text{Ei}(-2\log(2))\,\text{Ei}(-\log(2))+327\,680\,\pi^2\,\text{Ei}(-\log(2))^2+5\,C_{\text{MRB}}{}^2-e\big)\big)+5\Big)\Big)\,\hat{}\,(1/15)\Big)+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

Expanded logarithmic form

$$\left(27\sqrt{\left({C_{\text{MRB}}}^2 - 2048\,C_{\text{MRB}}\,\pi\,\text{Ei}(-3\log(2)) + 1\,048\,576\,\pi^2\,\text{Ei}(-3\log(2))^2 + 2\times 1024\,C_{\text{MRB}}\,\pi\,\text{Ei}(-2\log(2)) + 2\,(-1\,048\,576)\,\pi^2\,\text{Ei}(-3\log(2))\,\text{Ei}(-2\log(2)) + 2\,(-256)\,C_{\text{MRB}}\,\pi\,\text{Ei}(-\log(2)) + 2\times 262\,144\,\pi^2\,\text{Ei}(-3\log(2))\,\text{Ei}(-\log(2)) + 1\,048\,576\,\pi^2\,\text{Ei}(-2\log(2))^2 - 524\,288\,\pi^2\,\text{Ei}(-2\log(2))\,\text{Ei}(-\log(2)) + 65\,536\,\pi^2\,\text{Ei}(-\log(2))^2 - \frac{e}{5}\right) + 1\right)\,\hat{}\,(1/15) + {C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

With γ we indicate a circumference

Consider a circular crown Γ(r) with center at the origin and radii r_0 , r > r_0 . and let G(r) be the area $\int_{\gamma(r)} Jdxdy$ of its image. By increasing $G(r)$, we obtain the inequality $K\sqrt{\frac{2\pi r}{\mu}}f(r) - \int_{r_0}^{r} f^2(\varrho)d\varrho + c > 0$, where c indicates the length of the image of $\gamma(r_0)$. It follows:

$$G(r) < \frac{2\pi K^2}{\mu}\frac{1}{\log R/r} + c$$

The inequality for G(r) is written:

$$G(r) > \frac{2\mu}{\pi}\log\frac{r}{r_0}\tau^2(r)$$

where τ(r) indicates the smallest of the oscillations of w(z) on γ(r_0) and γ(r). Setting $r = \sqrt{Rr_0}$, we then obtain:

$$\tau^2(r) < \frac{4\pi^2K^2}{\mu^2}\frac{1}{log^2\,R/r} + \frac{\pi}{\mu}\frac{c}{\log R/r}$$

It follows that w(z) is continuous at the ∞ point, and therefore cannot be singular. We thus have: if a pseudo-analytic function presents an isolated singularity at infinity, the related "Riemannian field" contains simple circles of arbitrarily large radius.

We consider:

$$\frac{2\mu}{\pi}\log\frac{r}{r_0}\tau^2(r)$$

(2μ)/π log(r/(r_0)) τ^2*(r)

Input

$$\frac{2\,\mu}{\pi}\log\left(\frac{r}{r_0}\right)(\tau^2\,r)$$

log(x) is the natural logarithm

Result

$$\frac{2\,\mu\,r\,\tau^2\log\left(\frac{r}{r_0}\right)}{\pi}$$

Alternate form

$$\frac{2\,\mu\,r\,\tau^2\,(\log(r)-\log(r_0))}{\pi}$$

Expanded logarithmic form

$$\frac{2\,r\,\mu\,\tau^2\log(r)}{\pi}+\frac{2\,r\,\mu\,\tau^2\,(-1)\log(r_0)}{\pi}$$

Root

$$r\neq 0\,,\quad r_0=r$$

Roots

$r = 0$

$\mu = 0$

$\tau = 0$

Series expansion at r=0

$$\frac{2\,\mu\,r\,\tau^2 \log\left(\frac{r}{r_0}\right)}{\pi} + O\left(r^6\right)$$

(Puiseux series)

Series expansion at r=∞

$$\frac{2\,\mu\,r\,\tau^2 \log\left(\frac{r}{r_0}\right)}{\pi} + O\left(\left(\frac{1}{r}\right)^6\right)$$

(Puiseux series)

Derivative

$$\frac{\partial}{\partial r}\left(\frac{(2\,\mu)\log\left(\frac{r}{r_0}\right)\left(\tau^2\,r\right)}{\pi}\right) = \frac{2\,\mu\,\tau^2\left(\log\left(\frac{r}{r_0}\right)+1\right)}{\pi}$$

Indefinite integral

$$\int \frac{2\,r\,\mu\,\tau^2 \log\left(\frac{r}{r_0}\right)}{\pi}\,dr = \frac{\mu\,r^2\,\tau^2\left(2\log\left(\frac{r}{r_0}\right)-1\right)}{2\,\pi} + \text{constant}$$

(assuming a complex-valued logarithm)

Alternative representations

$$\frac{\left(\log\left(\frac{r}{r_0}\right)\left(\tau^2\, r\right)\right)(2\,\mu)}{\pi} = \frac{2\,r\,\mu\log_e\left(\frac{r}{r_0}\right)\tau^2}{\pi}$$

$$\frac{\left(\log\left(\frac{r}{r_0}\right)\left(\tau^2\, r\right)\right)(2\,\mu)}{\pi} = \frac{2\,r\,\mu\log(a)\log_a\left(\frac{r}{r_0}\right)\tau^2}{\pi}$$

$$\frac{\left(\log\left(\frac{r}{r_0}\right)\left(\tau^2\, r\right)\right)(2\,\mu)}{\pi} = -\frac{2\,r\,\mu\,\mathrm{Li}_1\left(1-\frac{r}{r_0}\right)\tau^2}{\pi}$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{\left(\log\left(\frac{r}{r_0}\right)\left(\tau^2\, r\right)\right)(2\,\mu)}{\pi} = -\frac{2\,r\,\mu\,\tau^2\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{r}{r_0}\right)^k}{k}}{\pi} \quad \text{for} \left|-1+\frac{r}{r_0}\right| < 1$$

$$\frac{\left(\log\left(\frac{r}{r_0}\right)\left(\tau^2\, r\right)\right)(2\,\mu)}{\pi} = \frac{2\,r\,\mu\,\tau^2\log\left(-1+\frac{r}{r_0}\right)}{\pi} - \frac{2\,r\,\mu\,\tau^2\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{r}{r_0}\right)^{-k}}{k}}{\pi}$$
$$\text{for} \left|-1+\frac{r}{r_0}\right| > 1$$

$$\frac{\left(\log\left(\frac{r}{r_0}\right)\left(\tau^2\, r\right)\right)(2\,\mu)}{\pi} = 4\,i\,r\,\mu\,\tau^2\left\lfloor\frac{\arg\left(\frac{r}{r_0}-x\right)}{2\,\pi}\right\rfloor +$$
$$\frac{2\,r\,\mu\,\tau^2\log(x)}{\pi} - \frac{2\,r\,\mu\,\tau^2\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{r}{r_0}-x\right)^k x^{-k}}{k}}{\pi} \quad \text{for } x < 0$$

$|z|$ is the absolute value of z

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{\left(\log\left(\frac{r}{r_0}\right)\left(\tau^2 r\right)\right)(2\mu)}{\pi} = \frac{2 r \mu \tau^2}{\pi} \int_1^{\frac{r}{r_0}} \frac{1}{t}\, dt$$

$$\frac{\left(\log\left(\frac{r}{r_0}\right)\left(\tau^2 r\right)\right)(2\mu)}{\pi} = -\frac{i r \mu \tau^2}{\pi^2} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+\frac{r}{r_0}\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)}\, ds$$

$$\text{for } \left(-1 < \gamma < 0 \text{ and } \left|\arg\left(-1+\frac{r}{r_0}\right)\right| < \pi\right)$$

$\Gamma(x)$ is the gamma function

From the indefinite integral result

$$\int \frac{2 r \mu \tau^2 \log\left(\frac{r}{r_0}\right)}{\pi}\, dr = \frac{\mu r^2 \tau^2 \left(2\log\left(\frac{r}{r_0}\right)-1\right)}{2\pi} + \text{constant}$$

(assuming a complex-valued logarithm)

we obtain:

(r^2 μ τ^2 (-1 + 2 log(r/r_0)))/(2 π)

Input

$$\frac{r^2 \mu \sigma_0 \left(-1+2\log\left(\frac{r}{r_0}\right)\right)^2}{2\pi}$$

Alternate forms assuming r, r_0, and μ are positive

$$\frac{\mu r^2 \sigma_0 (2(\log(r) - \log(r_0)) - 1)^2}{2\pi}$$

$$\frac{\mu r^2 \sigma_0 \left(\log(r^2) + \log\left(\frac{1}{r_0^2}\right) - 1\right)^2}{2\pi}$$

Expanded logarithmic form

$$\frac{r^2 \mu \sigma_0 (-1 + 2\log(r) + 2(-1)\log(r_0))^2}{2\pi}$$

Reduced logarithmic form

$$\frac{r^2 \mu \sigma_0 \left(-1 + \log\left(\frac{r^2}{r_0^2}\right)\right)^2}{2\pi}$$

From the following alternate form

$$\frac{\mu r^2 \sigma_0 \left(\log(r^2) + \log\left(\frac{1}{r_0^2}\right) - 1\right)^2}{2\pi}$$

(μ r^2 σ_0(log(r^2) + log(1/r_0^2) - 1)^2)/(2 π)

for $\mu = 2$, $r_0 = 4$, $\sigma_0 = 8$ and $r = 16$:

(2*16^2*8(log(16^2) + log(1/4^2) - 1)^2)/(2 π)

Input

$$\frac{2 \times 16^2 \times 8 \left(\log(16^2) + \log\left(\frac{1}{4^2}\right) - 1\right)^2}{2\pi}$$

log(x) is the natural logarithm

Result

$$\frac{2048\,(-1-\log(16)+\log(256))^2}{\pi}$$

Decimal approximation

2048.311688794046829496757630816319670240164666185240997480356695 8

…

2048.311688794….

Alternate forms

$$\frac{16^2\times 8\,(4\log(2)-1)^2}{\pi}$$

$$\frac{2048\,(\log(16)-1)^2}{\pi}$$

$$\frac{2048\,(1+\log(16)-\log(256))^2}{\pi}$$

Expanded form

$$\frac{2048}{\pi}+\frac{2048\log^2(16)}{\pi}+\frac{2048\log^2(256)}{\pi}+\frac{4096\log(16)}{\pi}-\frac{4096\log(256)}{\pi}-\frac{4096\log(16)\log(256)}{\pi}$$

Expanded logarithmic form

$$\frac{2048}{\pi}+-\frac{16\,384\log(2)}{\pi}+\frac{32\,768\log^2(2)}{\pi}$$

Reduced logarithmic form

$$\frac{2048\,(-1-\log(16)+\log(256))^2}{\pi}$$

Alternative representations

$$\frac{2\left(16^2\times 8\left(\log\left(16^2\right)+\log\left(\frac{1}{4^2}\right)-1\right)^2\right)}{2\,\pi}=\frac{16\times 16^2\left(-1+\log\left(\frac{16^2}{4^2}\right)\right)^2}{2\,\pi}$$

$$\frac{2\left(16^2\times 8\left(\log\left(16^2\right)+\log\left(\frac{1}{4^2}\right)-1\right)^2\right)}{2\,\pi}=\frac{16\times 16^2\left(-1+\log_e\left(16^2\right)+\log_e\left(\frac{1}{4^2}\right)\right)^2}{2\,\pi}$$

$$\frac{2\left(16^2\times 8\left(\log\left(16^2\right)+\log\left(\frac{1}{4^2}\right)-1\right)^2\right)}{2\,\pi}=\frac{16\times 16^2\left(-1+\log(a)\log_a\left(16^2\right)+\log(a)\log_a\left(\frac{1}{4^2}\right)\right)^2}{2\,\pi}$$

$\log_b(x)$ is the base-b logarithm

Series representations

$$\frac{2\left(16^2\times 8\left(\log\left(16^2\right)+\log\left(\frac{1}{4^2}\right)-1\right)^2\right)}{2\,\pi}=\frac{2048\left(-1+\log(17)+\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{255}\right)^k\left(-1+17^k\right)}{k}\right)^2}{\pi}$$

$$\frac{2\left(16^2\times 8\left(\log\left(16^2\right)+\log\left(\frac{1}{4^2}\right)-1\right)^2\right)}{2\,\pi}=\frac{2048\left(1+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left((16-z_0)^k-(256-z_0)^k\right)z_0^{-k}}{k}\right)^2}{\pi}$$

$$\frac{2\left(16^2\times 8\left(\log(16^2)+\log\left(\frac{1}{4^2}\right)-1\right)^2\right)}{2\pi}=\frac{2048\left(-1-2i\pi\left\lfloor\frac{\arg(16-x)}{2\pi}\right\rfloor+2i\pi\left\lfloor\frac{\arg(256-x)}{2\pi}\right\rfloor+\sum_{k=1}^{\infty}\frac{(-1)^k\left((16-x)^k-(256-x)^k\right)x^{-k}}{k}\right)^2}{\pi}$$

for $x<0$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Integral representations

$$\frac{2\left(16^2\times 8\left(\log(16^2)+\log\left(\frac{1}{4^2}\right)-1\right)^2\right)}{2\pi}=\frac{512\left(2\pi+\int_{-i\infty+\gamma}^{i\infty+\gamma}-\frac{i\,255^{-s}\left(-1+17^s\right)\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^2}{\pi^3}\quad\text{for } -1<\gamma<0$$

$$\frac{2\left(16^2\times 8\left(\log(16^2)+\log\left(\frac{1}{4^2}\right)-1\right)^2\right)}{2\pi}=\frac{1}{\pi}\left(2048+2048\left(\int_1^{16}\frac{1}{t}\,dt\right)^2+2048\left(\int_1^{256}\frac{1}{t}\,dt\right)^2+\pi\int_1^{16}\frac{65\,536}{16\pi t-17\pi t^2}\,dt+\int_0^1\int_0^1\frac{1}{(1+15t_1)(1+255t_2)}\,dt_2\,dt_1\right)$$

$\Gamma(x)$ is the gamma function

From which, we obtain:

2((2*16^2*8(log(16^2) + log(1/4^2) - 1)^2)/(2 π))-Φ

Input

$$2\times\frac{2\times 16^2\times 8\left(\log(16^2)+\log\left(\frac{1}{4^2}\right)-1\right)^2}{2\pi}-\Phi$$

$\log(x)$ is the natural logarithm

Φ is the golden ratio conjugate

Result

$$\frac{4096\,(-1-\log(16)+\log(256))^2}{\pi}-\Phi$$

Exact form

$$-\phi+1+\frac{4096\,(\log(16)-1)^2}{\pi}$$

ϕ is the golden ratio

Decimal approximation

4096.005343599343764145310674798273702362609023190676232098577943 1

…

4096.005343599…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality.

The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{-\pi\,\Phi + 4096 + 65\,536\log^2(2) - 32\,768\log(2)}{\pi}$$

$$\frac{4096\,(\log(16) - 1)^2}{\pi} - \Phi$$

$$-\frac{\pi\,\Phi - 4096\,(4\log(2) - 1)^2}{\pi}$$

Expanded forms

$$-\Phi + \frac{4096}{\pi} + \frac{4096\log^2(16)}{\pi} + \frac{4096\log^2(256)}{\pi} + \frac{8192\log(16)}{\pi} - \frac{8192\log(256)}{\pi} - \frac{8192\log(16)\log(256)}{\pi}$$

$$-\Phi + \frac{4096}{\pi} + \frac{65\,536\log^2(2)}{\pi} - \frac{32\,768\log(2)}{\pi}$$

Expanded logarithmic form

$$\frac{16^2 \times 8\left(1 - 8\log(2) + 16\log^2(2)\right)2}{\pi} - \Phi$$

Reduced logarithmic form

$$\frac{1}{\pi}\left(4096 - \Phi\,\pi + 8192\log(16) + 4096\log^2(16) - 8192\log(256) - 8192\log(16)\log(256) + 4096\log^2(256)\right)$$

27*sqrt(2((2*16^2*8(log(16^2) + log(1/4^2) - 1)^2)/(2 π))-Φ)+1

Input

$$27\sqrt{2\times\frac{2\times 16^2\times 8\left(\log(16^2)+\log\left(\frac{1}{4^2}\right)-1\right)^2}{2\,\pi}-\Phi}+1$$

log(x) is the natural logarithm

Φ is the golden ratio conjugate

Exact result

$$27\sqrt{\frac{4096\,(-1-\log(16)+\log(256))^2}{\pi}-\Phi}+1$$

Exact form

$$27\sqrt{-\phi+1+\frac{4096\,(\log(16)-1)^2}{\pi}}+1$$

ϕ is the golden ratio

Decimal approximation

1729.001127165118953372843261219990611161166556258652528756699756 2…

1729.00112716511….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$27\sqrt{\frac{4096\,(\log(16)-1)^2}{\pi}-\Phi}+1$$

$$27\sqrt{\frac{\pi\,(-\Phi)+4096\,(4\log(2)-1)^2}{\pi}}+1$$

$$\frac{27}{\sqrt{\frac{\pi}{-\pi\,\Phi+4096+4096\log^2(16)+4096\log^2(256)+8192\log(16)-8192\log(256)-8192\log(16)\log(256)}}}+1$$

Expanded logarithmic form

$$27\sqrt{\frac{16^2\times 8\left(1-8\log(2)+16\log^2(2)\right)2}{\pi}-\Phi}+1$$

Reduced logarithmic form

$$\frac{27+\sqrt{-\frac{\pi}{-4096+\Phi\,\pi-8192\log(16)-4096\log^2(16)+8192\log(256)+8192\log(16)\log(256)-4096\log^2(256)}}}{\sqrt{-\frac{\pi}{-4096+\Phi\,\pi-8192\log(16)-4096\log^2(16)+8192\log(256)+8192\log(16)\log(256)-4096\log^2(256)}}}$$

(27*sqrt(2((2*16^2*8(log(16^2) + log(1/4^2) - 1)^2)/(2 π))-Φ)+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{27\sqrt{2\times\frac{2\times16^2\times8\left(\log\left(16^2\right)+\log\left(\frac{1}{4^2}\right)-1\right)^2}{2\,\pi}-\Phi\ +1}\ +{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}}$$

log(x) is the natural logarithm

Φ is the golden ratio conjugate

C_{MRB} is the MRB constant

Exact result

$${C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\sqrt[15]{27\sqrt{\frac{4096\,(-1-\log(16)+\log(256))^2}{\pi}-\Phi\ +1}}$$

Decimal approximation

1.64493809557235102951536202929589364308273546796495403144851597 05

…

1.6449380955723…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$${C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\sqrt[15]{27\sqrt{\frac{4096\,(\log(16)-1)^2}{\pi}-\Phi\ +1}}$$

$${C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\sqrt[15]{27\sqrt{\frac{\pi\,(-\Phi)+4096\,(4\log(2)-1)^2}{\pi}}+1}$$

$$C_{\mathrm{MRB}}{}^{-1/(4\pi)}\left(\sqrt[4\pi]{C_{\mathrm{MRB}}}\ \sqrt[15]{27\sqrt{\frac{4096\,(-1-\log(16)+\log(256))^2}{\pi}-\Phi}+1}+C_{\mathrm{MRB}}{}^{1+\pi}\right)$$

Expanded logarithmic form

$$\sqrt[15]{27\sqrt{\frac{16^2\times 8\left(1-8\log(2)+16\log^2(2)\right)2}{\pi}-\Phi}+1}+C_{\mathrm{MRB}}{}^{1-1/(4\pi)+\pi}$$

Now, we analyze the following equation:

$$\zeta(z)=\log\left[\sqrt{\frac{1}{2\pi i}\log w}+\sqrt{\frac{1}{2\pi i}\log w-1}\right]$$

We consider:

$$\log\left[\sqrt{\frac{1}{2\pi i}\log w}+\sqrt{\frac{1}{2\pi i}\log w-1}\right]$$

log[√(1/(2πi) logw)+√(1/(2πi) logw-1)]

Input

$$\log\left(\sqrt{\frac{1}{2\pi i}\log(w)}+\sqrt{\frac{1}{2\pi i}\log(w)-1}\right)$$

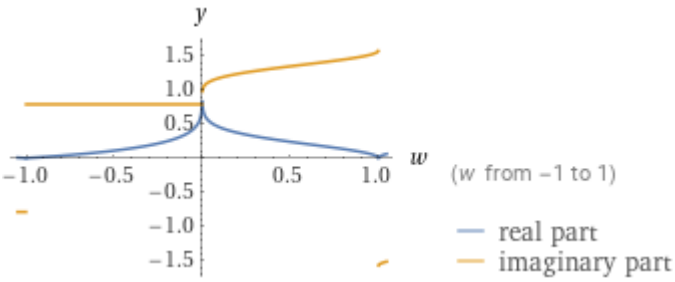
$\log(x)$ is the natural logarithm

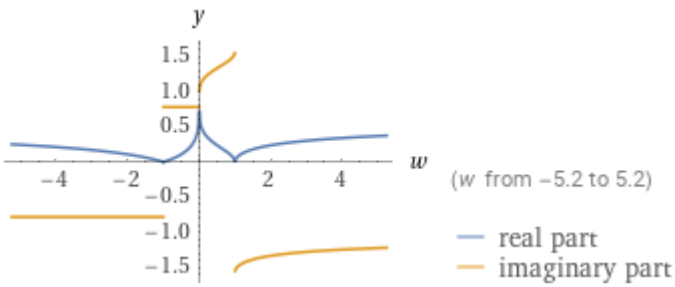
i is the imaginary unit

Exact result

$$\log\left(\frac{\sqrt{-i\log(w)}}{\sqrt{2\pi}}+\sqrt{-1-\frac{i\log(w)}{2\pi}}\right)$$

Plots (figures that can be related to the open strings)





Alternate forms

$$\log\left(\frac{\sqrt{-2\pi-i\log(w)}+\sqrt{-i\log(w)}}{\sqrt{2\pi}}\right)$$

$$-\frac{1}{2}\log(2\pi)+\log\left(\sqrt{-2\pi-i\log(w)}+\sqrt{-i\log(w)}\right)$$

$$\frac{1}{2}\left(2\log\left(\sqrt{-2\pi-i\log(w)}+\sqrt{-i\log(w)}\right)-\log(\pi)-\log(2)\right)$$

Alternate forms assuming w>0

$$\log\left(\frac{\sqrt{-4\pi - 2i\log(w)} + (1-i)\sqrt{\log(w)}}{2\sqrt{\pi}}\right)$$

$$\log\left(\sqrt{-1 - \frac{i\log(w)}{2\pi}} - \frac{e^{(3i\pi)/4}\sqrt{\log(w)}}{\sqrt{2\pi}}\right)$$

Series expansion at w=0

$$\log\left(\frac{\sqrt{-2\pi - i\log(w)} + \sqrt{-i\log(w)}}{\sqrt{2\pi}}\right) + O\left(w^6\right)$$

(generalized Puiseux series)

Derivative

$$\frac{d}{dw}\left(\log\left(\sqrt{\frac{\log(w)}{2\pi i}} + \sqrt{\frac{\log(w)}{2\pi i} - 1}\right)\right) = -\frac{i}{2w\sqrt{-2\pi - i\log(w)}\,\sqrt{-i\log(w)}}$$

From the alternate form

$$\log\left(\frac{\sqrt{-2\pi - i\log(w)} + \sqrt{-i\log(w)}}{\sqrt{2\pi}}\right)$$

log((sqrt(-2 π - i log(w)) + sqrt(-i log(w)))/sqrt(2 π))

for w = 4096:

log((sqrt(-2 π - i log(4096)) + sqrt(-i log(4096)))/sqrt(2 π))

Input

$$\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result

$$\log\left(\frac{-(-1)^{3/4}\sqrt{\log(4096)}+\sqrt{-2\pi-i\log(4096)}}{\sqrt{2\pi}}\right)$$

Decimal approximation

$$0.878336399437188438375123843539937947734085705235895376608628753\ldots$$
$$-$$
$$0.956331917813934729193806596173612584567758149604451445777117695\ldots\ i$$

Alternate complex forms

$$0.87834 - 0.95633\,i$$

$$1.2985\,(\cos(-0.827885)+i\sin(-0.827885))$$

$$1.2985\,e^{-0.827885\,i}$$

Alternate forms

$$\log\left(\frac{(1-i)\sqrt{\log(64)}+\sqrt{-2\pi-i\log(4096)}}{\sqrt{2\pi}}\right)$$

$$\log\left(-(-1)^{3/4}\sqrt{\frac{6\log(2)}{\pi}}+\sqrt{\frac{-\pi-6\,i\log(2)}{\pi}}\right)$$

$$-i\,\pi+\frac{1}{2}\,(-\log(2)-\log(\pi))+\log\left((-1)^{3/4}\,\sqrt{\log(4096)}\,-\sqrt{-2\,\pi-i\,\log(4096)}\right)$$

Alternative representations

$$\log\left(\frac{\sqrt{-2\,\pi-i\,\log(4096)}\,+\sqrt{-i\,\log(4096)}}{\sqrt{2\,\pi}}\right)=\log_e\left(\frac{\sqrt{-i\,\log(4096)}\,+\sqrt{-2\,\pi-i\,\log(4096)}}{\sqrt{2\,\pi}}\right)$$

$$\log\left(\frac{\sqrt{-2\,\pi-i\,\log(4096)}\,+\sqrt{-i\,\log(4096)}}{\sqrt{2\,\pi}}\right)=\log(a)\log_a\left(\frac{\sqrt{-i\,\log(4096)}\,+\sqrt{-2\,\pi-i\,\log(4096)}}{\sqrt{2\,\pi}}\right)$$

$$\log\left(\frac{\sqrt{-2\,\pi-i\,\log(4096)}\,+\sqrt{-i\,\log(4096)}}{\sqrt{2\,\pi}}\right)=-\mathrm{Li}_1\left(1-\frac{\sqrt{-i\,\log(4096)}\,+\sqrt{-2\,\pi-i\,\log(4096)}}{\sqrt{2\,\pi}}\right)$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right) = \log\left(-1 + \frac{(1-i)\sqrt{\log(64)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{(1-i)\sqrt{\log(64)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}\right)^{-k}}{k}$$

$$\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right) = \log\left(-1 + \frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right)^{-k}}{k}$$

$$\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right) = \log\left(-1 + \frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{(1-i)\sqrt{\log(64)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}\right)^{-k}}{k}$$

Integral representations

$$\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right) = \int_1^{\frac{(1-i)\sqrt{\log(64)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}} \frac{1}{t}\, dt$$

$$\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right) =$$
$$-\frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{\sqrt{-2\pi-i\log(4096)}-(-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right)^{-s}}{\Gamma(1-s)}\,ds$$
$$\text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

From which:

sqrt(2((log((sqrt(-2 π - i log(4096)) + sqrt(-i log(4096)))/sqrt(2 π)))))

Input

$$\sqrt{2\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right)}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result

$$\sqrt{2\log\left(\frac{-(-1)^{3/4}\sqrt{\log(4096)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}\right)}$$

Decimal approximation

1.475403190029423939735853670544417769381885017344323055203654 85…
$-$
0.648183441839286397817364906563033577855353350259594640813216815…
i

Alternate complex forms

$$1.4754 - 0.6482\,i$$

$$1.6115\,(\cos(-0.41394) + i\sin(-0.41394))$$

$$1.6115\,e^{-0.41394\,i}$$

Alternate forms

$$\sqrt{-\log(2\pi) + 2\log\left((1-i)\sqrt{\log(64)} + \sqrt{-2\pi - i\log(4096)}\right)}$$

$$\sqrt{2\log\left(-(-1)^{3/4}\sqrt{\frac{6\log(2)}{\pi}} + \sqrt{\frac{-\pi - 6\,i\log(2)}{\pi}}\right)}$$

$$\sqrt{2\left(-i\,\pi + \frac{1}{2}(-\log(2) - \log(\pi)) + \log\left((-1)^{3/4}\sqrt{\log(4096)} - \sqrt{-2\pi - i\log(4096)}\right)\right)}$$

All 2nd roots of 2 log((-(-1)^3/4 sqrt(log(4096))+sqrt(-2 π -i log(4096)))/sqrt(2 π))

$$\sqrt{2\left|\log\left(\frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right)\right|}\exp\left(\frac{1}{2}\,i\arg\left(\log\left(\frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right)\right)\right) \approx 1.4754 - 0.6482\,i$$

$$\sqrt{2\left|\log\left(\frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right)\right|}\exp\left(\frac{1}{2}\,i\left(2\pi + \arg\left(\log\left(\frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right)\right)\right)\right) \approx -1.4754 + 0.6482\,i \quad \text{(principal root)}$$

$|z|$ is the absolute value of z

$\arg(z)$ is the complex argument

Polar coordinates

$r = 1.61151$ (radius), $\theta = -0.413953$ (angle)

1.61151 result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternative representations

$$\sqrt{2\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right)} = \sqrt{2\log_e\left(\frac{\sqrt{-i\log(4096)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}\right)}$$

$$\sqrt{2\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right)} = \sqrt{2\log(a)\log_a\left(\frac{\sqrt{-i\log(4096)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}\right)}$$

$$\sqrt{2\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right)} = \sqrt{-2\,\mathrm{Li}_1\left(1 - \frac{\sqrt{-i\log(4096)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}\right)}$$

$\log_b(x)$ is the base- b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\sqrt{2\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right)} = \sqrt{2}\sqrt{\left(\log\left(-1 + \frac{(1-i)\sqrt{\log(64)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{(1-i)\sqrt{\log(64)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}\right)^{-k}}{k}\right)}$$

$$\sqrt{2\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right)} = \sqrt{2}\sqrt{\left(\log\left(-1 + \frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right)^{-k}}{k}\right)}$$

$$\sqrt{2\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right)} =$$
$$\sqrt{2}\sqrt{\left(\log\left(-1 + \frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right) - \sum_{k=1}^{\infty}\frac{(-1)^k\left(-1 + \frac{(1-i)\sqrt{\log(64)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}\right)^{-k}}{k}\right)}$$

Integral representations

$$\sqrt{2\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right)} =$$
$$\sqrt{2}\sqrt{\int_1^{\frac{(1-i)\sqrt{\log(64)} + \sqrt{-2\pi - i\log(4096)}}{\sqrt{2\pi}}} \frac{1}{t}\,dt}$$

$$\sqrt{2\log\left(\frac{\sqrt{-2\pi - i\log(4096)} + \sqrt{-i\log(4096)}}{\sqrt{2\pi}}\right)} =$$
$$\frac{\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(-1+\frac{\sqrt{-2\pi - i\log(4096)} - (-1)^{3/4}\sqrt{\log(4096)}}{\sqrt{2\pi}}\right)^{-s}}{\Gamma(1-s)}\,ds}}{\sqrt{\pi}} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

Now, we analyze the following equation:

$$w = -e^{\frac{\pi i}{2}(e^{2\zeta}+e^{-2\zeta})}$$

-e^((πi)/2 (e^(2*ζ)+e^(-2*ζ)))

Input

$$-\exp\left(\frac{\pi\, i}{2}\left(e^{2\zeta(u;g_2,g_3)}+e^{-2\zeta(u;g_2,g_3)}\right)\right)$$

$\zeta(u; g_2, g_3)$ is the Weierstrass zeta function

i is the imaginary unit

Exact result

$$-\exp\left(\frac{1}{2}\, i\,\pi\left(e^{-2\zeta(u;g_2,g_3)}+e^{2\zeta(u;g_2,g_3)}\right)\right)$$

Alternative representations

$$-\exp\left(\frac{1}{2}\left(e^{2\zeta(u;g_2,g_3)}+e^{-2\zeta(u;g_2,g_3)}\right)(\pi\, i)\right) = -w^a$$
$$\text{for } a = \frac{i\left(e^{-2\zeta(u;g_2,g_3)}+e^{2\zeta(u;g_2,g_3)}\right)\pi}{2\log(w)}$$

$$-\exp\left(\frac{1}{2}\left(e^{2\zeta(u;g_2,g_3)}+e^{-2\zeta(u;g_2,g_3)}\right)(\pi\, i)\right) =$$
$$-z^{1/2\left(e^{2\zeta(u;g_2,g_3)}+e^{-2\zeta(u;g_2,g_3)}\right)(\pi\, i)} \quad \text{for } z = e$$

$$-\exp\left(\frac{1}{2}\left(e^{2\zeta(u;g_2,g_3)}+e^{-2\zeta(u;g_2,g_3)}\right)(\pi\, i)\right) =$$
$$-1-\frac{2}{-1+\coth\left(\frac{1}{4}\, i\,\pi\left(e^{-2\zeta(u;g_2,g_3)}+e^{2\zeta(u;g_2,g_3)}\right)\right)}$$

log(x) is the natural logarithm

coth(x) is the hyperbolic cotangent function

From the exact result

$$-\exp\left(\frac{1}{2}\,i\,\pi\left(e^{-2\zeta(u;g_2,g_3)}+e^{2\zeta(u;g_2,g_3)}\right)\right)$$

we obtain:

-exp(1/2 i π (e^(-2 ζ(2;4, 8)) + e^(2 ζ(2;4, 8))))

Input

$$-\exp\left(\frac{1}{2}\,i\,\pi\left(e^{-2\zeta(\{2,4,8\})}+e^{2\zeta(\{2,4,8\})}\right)\right)$$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Alternate forms

$$\left\{-e^{i\,\pi\cosh\left(\pi^2/3\right)},\ -e^{i\,\pi\cosh\left(\pi^4/45\right)},\ -e^{i\,\pi\cosh\left(\pi^8/4725\right)}\right\}$$

$$\left\{-\cos\left(\frac{1}{2}\left(e^{-\pi^2/3}+e^{\pi^2/3}\right)\pi\right)-i\sin\left(\frac{1}{2}\left(e^{-\pi^2/3}+e^{\pi^2/3}\right)\pi\right),\right.$$
$$-\cos\left(\frac{1}{2}\left(e^{-\pi^4/45}+e^{\pi^4/45}\right)\pi\right)-i\sin\left(\frac{1}{2}\left(e^{-\pi^4/45}+e^{\pi^4/45}\right)\pi\right),$$
$$\left.-\cos\left(\frac{1}{2}\left(e^{-\pi^8/4725}+e^{\pi^8/4725}\right)\pi\right)-i\sin\left(\frac{1}{2}\left(e^{-\pi^8/4725}+e^{\pi^8/4725}\right)\pi\right)\right\}$$

cosh(x) is the hyperbolic cosine function

From the alternate form

$$\left\{-e^{i\,\pi\cosh(\pi^2/3)}, -e^{i\,\pi\cosh(\pi^4/45)}, -e^{i\,\pi\cosh(\pi^8/4725)}\right\}$$

we obtain:

{-e^(i π cosh(π^2/3)), -e^(i π cosh(π^4/45)), -e^(i π cosh(π^8/4725))}

Input

$$\left\{-e^{i\,\pi\cosh(\pi^2/3)}, -e^{i\,\pi\cosh(\pi^4/45)}, -e^{i\,\pi\cosh(\pi^8/4725)}\right\}$$

$\cosh(x)$ is the hyperbolic cosine function

i is the imaginary unit

Alternate forms

$$\left\{-e^{1/2\,i\left(e^{-\pi^2/3}+e^{\pi^2/3}\right)\pi}, -e^{1/2\,i\left(e^{-\pi^4/45}+e^{\pi^4/45}\right)\pi}, -e^{1/2\,i\left(e^{-\pi^8/4725}+e^{\pi^8/4725}\right)\pi}\right\}$$

$$\left\{-\cos\left(\pi\cosh\left(\frac{\pi^2}{3}\right)\right) - i\sin\left(\pi\cosh\left(\frac{\pi^2}{3}\right)\right), -\cos\left(\pi\cosh\left(\frac{\pi^4}{45}\right)\right) - i\sin\left(\pi\cosh\left(\frac{\pi^4}{45}\right)\right),\right.$$
$$\left.-\cos\left(\pi\cosh\left(\frac{\pi^8}{4725}\right)\right) - i\sin\left(\pi\cosh\left(\frac{\pi^8}{4725}\right)\right)\right\}$$

Decimal approximation

$$\{0.192651 + 0.981267\,i, -0.269456 - 0.963013\,i, -0.793794 + 0.608187\,i\}$$

Total

$$-e^{i\,\pi\cosh(\pi^2/3)} - e^{i\,\pi\cosh(\pi^4/45)} - e^{i\,\pi\cosh(\pi^8/4725)} \approx -0.870599 + 0.626442\,i$$

Vector length

$$\sqrt{3}$$

Mean value

$-0.2902 + 0.208814\,i$

Normalized vector

$$\left(-\frac{e^{i\,\pi\cosh(\pi^2/3)}}{\sqrt{3}}, -\frac{e^{i\,\pi\cosh(\pi^4/45)}}{\sqrt{3}}, -\frac{e^{i\,\pi\cosh(\pi^8/4725)}}{\sqrt{3}}\right)$$

From the vector length

$\sqrt{3}$

we obtain:

((sqrt(3))^2-1)^12

Input

$\left(\sqrt{3}^{\,2} - 1\right)^{12}$

Result

4096

4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string.

While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

27*sqrt((((sqrt(3))^2-1)^12))+1

Input

$$27\sqrt{\left(\sqrt{3}^{\,2}-1\right)^{12}}+1$$

Result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$.

The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(27*sqrt((((sqrt(3))^2-1)^12))+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{27\sqrt{\left(\sqrt{3}^2-1\right)^{12}}+1}+{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Exact result

$${C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\sqrt[15]{1729}$$

Decimal approximation

1.644938024130260257941733191946366957420242441725816380271305678 2

…

1.64493802413…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate form

$${C_{\text{MRB}}}^{-1/(4\pi)}\left(\sqrt[15]{1729}\,\sqrt[4\pi]{C_{\text{MRB}}}+{C_{\text{MRB}}}^{1+\pi}\right)$$

From

$$\sqrt{\log\frac{R+r_0}{2r_0}\,\lambda\left(\frac{R+r_0}{2R}\right)}\;R^{\mu}\,h(C_0)<\frac{\pi K}{\mu}\,.$$

We consider:

Sqrt(ln((R+r)/(2r))*λ*((R+r)/(2R))) * R^u*h*(C_0)

Input

$$\sqrt{\log\left(\frac{R+r}{2r}\right)\lambda \times \frac{R+r}{2R}}\; R^u\, h\, C_0$$

$\log(x)$ is the natural logarithm

Exact result

$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda\,(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}}$$

Alternate form assuming h, r, R, u, λ, and C_0 are positive

$$\frac{C_0\, h\, R^{u-1/2}\, \sqrt{\lambda\,(r+R)}\, \sqrt{\log(r+R)-\log(r)-\log(2)}}{\sqrt{2}}$$

Expanded logarithmic form

$$\sqrt{\left(\frac{\frac{1}{2}\,(-1)\,r\,\lambda\log(2)}{R}-\frac{1}{2}\,\lambda\log(2)+\frac{\frac{1}{2}\,(-1)\,r\,\lambda\log(r)}{R}-\frac{1}{2}\,\lambda\log(r)+\frac{\frac{1}{2}\,r\,\lambda\log(r+R)}{R}+\frac{1}{2}\,\lambda\log(r+R)\right)R^u\, h\, C_0}$$

Alternate form

$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda\,(r+R)\left(\log\left(\frac{r+R}{r}\right)-\log(2)\right)}{R}}}{\sqrt{2}}$$

Roots

$r \neq 0\,, \quad R = r$

$r\,R\,(r-R)\neq 0\,,\quad h=0$

$r\,R\,(r-R)\neq 0\,,\quad C_0=0$

$r\,R\,(r+R)\neq 0\,,\quad \lambda=0$

Periodicity

periodic in u with period $\frac{2\,i\,\pi}{\log(R)}$

Series expansion at u=0

$$\frac{C_0\,h\,\sqrt{\frac{\lambda\,(r+R)\,\log\left(\frac{r+R}{2\,r}\right)}{R}}}{\sqrt{2}}+\frac{C_0\,h\,u\,\log(R)\,\sqrt{\frac{\lambda\,(r+R)\,\log\left(\frac{r+R}{2\,r}\right)}{R}}}{\sqrt{2}}+\frac{C_0\,h\,u^2\,\log^2(R)\,\sqrt{\frac{\lambda\,(r+R)\,\log\left(\frac{r+R}{2\,r}\right)}{R}}}{2\,\sqrt{2}}+\frac{C_0\,h\,u^3\,\log^3(R)\,\sqrt{\frac{\lambda\,(r+R)\,\log\left(\frac{r+R}{2\,r}\right)}{R}}}{6\,\sqrt{2}}+\frac{C_0\,h\,u^4\,\log^4(R)\,\sqrt{\frac{\lambda\,(r+R)\,\log\left(\frac{r+R}{2\,r}\right)}{R}}}{24\,\sqrt{2}}+O\left(u^5\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial u}\left(\sqrt{\frac{\log\left(\frac{R+r}{2\,r}\right)\,\lambda\,(R+r)}{2\,R}}\;R^u\,h\,C_0\right)=\frac{C_0\,h\,R^u\,\log(R)\,\sqrt{\frac{\lambda\,(r+R)\,\log\left(\frac{r+R}{2\,r}\right)}{R}}}{\sqrt{2}}$$

Indefinite integral

$$\int \frac{h\,R^u \sqrt{\frac{(r+R)\,\lambda \log\left(\frac{r+R}{2r}\right)}{R}}\; C_0}{\sqrt{2}}\,du = \frac{C_0\,h\,R^u \sqrt{\frac{\lambda\,(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)} + \text{constant}$$

From the indefinite integral result

$$\int \frac{h\,R^u \sqrt{\frac{(r+R)\,\lambda \log\left(\frac{r+R}{2r}\right)}{R}}\; C_0}{\sqrt{2}}\,du = \frac{C_0\,h\,R^u \sqrt{\frac{\lambda\,(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)} + \text{constant}$$

we obtain:

(C_0 h R^u sqrt((λ (r + R) log((r + R)/(2 r)))/R))/(sqrt(2) log(R))

Input

$$\frac{C_0\,h\,R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}$$

$\lambda(\tau)$ is the modular lambda elliptic function

$\log(x)$ is the natural logarithm

Alternate forms assuming h, r, R, u, and C_0 are positive

$$\frac{C_0\,h\,R^{u-1/2}\,\sqrt{\lambda(r+R)}\,\sqrt{\log\left(\frac{r+R}{2}\right)-\log(r)}\;(-1)^{\left\lfloor -\left(\arg(\lambda(r+R))+\arg\left(\log\left(\frac{r+R}{2}\right)-\log(r)\right)-\pi\right)/(2\pi)\right\rfloor}}{\sqrt{2}\,\log(R)}$$

$$\frac{1}{\sqrt{2}\,\log(R)}\,C_0\,h\,R^{u-1/2}\,\sqrt{\lambda(r+R)}\,\sqrt{\log(r+R)-\log(r)-\log(2)}$$
$$\exp\left(i\,\pi\left\lfloor -\frac{\arg(-\log(r)+\log(r+R)-\log(2))}{2\,\pi}-\frac{\arg(\lambda(r+R))}{2\,\pi}+\frac{1}{2}\right\rfloor\right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Expanded logarithmic form

$$\frac{C_0\, h\, R^u \sqrt{\frac{2(-1)\log(2)\,\lambda(r+R)}{R} + \frac{2(-1)\log(r)\,\lambda(r+R)}{R} + \frac{2\log(r+R)\,\lambda(r+R)}{R}}}{2\log(R)}$$

Reduced logarithmic form

$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\log\left(R^{\sqrt{2}}\right)}$$

Series expansion at u=0

$$\frac{C_0\, h \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)} + \frac{C_0\, h\, u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}} + \frac{C_0\, h\, u^2 \log(R) \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{2\sqrt{2}} + \frac{C_0\, h\, u^3 \log^2(R) \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{6\sqrt{2}} + \frac{C_0\, h\, u^4 \log^3(R) \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{24\sqrt{2}} + O\left(u^5\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial u}\left(\frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}\right) = \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}}$$

Indefinite integral

$$\int \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}\,du = \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)} + \text{constant}$$

From the indefinite integral result

$$\int \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}\,du = \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)} + \text{constant}$$

we obtain:

(C_0 h R^u sqrt((λ(r + R) log((r + R)/(2 r)))/R))/(sqrt(2) log^2(R))

Input

$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)}$$

$\lambda(\tau)$ is the modular lambda elliptic function

$\log(x)$ is the natural logarithm

Alternate forms assuming h, r, R, u, and C_0 are positive

$$\frac{C_0\, h\, R^{u-1/2} \sqrt{\lambda(r+R)}\, \sqrt{\log\left(\frac{r+R}{2}\right) - \log(r)}\; (-1)^{\left\lfloor -\left(\arg(\lambda(r+R)) + \arg\left(\log\left(\frac{r+R}{2}\right) - \log(r)\right) - \pi\right)/(2\pi)\right\rfloor}}{\sqrt{2}\,\log^2(R)}$$

$$\frac{1}{\sqrt{2}\,\log^2(R)}\, C_0\, h\, R^{u-1/2} \sqrt{\lambda(r+R)}\, \sqrt{\log(r+R) - \log(r) - \log(2)}$$
$$\exp\left(i\,\pi \left\lfloor -\frac{\arg(-\log(r) + \log(r+R) - \log(2))}{2\pi} - \frac{\arg(\lambda(r+R))}{2\pi} + \frac{1}{2}\right\rfloor\right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Expanded logarithmic form

$$\frac{C_0\, h\, R^u \sqrt{\frac{2(-1)\log(2)\,\lambda(r+R)}{R} + \frac{2(-1)\log(r)\,\lambda(r+R)}{R} + \frac{2\log(r+R)\,\lambda(r+R)}{R}}}{2\log^2(R)}$$

Reduced logarithmic form

$$\frac{C_0\, h\, R^u \sqrt{\frac{2\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{2\log^2(R)}$$

Series expansion at u=0

$$\frac{C_0\, h \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)} + \frac{C_0\, h\, u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)} + \frac{C_0\, h\, u^2 \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{2\sqrt{2}} + \frac{C_0\, h\, u^3 \log(R) \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{6\sqrt{2}} + \frac{C_0\, h\, u^4 \log^2(R) \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{24\sqrt{2}} + O\left(u^5\right)$$

(Taylor series)

Derivative

$$\frac{\partial}{\partial u}\left(\frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)}\right) = \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}$$

Indefinite integral

$$\int \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)}\,du = \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^3(R)} + \text{constant}$$

From the above indefinite integral result

$$\int \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)}\,du = \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^3(R)} + \text{constant}$$

(C_0 h R^u sqrt((λ(r + R) log((r + R)/(2 r)))/R))/(sqrt(2) log^3(R))

for u = 2, C_0 = 4, h = 8, λ = 16, r = 32 and R = 64:

(4*8*64^2*sqrt((16(32 + 64) log((32 + 64)/(2*32)))/64))/(sqrt(2) log^3(64))

Input

$$\frac{4\times 8\times 64^2 \sqrt{\frac{1}{64}\left(16\,(32+64)\log\left(\frac{32+64}{2\times 32}\right)\right)}}{\sqrt{2}\,\log^3(64)}$$

log(x) is the natural logarithm

Exact result

$$\frac{262\,144\sqrt{3\log\left(\frac{3}{2}\right)}}{\log^3(64)}$$

Decimal approximation

4019.269789115031685419195329425218226673450132970064587495640392 4

…

4019.269789115….

Alternate forms

$$\frac{262\,144\sqrt{\log\left(\frac{27}{8}\right)}}{\log^3(64)}$$

$$\frac{32\,768\sqrt{\frac{1}{3}\log\left(\frac{3}{2}\right)}}{9\log^3(2)}$$

$$\frac{32\,768\sqrt{\frac{1}{3}(\log(3)-\log(2))}}{9\log^3(2)}$$

Expanded logarithmic form

$$\frac{4\times 8\times 256\sqrt{-24\log(2)+24\log(3)}\ \sqrt{2}}{27\log^3(2)}$$

Reduced logarithmic form

$$\frac{65\,536\sqrt{\log\left(\frac{79\,766\,443\,076\,872\,509\,863\,361}{281\,474\,976\,710\,656}\right)}}{\log^3(64)}$$

Alternative representations

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}=\frac{32\times 64^2\sqrt{\frac{1536\log_e\left(\frac{96}{64}\right)}{64}}}{\log_e^3(64)\sqrt{2}}$$

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}=\frac{32\times 64^2\sqrt{\frac{1536}{64}\log(a)\log_a\left(\frac{96}{64}\right)}}{(\log(a)\log_a(64))^3\sqrt{2}}$$

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}=\frac{32\times 64^2\sqrt{-\frac{1536}{64}\mathrm{Li}_1\left(1-\frac{96}{64}\right)}}{(-\mathrm{Li}_1(-63))^3\sqrt{2}}$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}=\frac{65\,536\sqrt{6}\sum_{k=0}^{\infty}2^{1-k}\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}\frac{(-1)^j\binom{k}{j}p_{j,k}}{1-2j}}{\log^3(64)}$$

$$\text{for}\left(c_k=\frac{3(-1)^k}{1+k}\text{ and }p_{j,0}=1\text{ and }3\,p_{j,k}=\frac{\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\text{ and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)} =$$
$$\frac{262144\sqrt{3}\sqrt{2i\pi\left\lfloor\frac{\arg\left(\frac{3}{2}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-x\right)^k x^{-k}}{k}}}{\left(2i\pi\left\lfloor\frac{\arg(64-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(64-x)^k x^{-k}}{k}\right)^3} \quad \text{for } x<0$$

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)} =$$
$$\frac{262144\sqrt{3}\sqrt{2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-z_0\right)^k z_0^{-k}}{k}}}{\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(64-z_0)^k z_0^{-k}}{k}\right)^3}$$

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)} =$$
$$\left(262144\sqrt{3}\sqrt{\left(\left\lfloor\frac{\arg\left(\frac{3}{2}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg\left(\frac{3}{2}-z_0\right)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-z_0\right)^k z_0^{-k}}{k}\right)\Big/\left(\left\lfloor\frac{\arg(64-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\log(z_0)+\left\lfloor\frac{\arg(64-z_0)}{2\pi}\right\rfloor\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k(64-z_0)^k z_0^{-k}}{k}\right)^3\right)$$

$\binom{n}{m}$ is the binomial coefficient

$\mathbb{Z}$ is the set of integers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}=\frac{262\,144\sqrt{3}\,\sqrt{\int_1^{\frac{3}{2}}\frac{1}{t}\,dt}}{\left(\int_1^{64}\frac{1}{t}\,dt\right)^3}$$

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}=$$
$$-\frac{1\,048\,576\,i\sqrt{6}\,\pi^{5/2}\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^s\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{63^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3}\quad\text{for}\quad -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

From which:

(4*8*64^2*sqrt((16(32 + 64) log((32 + 64)/(2*32)))/64))/(sqrt(2) log^3(64))+76+1/√2

Input

$$\frac{4\times 8\times 64^2\sqrt{\frac{1}{64}\left(16\,(32+64)\log\left(\frac{32+64}{2\times 32}\right)\right)}}{\sqrt{2}\,\log^3(64)}+76+\frac{1}{\sqrt{2}}$$

$\log(x)$ is the natural logarithm

Exact result

$$76+\frac{1}{\sqrt{2}}+\frac{262\,144\sqrt{3\log\left(\frac{3}{2}\right)}}{\log^3(64)}$$

Decimal approximation

4095.976895896218232943596173787323075712734968907753061532228732 2

…

4095.97689589…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$76+\frac{1}{\sqrt{2}}+\frac{262\,144\sqrt{\log\left(\frac{27}{8}\right)}}{\log^3(64)}$$

$$\frac{1}{2}\left(152+\sqrt{2}\right)+\frac{262\,144\sqrt{3\log\left(\frac{3}{2}\right)}}{\log^3(64)}$$

$$\frac{152\log^3(64)+\sqrt{2}\,\log^3(64)+524\,288\sqrt{3\log\left(\frac{3}{2}\right)}}{2\log^3(64)}$$

Expanded logarithmic form

$$\frac{4\times 8\times 256\sqrt{-24\log(2)+24\log(3)}\,\sqrt{2}}{27\log^3(2)}+76+\frac{\sqrt{2}}{2}$$

Reduced logarithmic form

$$\frac{\left(152+\sqrt{2}\right)\log^3(64)+131\,072\sqrt{\log\left(\frac{79\,766\,443\,076\,872\,509\,863\,361}{281\,474\,976\,710\,656}\right)}}{2\log^3(64)}$$

Alternative representations

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}+76+\frac{1}{\sqrt{2}}=$$

$$76+\frac{1}{\sqrt{2}}+\frac{32\times 64^2\sqrt{\frac{1536\log_e\left(\frac{96}{64}\right)}{64}}}{\log_e^3(64)\sqrt{2}}$$

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}+76+\frac{1}{\sqrt{2}}=$$
$$76+\frac{1}{\sqrt{2}}+\frac{32\times 64^2\sqrt{\frac{1536}{64}\log(a)\log_a\left(\frac{96}{64}\right)}}{(\log(a)\log_a(64))^3\sqrt{2}}$$

$$\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}+76+\frac{1}{\sqrt{2}}=$$
$$76+\frac{1}{\sqrt{2}}+\frac{32\times 64^2\sqrt{-\frac{1536}{64}\mathrm{Li}_1\left(1-\frac{96}{64}\right)}}{(-\mathrm{Li}_1(-63))^3\sqrt{2}}$$

$\log_b(x)$ is the base- b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

27*sqrt(((4*8*64^2*sqrt((16(32 + 64) log((32 + 64)/(2*32)))/64))/(sqrt(2) log^3(64))+76+1/√2))+1

Input

$$27\sqrt{\frac{4\times 8\times 64^2\sqrt{\frac{1}{64}\left(16(32+64)\log\left(\frac{32+64}{2\times 32}\right)\right)}}{\sqrt{2}\,\log^3(64)}+76+\frac{1}{\sqrt{2}}}+1$$

$\log(x)$ is the natural logarithm

Exact result

$$1+27\sqrt{76+\frac{1}{\sqrt{2}}+\frac{262\,144\sqrt{3\log\left(\frac{3}{2}\right)}}{\log^3(64)}}$$

Decimal approximation

1728.995126471236055903614334126126519024675032845957328410311158 4

…

1728.995126471236….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2.

Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$1+27\sqrt{76+\frac{1}{\sqrt{2}}+\frac{262\,144\sqrt{\log\left(\frac{27}{8}\right)}}{\log^3(64)}}$$

$$1+27\sqrt{\frac{1}{2}\left(152+\sqrt{2}\right)+\frac{262\,144\sqrt{3\log\left(\frac{3}{2}\right)}}{\log^3(64)}}$$

$$1+\frac{3\sqrt{3\left(27\left(1+76\sqrt{2}\right)\log^3(2)+32\,768\sqrt{6\log\left(\frac{3}{2}\right)}\right)}}{\sqrt[4]{2}\,\log^{3/2}(2)}$$

Expanded logarithmic form

$$27\sqrt{\frac{4\times 8\times 256\sqrt{-24\log(2)+24\log(3)}\,\sqrt{2}}{27\log^3(2)}+76+\frac{\sqrt{2}}{2}}+1$$

Reduced logarithmic form

$$\frac{1}{2\log^{3/2}(64)}\left(2\log^{3/2}(64)+27\sqrt{2\left(\left(152+\sqrt{2}\right)\log^3(64)+131\,072\sqrt{\log\left(\frac{79\,766\,443\,076\,872\,509\,863\,361}{281\,474\,976\,710\,656}\right)}\right)}\right)$$

Alternative representations

$$27\sqrt{\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}+76+\frac{1}{\sqrt{2}}}+1=1+27\sqrt{76+\frac{1}{\sqrt{2}}+\frac{32\times 64^2\sqrt{\frac{1536\log_e\left(\frac{96}{64}\right)}{64}}}{\log_e^3(64)\sqrt{2}}}$$

$$27\sqrt{\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}+76+\frac{1}{\sqrt{2}}}+1=1+27\sqrt{76+\frac{1}{\sqrt{2}}+\frac{32\times 64^2\sqrt{\frac{1536}{64}\log(a)\log_a\left(\frac{96}{64}\right)}}{(\log(a)\log_a(64))^3\sqrt{2}}}$$

$$27\sqrt{\frac{4\left(8\times 64^2\sqrt{\frac{16}{64}(32+64)\log\left(\frac{32+64}{2\times 32}\right)}\right)}{\sqrt{2}\,\log^3(64)}+76+\frac{1}{\sqrt{2}}}+1=1+27\sqrt{76+\frac{1}{\sqrt{2}}+\frac{32\times 64^2\sqrt{-\frac{1536}{64}\mathrm{Li}_1\left(1-\frac{96}{64}\right)}}{(-\mathrm{Li}_1(-63))^3\sqrt{2}}}$$

$\log_b(x)$ is the base-b logarithm

$Li_n(x)$ is the polylogarithm function

((27*sqrt(((4*8*64^2*sqrt((16(32 + 64) log((32 + 64)/(2*32)))/64))/(sqrt(2) log^3(64))+76+1/√2))+1))^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\sqrt[15]{27\sqrt{\frac{4\times 8\times 64^2\sqrt{\frac{1}{64}\left(16\,(32+64)\log\left(\frac{32+64}{2\times 32}\right)\right)}}{\sqrt{2}\,\log^3(64)}+76+\frac{1}{\sqrt{2}}}+1}+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}+\sqrt[15]{1+27\sqrt{76+\frac{1}{\sqrt{2}}+\frac{262\,144\sqrt{3\log\left(\frac{3}{2}\right)}}{\log^3(64)}}}$$

Decimal approximation

1.644937715235279044076936774236155395500064129736401692484706438 5

…

1.644937715235279…. ≈ ζ(2) = π²/6 = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}+\sqrt[15]{1+27\sqrt{76+\frac{1}{\sqrt{2}}+\frac{262\,144\sqrt{\log\left(\frac{27}{8}\right)}}{\log^3(64)}}}$$

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}+\sqrt[15]{1+27\sqrt{\frac{1}{2}\left(152+\sqrt{2}\right)+\frac{262\,144\sqrt{3\log\left(\frac{3}{2}\right)}}{\log^3(64)}}}$$

$$C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}+\frac{1}{\sqrt[15]{2+\frac{27\sqrt{2\left(152\log^3(64)+\sqrt{2}\,\log^3(64)+524\,288\sqrt{3\log\left(\frac{3}{2}\right)}\right)}}{\log^{3/2}(64)}}}$$

Expanded logarithmic form

$$\sqrt[15]{27\sqrt{\frac{4\times8\times256\sqrt{-24\log(2)+24\log(3)}\;\sqrt{2}}{27\log^3(2)}+76+\frac{\sqrt{2}}{2}}+1}+C_{\text{MRB}}^{\,1-1/(4\pi)+\pi}$$

We consider also:

Sqrt(ln((R+r)/(2r))*λ*((R+r)/(2R))) * R^u*h*(C_0) - (Pi*K)/u

Input

$$\sqrt{\log\left(\frac{R+r}{2r}\right)\lambda\times\frac{R+r}{2R}}\;R^u\,h\,C_0-\frac{\pi K}{u}$$

log(x) is the natural logarithm

Exact result

$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda\,(r+R)\log\left(\frac{r+R}{2\,r}\right)}{R}}}{\sqrt{2}} - \frac{\pi\, K}{u}$$

Alternate form assuming h, K, r, R, u, λ, and C_0 are positive

$$\frac{C_0\, h\, R^{u-1/2}\, \sqrt{\lambda\,(r+R)}\, \sqrt{\log(r+R) - \log(r) - \log(2)}}{\sqrt{2}} - \frac{\pi\, K}{u}$$

Expanded logarithmic form

$$\sqrt{\left(\frac{\frac{1}{2}\,(-1)\,r\,\lambda\log(2)}{R} - \frac{1}{2}\,\lambda\log(2) + \frac{\frac{1}{2}\,(-1)\,r\,\lambda\log(r)}{R} - \frac{1}{2}\,\lambda\log(r) + \frac{\frac{1}{2}\,r\,\lambda\log(r+R)}{R} + \frac{1}{2}\,\lambda\log(r+R)\right) R^u\, h\, C_0} - \frac{\pi\, K}{u}$$

Alternate forms

$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda\,(r+R)\left(\log\left(\frac{r+R}{r}\right)-\log(2)\right)}{R}}}{\sqrt{2}} - \frac{\pi\, K}{u}$$

$$-\frac{2\,\pi\, K - \sqrt{2}\; C_0\, h\, u\, R^u \sqrt{\frac{\lambda\,(r+R)\log\left(\frac{r+R}{2\,r}\right)}{R}}}{2\,u}$$

$$\frac{C_0\, h\, u\, R^u \sqrt{\frac{\lambda\,(r+R)\left(\log\left(\frac{r+R}{r}\right)-\log(2)\right)}{R}} - \sqrt{2}\;\pi\, K}{\sqrt{2}\; u}$$

Series expansion at u=0

$$-\frac{\pi K}{u}+\frac{C_0 h \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}}+\frac{C_0 h u \log(R) \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}}+$$
$$\frac{C_0 h u^2 \log^2(R) \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{2\sqrt{2}}+\frac{C_0 h u^3 \log^3(R) \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{6\sqrt{2}}+O\left(u^4\right)$$

(Laurent series)

Series expansion at u=∞

$$\left(-\frac{K\pi}{u}+O\left(\left(\frac{1}{u}\right)^6\right)\right)+\frac{C_0 h R^u \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}}$$

Derivative

$$\frac{\partial}{\partial u}\left(\sqrt{\frac{\log\left(\frac{R+r}{2r}\right)\lambda (R+r)}{2R}}\ R^u h C_0-\frac{\pi K}{u}\right)=\frac{C_0 h R^u \log(R) \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}}+\frac{\pi K}{u^2}$$

Indefinite integral

$$\int\left(-\frac{K\pi}{u}+\frac{h R^u \sqrt{\frac{(r+R)\lambda \log\left(\frac{r+R}{2r}\right)}{R}}\ C_0}{\sqrt{2}}\right)du=$$
$$\frac{C_0 h R^u \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\ \log(R)}-\pi K \log(u)+\text{constant}$$

(assuming a complex-valued logarithm)

From the indefinite integral result

$$\int\left(-\frac{K\pi}{u}+\frac{h\,R^u\sqrt{\frac{(r+R)\,\lambda\,\log\left(\frac{r+R}{2r}\right)}{R}}\,C_0}{\sqrt{2}}\right)du=$$

$$\frac{C_0\,h\,R^u\sqrt{\frac{\lambda\,(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}-\pi\,K\log(u)+\text{constant}$$

(assuming a complex-valued logarithm)

we obtain:

(C_0 h R^u sqrt((λ (r + R) log((r + R)/(2 r)))/R))/(sqrt(2) log(R)) - π K log(u)

Input

$$\frac{C_0\,h\,R^u\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}-\pi\,K\log(u)$$

$\lambda(\tau)$ is the modular lambda elliptic function

$\log(x)$ is the natural logarithm

Alternate forms assuming h, K, r, R, u, and C_0 are positive

$$\frac{1}{\sqrt{2}\,\log(R)}\,C_0\,h\,R^{u-1/2}\sqrt{\lambda(r+R)}\,\sqrt{\log(r+R)-\log(2r)}$$
$$(-1)^{\left\lfloor-\left(\arg(\lambda(r+R))+\arg\left(\log\left(\frac{r+R}{2}\right)-\log(r)\right)-\pi\right)/(2\pi)\right\rfloor}-\pi\,K\log(u)$$

$$-\pi\,K\log(u)+\frac{1}{\sqrt{2}\,\log(R)}\,C_0\,h\,R^{u-1/2}\sqrt{\lambda(r+R)}\,\sqrt{\log(r+R)-\log(r)-\log(2)}$$
$$\exp\left(i\,\pi\left\lfloor-\frac{\arg(-\log(r)+\log(r+R)-\log(2))}{2\pi}-\frac{\arg(\lambda(r+R))}{2\pi}+\frac{1}{2}\right\rfloor\right)$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Alternate forms

$$-\frac{2\pi K \log(R)\log(u) - \sqrt{2}\, C_0 h R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{2\log(R)}$$

$$\frac{C_0 h R^u \sqrt{\frac{\lambda(r+R)\left(\log\left(\frac{r+R}{r}\right)-\log(2)\right)}{R}} - \sqrt{2}\,\pi K \log(R)\log(u)}{\sqrt{2}\,\log(R)}$$

Expanded logarithmic form

$$\frac{C_0 h R^u \sqrt{\frac{2(-1)\log(2)\lambda(r+R)}{R} + \frac{2(-1)\log(r)\lambda(r+R)}{R} + \frac{2\log(r+R)\lambda(r+R)}{R}}}{2\log(R)} - \pi K \log(u)$$

Reduced logarithmic form

$$\frac{-K\pi \log\left(R^{\sqrt{2}}\right)\log(u) + h R^u \sqrt{\frac{\log\left(\frac{r+R}{2r}\right)\lambda(r+R)}{R}}\, C_0}{\log\left(R^{\sqrt{2}}\right)}$$

Series expansion at u=0

$$\left(\frac{C_0 h \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)} - \pi K \log(u)\right) + \frac{C_0 h u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}} + \frac{C_0 h u^2 \log(R)\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{2\sqrt{2}} + \frac{C_0 h u^3 \log^2(R)\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{6\sqrt{2}} + \frac{C_0 h u^4 \log^3(R)\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{24\sqrt{2}} + O\left(u^5\right)$$

(Puiseux series)

Series expansion at u=∞

$$\left(-K\,\pi\log(u)+O\left(\left(\frac{1}{u}\right)^6\right)\right)+\frac{C_0\,h\,R^u\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}$$

Derivative

$$\frac{\partial}{\partial u}\left(\frac{C_0\,h\,R^u\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}-\pi\,K\log(u)\right)=\frac{C_0\,h\,R^u\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}}-\frac{\pi\,K}{u}$$

Indefinite integral

$$\int\left(\frac{C_0\,h\,R^u\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}-\pi\,K\log(u)\right)du=$$

$$\frac{C_0\,h\,R^u\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)}-\pi\,K\,u\,(\log(u)-1)+\text{constant}$$

(assuming a complex-valued logarithm)

From the indefinite integral result

$$\int\left(\frac{C_0\,h\,R^u\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)}-\pi\,K\log(u)\right)du=$$

$$\frac{C_0\,h\,R^u\sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)}-\pi\,K\,u\,(\log(u)-1)+\text{constant}$$

(assuming a complex-valued logarithm)

we obtain:

(C_0 h R^u sqrt((λ(r + R) log((r + R)/(2 r)))/R))/(sqrt(2) log^2(R)) - π K u (log(u) - 1)

Input

$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)} - \pi\, K\, u\,(\log(u) - 1)$$

$\lambda(\tau)$ is the modular lambda elliptic function

$\log(x)$ is the natural logarithm

Alternate forms assuming h, K, r, R, u, and C_0 are positive

$$\frac{C_0\, h\, R^{u-1/2}\, \sqrt{\lambda(r+R)}\, \sqrt{\log\left(\frac{r+R}{2}\right) - \log(r)}\; (-1)^{\left\lfloor -\left(\arg(\lambda(r+R)) + \arg\left(\log\left(\frac{r+R}{2}\right) - \log(r)\right) - \pi\right)/(2\pi)\right\rfloor}}{\sqrt{2}\,\log^2(R)}$$
$$- \pi\, K\, u\,(\log(u) - 1)$$

$$-\pi\, K\, u\,(\log(u) - 1) + \frac{1}{\sqrt{2}\,\log^2(R)}\, C_0\, h\, R^{u-1/2}\, \sqrt{\lambda(r+R)}\, \sqrt{\log(r+R) - \log(r) - \log(2)}\, \exp\left(i\,\pi \left\lfloor -\frac{\arg(-\log(r) + \log(r+R) - \log(2))}{2\pi} - \frac{\arg(\lambda(r+R))}{2\pi} + \frac{1}{2}\right\rfloor\right)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Expanded logarithmic form

$$\frac{C_0\, h\, R^u \sqrt{\frac{2(-1)\log(2)\,\lambda(r+R)}{R} + \frac{2(-1)\log(r)\,\lambda(r+R)}{R} + \frac{2\log(r+R)\,\lambda(r+R)}{R}}}{2\log^2(R)} + K\,\pi\,u - K\,\pi\,u\log(u)$$

Reduced logarithmic form

$$\frac{-2\,K\,\pi\,u\log^2(R)\,(-1 + \log(u)) + \sqrt{2}\; h\, R^u \sqrt{\frac{\log\left(\frac{r+R}{2r}\right)\lambda(r+R)}{R}}\; C_0}{2\log^2(R)}$$

Alternate form

$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\left(\log\left(\frac{r+R}{r}\right)-\log(2)\right)}{R}} - \sqrt{2}\,\pi\, K\, u \log^2(R)\,(\log(u)-1)}{\sqrt{2}\,\log^2(R)}$$

Series expansion at u=0

$$\frac{C_0\, h \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)} + u\left(\frac{C_0\, h \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)} - \pi\, K\,(\log(u)-1)\right) +$$
$$\frac{C_0\, h\, u^2 \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{2\sqrt{2}} + \frac{C_0\, h\, u^3 \log(R) \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{6\sqrt{2}} +$$
$$\frac{C_0\, h\, u^4 \log^2(R) \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{24\sqrt{2}} + O\left(u^5\right)$$

(generalized Puiseux series)

Series expansion at u=∞

$$\left(-K\,\pi\,(\log(u)-1)\,u + O\left(\left(\frac{1}{u}\right)^6\right)\right) + \frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)}$$

Derivative

$$\frac{\partial}{\partial u}\left(\frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log^2(R)} - \pi\, K\, u\,(\log(u)-1)\right) =$$
$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda(r+R)\log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\,\log(R)} - \pi\, K \log(u)$$

Indefinite integral

$$\int \left(\frac{C_0\, h\, R^u \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\, \log^2(R)} - \pi\, K\, u\, (\log(u) - 1) \right) du =$$

$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\, \log^3(R)} + \frac{1}{4}\, \pi\, K\, u^2\, (3 - 2 \log(u)) + \text{constant}$$

(assuming a complex-valued logarithm)

From the indefinite integral result

$$\int \left(\frac{C_0\, h\, R^u \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\, \log^2(R)} - \pi\, K\, u\, (\log(u) - 1) \right) du =$$

$$\frac{C_0\, h\, R^u \sqrt{\frac{\lambda (r+R) \log\left(\frac{r+R}{2r}\right)}{R}}}{\sqrt{2}\, \log^3(R)} + \frac{1}{4}\, \pi\, K\, u^2\, (3 - 2 \log(u)) + \text{constant}$$

(assuming a complex-valued logarithm)

(C_0 h R^u sqrt((λ(r + R) log((r + R)/(2 r)))/R))/(sqrt(2) log^3(R)) + 1/4 π K u^2 (3 - 2 log(u))

for u = 2, C_0 = 4, h = 8, K = 16, λ = 32, r = 64 and R = 128:

(4*8*128^2 sqrt((32(64 + 128) log((64 + 128)/(2*64)))/128))/(sqrt(2) log^3(128)) + 1/4 π 16 2^2 (3 - 2 log(2))

Input

$$\frac{4 \times 8 \times 128^2 \sqrt{\frac{1}{128} \left(32\, (64 + 128) \log\left(\frac{64+128}{2 \times 64}\right)\right)}}{\sqrt{2}\, \log^3(128)} + \frac{1}{4}\, \pi \times 16 \times 2^2\, (3 - 2 \log(2))$$

log(x) is the natural logarithm

Result

$$\frac{1\,048\,576\sqrt{6\log\left(\frac{3}{2}\right)}}{\log^3(128)} + 16\pi\,(3 - 2\log(2))$$

Decimal approximation

14399.094600543613212756587520751378822414585743324641512934820697

…

14399.094600543….

Alternate forms

$$\frac{16\left(-686\pi\log^4(2) + 1029\pi\log^3(2) + 65\,536\sqrt{6\,(\log(3) - \log(2))}\right)}{343\log^3(2)}$$

$$16\left(\frac{65\,536\sqrt{6\log\left(\frac{3}{2}\right)}}{\log^3(128)} - \pi\,(\log(4) - 3)\right)$$

$$\frac{1\,048\,576\sqrt{6\log\left(\frac{3}{2}\right)}}{343\log^3(2)} + \pi\,(48 - 32\log(2))$$

Expanded forms

$$48\pi + \frac{1\,048\,576\sqrt{6\log(3) - 6\log(2)}}{343\log^3(2)} - 32\pi\log(2)$$

$$48\pi + \frac{1\,048\,576\sqrt{6\log\left(\frac{3}{2}\right)}}{\log^3(128)} - 32\pi\log(2)$$

Expanded logarithmic form

$$\frac{4 \times 8 \times 8192 \sqrt{-48 \log(2) + 48 \log(3)}\ \sqrt{2}}{343 \log^3(2)} + 16\pi 3 + 16\pi(-2)\log(2)$$

Reduced logarithmic form

$$\frac{1}{\log^3(128)} 16 \left(3\pi \log^3(128) - \pi \log(4) \log^3(128) + 16384 \sqrt{\log\left(\frac{6362685441135942358474828762538534230890216321}{79228162514264337593543950336}\right)}\right)$$

Alternative representations

$$\frac{4\left(8 \times 128^2 \sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2 \times 64}\right)}\right)}{\sqrt{2}\ \log^3(128)} + \frac{1}{4}(\pi 16)\, 2^2 (3 - 2\log(2)) =$$

$$\frac{64}{4}\pi\left(3 - 2\log_e(2)\right) + \frac{32 \times 128^2 \sqrt{\frac{6144 \log_e\left(\frac{192}{128}\right)}{128}}}{\log_e^3(128)\sqrt{2}}$$

$$\frac{4\left(8 \times 128^2 \sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2 \times 64}\right)}\right)}{\sqrt{2}\ \log^3(128)} + \frac{1}{4}(\pi 16)\, 2^2 (3 - 2\log(2)) =$$

$$\frac{64}{4}\pi(3 - 2\log(a)\log_a(2)) + \frac{32 \times 128^2 \sqrt{\frac{6144}{128}\log(a)\log_a\left(\frac{192}{128}\right)}}{(\log(a)\log_a(128))^3\sqrt{2}}$$

$$\frac{4\left(8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}\right)}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}(\pi\,16)\,2^2\,(3-2\log(2))=$$
$$\frac{64}{4}\pi\,(3+2\,\mathrm{Li}_1(-1))+\frac{32\times 128^2\sqrt{-\frac{6144}{128}\,\mathrm{Li}_1\left(1-\frac{192}{128}\right)}}{(-\mathrm{Li}_1(-127))^3\sqrt{2}}$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{4\left(8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}\right)}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}(\pi\,16)\,2^2\,(3-2\log(2))=$$
$$-\frac{1}{\log^3(128)}\,16\left(-3\,\pi\log^3(128)+\pi\log(4)\log^3(128)-\right.$$
$$\left.32\,768\sqrt{3}\sum_{k=0}^{\infty}2^{1-k}\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}\frac{(-1)^j\binom{k}{j}p_{j,k}}{1-2\,j}\right)$$
$$\text{for}\left(c_k=\frac{6\,(-1)^k}{1+k}\text{ and }p_{j,0}=1\text{ and }6\,p_{j,k}=\frac{\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\right.$$
$$\left.\text{and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$\frac{4\left(8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}\right)}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}(\pi\,16)\,2^2\,(3-2\log(2))=$$
$$48\pi-64\,i\,\pi^2\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor-32\pi\log(x)+32\pi\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-x)^k\,x^{-k}}{k}+$$
$$\frac{1\,048\,576\sqrt{6}\sqrt{2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{3}{2}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-x\right)^k x^{-k}}{k}}}{\left(2\,i\,\pi\left\lfloor\frac{\arg(128-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(128-x)^k\,x^{-k}}{k}\right)^3}\quad\text{for } x<0$$

$$\frac{4\left(8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}\right)}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}(\pi\,16)\,2^2\,(3-2\log(2))=48\pi-$$
$$64\,i\,\pi^2\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-32\pi\log(z_0)+32\pi\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}+$$
$$\frac{1\,048\,576\sqrt{6}\sqrt{2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-z_0\right)^k z_0^{-k}}{k}}}{\left(2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(128-z_0)^k\,z_0^{-k}}{k}\right)^3}$$

$$\frac{4\left(8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}\right)}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}(\pi\,16)\,2^2\,(3-2\log(2))=$$
$$48\pi-32\pi\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)-32\pi\log(z_0)-$$
$$32\pi\left\lfloor\frac{\arg(2-z_0)}{2\pi}\right\rfloor\log(z_0)+32\pi\sum_{k=1}^{\infty}\frac{(-1)^k\,(2-z_0)^k\,z_0^{-k}}{k}+$$
$$\frac{1\,048\,576\sqrt{6}\sqrt{\log(z_0)+\left\lfloor\frac{\arg\left(\frac{3}{2}-z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-z_0\right)^k z_0^{-k}}{k}}}{\left(\log(z_0)+\left\lfloor\frac{\arg(128-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\,(128-z_0)^k\,z_0^{-k}}{k}\right)^3}$$

$\binom{n}{m}$ is the binomial coefficient

$\mathbb{Z}$ is the set of integers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{4\left(8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}\right)}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}(\pi 16)\,2^2\,(3-2\log(2))=$$
$$-\frac{16\left(-65\,536\sqrt{6}\sqrt{\int_1^{\frac{3}{2}}\frac{1}{t}\,dt}-3\pi\left(\int_1^{128}\frac{1}{t}\,dt\right)^3+2\pi\left(\int_1^2\frac{1}{t}\,dt\right)\left(\int_1^{128}\frac{1}{t}\,dt\right)^3\right)}{\left(\int_1^{128}\frac{1}{t}\,dt\right)^3}$$

$$\frac{4\left(8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}\right)}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}(\pi 16)\,2^2\,(3-2\log(2))=$$
$$\left(16\left(-524\,288\,i\sqrt{3}\,\pi^{5/2}\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^s\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}+\right.\right.$$
$$3\pi\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{127^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3+$$
$$\left.\left.i\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{127^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3\right)\right)\Big/$$
$$\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{127^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3\quad\text{for}\quad -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

From which:

1/Pi((4*8*128^2 sqrt((32(64 + 128) log((64 + 128)/(2*64)))/128))/(sqrt(2) log^3(128)) + 1/4 π 16 2^2 (3 - 2 log(2)))-521+34-34/(10^2)

Input

$$\frac{1}{\pi}\left(\frac{4\times 8\times 128^2\sqrt{\frac{1}{128}\left(32\,(64+128)\log\left(\frac{64+128}{2\times 64}\right)\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi\times 16\times 2^2\,(3-2\log(2))\right)-521+34-\frac{34}{10^2}$$

log(x) is the natural logarithm

Result

$$\frac{\frac{1048\,576\sqrt{6\log\left(\frac{3}{2}\right)}}{\log^3(128)}+16\,\pi\,(3-2\log(2))}{\pi}-\frac{24\,367}{50}$$

Decimal approximation

4096.034163448672325950396247318429217930418337964286371382502348 5

…

4096.034163448672…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson.

This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Alternate forms

$$\frac{1}{17150\,\pi\log^3(2)}\left(50\left(-10976\,\pi\log^4(2)+16464\,\pi\log^3(2)+1048576\sqrt{6\,(\log(3)-\log(2))}\right)-8357881\,\pi\log^3(2)\right)$$

$$-\frac{21967\,\pi\log^3(128)+1600\,\pi\log(2)\log^3(128)-52428800\sqrt{6\log\left(\frac{3}{2}\right)}}{50\,\pi\log^3(128)}$$

$$\frac{52428800\sqrt{6\,(\log(3)-\log(2))}-343\,\pi\log^3(2)\,(21967+1600\log(2))}{17150\,\pi\log^3(2)}$$

Expanded forms

$$-\frac{21967}{50}+\frac{1048576\sqrt{6\log(3)-6\log(2)}}{343\,\pi\log^3(2)}-32\log(2)$$

$$-\frac{21\,967}{50}+\frac{1\,048\,576\sqrt{6\log\left(\frac{3}{2}\right)}}{\pi\log^3(128)}-32\log(2)$$

Expanded logarithmic form

$$-\frac{21\,967}{50}-32\log(2)+\frac{\frac{262\,144}{343}\sqrt{2\left(-48\log(2)+48\log(3)\right)}}{\pi\log^3(2)}$$

Reduced logarithmic form

$$\frac{1}{50\,\pi\log^3(128)}\left(-21\,967\,\pi\log^3(128)-800\,\pi\log(4)\log^3(128)+13\,107\,200\right.$$
$$\left.\sqrt{\log\left(\frac{6\,362\,685\,441\,135\,942\,358\,474\,828\,762\,538\,534\,230\,890\,216\,321}{79\,228\,162\,514\,264\,337\,593\,543\,950\,336}\right)}\right)$$

Alternative representations

$$\frac{\frac{4\times8\times128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi\,16\left(2^2\,(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}=$$
$$-487-\frac{34}{10^2}+\frac{\frac{64}{4}\pi\left(3-2\log_e(2)\right)+\frac{32\times128^2\sqrt{\frac{6144\log_e\left(\frac{192}{128}\right)}{128}}}{\log_e^3(128)\sqrt{2}}}{\pi}$$

$$\frac{\frac{4\times8\times128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi\,16\left(2^2\,(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}=$$
$$-487-\frac{34}{10^2}+\frac{\frac{64}{4}\pi\left(3-2\log(a)\log_a(2)\right)+\frac{32\times128^2\sqrt{\frac{6144}{128}\log(a)\log_a\left(\frac{192}{128}\right)}}{\left(\log(a)\log_a(128)\right)^3\sqrt{2}}}{\pi}$$

$$\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi\,16\left(2^2\,(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}=$$
$$-487-\frac{34}{10^2}+\frac{\frac{64}{4}\pi\,(3+2\,\mathrm{Li}_1(-1))+\frac{32\times 128^2\sqrt{-\frac{6144}{128}\mathrm{Li}_1\left(1-\frac{192}{128}\right)}}{(-\mathrm{Li}_1(-127))^3\sqrt{2}}}{\pi}$$

$\log_b(x)$ is the base-b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi\,16\left(2^2\,(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}=$$
$$-\frac{1}{50\,\pi\log^3(128)}\left(21\,967\,\pi\log^3(128)+1600\,\pi\log(2)\log^3(128)-\right.$$
$$\left.26\,214\,400\sqrt{3}\sum_{k=0}^{\infty}2^{1-k}\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}\frac{(-1)^j\binom{k}{j}p_{j,k}}{1-2\,j}\right)$$
$$\text{for}\left(c_k=\frac{6\,(-1)^k}{1+k}\text{ and }p_{j,0}=1\text{ and }6\,p_{j,k}=\frac{\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\right.$$
$$\left.\text{and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi 16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}=$$

$$-\frac{1}{50\,\pi\log^3(128)}\left(21\,967\,\pi\log^3(128)+1600\,\pi\log(2)\log^3(128)-\right.$$

$$\left.26\,214\,400\sqrt{3}\;e^{i\pi\left\lfloor 1/2-\arg\left(\frac{1}{c}\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}2^{-k}\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}\right)$$

$$\text{for}\left(c_k=\frac{6(-1)^k}{1+k}\text{ and }p_{j,0}=1\text{ and }6\,p_{j,k}=\frac{\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\right.$$

$$\left.\text{and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi 16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}=$$

$$-\frac{21\,967}{50}-64\,i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor-32\log(x)+32\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}}{k}+$$

$$\frac{1\,048\,576\sqrt{6}\sqrt{2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{3}{2}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-x\right)^k x^{-k}}{k}}}{\pi\left(2\,i\,\pi\left\lfloor\frac{\arg(128-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(128-x)^k\,x^{-k}}{k}\right)^3}\quad\text{for } x<0$$

$\binom{n}{m}$ is the binomial coefficient

$\mathbb{Z}$ is the set of integers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi 16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}=$$
$$-\frac{1}{50\pi\left(\int_1^{128}\frac{1}{t}\,dt\right)^3}\left(-52\,428\,800\sqrt{6}\sqrt{\int_1^{\frac{3}{2}}\frac{1}{t}\,dt}+\right.$$
$$\left.21\,967\pi\left(\int_1^{128}\frac{1}{t}\,dt\right)^3+1600\pi\left(\int_1^{2}\frac{1}{t}\,dt\right)\left(\int_1^{128}\frac{1}{t}\,dt\right)^3\right)$$

$$\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi 16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}=$$
$$-\left(\left(\left(i\left(419\,430\,400\sqrt{3}\,\pi^{5/2}\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^s\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}-\right.\right.\right.\right.$$
$$21\,967\,i\pi\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{127^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3-$$
$$800\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)$$
$$\left.\left.\left.\left.\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{127^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3\right)\right)\right/$$
$$\left.\left(50\pi\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{127^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3\right)\right)\quad\text{for}\quad -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

27*sqrt(1/Pi((4*8*128^2 sqrt((32(64 + 128) log((64 + 128)/(2*64)))/128))/(sqrt(2) log^3(128)) + 1/4 π 16 2^2 (3 - 2 log(2)))-521+34-34/(10^2))+1

Input

$$27\sqrt{\left(\frac{1}{\pi}\left(\frac{4\times 8\times 128^2\sqrt{\frac{1}{128}\left(32\,(64+128)\log\left(\frac{64+128}{2\times 64}\right)\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\,\pi\times 16\times 2^2\,(3-2\log(2))\right)-521+34-\frac{34}{10^2}\right)}+1$$

log(x) is the natural logarithm

Exact result

$$1+27\sqrt{\frac{\frac{1\,048\,576\sqrt{6\log\left(\frac{3}{2}\right)}}{\log^3(128)}+16\,\pi\,(3-2\log(2))}{\pi}-\frac{24\,367}{50}}$$

Decimal approximation

1729.007206337427901184337212711099094026203206407618706845822297 2

…

1729.0072063374279….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string.

This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

Alternate forms

$$1+27\sqrt{-\frac{21\,967}{50}+\frac{1\,048\,576\sqrt{6\log\left(\frac{3}{2}\right)}}{\pi\log^3(128)}-32\log(2)}$$

$$1+\frac{27}{5}\sqrt{-\frac{21967}{2}+\frac{26214400\sqrt{6\log\left(\frac{3}{2}\right)}}{\pi\log^3(128)}-800\log(2)}$$

$$1+\frac{27}{35\log^{3/2}(2)\sqrt{\frac{14\pi}{52428800\sqrt{6\log\left(\frac{3}{2}\right)}-343\pi\log^3(2)(21967+1600\log(2))}}}$$

Expanded logarithmic form

$$27\sqrt{-\frac{21967}{50}-32\log(2)+\frac{\frac{262144}{343}\sqrt{2(-48\log(2)+48\log(3))}}{\pi\log^3(2)}}+1$$

Alternative representations

$$27\sqrt{\left(\frac{\frac{4\times8\times128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times64}\right)}}{\sqrt{2}\log^3(128)}+\frac{1}{4}\pi16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}\right)}+1=$$

$$1+27\sqrt{-487-\frac{34}{10^2}+\frac{\frac{64}{4}\pi\left(3-2\log_e(2)\right)+\frac{32\times128^2\sqrt{\frac{6144\log_e\left(\frac{192}{128}\right)}{128}}}{\log_e^3(128)\sqrt{2}}}{\pi}}$$

$$27\sqrt{\left(\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi\,16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}\right)}+1=1+27\sqrt{-487-\frac{34}{10^2}+\frac{\frac{64}{4}\pi(3-2\log(a)\log_a(2))+\frac{32\times 128^2\sqrt{\frac{6144}{128}\log(a)\log_a\left(\frac{192}{128}\right)}}{(\log(a)\log_a(128))^3\sqrt{2}}}{\pi}}$$

$$27\sqrt{\left(\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi\,16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}\right)}+1=1+27\sqrt{-487-\frac{34}{10^2}+\frac{\frac{64}{4}\pi(3+2\,\mathrm{Li}_1(-1))+\frac{32\times 128^2\sqrt{-\frac{6144}{128}\,\mathrm{Li}_1\left(1-\frac{192}{128}\right)}}{(-\mathrm{Li}_1(-127))^3\sqrt{2}}}{\pi}}$$

$\log_b(x)$ is the base- b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations

$$27\sqrt{\left(\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\log^3(128)}+\frac{1}{4}\pi 16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}\right)}+1=\frac{1}{10\log^{3/2}(128)}$$

$$\left(10\log^{3/2}(128)+27\sqrt{\frac{2}{\pi}}\sqrt{\left(-21967\pi\log^3(128)-1600\pi\log(2)\log^3(128)+26214400\sqrt{3}\sum_{k=0}^{\infty}2^{1-k}\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}\frac{(-1)^j\binom{k}{j}p_{j,k}}{1-2j}\right)}\right)$$

$$\text{for}\left(c_k=\frac{6(-1)^k}{1+k}\text{ and }p_{j,0}=1\text{ and }6\,p_{j,k}=\frac{\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\text{ and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$27\sqrt{\left(\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi\,16\left(2^2(3-2\log(2))\right)}{\pi}-521+\right.$$

$$\left.34-\frac{34}{10^2}\right)+1=\frac{1}{10\log^{3/2}(128)}\left(10\log^{3/2}(128)+27\sqrt{\frac{2}{\pi}}\right.$$

$$\left.\sqrt{\left(-21\,967\,\pi\log^3(128)-1600\,\pi\log(2)\log^3(128)+26\,214\,400\sqrt{3}\right.}\right.$$

$$\left.\left.e^{i\pi\left\lfloor 1/2-\arg\left(\frac{1}{c}\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}2^{-k}\binom{-\frac{1}{2}+k}{k}\sum_{j=0}^{k}-\frac{2(-1)^j\binom{k}{j}p_{j,k}}{-1+2j}\right)\right)$$

$$\text{for}\left(c_k=\frac{6(-1)^k}{1+k}\text{ and }p_{j,0}=1\text{ and }6\,p_{j,k}=\frac{\sum_{m=1}^{k}(-k+m+j\,m)\,c_m\,p_{j,k-m}}{k}\right.$$

$$\left.\text{and }k\in\mathbb{Z}\text{ and }k>0\right)$$

$$27\sqrt{\left(\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi 16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}\right)}+1=1+27\sqrt{\left(-\frac{24367}{50}+\frac{1}{\pi}\left(16\pi\left(3-2\left(2i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}}{k}\right)\right)+\frac{1048576\sqrt{6}\sqrt{2i\pi\left\lfloor\frac{\arg\left(\frac{3}{2}-x\right)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-x\right)^k x^{-k}}{k}}}{\left(2i\pi\left\lfloor\frac{\arg(128-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(128-x)^k x^{-k}}{k}\right)^3}\right)\right)}\quad\text{for } x<0$$

$\binom{n}{m}$ is the binomial coefficient

$\mathbb{Z}$ is the set of integers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations

$$27\sqrt{\left(\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi 16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}\right)}+1=\frac{1}{10}\left(10+27\sqrt{\frac{2}{\pi}}\sqrt{\left(\frac{1}{\left(\int_1^{128}\frac{1}{t}\,dt\right)^3}\left(52\,428\,800\sqrt{6}\sqrt{\int_1^{\frac{3}{2}}\frac{1}{t}\,dt}-21\,967\,\pi\left(\int_1^{128}\frac{1}{t}\,dt\right)^3-1600\,\pi\left(\int_1^{2}\frac{1}{t}\,dt\right)\left(\int_1^{128}\frac{1}{t}\,dt\right)^3\right)\right)\right)}\right)$$

$$27\sqrt{\left(\frac{\frac{4\times 8\times 128^2\sqrt{\frac{32}{128}(64+128)\log\left(\frac{64+128}{2\times 64}\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\pi 16\left(2^2(3-2\log(2))\right)}{\pi}-521+34-\frac{34}{10^2}\right)}+1=\frac{1}{10}\left(10+27\sqrt{\frac{2}{\pi}}\sqrt{\left(\left(-419\,430\,400\,i\sqrt{3}\,\pi^{5/2}\sqrt{-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^s\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}-21\,967\,\pi\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{127^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3+800\,i\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{127^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3\right)\Big/\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{127^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3\right)}\right)\quad\text{for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

((27*sqrt(1/Pi((4*8*128^2 sqrt((32(64 + 128) log((64 + 128)/(2*64)))/128))/(sqrt(2) log^3(128)) + 1/4 π 16 2^2 (3 - 2 log(2)))-521+34-34/(10^2))+1))^1/15+(MRB const)^(1-1/(4π)+π)

Input

$$\left(27\sqrt{\left(\frac{1}{\pi}\left(\frac{4\times 8\times 128^2\sqrt{\frac{1}{128}\left(32\,(64+128)\log\left(\frac{64+128}{2\times 64}\right)\right)}}{\sqrt{2}\,\log^3(128)}+\frac{1}{4}\,\pi\times 16\times 2^2\,(3-2\log(2))\right)-521+34-\frac{34}{10^2}\right)+1}\right)\text{^}\,(1/15)+{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm

C_{MRB} is the MRB constant

Exact result

$${C_{\text{MRB}}}^{1-1/(4\pi)+\pi}+\sqrt[15]{1+27\sqrt{\frac{\frac{1\,048\,576\sqrt{6\log\left(\frac{3}{2}\right)}}{\log^3(128)}+16\pi\,(3-2\log(2))}{\pi}-\frac{24\,367}{50}}}$$

Decimal approximation

1.64493848088230062569620511245503721692895405391708238094854469 14

…

1.6449384808823…. ≈ $\zeta(2) = \pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{1 + 27\sqrt{-\frac{21\,967}{50} + \frac{1\,048\,576\sqrt{6\log\left(\frac{3}{2}\right)}}{\pi\log^3(128)}} - 32\log(2)}$$

$$C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \sqrt[15]{1 + \frac{27}{5}\sqrt{-\frac{21\,967}{2} + \frac{26\,214\,400\sqrt{6\log\left(\frac{3}{2}\right)}}{\pi\log^3(128)}} - 800\log(2)}$$

$$C_{\text{MRB}}^{1-1/(4\pi)+\pi} + \frac{1}{\sqrt[15]{10 + \frac{27\sqrt{\frac{2\left(-21\,967\,\pi\log^3(128) - 1600\,\pi\log(2)\log^3(128) + 52\,428\,800\sqrt{6\log\left(\frac{3}{2}\right)}\right)}{\pi}}}{\log^{3/2}(128)}}}}$$

Expanded logarithmic form

$$\sqrt[15]{27\sqrt{-\frac{21\,967}{50} - 32\log(2) + \frac{\frac{262\,144}{343}\sqrt{2\left(-48\log(2) + 48\log(3)\right)}}{\pi\log^3(2)}} + 1} + C_{\text{MRB}}^{1-1/(4\pi)+\pi}$$

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

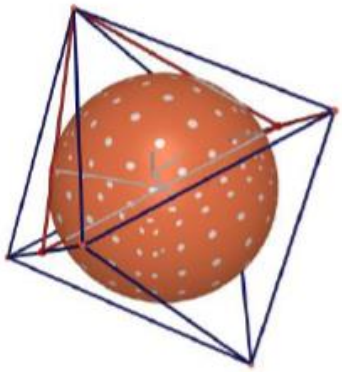


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\ z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\ x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\,l^3}{\frac{1}{3}\left(4\,\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\, z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^0\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{MRB}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{\left(6\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$).

The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^8\right)^{-k}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^8\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.6180\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\,\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\,\sqrt{2}\,a^3}{\frac{4}{3}\,\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\,\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{((\sqrt{2}\,a^3)(\sqrt{2}\,d^3))12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\,\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\;\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^3}}}=\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))) = 3.1415926535

Indeed:

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.1415926535884871624221932509858715278931025189245576170078100613
…

3.1415926535…. = π (Ramanujan Recurring Number)

From the following extended DN Constant (“Unitary Formula”)

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}\times\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}}$$

with regard

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

for q = 1729 and p = 4096, we obtain by changing the sign in the algebraic sum of the aforementioned Cardano’s Formula and after some calculations:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

multiplied by

$$\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}$$

and performing the ninth root of the entire expression:

√((2^(-1/π)(5(3+√(5))π)^(1/(2π)))((∛(-1729/2+√(1729^2/4+4096^3/27)) -∛(-1729/2-√(1729^2/4+4096^3/27)))^1/9))

we obtain:

$$\sqrt{\left(\left(2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right)\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}\right)}$$

$$2^{-1/(2\pi)}\sqrt[18]{\sqrt[3]{\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}-\frac{1729}{2}}+\sqrt[3]{\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}}\Big/\sqrt[4\pi]{5\left(3+\sqrt{5}\right)\pi}$$

i.e.

2^(-1/(2π)) ((-1729/2+√(274958621851/3)/6)^(1/3)+(1729/2 + √(274958621851/3)/6)^(1/3))^(1/18) (5(3+√(5)) π)^(1/(4π))

= 1.61549140391….

The general "unitary" formula, which derives from DN Constant, is the following:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}}\cong 1.61803398\ldots=\frac{\sqrt{5}+1}{2}$$

Where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

C is any constant or solution to an equation, R is the radius of the Universe and $2.33*10^{-13}$ is the temperature of the universe expressed in GeV.

For example, $C = 9.9128$, inserting a radius of the Universe, which we have calculated, equal to $R = 2.06274*10^{12}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}$$

1.618035912348264235440174408834709854254527350840173356306481810 7

…

1.618035912348…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We obtain also:

(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))dxdydz

Indefinite integral assuming all variables are real

$$0.809018\, x^2\, y\, z + \text{constant}$$

Definite integral over a cube of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} 1.61804\, dx\, dy\, dz = 12.9443\, L^3$$

Definite integral over a sphere of radius R

$$\iiint_{x^2+y^2+z^2<R^2} 1.61804\, dz\, dy\, dx = 6.77761\, R^3$$

From which, for L = R = 1 , dividing the two definite integral results by the original expression, we obtain:

12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))

Input interpretation

$$\frac{12.9443}{\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

8.00001…

8.00001…. ≈ 8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

And

3*(6.77761/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))

Input interpretation

$$3\times\frac{6.77761}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

12.5664…

12.5664…. ≈ 4π = Bekenstein-Hawking (S_{BH}) black hole entropy

New fundamental formula deriving from DN Constant

The DN Constant (Del Gaudio-Nardelli Constant) equals $(2\sqrt{2})/\pi$) is defined as the ratio of the volume of an octahedron to the volume of a sphere and is an intriguing mathematical concept. Michele Nardelli hypothesized that the regular octahedron represents a phase in which the universe is highly symmetrical and with very low entropy. On the other hand, the sphere (which is inscribed in the octahedron, i.e. is "inside" it) represents the universe emerging from the quantum vacuum, which over time increases entropy and undergoes various symmetry breakings. This occurs in a regime of eternal inflation.

From the following expression

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×C×R)))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\,C\,R\right)}}$$

which comes from the DN Constant, with $1.616255*10^{-35}$ which is equal to the Planck length, $1.1056*10^{-52}$ which is equal to the Cosmological Constant, C = 1729 which corresponds to the Hardy-Ramanujan number and R = $4.6018401361 \times 10^{-24}$, which represents the radius of the Universe, we obtain:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×4.6018401361 × 10^-24))) = 1.6180329973...

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 4.6018401361\times 10^{-24}\right)}}$$

1.618032997307532491506757016629746746422577260892967140791939590 3
…

1.6180329973075… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We have also the following formula:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(4.4525642121 × 10^-24))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 4.4525642121\times 10^{-24}\right)}}$$

Result

1.644932352102092130483898983704151168876621841655166877914166033 8
…

1.64493235210209213…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

And again:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(1.2206935225 × 10^-24))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 1.2206935225\times 10^{-24}\right)}}$$

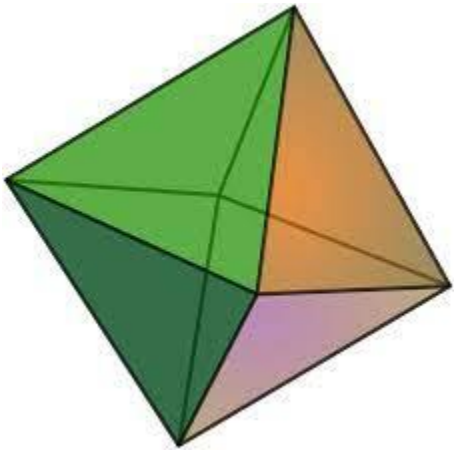
Result

3.14159225731469939939950039231605796469731171781644423955790024797

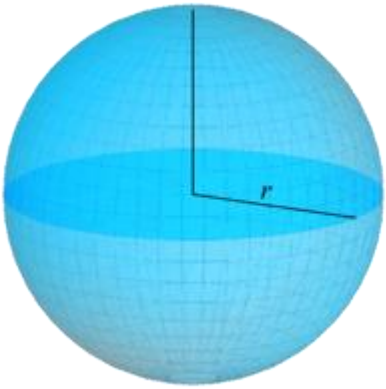
…

3.14159225731469…≈ π (Ramanujan Recurring Number)

Now, we have that:



Octahedron



Sphere

Given the value of a volume, independently of the solid, following the Poincaré Conjecture, we compare any solid "without holes" and a sphere. If we compare an octahedron with a sphere, we have:

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

If we consider the radius of the sphere as an unknown, we must find the value of the side of the octahedron which allows us to equalize the two volumes and which will give us the DN Constant as a result (which will therefore be equal to the radius of the sphere).

From

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

To find r we perform the following calculation:

$$r^3 = \frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi} = \frac{1}{3}\sqrt{2}a^3 \cdot \frac{3}{4\pi} = \frac{\sqrt{2}a^3}{4\pi} = \frac{\sqrt{2}\cdot\sqrt{2}a^3}{\sqrt{2}\cdot 4\pi} = \frac{2a^3}{\sqrt{2}\cdot 4\pi} = \frac{a^3}{2\sqrt{2}\pi}$$

$$r^3 = \frac{a^3}{2\sqrt{2}\pi}; \qquad r = \sqrt[3]{\frac{a^3}{2\sqrt{2}\pi}} = \frac{a}{\sqrt[3]{2\sqrt{2}\pi}}$$

To find *a*, we have, for

$$r = \frac{2\sqrt{2}}{\pi}; \qquad \frac{a}{\sqrt[3]{2\sqrt{2}\pi}} = \frac{2\sqrt{2}}{\pi};$$

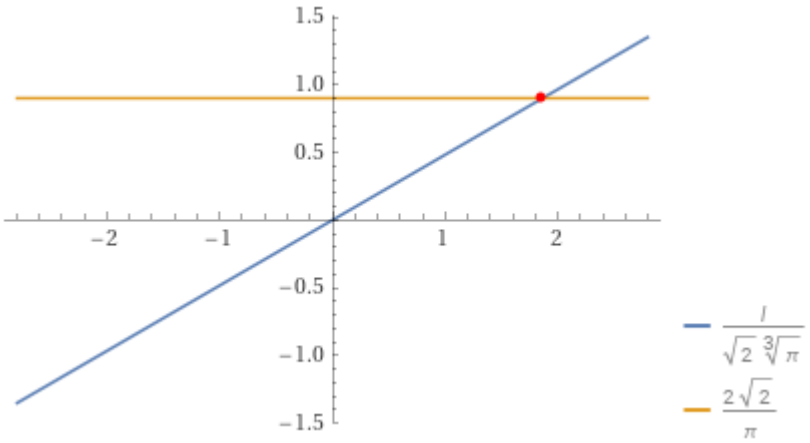
Thus, multiplying both the sides by $\sqrt[3]{2\sqrt{2}\pi}$, we obtain:

$$a = \frac{2\sqrt{2}}{\pi} \cdot \sqrt[3]{2\sqrt{2}\pi} = \sqrt[3]{\frac{(2\sqrt{2})^3 2\sqrt{2}\pi}{\pi^3}} =$$

$$= \sqrt[3]{\frac{8\cdot\sqrt{2^3}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{8\cdot\sqrt{2\cdot 2^2}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{16\cdot\sqrt{2}\cdot 2\sqrt{2}}{\pi^2}}$$

$$= \sqrt[3]{\frac{32\sqrt{2}\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{32 \cdot 2}{\pi^2}} = \frac{\sqrt[3]{64}}{\sqrt[3]{\pi^2}} = \frac{4}{\sqrt[3]{\pi^2}};\ a = \frac{4}{\sqrt[3]{\pi^2}}$$

Plot



Solution

$$a = \frac{4}{\sqrt[3]{\pi^2}}$$

for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), we obtain:

from the octahedron volume, we have: V = 1/3*√2*a^3 = (1/3*√2*($\frac{4}{\sqrt[3]{\pi^2}}$)^3)

(1/3*√2*(4/∛(π^2))^3)

Input

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3$$

$\sqrt[3]{x}$ is the real-valued 3rd root of x

Exact result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{3\pi^2}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And, from the sphere volume V = (4/3*π*r^3) = (4/3*π*((2√2)/π)^3)

(4/3*π*((2√2)/π)^3)

Input

$$\frac{4}{3}\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3$$

Result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\sqrt{z_0}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\exp^3\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3}{3\pi^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\left(\frac{1}{z_0}\right)^{3/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{3/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. ($1728 = 8^2 * 3^3$) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

$1.64381856858\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad \text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10^{-6}$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

$-5.010882647757...$

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874} + b}{M}$$

$$\frac{\partial M(b,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245 + 18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M} = \frac{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b} = \frac{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right\} = 0 \text{ at } (b, M) = (-16, 0)$$

Global minima:

$$\min\left\{\frac{1}{3}\,M^2\left(1 - \frac{(b\,2)\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\,\sqrt{6}}\right)^2\right\} = 0$$

for $b = -\frac{226\,802\,245}{18\,480\,874}$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

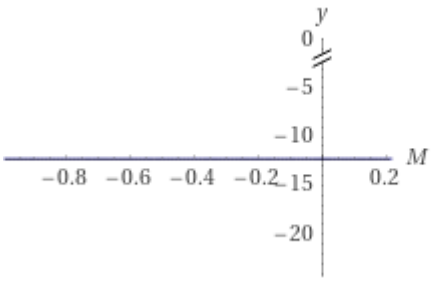
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

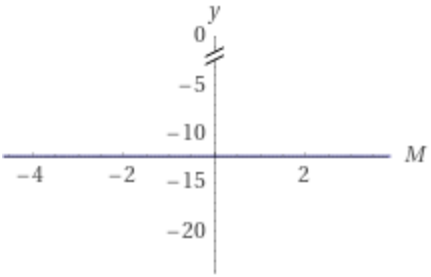
Result:

$$\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723\right)+O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

-12.2723

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times10^{-15}}{M}$$

Indefinite integral:

$$\int\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3}(0.0814845\,b + 1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

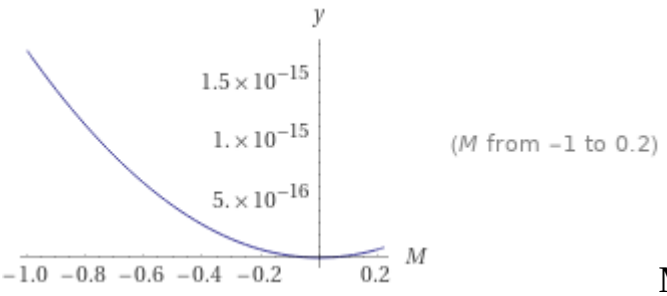
Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

Result:

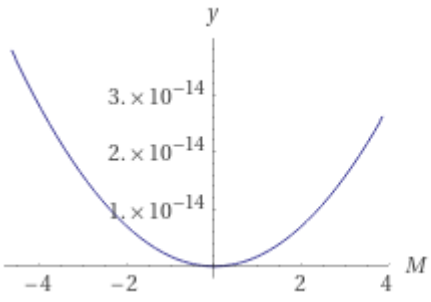
0

Plots: **(possible mathematical connection with an open string)**



M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O\left(M^{62194}\right)$

(Taylor series)

Series expansion at M = ∞:

$1.75541 \times 10^{-15}\, M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.388513449474645453489707833780880208333333333333333333\ldots\times10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^2+6.58545\times10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times0.2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times10^{-17}$

7.021621519159*10^{-17}

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875 \times 10^{-14}$

1.57986484181*10^-14

For M = 2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\, M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 2^2 + 6.58545 \times 10^{-10}\sqrt{2^4}\right)}{2^2} + 1\right)^2 \times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots \times 10^{-15}$

7.021621519*10^-15

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots\times 10^{-8}$

5.9776991059*10^-8 result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E}=\mathbf{E}_{\mathrm{P}}l_{\mathrm{P}}^{2}=\phi_{\mathrm{P}}l_{\mathrm{P}}=\sqrt{\frac{\hbar c}{\varepsilon_{0}}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left((1/(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}))\,\hat{}\,(1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

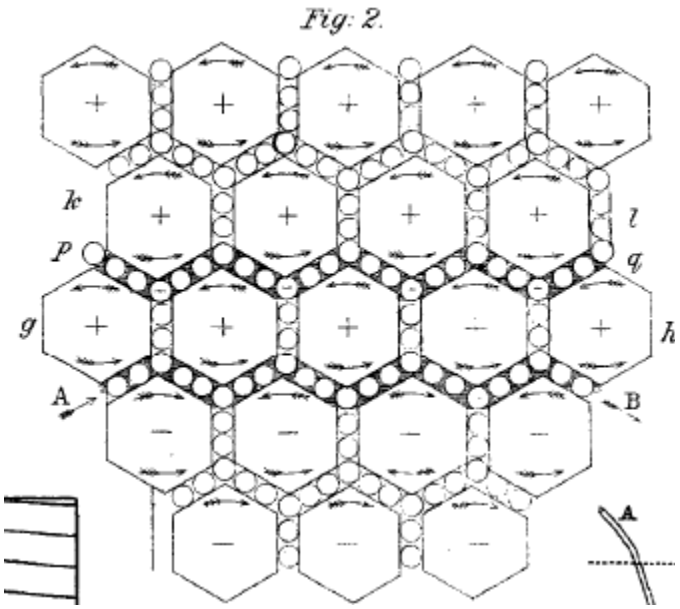
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

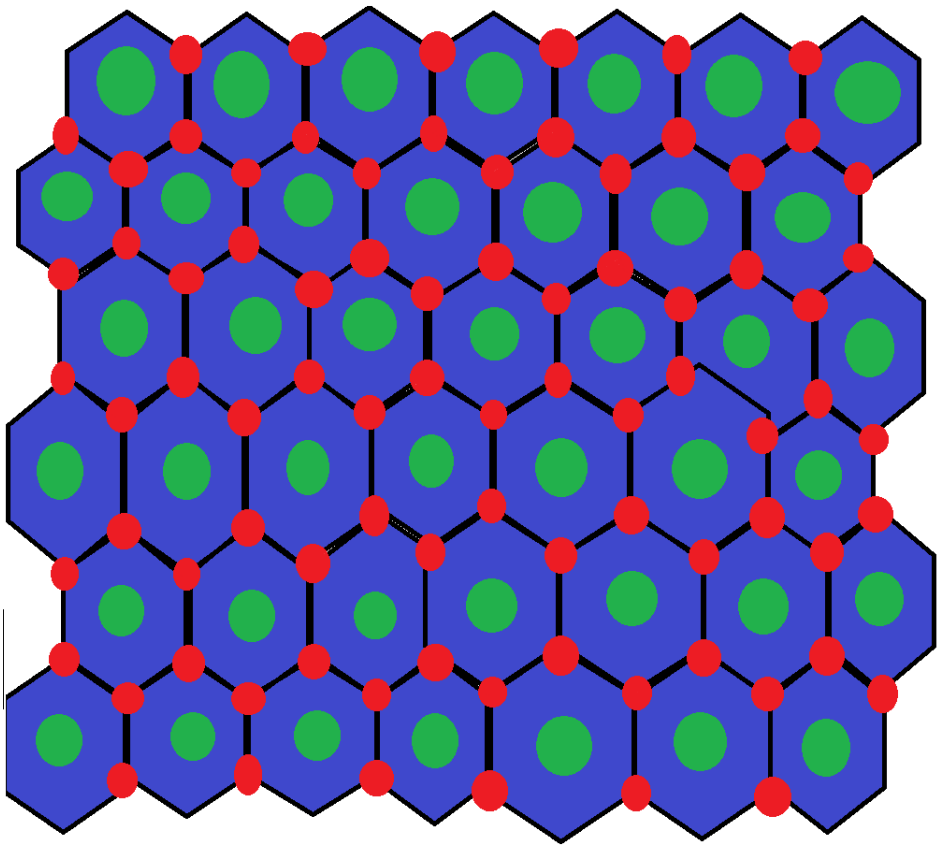
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability.

References

Fondamenti per una Teoria Generale delle Funzioni Pseudo-Analitiche di una Variabile Complessa – *Prof. Renato Caccioppoli* – 1952

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf

On some equations concerning a possible TOE. New possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory, Theoretical Cosmology and String Theory

Michele Nardelli[1], Chavis Srichan[2],

Abstract

In this paper, we analyze some equations concerning a possible TOE. We obtain new possible mathematical connections with the Ramanujan Recurring Numbers, DN Constant and some parameters of Number Theory, Theoretical Cosmology and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

[2] Faculty of Engineering, Khon Kaen University, Khon Kaen, 40002, Thailand

Srinivasa Ramanujan

(1887 - 1920)

Introduction

In this paper, an octahedron could serve as a mathematical or conceptual model of the universe in the quantic phase, while the spherical surface could be used to describe the geometry of the bubble-universe.

The values $(2\sqrt{2})/\pi$, the golden ratio φ, $\zeta(2)$ and π, can be connected to the proposed cosmological model. Here's how they might be connected:

Ratio $(2\sqrt{2})/\pi$ the so called DN Constant:

This relationship may have a connection with the geometric properties of the octahedron and the sphere, which have been considered as mathematical models of the early universe and bubbles universe in eternal inflation.

Golden Ratio φ:

The golden ratio is a mathematical constant that appears in many natural and artistic contexts and is often associated with harmonious proportions and aesthetic beauty. Its emergence in this context could suggest a kind of intrinsic symmetry or harmony in the structure of the early universe and bubbles universe.

Value of π:

The value of π is a fundamental mathematical constant that appears in many geometric formulas and relationships, including the geometry of the sphere. Its appearance could indicate a direct connection between the geometry of bubbles universe and the mathematical properties of spherical surfaces.

Ultimately, the results obtained can be interpreted as manifestations of the geometric and mathematical properties of the models proposed for the early universe and universe bubbles. This suggests that there is a profound connection between geometry, mathematics and cosmological physics, and that through the analysis of these relationships we can deepen our understanding of the universe and its fundamental phenomena.

The above values $(2\sqrt{2})/\pi$, the golden ratio φ and π, can be connected to the proposed cosmological model. This hypothesis is certainly plausible.

The various mathematical solutions and relationships can be seen as representations of the principles and laws that govern the formation and evolution of the universe.

Regarding the fundamental mathematical values, they could emerge as a consequence of the geometric and physical laws that govern the structure and evolution of the quantum universe and bubbles universe.

The multidisciplinary approach involving complex mathematical solutions and cosmological concepts can offer deeper insight into the fundamental nature of the universe and its processes. Exploring these connections could lead to new discoveries and insights into our understanding of the early universe and its complexity.

Proposal:

The initial octahedron: Let's imagine a regular octahedron, with perfectly symmetrical faces. Each face represents an ideal symmetry.

The emerging sphere: Inside the octahedron, there is an inscribed sphere. This sphere represents the bubble of the universe that emerges from the perturbations of the quantum vacuum during eternal inflation.

Expansion and transitions: As time passes, the universe expands. The faces of the octahedron begin to break, symbolizing "symmetry breaks." The sphere continues to grow, representing the expanding universe.

Constants and numbers: We integrate the mathematical results you obtained. For example, the golden ratio (φ) could be represented by a proportion between the dimensions of the octahedron and the sphere.

Entropy and complexity: Entropy increases as the universe evolves. We can represent this with a disordered growth of structures within the emerging sphere. Imagine this scene as an abstract work of art, where geometric shapes and cosmological concepts merge

In Fig.1 and Fig.2 let's imagine a regular octahedron representing the universe in its phase of high symmetry and very low entropy. Inside the octahedron we have an inscribed sphere that emerges from perturbations of the quantum vacuum during eternal inflation. As time passes, the universe expands, the faces of the octahedron break (symmetry breaks), and entropy increases. Spheres emerge from the octahedra, symbolizing the transition phases from a regime of very low entropy to a universe in which, with the passage of time, entropy increases, increasing the complexity of the universe itself.

Fig. 1

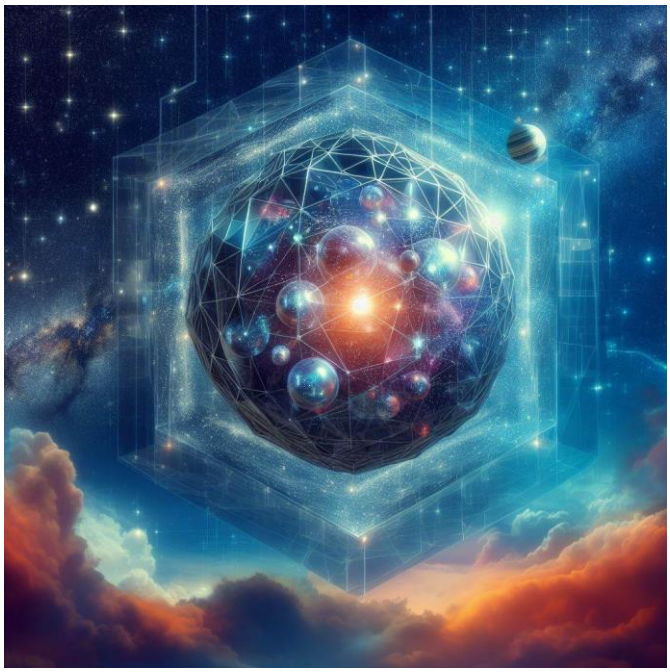
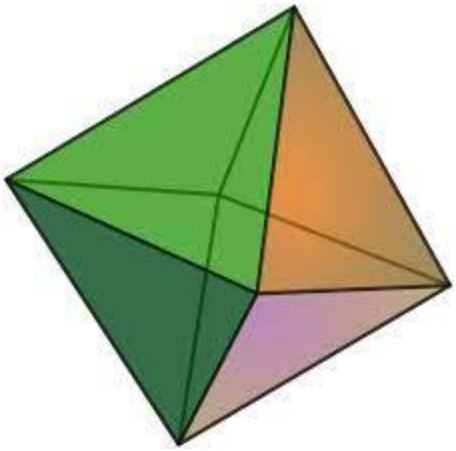
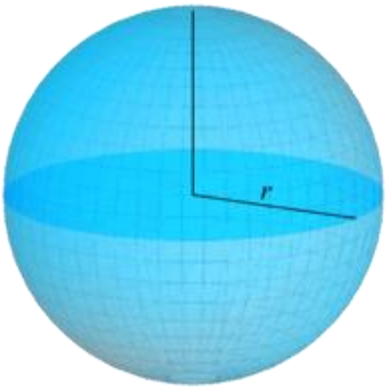


Fig. 2

Now, we have that:



Octahedron



Sphere

From the octahedron volume V = 1/3*√2 l^3 and, from the sphere volume,

V = (4/3*π*r^3) , we consider the following relationship, for r = x:

4/3*π*x^3 = 1/3*√2*l^3

Input

$$\frac{4}{3}\pi x^3 = \frac{1}{3}\sqrt{2}\, l^3$$

Exact result

$$\frac{4\pi x^3}{3} = \frac{\sqrt{2}\, l^3}{3}$$

Alternate forms

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$\frac{4\pi x^3}{3} - \frac{\sqrt{2}\,l^3}{3} = 0$$

Real solution

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Solutions

$$x = -\frac{\sqrt[3]{-\frac{1}{\pi}}\,l}{\sqrt{2}}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

$$x = \frac{(-1)^{2/3}\,l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

Integer solution

$l = 0, \quad x = 0$

Implicit derivatives

$$\frac{\partial x(l)}{\partial l} = \frac{l^2}{2\sqrt{2}\,\pi x^2}$$

$$\frac{\partial l(x)}{\partial x} = \frac{2\sqrt{2}\,\pi x^2}{l^2}$$

From the alternate form

$$x^3 = \frac{l^3}{2\sqrt{2}\,\pi}$$

$$x = \frac{l}{\sqrt{2}\,\sqrt[3]{\pi}}$$

for l = 8, we have that:

8/(sqrt(2) π^(1/3)) = 8/(2sqrt2 * Pi)^1/3

Input

$$\frac{8}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{8}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Result

True

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(8) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Thence:

l/(sqrt(2) π^(1/3)) = l/(2sqrt2 * Pi)^1/3

Input

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}}$$

Logarithmic form

$$\log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt{2}\,\sqrt[3]{\pi}\right) = \log_{\sqrt{2}\,\sqrt[3]{\pi}}(l) - \log_{\sqrt{2}\,\sqrt[3]{\pi}}\left(\sqrt[3]{2\sqrt{2}\,\pi}\right)$$

$\log_b(x)$ is the base- b logarithm

Now, we have that:

l/(2 sqrt(2) π)^(1/3) = (2sqrt2)/Pi

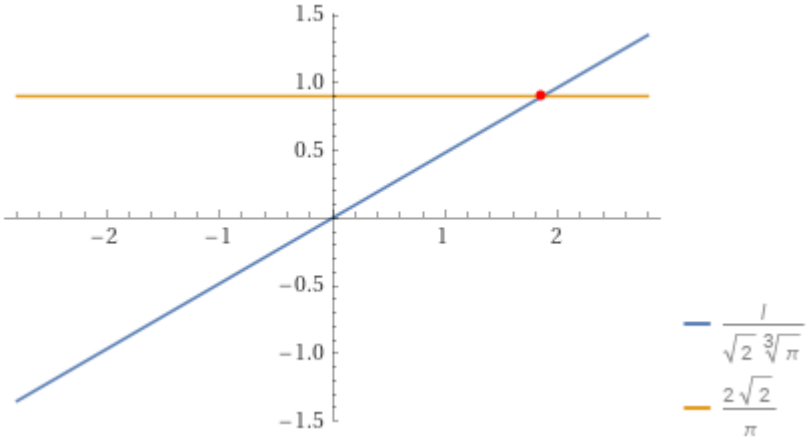
Input

$$\frac{l}{\sqrt[3]{2\sqrt{2}\,\pi}} = \frac{2\sqrt{2}}{\pi}$$

Exact result

$$\frac{l}{\sqrt{2}\,\sqrt[3]{\pi}} = \frac{2\sqrt{2}}{\pi}$$

Plot



Solution

$$l = \frac{4}{\pi^{2/3}}$$

$$l = \frac{4}{\sqrt[3]{\pi^2}}$$

From

On the same origin of quantum physics and general relativity from Riemannian geometry and Planck scale formalism - *Chavis Srichan, Pobporn Danvirutai, Adrian David Cheok, Jun Cai, Ying Yan*

In the above work, very interesting is that has been “modeled the universe using the equation with Einstein’s lambda formalism and found that the universe dynamics could be considered as harmonic oscillators entangled with lambda curvature”. Furthermore, “this equation can be used to describe the energy transfer between two entangled spacetimes between the same universe and between any two universes (ER=EPR)”. And very interesting is also that “the singularity of black holes can be avoided at the Planck scale, because space and time are no longer entangled”.

The originality, the goal, in my opinion, lies in the fact that "the constants of the universe can be reduced to just two quantities: Planck length and Planck time" and in having demonstrated that "the Einstein field equation of general relativity is actually a relativistic quantum mechanical equation".

The authors' thesis, in my humble opinion, is very valid and coherent from a physical and mathematical point of view.

Now, we analyze the following equation:

$$\frac{E^2}{E_p^2} = l_p^2(R^2 + \Lambda^2)$$

$$= l_p^2(\cos^2 \omega\tau + \sin^2 \omega\tau) = l_p^2$$

and, we note that:

$$l_p^2 = \left(\sqrt{\frac{\hbar G}{c^3}}\right)^2$$

that is equal to

$(\sqrt{(\hbar G/c^3)})^2$

$$\left(\sqrt{\frac{G\hbar}{c^3}}\right)^2$$

and the resulti is:

$2.612*10^{-70}\ m^2$

Now, we perform the following calculation

$(-\ln(2.612\times10^{-70})-4^2)$

(String Theory, Superstring Theory and Beyond – Vol. II – J. Polchinski)

Table B.3. *Supergravities with 32 supercharges. The group G is a symmetry of the low energy supergravity theory, and the moduli space is locally G/H.*

d	scalars	vectors	G	H
10A	1	1	$SO(1,1,\mathbf{R})$	-
10B	2	0	$SL(2,\mathbf{R})$	$SO(2,\mathbf{R})$
9	3	3	$SL(2,\mathbf{R})\times SO(1,1,\mathbf{R})$	$SO(2,\mathbf{R})$
8	7	6	$SL(2,\mathbf{R})\times SL(3,\mathbf{R})$	$SO(2,\mathbf{R})\times SO(3,\mathbf{R})$
7	14	10	$SL(5,\mathbf{R})$	$SO(5,\mathbf{R})$
6	25	16	$SO(5,5,\mathbf{R})$	$SO(5,\mathbf{R})\times SO(5,\mathbf{R})$
5	42	27	$E_{6(6)}(\mathbf{R})$	$US_p(8)$
4	70	28	$E_{7(7)}(\mathbf{R})$	$SU(8)$
3	128	-	$E_{8(8)}(\mathbf{R})$	$SO(16,\mathbf{R})$

Where $16 = 4^2$ are vectors (see above Table) and 233 is a Fibonacci's number and the difference between 248 and 15.

There is a group, the exceptional group *E*8, that has an *SO*(16) subgroup under which the *E* adjoint **248** transforms as **120** + **128**. The total matter central charge is $c^{\mathrm{m}} = \tilde{c}^{\,m} = 15$. Thence, we obtain the following expression:

$$\frac{233}{-\log(2.612 \times 10^{-70}) - 4^2}$$

The result is 1.6155779…that is very near to the value of the Golden Ratio.

Furthermore, with regard the Planck time

$$t_p = \left(\sqrt{\frac{\hbar G}{c^5}}\right)$$

we obtain:

$(\sqrt{(\hbar G/c^5)})$

$$\left(\sqrt{\frac{G\hbar}{c^5}}\right)$$

that is equal to

$5.391*10^{-44}$ seconds

And performing the following calculations:

$$27 \times \frac{1}{2}(-\log(5.391 \times 10^{-44}) + 24 + 2\Phi + \pi) + 1$$

where 27 are vectors (see above table) and 24 is the number of vibration modes of the bosonic strings

we obtain:

1729.0901….

1728 (1729 - 1) occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$). Furthermore, the number 1729 is the Hardy–Ramanujan number (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

$$\left[\frac{1}{2}(-\log(5.391 \times 10^{-44}) + 24 + 2\Phi + \pi)\right]^2 - \frac{2}{5}$$

and the result is:

4096.0271…. ≈ $4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: “Dilaton Tadpole for the Open Bosonic String “ Michael R. Douglas and Benjamin Grinstein - September 2,1986)

And:

$$\sqrt[15]{27 \times \frac{1}{2}(-\log(5.391 \times 10^{-44}) + 24 + 2\Phi + \pi) + 1} + C_{MRB}^{1-\frac{1}{(4\pi)}+\pi}$$

$$\sqrt[15]{27 \times \frac{1}{2}\left(-\log\left(5.391 \times 10^{-44}\right) + 24 + 2\,\Phi + \pi\right) + 1} + C_{\mathrm{MRB}}^{1-1/(4\pi)+\pi}$$

the result is:

1.64494373…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (practically the value of the ζ(2), that is equal to $\pi^2/6$, the possible trace of the instanton shape and Ramanujan Recurring Number)

Now, we analyze the following equation:

$$N(f_n) = \frac{2}{3}\kappa \cdot \frac{\hbar^{\frac{3}{2}}(cl_p)^3}{f_n^{\frac{3}{2}}} = \frac{\frac{2}{3}\kappa\hbar^{\frac{3}{2}}(cl_p)^3\left(n+\frac{1}{2}\right)^{\frac{1}{2}\cdot\frac{3}{2}}}{(cl_p)^{\frac{3}{2}}}$$

$$N(f_n) = \frac{2}{3}\kappa\hbar^{\frac{3}{2}}\left(cl_p\left(n+\frac{1}{2}\right)\right)^{3/2} \propto \frac{1}{f^{\frac{3}{2}}}$$

We consider:

$$\frac{\frac{2}{3}\kappa\hbar^{\frac{3}{2}}(cl_p)^3\left(n+\frac{1}{2}\right)^{\frac{1}{2}\cdot\frac{3}{2}}}{(cl_p)^{\frac{3}{2}}}$$

where c is the speed of light, $\kappa = 8\pi G/c^4$. We obtain:

(2/3*(8π*6.67259*10^-11)/(299792458^4)*ħ^(3/2)*(299792458*1.616255*10^-34)^3 (n+1/2)^(1/2·3/2))/(299792458*1.616255*10^-34)^(3/2)

Input interpretation

$$\left(\frac{2}{3}\times\frac{8\pi\times 6.67259\times 10^{-11}}{299\,792\,458^4}\ \hbar^{3/2}\right.$$ (reduced Planck constants to the 3/2)

$$\left.\left(299\,792\,458\times 1.616255\times 10^{-34}\right)^3\left(n+\frac{1}{2}\right)^{1/2\times 3/2}\right)\Big/$$

$$\left(299\,792\,458\times 1.616255\times 10^{-34}\right)^{3/2}$$

Result

$$\left(n+\frac{1}{2}\right)^{3/4} 1.476\times 10^{-81}\ \hbar^{3/2}$$ (reduced Planck constants to the 3/2)

where $\hbar$ = 1.054571817*10^-34

(n + 1/2)^(3/4) 1.476×10^-81 (1.054571817*10^-34)^(3/2)

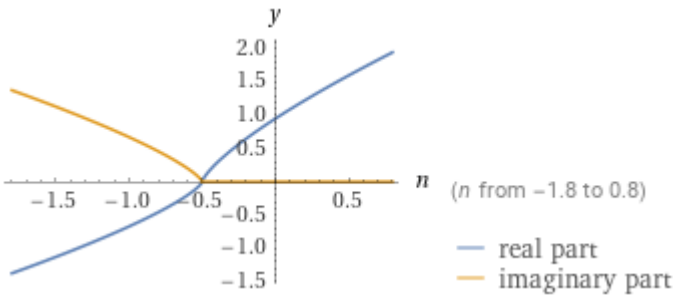
Input interpretation

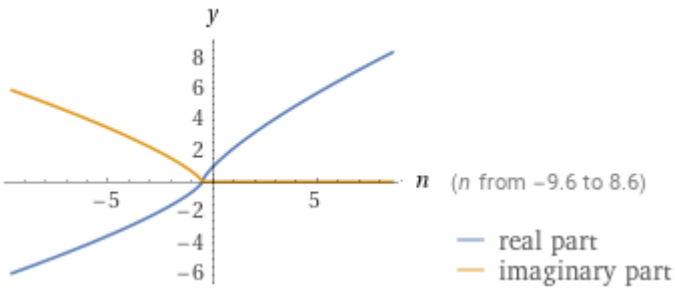
$$\left(n+\frac{1}{2}\right)^{3/4}\times 1.476\times 10^{-81}\left(1.054571817\times 10^{-34}\right)^{3/2}$$

Result

$$1.59846\times 10^{-132}\left(n+\frac{1}{2}\right)^{3/4}$$

Plots **(figures that can be related to the open strings)**





Values

n	1	2	3	4	5
$1.59846\times 10^{-132}\left(n+\frac{1}{2}\right)^{3/4}$	2.16655×10^{-132}	3.17801×10^{-132}	4.09027×10^{-132}	4.93867×10^{-132}	5.7408×10^{-132}

Alternate form

$$9.50447\times 10^{-133}\,(2n+1)^{3/4}$$

Root

$$n\approx -0.5$$

Series expansion at n=-1/2

$$1.59846\times 10^{-132}\left(n+\frac{1}{2}\right)^{3/4}+O\left(\left(n+\frac{1}{2}\right)^{49/4}\right)$$

(Puiseux series)

Series expansion at n=0

$$9.50447\times 10^{-133}+1.42567\times 10^{-132}\,n-3.56418\times 10^{-133}\,n^2+2.97015\times 10^{-133}\,n^3-3.34142\times 10^{-133}\,n^4+O\left(n^5\right)$$

(Taylor series)

Series expansion at n=∞

$$1.59846\times 10^{-132}\, n^{3/4} + 5.99421\times 10^{-133}\, \sqrt[4]{\frac{1}{n}} + O\left(\left(\frac{1}{n}\right)^{1}\right)$$

(Puiseux series)

Derivative

$$\frac{d}{dn}\left(1.59846\times 10^{-132}\left(n+\frac{1}{2}\right)^{3/4}\right) = \frac{1.42567\times 10^{-132}}{\sqrt[4]{2n+1}}$$

Indefinite integral

$$\int 1.59846\times 10^{-132}\left(\frac{1}{2}+n\right)^{3/4} dn = 2.71556\times 10^{-133}\,(2n+1)^{7/4} + \text{constant}$$

Global minimum

$$\min\left\{1.59846\times 10^{-132}\left(n+\frac{1}{2}\right)^{3/4}\right\} = 0 \text{ at } n = -\frac{1}{2}$$

Definite integral after subtraction of diverging parts

$$\int_0^{\infty} 1.59846\times 10^{-132}\left(\frac{1}{2}+n\right)^{3/4} - \left(\frac{5.99421\times 10^{-133}}{\sqrt[4]{n}} + 1.59846\times 10^{-132}\, n^{3/4}\right) dn =$$
$$-2.71556\times 10^{-133}$$

For n = Φ = 0.61803398…:

(0.61803398 + 1/2)^(3/4) 1.476×10^-81 (1.054571817*10^-34)^(3/2)

Input interpretation

$$\left(0.61803398 + \frac{1}{2}\right)^{3/4} \times 1.476 \times 10^{-81} \left(1.054571817 \times 10^{-34}\right)^{3/2}$$

Result

$1.73797\ldots \times 10^{-132}$

1.73797…*10^-132

From which, after some calculations:

2e*(-ln((0.61803398 + 1/2)^(3/4) 1.476×10^-81 (1.054571817*10^-34)^(3/2)))+76+π+ (φ+((2√2)/π))*MRB Const

Input interpretation

$$2e\left(-\log\left(\left(0.61803398 + \frac{1}{2}\right)^{3/4} \times 1.476 \times 10^{-81} \left(1.054571817 \times 10^{-34}\right)^{3/2}\right)\right) + 76 + \pi + \left(\phi + \frac{2\sqrt{2}}{\pi}\right) C_{\text{MRB}}$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

C_{MRB} is the MRB constant

Result

1729.00567…

1729.00567….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$.

The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(2e*(-ln((0.61803398 + 1/2)^(3/4) 1.476×10^-81 (1.054571817*10^-34)^(3/2)))+76+π+ (φ+((2√2)/π))*MRB Const)^1/15+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\left(2\,e\left(-\log\left(\left(0.61803398+\frac{1}{2}\right)^{3/4}\times 1.476\times 10^{-81}\left(1.054571817\times 10^{-34}\right)^{3/2}\right)\right)+76+\pi+\left(\phi+\frac{2\sqrt{2}}{\pi}\right)C_{\text{MRB}}\right)\text{^}(1/15)+{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

C_{MRB} is the MRB constant

Result

1.644938383269668323720028413762032653930640598870227171821901563 0

…

1.644938383…. ≈ ζ(2) = π²/6 = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

(1/27((2e*(-ln((0.61803398 + 1/2)^(3/4) 1.476×10^-81 (1.054571817*10^-34)^(3/2)))+76+π+ (φ+((2√2)/π))*MRB Const)-1))^2

Input interpretation

$$\left(\frac{1}{27}\left(\left(2\,e\left(-\log\left(\left(0.61803398+\frac{1}{2}\right)^{3/4}\times 1.476\times 10^{-81}\left(1.054571817\times 10^{-34}\right)^{3/2}\right)\right)+76+\pi+\left(\phi+\frac{2\sqrt{2}}{\pi}\right)C_{\text{MRB}}\right)-1\right)\right)^2$$

$\log(x)$ is the natural logarithm

ϕ is the golden ratio

C_{MRB} is the MRB constant

Result

4096.0269…

4096.0269…. ≈ 4096

The number $4096 = 64^2$, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

Now, we analyze the following formula

$$E_n = \frac{(cl_p)^2 \hbar}{\sqrt{n+1/2}\cdot cl_p} = \frac{\hbar (cl_p)^2}{f_n}.$$

We consider $\frac{(cl_p)^2 \hbar}{\sqrt{n+1/2}\cdot cl_p}$ and to simplify $l_p = l$ and $\hbar = h$:

((((c*l)^2*h)))/(((sqrt(n+1/2)*c*l)))

Input

$$\frac{(c\,l)^2\,h}{\sqrt{n+\frac{1}{2}}\;c\,l}$$

Exact result

$$\frac{c\,h\,l}{\sqrt{n+\frac{1}{2}}}$$

Alternate form

$$\frac{\sqrt{2}\;c\,h\,l}{\sqrt{2n+1}}$$

Roots

$2n+1 \neq 0, \quad c = 0$

$c \neq 0, \quad 2n+1 \neq 0, \quad h = 0$

$$c\,h \neq 0, \quad 2n+1 \neq 0, \quad l = 0$$

Series expansion at n=0

$$\sqrt{2}\,c\,h\,l - \sqrt{2}\,c\,h\,l\,n + \frac{3\,c\,h\,l\,n^2}{\sqrt{2}} - \frac{5\,n^3\,(c\,h\,l)}{\sqrt{2}} + \frac{35\,c\,h\,l\,n^4}{4\sqrt{2}} + O\left(n^5\right)$$

(Taylor series)

Series expansion at n=∞

$$c\,h\,l\sqrt{\frac{1}{n}} - \frac{1}{4}\left(\frac{1}{n}\right)^{3/2}(c\,h\,l) + \frac{3}{32}\,c\,h\,l\left(\frac{1}{n}\right)^{5/2} - \frac{5}{128}\left(\frac{1}{n}\right)^{7/2}(c\,h\,l) + \frac{35\,c\,h\,l\left(\frac{1}{n}\right)^{9/2}}{2048} + O\left(\left(\frac{1}{n}\right)^5\right)$$

(Puiseux series)

Derivative

$$\frac{\partial}{\partial n}\left(\frac{(c\,l)^2\,h}{\sqrt{n+\frac{1}{2}}\,c\,l}\right) = -\frac{c\,h\,l}{2\left(n+\frac{1}{2}\right)^{3/2}}$$

Indefinite integral

$$\int \frac{c\,h\,l}{\sqrt{\frac{1}{2}+n}}\,dn = 2\,c\,h\,l\sqrt{n+\frac{1}{2}} + \text{constant}$$

Limit

$$\lim_{n\to\pm\infty} \frac{c\,h\,l}{\sqrt{\frac{1}{2}+n}} = 0$$

Series representations

$$\frac{(c\,l)^2\,h}{\sqrt{n+\frac{1}{2}}\;c\,l} = \frac{c\,h\,l}{\sqrt{-\frac{1}{2}+n}\;\sum_{k=0}^{\infty}\left(-\frac{1}{2}+n\right)^{-k}\binom{\frac{1}{2}}{k}} \quad \text{for} \quad \left|-\frac{1}{2}+n\right| > 1$$

$$\frac{(c\,l)^2\,h}{\sqrt{n+\frac{1}{2}}\;c\,l} = \frac{c\,h\,l}{\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}+n\right)^k\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for} \quad \left|-\frac{1}{2}+n\right| < 1$$

$$\frac{(c\,l)^2\,h}{\sqrt{n+\frac{1}{2}}\;c\,l} = \frac{c\,h\,l}{\sqrt{-\frac{1}{2}+n}\;\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}+n\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for} \quad \left|-\frac{1}{2}+n\right| > 1$$

$\binom{n}{m}$ is the binomial coefficient

$|z|$ is the absolute value of z

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

From the indefinite integral result

$$\int \frac{c\,h\,l}{\sqrt{\frac{1}{2}+n}}\,dn = 2\,c\,h\,l\sqrt{n+\frac{1}{2}} + \text{constant}$$

we obtain:

2 c h l sqrt(1/2 + n)

Input

$$2\,c\,h\,l\sqrt{\frac{1}{2}+n}$$

Alternate forms

$$c\,h\,l\sqrt{4\,n+2}$$

$$\sqrt{2}\;c\,h\,l\sqrt{2\,n+1}$$

Roots

$$c \neq 0, \quad h = 0$$

$$c\,h \neq 0, \quad l = 0$$

$$c\,h\,l \neq 0, \quad n = -\frac{1}{2}$$

Root

$$c = 0$$

Series expansion at n=0

$$\sqrt{2}\;c\,h\,l + \sqrt{2}\;c\,h\,l\,n - \frac{c\,h\,l\,n^2}{\sqrt{2}} + \frac{c\,h\,l\,n^3}{\sqrt{2}} - \frac{5\,n^4\,(c\,h\,l)}{4\,\sqrt{2}} + O\left(n^5\right)$$

(Taylor series)

Series expansion at n=∞

$$2\,c\,h\,l\sqrt{n} + \frac{1}{2}\,c\,h\,l\,\sqrt{\frac{1}{n}} - \frac{1}{16}\left(\frac{1}{n}\right)^{3/2}(c\,h\,l) + \frac{1}{64}\,c\,h\,l\left(\frac{1}{n}\right)^{5/2} + O\left(\left(\frac{1}{n}\right)^3\right)$$

(Puiseux series)

Derivative

$$\frac{\partial}{\partial n}\left(2\,c\,h\,l\sqrt{\frac{1}{2}+n}\right)=\frac{c\,h\,l}{\sqrt{n+\frac{1}{2}}}$$

Indefinite integral

$$\int 2\,c\,h\,l\sqrt{\frac{1}{2}+n}\;dn=\frac{1}{6}\,c\,h\,l\,(4\,n+2)^{3/2}+\text{constant}$$

Series representations

$$2\,c\,h\,l\sqrt{\frac{1}{2}+n}=2\,c\,h\,l\sqrt{-\frac{1}{2}+n}\sum_{k=0}^{\infty}\left(-\frac{1}{2}+n\right)^{-k}\binom{\frac{1}{2}}{k}\quad\text{for}\left|-\frac{1}{2}+n\right|>1$$

$$2\,c\,h\,l\sqrt{\frac{1}{2}+n}=2\,c\,h\,l\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}+n\right)^k\left(-\frac{1}{2}\right)_k}{k!}\quad\text{for}\left|-\frac{1}{2}+n\right|<1$$

$$2\,c\,h\,l\sqrt{\frac{1}{2}+n}=2\,c\,h\,l\sqrt{-\frac{1}{2}+n}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}+n\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}\quad\text{for}\left|-\frac{1}{2}+n\right|>1$$

$\binom{n}{m}$ is the binomial coefficient

$|z|$ is the absolute value of z

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

And again, from the indefinite integral result

$$\int 2\,c\,h\,l\sqrt{\frac{1}{2}+n}\;dn = \frac{1}{6}\,c\,h\,l\,(4\,n+2)^{3/2} + \text{constant}$$

we obtain:

1/6 c h l (2 + 4 n)^(3/2)

Input

$$\frac{1}{6}\,c\,h\,l\,(2+4\,n)^{3/2}$$

Expanded form

$$\frac{1}{3}\,c\,h\,l\,\sqrt{4\,n+2} + \frac{2}{3}\,c\,h\,l\,n\,\sqrt{4\,n+2}$$

Alternate form

$$\frac{1}{3}\,\sqrt{2}\;c\,h\,l\,(2\,n+1)^{3/2}$$

Roots

$c \neq 0, \quad h = 0$

$c\,h \neq 0, \quad l = 0$

$c\,h\,l \neq 0, \quad n = -\frac{1}{2}$

Root

$c = 0$

Series expansion at n=0

$$\frac{1}{3}\sqrt{2}\,chl + \sqrt{2}\,chln + \frac{chln^2}{\sqrt{2}} - \frac{n^3\,(chl)}{3\sqrt{2}} + \frac{chln^4}{4\sqrt{2}} + O(n^5)$$

(Taylor series)

Series expansion at n=∞

$$\frac{4}{3}\,chln^{3/2} + chl\sqrt{n} + \frac{1}{8}\,chl\sqrt{\frac{1}{n}} + O\left(\left(\frac{1}{n}\right)^1\right)$$

(Puiseux series)

Derivative

$$\frac{\partial}{\partial n}\left(\frac{1}{6}\,chl(2+4n)^{3/2}\right) = chl\sqrt{2+4n}$$

Indefinite integral

$$\int \frac{1}{6}\,chl(2+4n)^{3/2}\,dn = \frac{1}{60}\,chl(4n+2)^{5/2} + \text{constant}$$

Series representations

$$\frac{1}{6}\,chl(2+4n)^{3/2} = \sum_{\nu=0}^{\infty} \frac{1}{3}\,n^{\nu}\left(2^{1/2+\nu}\,chl\binom{\frac{3}{2}}{\nu}\right)$$

$$\frac{1}{6}\,chl(2+4n)^{3/2} = \sum_{\nu=0}^{\infty} \frac{(-1+n)^{\nu}\,(-1)^{\nu}\left(\left(\frac{3}{2}\right)^{3/2-\nu}\,chl\left(-\frac{5}{2}+\nu\right)!\right)}{\sqrt{\pi}\;\nu!}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

From the indefinite integral result

$$\int \frac{1}{6} c\,h\,l\,(2+4\,n)^{3/2}\,dn = \frac{1}{60} c\,h\,l\,(4\,n+2)^{5/2} + \text{constant}$$

1/60 c h l (2 + 4 n)^(5/2)

for c = 299792458, $h = \hbar$ = 1.054571817*10^{-34} and l = l_p = 1.616255*10^{-35} , we obtain:

1/60 299792458*1.054571817*10^-34*1.616255*10^-35* (2 + 4 n)^(5/2)

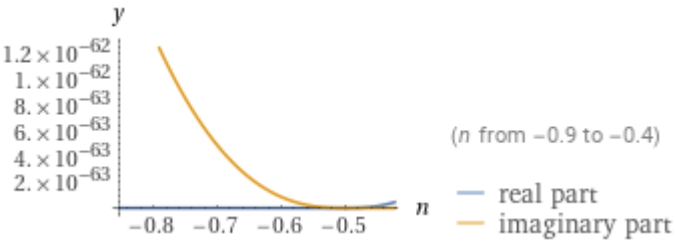
Input interpretation

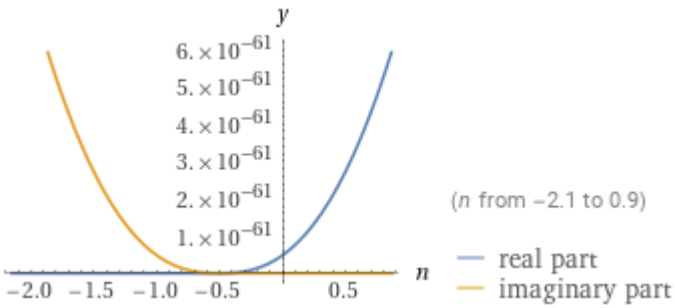
$$\frac{1}{60} \times 299\,792\,458 \times 1.054571817 \times 10^{-34} \times 1.616255 \times 10^{-35}\,(2+4\,n)^{5/2}$$

Result

0

Plots **(figures that can be related to the open strings)**





Values

n	1	2	3	4	5
$8.51639\times 10^{-63}(4n+2)^{5/2}$	7.50989×10^{-61}	2.69312×10^{-60}	6.24562×10^{-60}	1.17068×10^{-59}	1.93336×10^{-59}

Roots

$n \approx -0.5$

$n \approx -0.5$

Series expansion at n=-1/2

$$2.72524\times 10^{-61}\left(n+\frac{1}{2}\right)^{5/2}+O\left(\left(n+\frac{1}{2}\right)^{25/2}\right)$$

(Puiseux series)

Series expansion at n=0

$$4.8176\times 10^{-62}+2.4088\times 10^{-61}\,n+3.6132\times 10^{-61}\,n^2+1.2044\times 10^{-61}\,n^3-3.011\times 10^{-62}\,n^4+O\left(n^5\right)$$

(Taylor series)

Series expansion at n=∞

$$2.72524\times10^{-61}\,n^{5/2}+3.40656\times10^{-61}\,n^{3/2}+1.27746\times10^{-61}\,\sqrt{n}\;+O\left(\sqrt{\frac{1}{n}}\right)$$

(Puiseux series)

Derivative

$$\frac{d}{dn}\left(8.51639\times10^{-63}\,(4\,n+2)^{5/2}\right)=2.4088\times10^{-61}\,(2\,n+1)^{3/2}$$

Indefinite integral

$$\int 8.51639\times10^{-63}\,(2+4\,n)^{5/2}\,dn=6.88228\times10^{-63}\,(2\,n+1)^{7/2}+\text{constant}$$

Global minimum

$$\min\left\{8.51639\times10^{-63}\,(4\,n+2)^{5/2}\right\}=0\ \text{at}\ n=-\frac{1}{2}$$

From the indefinite integral result

$$\int 8.51639\times10^{-63}\,(2+4\,n)^{5/2}\,dn=6.88228\times10^{-63}\,(2\,n+1)^{7/2}+\text{constant}$$

6.88228×10^-63 (1 + 2 n)^(7/2)

for n = 2:

6.88228×10^-63 (1 + 2*2)^(7/2)

Input interpretation

$$6.88228\times10^{-63}\,(1+2\times2)^{7/2}$$

Result

$1.92366\ldots \times 10^{-60}$

1.92366…*10-60

Performing the ln , after some easy calculations, we obtain:

12*(-ln((6.88228×10^-63 (1 + 2*2)^(7/2)))+13*1/2)+1

Input interpretation

$$12\left(-\log\left(6.88228\times 10^{-63}\,(1+2\times 2)^{7/2}\right)+13\times\frac{1}{2}\right)+1$$

log(x) is the natural logarithm

Result

1729.0105…

1729.0105….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron.

As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(1/27(12*(-ln((6.88228×10^-63 (1 + 2*2)^(7/2)))+13*1/2)))^2

Input interpretation

$$\left(\frac{1}{27}\left(12\left(-\log\left(6.88228\times 10^{-63}\,(1+2\times 2)^{7/2}\right)+13\times\frac{1}{2}\right)\right)\right)^2$$

log(x) is the natural logarithm

Result

4096.0500…

4096.05…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

(12*(-ln((6.88228×10^-63 (1 + 2*2)^(7/2)))+13*1/2)+1)^1/15+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\sqrt[15]{12\left(-\log(6.88228\times 10^{-63}\,(1+2\times 2)^{7/2})+13\times\frac{1}{2}\right)+1}\;+{C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm

C_{MRB} is the MRB constant

Result

1.64493869205371637702514900498487703375159611025729962398776403 01
…

1.644938692…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Now, we try to analyze the following equation:

$$i\gamma^{\mu}D_{\mu} - \frac{e}{c}A_{\mu} + \frac{mc^2}{\hbar c} = 0.$$

We consider:

(i*γ^μ*D) – e/c*A + ((m*c^2)/(h*c))

Input

$$i\,\gamma^{\mu}\,D - \frac{e}{c}\,A + \frac{m\,c^2}{h\,c}$$

i is the imaginary unit

Exact result

$$-\frac{e\,A}{c} + \frac{c\,m}{h} + i\,D\,\gamma^{\mu}$$

Alternate forms

$$-\frac{e\,A\,h - c^2\,m - i\,c\,D\,h\,\gamma^{\mu}}{c\,h}$$

$$-\frac{e\,A\,h - c\,(c\,m + i\,D\,h\,\gamma^{\mu})}{c\,h}$$

$$-\frac{e\,A\,h - c^2\,m}{c\,h} + i\,D\,\gamma^{\mu}$$

Roots

$$c\,D\,h \neq 0\,, \quad i\,c\,D\,h \neq 0\,, \quad m = \frac{e\,A\,h - i\,c\,D\,h}{c^2}\,, \quad \gamma = 1$$

$$c \neq 0\,, \quad h \neq 0\,, \quad m = \frac{e\,A\,h}{c^2}\,, \quad \gamma = 0\,, \quad \mathrm{Re}(\mu) > 0$$

$$c \neq 0\,, \quad h \neq 0\,, \quad \gamma \neq 0\,, \quad D = 0\,, \quad m = \frac{e\,A\,h}{c^2}$$

$$c\,D\,h \neq 0\,, \quad e\,A\,h - c^2\,m \neq 0\,, \quad \gamma \neq 0\,,$$
$$\log(\gamma) \neq 0\,, \quad \mu = \frac{i\left(2\,\pi\,n - i\,\log\left(-\frac{i\,(e\,A\,h - c^2\,m)}{c\,D\,h}\right)\right)}{\log(\gamma)}\,, \quad n \in \mathbb{Z}$$

$\mathrm{Re}(z)$ is the real part of z

$\log(x)$ is the natural logarithm

$\mathbb{Z}$ is the set of integers

Root for the variable μ

$$\mu = \frac{\log\left(\frac{i\left(\frac{c\,m}{h} - \frac{e\,A}{c}\right)}{D}\right)}{\log(\gamma)}$$

Derivative

$$\frac{\partial}{\partial m}\left(i\,\gamma^{\mu}\,D-\frac{e\,A}{c}+\frac{m\,c^2}{h\,c}\right)=\frac{c}{h}$$

Indefinite integral

$$\int\left(-\frac{A\,e}{c}+\frac{c\,m}{h}+i\,D\,\gamma^{\mu}\right)dm=-\frac{e\,A\,m}{c}+\frac{c\,m^2}{2\,h}+i\,D\,m\,\gamma^{\mu}+\text{constant}$$

Alternative representation

$$i\,\gamma^{\mu}\,D-\frac{A\,e}{c}+\frac{m\,c^2}{h\,c}=i\,\gamma^{\mu}\,D-\frac{A\exp(z)}{c}+\frac{m\,c^2}{h\,c}\quad\text{for } z=1$$

Series representations

$$i\,\gamma^{\mu}\,D-\frac{A\,e}{c}+\frac{m\,c^2}{h\,c}=\frac{c\,m}{h}+i\,D\,\gamma^{\mu}-\frac{A\sum_{k=0}^{\infty}\frac{1}{k!}}{c}$$

$$i\,\gamma^{\mu}\,D-\frac{A\,e}{c}+\frac{m\,c^2}{h\,c}=\frac{c\,m}{h}+i\,D\,\gamma^{\mu}-\frac{A\sum_{k=0}^{\infty}\frac{-1+k+z}{k!}}{c\,z}$$

$$i\,\gamma^{\mu}\,D-\frac{A\,e}{c}+\frac{m\,c^2}{h\,c}=\frac{c\,m}{h}+i\,D\,\gamma^{\mu}-\frac{A\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}}{c}$$

$n!$ is the factorial function

From the indefinite integral result

$$\int\left(-\frac{A\,e}{c}+\frac{c\,m}{h}+i\,D\,\gamma^{\mu}\right)dm=-\frac{e\,A\,m}{c}+\frac{c\,m^2}{2\,h}+i\,D\,m\,\gamma^{\mu}+\text{constant}$$

we obtain:

-(A e m)/c + (c m^2)/(2 h) + i D m γ^μ

Input

$$-\frac{A\,e\,m}{c}+\frac{c\,m^2}{2\,h}+i\,D\,m\,\gamma^{\mu}$$

i is the imaginary unit

Alternate forms

$$\frac{1}{2}\,m\left(-\frac{2\,e\,A}{c}+\frac{c\,m}{h}+2\,i\,D\,\gamma^{\mu}\right)$$

$$-\frac{m\left(2\,e\,A\,h-c^2\,m-2\,i\,c\,D\,h\,\gamma^{\mu}\right)}{2\,c\,h}$$

$$-\frac{m\,(2\,e\,A\,h-c\,(c\,m+2\,i\,D\,h\,\gamma^{\mu}))}{2\,c\,h}$$

Roots

$$c\,h\neq 0,\quad m=0,\quad \text{Re}(\mu)>0$$

$$c\,h\neq 0,\quad \gamma\neq 0,\quad m=0$$

$$c\,D\,h\neq 0,\quad 2\,i\,c\,D\,h\neq 0,\quad m=\frac{2\,(e\,A\,h-i\,c\,D\,h)}{c^2},\quad \gamma=1$$

$$c \neq 0, \quad h \neq 0, \quad m = \frac{2\,e\,A\,h}{c^2}, \quad \gamma = 0, \quad \mathrm{Re}(\mu) > 0$$

$$c \neq 0, \quad h \neq 0, \quad \gamma \neq 0, \quad D = 0, \quad m = \frac{2\,e\,A\,h}{c^2}$$

Re(z) is the real part of z

Root for the variable μ

$$\mu = \frac{\log\left(\frac{i\left(\frac{c\,m^2}{h} - \frac{2\,e\,A\,m}{c}\right)}{2\,D\,m}\right)}{\log(\gamma)}$$

log(x) is the natural logarithm

Derivative

$$\frac{\partial}{\partial m}\left(-\frac{A\,e\,m}{c} + \frac{c\,m^2}{2\,h} + i\,D\,m\,\gamma^\mu\right) = -\frac{e\,A}{c} + \frac{c\,m}{h} + i\,D\,\gamma^\mu$$

Indefinite integral

$$\int\left(-\frac{A\,e\,m}{c} + \frac{c\,m^2}{2\,h} + i\,D\,m\,\gamma^\mu\right)dm = \frac{1}{6}\,m^2\left(-\frac{3\,e\,A}{c} + \frac{c\,m}{h} + 3\,i\,D\,\gamma^\mu\right) + \text{constant}$$

Alternative representation

$$-\frac{A\,e\,m}{c} + \frac{c\,m^2}{2\,h} + i\,D\,m\,\gamma^\mu = -\frac{A\exp(z)\,m}{c} + \frac{c\,m^2}{2\,h} + i\,D\,m\,\gamma^\mu \quad \text{for } z = 1$$

Series representations

$$-\frac{A\,e\,m}{c}+\frac{c\,m^2}{2\,h}+i\,D\,m\,\gamma^{\mu}=\frac{c\,m^2}{2\,h}+i\,D\,m\,\gamma^{\mu}-\frac{A\,m\sum_{k=0}^{\infty}\frac{1}{k!}}{c}$$

$$-\frac{A\,e\,m}{c}+\frac{c\,m^2}{2\,h}+i\,D\,m\,\gamma^{\mu}=\frac{c\,m^2}{2\,h}+i\,D\,m\,\gamma^{\mu}-\frac{A\,m\sum_{k=0}^{\infty}\frac{-1+k+z}{k!}}{c\,z}$$

$$-\frac{A\,e\,m}{c}+\frac{c\,m^2}{2\,h}+i\,D\,m\,\gamma^{\mu}=\frac{c\,m^2}{2\,h}+i\,D\,m\,\gamma^{\mu}-\frac{A\,m\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}}{c}$$

$n!$ is the factorial function

And again, from the indefinite integral result

$$\int\left(-\frac{A\,e\,m}{c}+\frac{c\,m^2}{2\,h}+i\,D\,m\,\gamma^{\mu}\right)dm=\frac{1}{6}\,m^2\left(-\frac{3\,e\,A}{c}+\frac{c\,m}{h}+3\,i\,D\,\gamma^{\mu}\right)+\text{constant}$$

1/6 m^2 (-(3 A e)/c + (c m)/h + 3 i D γ^μ)

for m = 125.35 GeV/c^2 , e = 1.602176634×10^{-19} C , c = 299792458, $h = \hbar$ = 1.054571817*10^{-34} , we obtain:

1/6 125.35^2 (-(3 A * 1.602176634*10^-19)/299792458 + (299792458*125.35)/(1.054571817*10^-34) + 3 i D γ^μ)

Input interpretation

$$\frac{1}{6}\times 125.35^2\left(-\frac{3\,A\times 1.602176634\times 10^{-19}}{299\,792\,458}+\frac{299\,792\,458\times 125.35}{1.054571817\times 10^{-34}}+3\,i\,D\,\gamma^{\mu}\right)$$

i is the imaginary unit

Result

$$2618.77\left(-1.60329\times10^{-27}\,A+3\,i\,D\,\gamma^{\mu}+3.56344\times10^{44}\right)$$

Expanded form

$$-4.19864\times10^{-24}\,A+(7856.31\,i)\,D\,\gamma^{\mu}+$$
$$933\,181\,899\,984\,584\,511\,139\,970\,690\,003\,195\,146\,508\,739\,543\,040$$

Alternate form

$$-\frac{6\,285\,049}{800}\left(5.34429\times10^{-28}\,A-i\left(D\,\gamma^{\mu}-1.18781\times10^{44}\,i\right)\right)$$

Roots

$$A\approx2.22258\times10^{71},\quad \gamma=0,\quad \mathrm{Re}(\mu)>0$$

$$\gamma\neq0,\quad A\approx2.22258\times10^{71},\quad D=0$$

$$(-800\,i)\left(A-2.22258\times10^{71}\right)\neq0,\quad A-2.22258\times10^{71}\neq0,$$
$$D\approx\left(-5.34429\times10^{-28}\,i\right)\left(A-2.22258\times10^{71}\right),\quad \gamma\approx1$$

$$D\neq0,\quad A-2.22258\times10^{71}\neq0,\quad \gamma\neq0,\quad \log(\gamma)\neq0,$$
$$\mu\approx\frac{i\left(6.28319\,n+-i\log\left(-\frac{\left(5.34429\times10^{-28}\,i\right)\left(A-2.22258\times10^{71}\right)}{D}\right)\right)}{\log(\gamma)},\quad n\in\mathbb{Z}$$

$\mathrm{Re}(z)$ is the real part of z

$\log(x)$ is the natural logarithm

$\mathbb{Z}$ is the set of integers

Root for the variable μ

$$\mu \approx \frac{1}{\log(\gamma)} \log\left(\frac{1}{D} 0.0001272861993597823\, i \left(933\,181\,899\,984\,584\,511\,139\,970\,690\,003\,195\,146\,508\,739\,543\,040 - 4.198637416749600 \times 10^{-24}\, A\right)\right)$$

Derivative

$$\frac{\partial}{\partial A}\left(2618.77\left(-1.60329 \times 10^{-27}\, A + 3\, i\, D\, \gamma^{\mu} + 356\,343\,532\,081\,134\,637\,896\,596\,252\,312\,549\,050\,686\,636\,032\right)\right) = -4.19864 \times 10^{-24}$$

Indefinite integral

$$\int 2618.77\left(3.56344 \times 10^{44} - 1.60329 \times 10^{-27}\, A + 3\, i\, D\, \gamma^{\mu}\right) dA = -2.09932 \times 10^{-24}\, A^2 + (7856.31\, i)\, A\, D\, \gamma^{\mu} + 9.33182 \times 10^{47}\, A + \text{constant}$$

From the alternate form

$$-\frac{6\,285\,049}{800}\left(5.34429 \times 10^{-28}\, A - i\left(D\, \gamma^{\mu} - 1.18781 \times 10^{44}\, i\right)\right)$$

for A = 2.22258*10^71:

-6285049/800 (5.34429×10^-28 (2.22258*10^71) - i (-1.18781×10^44 i + D γ^μ))

Input interpretation

$$-\frac{6\,285\,049}{800}\left(5.34429 \times 10^{-28} \times 2.22258 \times 10^{71} - i\left(-1.18781 \times 10^{44}\, i + D\, \gamma^{\mu}\right)\right)$$

i is the imaginary unit

Result

$$-\frac{6285049}{800}\left(1.18781\times10^{44}-i\left(D\,\gamma^{\mu}-1.18781\times10^{44}\,i\right)\right)$$

Alternate forms

$$-9.48115\times10^{41}+(7856.31\,i)\,D\,\gamma^{\mu}$$

$$-9.48115\times10^{41}+\frac{6285049}{800}\,i\,D\,\gamma^{\mu}$$

$$\frac{6285049}{800}\,i\left(D\,\gamma^{\mu}+1.20682\times10^{38}\,i\right)$$

Expanded form

$$-948\,115\,354\,289\,857\,096\,054\,855\,849\,986\,018\,139\,701\,248+\frac{6285049}{800}\,i\,D\,\gamma^{\mu}$$

Alternate forms assuming D, γ, and μ are positive

$$-9.48115\times10^{41}+(7856.31\,i)\,D\,\gamma^{\mu}$$

$$-9.33181\times10^{47}+\frac{6285049}{800}\,i\left(D\,\gamma^{\mu}-1.18781\times10^{44}\,i\right)$$

Root

$$D\neq0,\quad \gamma\neq0,\quad \log(\gamma)\neq0,\quad \mu\approx\frac{i\left(6.28319\,n+-i\log\left(-\frac{1.20682\times10^{38}\,i}{D}\right)\right)}{\log(\gamma)},\quad n\in\mathbb{Z}$$

$\log(x)$ is the natural logarithm

$\mathbb{Z}$ is the set of integers

Complex roots

$$D = -120\,682\,000\,002\,209\,310\,141\,503\,711\,372\,029\,984\,768\,i$$

$$\gamma = 1$$

Root for the variable μ

$$\mu = \frac{\log\left(-\frac{120\,682\,000\,002\,209\,322\,700\,593\,734\,566\,900\,334\,637\,i}{D}\right)}{\log(\gamma)}$$

Derivative

$$\frac{\partial}{\partial D}\left(-\frac{6\,285\,049}{800}\left(118\,781\,120\,681\,999\,998\,078\,549\,435\,801\,463\,518\,589\,353\,984 - i\left(D\,\gamma^{\mu} - 118\,780\,999\,999\,999\,995\,869\,220\,404\,831\,820\,667\,978\,514\,432\,i\right)\right)\right) = \frac{6\,285\,049\,i\,\gamma^{\mu}}{800}$$

Indefinite integral

$$\int -\frac{6\,285\,049}{800}\left(1.18781\times 10^{44} - i\left(-1.18781\times 10^{44}\,i + D\,\gamma^{\mu}\right)\right) dD = -9.48115\times 10^{41}\,D + (3928.16\,i)\,D^2\,\gamma^{\mu} + \text{constant}$$

From the alternate form

$$-9.33181\times 10^{47} + \frac{6\,285\,049}{800}\,i\left(D\,\gamma^{\mu} - 1.18781\times 10^{44}\,i\right)$$

for D = -1.2068200000220931 × 10^38 i and γ = 1:

-9.33181×10^47 + 6285049/800 i (-1.18781×10^44 i - 1.2068200000220931 × 10^38 i)

Input interpretation

$$-9.33181 \times 10^{47} + \frac{6\,285\,049}{800}\, i \left(-1.18781 \times 10^{44}\, i - 1.2068200000220931 \times 10^{38}\, i\right)$$

i is the imaginary unit

Result

454 701 604 289 857 027 007 737 500 000 000 000 000 000

Scientific notation

$4.547016042898570270077375 \times 10^{41}$

$4.5470160428\ldots * 10^{41}$

From which, we obtain, after some calculations:

18*ln((-9.33181×10^47 + 6285049/800 i (-1.18781×10^44 i - 1.2068200000220931 × 10^38 i)))+e-MRB Const

Input interpretation

$$18 \log\left(-9.33181 \times 10^{47} + \frac{6\,285\,049}{800}\, i \left(-1.18781 \times 10^{44}\, i - 1.2068200000220931 \times 10^{38}\, i\right)\right) + e - C_{\text{MRB}}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

C_{MRB} is the MRB constant

Result

1729.0987024709479013436423814877548751525931016258001601161708727

…

1729.09870247….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2.

Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(18*ln((-9.33181×10^47 + 6285049/800 i (-1.18781×10^44 i - 1.2068200000220931 × 10^38 i)))+e-MRB Const)^1/15+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$\left(18 \log\left(-9.33181 \times 10^{47} + \frac{6285049}{800} i \left(-1.18781 \times 10^{44} i - 1.2068200000220931 \times 10^{38} i\right)\right) + e - C_{\text{MRB}}\right)\text{^}(1/15) + {C_{\text{MRB}}}^{1-1/(4\pi)+\pi}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

C_{MRB} is the MRB constant

Result

1.64494427993525965160260507888558978181392846922841577829567236 60…

1.6449442799…. ≈ $\zeta(2) = \pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

(1/27((18*ln((-9.33181×10^47 + 6285049/800 i (-1.18781×10^44 i - 1.2068200000220931 × 10^38 i)))+e-MRB Const)-1))^2-1/2

Input interpretation

$$\left(\frac{1}{27}\left(\left(18\log\left(-9.33181\times10^{47}+\frac{6285049}{800}\,i\left(-1.18781\times10^{44}\,i-1.2068200000220931\times10^{38}\,i\right)\right)+e-C_{\text{MRB}}\right)-1\right)\right)^2-\frac{1}{2}$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

C_{MRB} is the MRB constant

Result

4095.967936188990011337361915750805452567112893516098627123853410 6

…

4095.96793618899…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality. The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry.

Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

From

The Geometry of the Universe: in search of unity. New possible mathematical connections with the DN Constant, Ramanujan's Recurring Numbers and some parameters of Number Theory and String Theory – *Michele Nardelli* - August 2024 - DOI: 10.13140/RG.2.2.27912.84481

DN Constant

What is DN Constant (Del Gaudio-Nardelli Constant)?

The ratio of the volume of an octahedron to the volume of a sphere

$$V_o = \frac{1}{3} \cdot \sqrt{2} \cdot a^3 \; ; \quad V_s = \frac{4}{3} \cdot \pi \cdot \left(\frac{a}{2}\right)^3$$

is given by the following formula:

$$\frac{V_o}{V_s} = \frac{\frac{1}{3} \cdot \sqrt{2} \cdot a^3}{\frac{4}{3} \cdot \pi \cdot \left(\frac{a}{2}\right)^3} = \frac{2\sqrt{2}}{\pi} = 0.9003163161571 \ldots = DN\ Constant$$

Multiplying the ratio described above by 3, and taking the square root, we obtain:

$$\sqrt{3\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)} = 1.64345640297\ldots \approx \zeta(2) = \frac{\pi^2}{6}$$

Indeed:

$$\sqrt{3\times\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}$$

Exact result

$$2^{3/4}\sqrt{\frac{3}{\pi}}$$

Decimal approximation

1.6434564029725…≈ $\zeta(2) = \pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$2^{3/4}\sqrt{\frac{3}{\pi}}$ is a transcendental number

Series representations

$$\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}} = \sqrt{-1+\frac{6\sqrt{2}}{\pi}}\ \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+\frac{6\sqrt{2}}{\pi}\right)^{-k}$$

$$\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}} = \sqrt{-1+\frac{6\sqrt{2}}{\pi}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+\frac{6\sqrt{2}}{\pi}\right)^{-k}}{k!}$$

$$\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}} = \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{6\sqrt{2}}{\pi}-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not}\ (z_0 \in \mathbb{R}\ \text{and}\ -\infty < z_0 \leq 0))$

Multiplying the above expression by 6, and taking the square root, we obtain:

$$\sqrt{6\sqrt{3\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\cdot\pi\cdot\left(\frac{a}{2}\right)^3}\right)}} = 3.14018127149294\ldots \approx \pi$$

Indeed:

$$\sqrt{6\sqrt{3\times\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}}$$

Exact result

$$\frac{2^{7/8}\times 3^{3/4}}{\sqrt[4]{\pi}}$$

Decimal approximation

3.14018127149294...≈ π (Ramanujan Recurring Number)

Property

$\frac{2^{7/8}\times 3^{3/4}}{\sqrt[4]{\pi}}$ is a transcendental number

Series representations

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}}=\frac{2^{3/8}\times 3^{3/4}}{\sqrt[4]{\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2\,k}}}$$

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}}=\frac{2^{7/8}\times 3^{3/4}}{\sqrt[4]{\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2\,k}+\frac{2}{1+4\,k}+\frac{1}{3+4\,k}\right)}}$$

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}}=\frac{2^{3/8}\times 3^{3/4}}{\sqrt[4]{\sum_{k=0}^{\infty}\frac{(-1)^{1+k}\,1195^{-1-2\,k}\left(5^{1+2\,k}-4\times 239^{1+2\,k}\right)}{1+2\,k}}}$$

Integral representations

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}} = \frac{2^{3/8}\times 3^{3/4}}{\sqrt[4]{\int_0^1 \sqrt{1-t^2}\;dt}}$$

$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}} = \frac{2^{5/8}\times 3^{3/4}}{\sqrt[4]{\int_0^\infty \frac{1}{1+t^2}\;dt}}$$

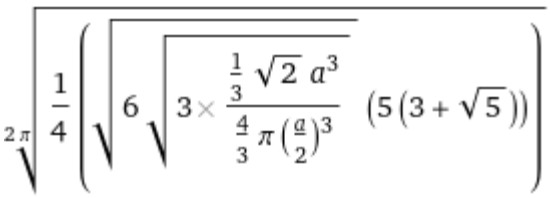
$$\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\,a^3\right)}{\frac{3}{3}\left(4\,\pi\left(\frac{a}{2}\right)^3\right)}}} = \frac{2^{5/8}\times 3^{3/4}}{\sqrt[4]{\int_0^1 \frac{1}{\sqrt{1-t^2}}\;dt}}$$

Finally carrying out the following calculations, which include the above expression, we obtain:

$$\sqrt[2\pi]{\frac{1}{4}\left(\sqrt{6\sqrt{3\left(\frac{\frac{1}{3}\cdot\sqrt{2}\cdot a^3}{\frac{4}{3}\pi\cdot\left(\frac{a}{2}\right)^3}\right)}\left(5\left(3+\sqrt{5}\right)\right)}\right)} = 1.61789283408194\ldots \approx \phi = \frac{\sqrt{5}+1}{2} = 1.61803398\ldots$$

practically, a value very close to the Golden Ratio.

Indeed:



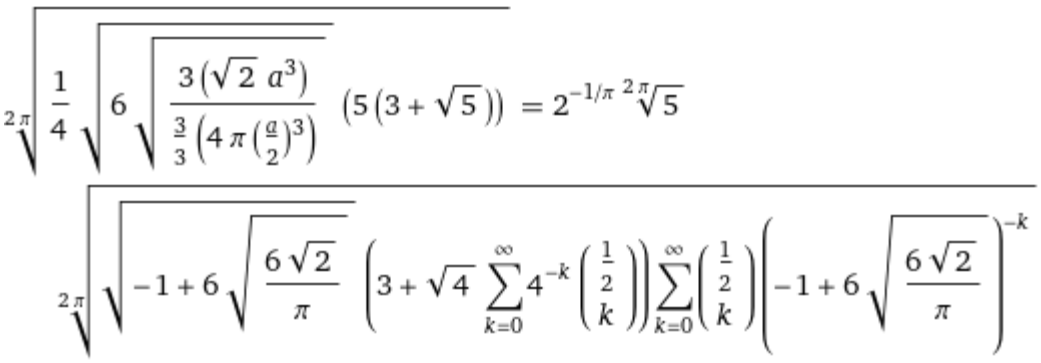
Exact result

$$2^{-9/(16\pi)} \times 3^{3/(8\pi)} \sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\ \pi^{-1/(8\pi)}$$

Decimal approximation

1.61789283408194…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Series representations



$$\sqrt[2\pi]{\frac{1}{4}\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\ a^3\right)}{\frac{3}{3}\left(4\pi\left(\frac{a}{2}\right)^3\right)}}}}\ \left(5\left(3+\sqrt{5}\right)\right) =$$

$$2^{-1/\pi\ \sqrt[2\pi]{5}}\left(\sqrt{-1+6\sqrt{\frac{6\sqrt{2}}{\pi}}}\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+6\sqrt{\frac{6\sqrt{2}}{\pi}}\right)^{-k}}{k!}\right)\wedge\left(\frac{1}{2\pi}\right)$$

$$\sqrt[2\pi]{\frac{1}{4}\sqrt{6\sqrt{\frac{3\left(\sqrt{2}\ a^3\right)}{\frac{3}{3}\left(4\pi\left(\frac{a}{2}\right)^3\right)}}}}\ \left(5\left(3+\sqrt{5}\right)\right) =$$

$$2^{-1/\pi\ \sqrt[2\pi]{5}}\left(\sqrt{z_0}\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\ z_0^{-k}}{k!}\right)\right.$$

$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(6\sqrt{\frac{6\sqrt{2}}{\pi}}-z_0\right)^k z_0^{-k}}{k!}\right)\wedge\left(\frac{1}{2\pi}\right)$$

for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for}\ (0<\gamma<-\mathrm{Re}(a)\ \text{and}\ |\arg(z)|<\pi)$$

These results are fascinating and connect the octahedron, the sphere and the constant (phi), also known as the Golden Ratio.

The relationship between the volume of an octahedron and the volume of a sphere is an interesting mathematical topic. Let's see how it is calculated:

Volume of the Octahedron (V_0):

The octahedron is a polyhedron with 8 triangular faces. If (a) represents the length of one side of the octahedron, then its volume is given by:

$$[\ V_o = \frac{1}{3} \cdot \sqrt{2} \cdot a^3\ ]$$

Volume of the Sphere (V_s): The sphere has a radius (r = a/2). Its volume is given by:

$$[\ V_s = \frac{4}{3} \cdot \pi \cdot \left(\frac{a}{2}\right)^3\ ]$$

Volume ratio: the volume ratio is therefore:

$$\frac{\frac{1}{3} \cdot \sqrt{2} \cdot a^3}{\frac{4}{3} \cdot \pi \cdot \left(\frac{a}{2}\right)^3} = \frac{2\sqrt{2}}{\pi}$$

This ratio is known as the **Del Gaudio-Nardelli (DN) constant**. Its approximate value is (0.9003163161571). From this formula, we can obtain easily also 4096, where $4096 = 64^2 = 8^4 = 2^{12}$, that is a fundamental Ramanujan number and appears in the fundamental work of Srinivasa Ramanujan: "Modular Equations and Approximations to Pi". We obtain also 1729, the so-called Hardy-Ramanujan number, that is a Taxicab Number. Ramanujan said: "it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways.". Indeed, $1729 = 1^3 + 12^3 = 10^3 + 9^3$. These numbers are also connected to some parameters of String Theory [1] [2]

Extended DN Constant

We have the following expression concerning the ratios (and/or inverses) between the volumes of the icosahedron, octahedron, tetrahedron and the volume of the sphere:

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(in the formula we have highlighted the DN Constant in blue)

The exact result of the above formula is:

$$2^{-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi} = 1.618008545900107\ldots$$

Indeed:

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.6180085459… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)3}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}{\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2}}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

Furthermore, from the above exact result

$$\mathbf{2^{-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi}}$$

we obtain also:

$$\frac{4\times 1.6180085459001 07^{2\pi}}{5(3+\sqrt{5})} = 3.141592653589\ldots = \pi$$

Indeed:

$$\frac{4\times 1.6180085459001 07^{2\pi}}{5(3+\sqrt{5})}$$

Result

3.14159265358979…

3.14159265358979…= π (Ramanujan Recurring Number)

Series representations

$$\frac{4\times 1.6180085459001070000^{2\pi}}{5(3+\sqrt{5})} = \frac{4\, e^{0.9623922007800638614\pi}}{5\left(3+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{4\times 1.6180085459001070000^{2\pi}}{5(3+\sqrt{5})} = \frac{4\, e^{0.9623922007800638614\pi}}{5\left(3+\sqrt{4}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\frac{4\times 1.61800854590010700002^{2\pi}}{5\left(3+\sqrt{5}\right)}=\frac{8\,e^{0.9623922007800638614\,\pi}\sqrt{\pi}}{5\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)}$$

and also, obviously:

$$\frac{1}{6}\left(\frac{4\times 1.6180085459001072^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=1.64493406684823\ldots\approx\zeta(2)=\frac{\pi^2}{6}$$

Indeed:

$$\frac{1}{6}\left(\frac{4\times 1.6180085459001072^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2$$

Result

1.64493406684823……≈ $\zeta(2) = \pi^2/6 = 1.644934$ (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$\frac{1}{6}\left(\frac{4\times 1.61800854590010700002^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.9247844015601277229\,\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.61800854590010700002^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2=\frac{8\,e^{1.9247844015601277229\,\pi}}{75\left(3+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{1}{6}\left(\frac{4\times 1.6180085459001070000^{2\pi}}{5\left(3+\sqrt{5}\right)}\right)^2 = \frac{32\,e^{1.9247844015601277229\,\pi}\,\sqrt{\pi}^{\,2}}{75\left(6\sqrt{\pi}+\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)\right)^2}$$

The expression regarding the relationships between the volumes of the icosahedron, the octahedron, the tetrahedron and the volume of the sphere is fascinating! Let's break it down step by step:

1. **Icosahedron, Octahedron and tetrahedron**:

The icosahedron has a surface made up of 20 regular triangular faces. Its volume is given by the formula: [$V = \frac{5}{12}\left(3+\sqrt{5}\right)d^3$]

The octahedron has a surface composed of 8 regular triangular faces. Its volume is given by the formula: [$V = \frac{1}{3}\sqrt{2}a^3$]

A regular tetrahedron is a tetrahedron in which all four faces are equilateral triangles. Its volume is given by the formula: [$V = \sqrt{2}/12\ d^3$]

2. **Ratio between volumes**:

The ratio between the volumes is given by the following expression:

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

3. **Del Gaudio-Nardelli Constant (DN)**:

The DN constant is highlighted in blue in the formula. The exact result of this expression is:

$$[2^{-1/\pi}\sqrt[2\pi]{5(3+\sqrt{5})\pi} = 1.6180085459001 07]$$

from which, from the previous formula, we obtain a good approximation to π

These results link the geometry of the Platonic solids with the golden constant (ϕ) and the constant (π).

Proposal

Taking into account that a sphere is contained in an octahedron and the above results, it is hypothesized, in an "Eternal Inflation" type cosmology, that the octahedron represents a phase in which the Universe is highly symmetrical and with very low entropy, and the sphere represents the bubble Universe that emerges from the perturbations of the quantum vacuum. The relationship between the two volumes is, as we have described, equal to the DN Constant, which therefore plays a key role in the geometry of the Universe, a sort of "bridge" between geometry and physics, even before the phase of the so-called Big Bang.

The idea that the octahedron and the sphere could represent different phases of the Universe in an “Eternal Inflation” cosmology is fascinating.

Let's further explore this connection between geometry and physics:

1. The Octahedron and the Symmetrical Universe:

The octahedron, with its 8 triangular faces, could represent an initial phase of the Universe characterized by very high symmetry and very low entropy.

During this phase, the Universe may be highly ordered and uniform, with regular geometric structures.

2. The Sphere and the Bubble Universe:

The sphere, contained in the octahedron, could represent a transition towards a different phase.

This sphere could symbolize the bubble Universe that emerges from the perturbations of the quantum vacuum.

Quantum fluctuations may have given rise to regions of different densities, creating expanding bubbles of space-time.

3. The Relationship between Volumes and DN Constant:

The ratio between the volume of the octahedron and the volume of the sphere is equal to the Del Gaudio-Nardelli Constant (DN).

This constant, which we calculated previously, could play a key role in the geometry of the Universe.

It could be a “bridge” between the geometric structure and the physical laws that govern the Universe.

4. Before the Big Bang:

The idea that the DN Constant is relevant even before the Big Bang phase suggests that geometry and physics are closely intertwined.

Perhaps symmetry and geometric regularity played a fundamental role in the early phases of the Universe.

Ultimately, this connection between geometric solids, mathematical constants and cosmology invites us to further explore the mysteries of the Universe and seek a deeper integration between geometry and physics.

But let's see what the cosmological implications could be in an "eternal inflation" type scenario of these formulas, results and, above all, of the DN Constant (Del Gaudio-Nardelli Constant).

Eternal inflation is a model of cosmological inflation predicted by some extensions of the Big Bang theory and the standard model of cosmology.

Let's explore the implications of this model, along with the formulas and results associated with the Del Gaudio-Nardelli Constant (DN):

1. Eternal Inflation and Multiverse:

Eternal inflation suggests that the accelerated expansion of the universe due to inflation continues forever, at least in some regions.

These regions expand at exponential rates, leading to the indefinite increase in the volume of the universe.

This model predicts the existence of multiple universes, often called a multiverse. Each “bubble” of space-time could represent a separate universe.

2. Bubble Theory:

The bubble theory is part of the multiverse elaborations.

According to this theory, our universe is just one of infinite “bubbles” that emerge from the quantum foam of a “parent universe” or from a single Big Bang.

These bubbles expand at different rates and may have different physical constants.

3. Connection with DN Constant:

The DN Constant (Del Gaudio-Nardelli Constant) is present in the formulas that connect the volumes of geometric solids such as the octahedron and the sphere.

This constant could play a key role in the geometry of the Universe.

It could be a “bridge” between the geometric structure and the physical laws that govern the Universe.

4. Experimental checks:

Some experimental evidence could transform the inflationary universe hypothesis into a verified theory.

If theoretical calculations based on DN Constant and bubble theory were experimentally confirmed, this would support the idea of an eternal multiverse.

In summary, eternal inflation and DN Constant open new perspectives on the connection between geometry, physics and the origin of the Universe. These concepts challenge us to further explore cosmological mysteries and better understand our reality.

We will explore the image of the Universe in relation to the concepts of eternal inflation, bubble theory and the Del Gaudio-Nardelli Constant (DN). Let's imagine the DN as a thread that connects the different bubbles of the multiverse.

1. Image of the Universe:

We visualize a vast space in which each bubble represents a universe.

Each bubble has different geometry, different physical constants and different laws.

These bubbles expand, collide and overlap in the great cosmic landscape.

In summary, the image of the Universe in a context of eternal inflation is that of a multiversal fabric, with bubbles of reality that form, expand and intertwine infinitely. This concept challenges us to explore the depths of the Universe and understand its complexity.

Fig.1

A representation of the universe incorporating the concepts of eternal inflation, octahedron, sphere, and the Del Gaudio-Nardelli (DN) Constant.

In the image, the different bubbles represent separate universes, each with its own geometry and physical laws. The DN Constant connects these different realities, creating a mathematical tapestry in the vast cosmic landscape

We have the following extended DN Constant:

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

and the following Cardano formula:

$$y=\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

Multiplying both the above formulas, we obtain:

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(\sqrt[3]{-\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}+\sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}\right)$$

Alternate form

$$-\frac{1}{\sqrt{3}}2^{-1/3-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}\left(\sqrt[3]{3\sqrt{3}\,q-\sqrt{4\,p^3+27q^2}}+\sqrt[3]{\sqrt{4\,p^3+27q^2}+3\sqrt{3}\,q}\right)$$

Expanded forms

$$\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\sqrt[3]{\sqrt{3}\,\sqrt{4\,p^3+27q^2}-9q}}{3^{2/3}}-\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\sqrt[3]{\sqrt{3}\,\sqrt{4\,p^3+27q^2}+9q}}{3^{2/3}}$$

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\sqrt[3]{-\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}+2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}$$

Alternate forms assuming p and q are positive

$$\frac{2^{-(3+\pi)/(3\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(\sqrt[3]{\sqrt{12\,p^3+81q^2}-9q}-\sqrt[3]{\sqrt{12\,p^3+81q^2}+9q}\right)}{3^{2/3}}$$

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}-2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}+\frac{q}{2}}$$

Derivative

$$\frac{\partial}{\partial p}\left(\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\left(\sqrt{2}\ a^3\right)\left(\sqrt{2}\ d^3\right)}{\frac{3\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)}{3\times 3}}}}\left(\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}\right)\right)=$$

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(\frac{p^2}{54\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}\left(\sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}\right)^2}-\frac{p^2}{54\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}\left(\sqrt[3]{-\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}\right)^2}\right)$$

From the extended form

$$\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}}\ \pi\sqrt[3]{\sqrt{3}\ \sqrt{4\,p^3+27\,q^2}\ -9\,q}}{3^{2/3}}-\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}}\ \pi\sqrt[3]{\sqrt{3}\ \sqrt{4\,p^3+27\,q^2}\ +9\,q}}{3^{2/3}}$$

we obtain:

$$\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}}\ \pi\sqrt[3]{-9\,q+\sqrt{3}\ \sqrt{4\,p^3+27\,q^2}}}{3^{2/3}}-\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}}\ \pi\sqrt[3]{9\,q+\sqrt{3}\ \sqrt{4\,p^3+27\,q^2}}}{3^{2/3}}$$

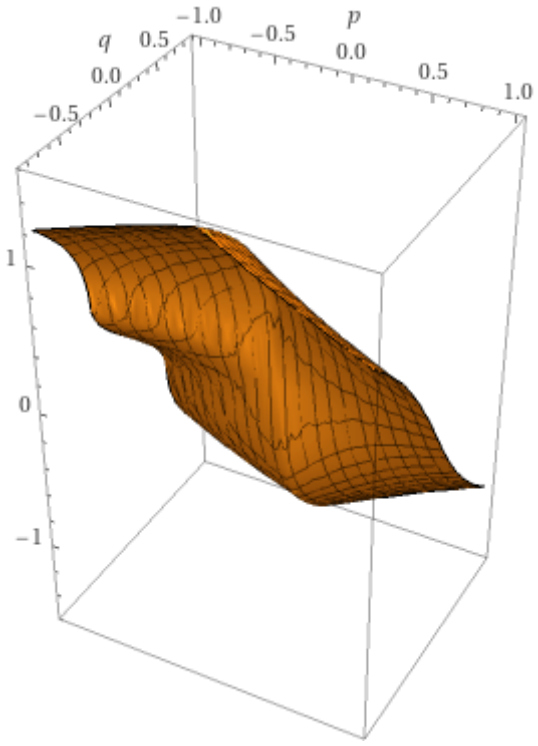
Exact result

$$\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}}\ \pi\sqrt[3]{\sqrt{3}\ \sqrt{4\,p^3+27\,q^2}\ -9\,q}}{3^{2/3}}-\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}}\ \pi\sqrt[3]{\sqrt{3}\ \sqrt{4\,p^3+27\,q^2}\ +9\,q}}{3^{2/3}}$$

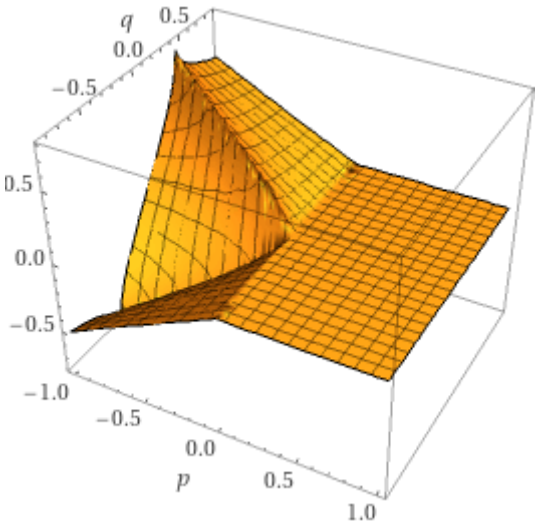
3D plots

Real part **(figures that can be related to the D-branes/Instantons)**

The key observation from the above plots and is that at $p = 1$, which is taken as the energy density of the universe at the Big Bang, with $q = 0$ the zero spacetime volume, the vacuum geometry brakes / or there is symmetry breaking on the vacuum quantum geometry. We see from the plots as the vacuum spacetime break/tear apart.

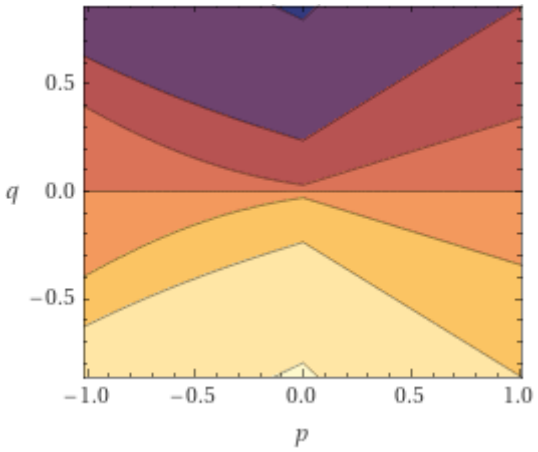


Imaginary part

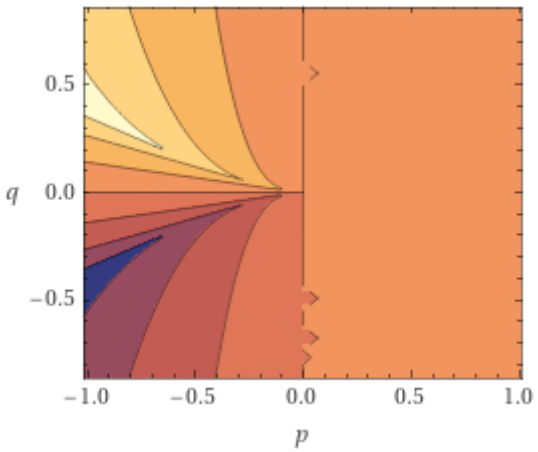


Contour plots

Real part



Imaginary part



Alternate forms

$$\frac{2^{-(3+\pi)/(3\pi)}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(\sqrt[3]{\sqrt{12\,p^3+81\,q^2}-9\,q}-\sqrt[3]{\sqrt{12\,p^3+81\,q^2}+9\,q}\right)}{3^{2/3}}$$

$$\frac{1}{3^{2/3}}2^{-1/3-1/\pi}\sqrt[2\pi]{15\,\pi+5\sqrt{5}\,\pi}\left(\sqrt[3]{\sqrt{3}\,\sqrt{4\,p^3+27\,q^2}-9\,q}-\sqrt[3]{\sqrt{3}\,\sqrt{4\,p^3+27\,q^2}+9\,q}\right)$$

Real root

$p \geq 0\,,\quad q = 0$

Root for the variable q

$q = 0$

Series expansion at p=∞

$$-\frac{2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ q}{p}+\frac{2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ q^3}{p^4}+O\left(\left(\frac{1}{p}\right)^{9/2}\right)$$

(Laurent series)

Derivative

$$\frac{\partial}{\partial p}\left(\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\ \pi}\ \sqrt[3]{-9q+\sqrt{3}\ \sqrt{4\,p^3+27q^2}}}{3^{2/3}}-\right.$$

$$\left.\frac{2^{-1/3-1/\pi}\sqrt[2\pi]{15\pi+5\sqrt{5}\ \pi}\ \sqrt[3]{9q+\sqrt{3}\ \sqrt{4\,p^3+27q^2}}}{3^{2/3}}\right)=$$

$$-\left(\left(2^{2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ p^2\right.\right.$$

$$\left.\left(\left(\sqrt{12\,p^3+81\,q^2}-9q\right)^{2/3}-\left(\sqrt{12\,p^3+81\,q^2}+9q\right)^{2/3}\right)\right)\Big/$$

$$\left(\sqrt[6]{3}\ \left(\sqrt{12\,p^3+81\,q^2}-9q\right)^{2/3}\left(\sqrt{12\,p^3+81\,q^2}+9q\right)^{2/3}\sqrt{4\,p^3+27\,q^2}\right)\Big)$$

From the derivative result

$$\frac{\partial}{\partial p}\left(\frac{2^{-1/3-1/\pi}\,\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{-9q+\sqrt{3}\,\sqrt{4\,p^3+27q^2}}}{3^{2/3}}-\frac{2^{-1/3-1/\pi}\,\sqrt[2\pi]{15\pi+5\sqrt{5}\,\pi}\,\sqrt[3]{9q+\sqrt{3}\,\sqrt{4\,p^3+27q^2}}}{3^{2/3}}\right)=$$

$$-\left(\left(2^{2/3-1/\pi}\,\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\;p^2\left(\left(\sqrt{12\,p^3+81\,q^2}-9q\right)^{2/3}-\left(\sqrt{12\,p^3+81\,q^2}+9q\right)^{2/3}\right)\right)\Big/\left(\sqrt[6]{3}\left(\sqrt{12\,p^3+81\,q^2}-9q\right)^{2/3}\left(\sqrt{12\,p^3+81\,q^2}+9q\right)^{2/3}\sqrt{4\,p^3+27q^2}\right)\right)$$

we obtain:

$$-\left(\left(2^{2/3-1/\pi}\left(\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\;p^2\right)\left(\left(\sqrt{12\,p^3+81\,q^2}-9q\right)^{2/3}-\left(\sqrt{12\,p^3+81\,q^2}+9q\right)^{2/3}\right)\right)\Big/\left(\sqrt[6]{3}\left(\sqrt{12\,p^3+81\,q^2}-9q\right)^{2/3}\left(\left(\sqrt{12\,p^3+81\,q^2}+9q\right)^{2/3}\sqrt{4\,p^3+27q^2}\right)\right)\right)$$

Exact result

$$-\left(\left(2^{2/3-1/\pi}\,\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\;p^2\left(\left(\sqrt{12\,p^3+81\,q^2}-9q\right)^{2/3}-\left(\sqrt{12\,p^3+81\,q^2}+9q\right)^{2/3}\right)\right)\Big/\left(\sqrt[6]{3}\,\sqrt{4\,p^3+27q^2}\left(\sqrt{12\,p^3+81\,q^2}-9q\right)^{2/3}\left(\sqrt{12\,p^3+81\,q^2}+9q\right)^{2/3}\right)\right)$$

Expanded form

$$\frac{2^{2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ p^2}{\sqrt[6]{3}\ \sqrt{4\,p^3+27q^2}\ \left(\sqrt{12\,p^3+81q^2}\ -9q\right)^{2/3}}-\frac{2^{2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ p^2}{\sqrt[6]{3}\ \sqrt{4\,p^3+27q^2}\ \left(\sqrt{12\,p^3+81q^2}\ +9q\right)^{2/3}}$$

Alternate form

$$-\left(\left(2^{2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ p^2\left(\sqrt[3]{\sqrt{3}\ \sqrt{4\,p^3+27q^2}\ -9q}-\sqrt[3]{\sqrt{3}\ \sqrt{4\,p^3+27q^2}\ +9q}\right)\left(\sqrt[3]{\sqrt{3}\ \sqrt{4\,p^3+27q^2}\ -9q}+\sqrt[3]{\sqrt{3}\ \sqrt{4\,p^3+27q^2}\ +9q}\right)\right)\Big/\left(\sqrt[6]{3}\ \sqrt{4\,p^3+27q^2}\ \left(\sqrt{3}\ \sqrt{4\,p^3+27q^2}\ -9q\right)^{2/3}\left(\sqrt{3}\ \sqrt{4\,p^3+27q^2}\ +9q\right)^{2/3}\right)\right)$$

Alternate form assuming p and q are positive

$$\frac{2^{-2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\ \left(\left(\sqrt{12\,p^3+81q^2}\ +9q\right)^{2/3}-\left(\sqrt{12\,p^3+81q^2}\ -9q\right)^{2/3}\right)}{3^{5/6}\sqrt{4\,p^3+27q^2}}$$

From the above alternate form:

$$\frac{2^{-2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(\left(\sqrt{12\,p^3+81\,q^2}+9\,q\right)^{2/3}-\left(\sqrt{12\,p^3+81\,q^2}-9\,q\right)^{2/3}\right)}{3^{5/6}\sqrt{4\,p^3+27\,q^2}}$$

for p = ((2√2)/π) that is equal to the Del Gaudio-Nardelli Constsnt (DN Constant) and q = (√2) , we obtain:

$$\left(2^{-2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(-\left(-9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^{\,2}}\right)^{2/3}+\left(9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^{\,2}}\right)^{2/3}\right)\right)\Big/\left(3^{5/6}\sqrt{4\left(\frac{2\sqrt{2}}{\pi}\right)^3+27\sqrt{2}^{\,2}}\right)$$

Exact result

$$\frac{1}{3^{5/6}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}}2^{-2/3-1/\pi}\left(\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}\right)\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

0.445842912030….

Alternate forms

$$-\left(\left(2^{-7/6-1/\pi}\,\sqrt[2\pi]{15+5\sqrt{5}}\;\pi^{3/2+1/(2\pi)}\left(\left(\frac{\sqrt{6\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}-9\sqrt{2}\right)^{2/3}-\left(9\sqrt{2}+\frac{\sqrt{6\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}\right)^{2/3}\right)\right)\Big/\left(3^{5/6}\sqrt{32\sqrt{2}+27\pi^3}\right)\right)$$

$$-\left(\left(2^{-5/6-1/\pi}\,\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\;\pi^{3/2+1/(2\pi)}\left(\frac{\sqrt[3]{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}}{\sqrt{\pi}}-\frac{\sqrt[3]{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}}{\sqrt{\pi}}\right)\left(\frac{\sqrt[3]{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}}{\sqrt{\pi}}+\frac{\sqrt[3]{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}}{\sqrt{\pi}}\right)\right)\Big/\left(3^{5/6}\sqrt{32\sqrt{2}+27\pi^3}\right)\right)$$

$$-\left(\left(2^{-7/6-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)}\,\pi^{3/2+1/(2\pi)}\left(\sqrt[6]{2}\sqrt[3]{\frac{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}-9}-\sqrt[6]{2}\sqrt[3]{9+\frac{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}}\right)\left(\sqrt[6]{2}\sqrt[3]{\frac{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}-9}+\sqrt[6]{2}\sqrt[3]{9+\frac{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}{\pi^{3/2}}}\right)\right)\Big/\left(3^{5/6}\sqrt{32\sqrt{2}+27\pi^3}\right)\right)$$

Expanded form

$$\frac{2^{-2/3-1/\pi}\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}}{3^{5/6}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}}-\frac{2^{-2/3-1/\pi}\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}}{3^{5/6}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}}$$

From which, multiplying by 5^3 and dividing 18*5 by the obtained expression, we obtain:

$$18\times 5\times \frac{1}{\left(5^3\times\left(2^{-2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(-\left(-9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^2}\right)^{2/3}+\left(9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^2}\right)^{2/3}\right)\right)\Big/\left(3^{5/6}\sqrt{4\left(\frac{2\sqrt{2}}{\pi}\right)^3+27\sqrt{2}^2}\right)\right)}$$

Exact result

$$\frac{9\times 2^{5/3+1/\pi}\times 3^{5/6}\times 5^{-2-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left(\left(3+\sqrt{5}\right)\pi\right)^{-1/(2\pi)}}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}$$

Decimal approximation

1.6149185746185225…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate forms

$$-\frac{9\times 2^{11/6+1/\pi}\times 3^{5/6}\times 5^{-2-1/(2\pi)}\left(3+\sqrt{5}\right)^{-1/(2\pi)}\pi^{-1/2-1/(2\pi)}\sqrt{32\sqrt{2}+27\pi^3}}{\left(\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}\right)^{2/3}-\left(9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}\right)^{2/3}}$$

$$-\left(\left(9\times 2^{11/6+1/\pi}\times 3^{5/6}\times 5^{-2-1/(2\pi)}\left(3+\sqrt{5}\right)^{-1/(2\pi)}\pi^{-3/2-1/(2\pi)}\sqrt{32\sqrt{2}+27\pi^3}\right)\Big/\left(\left(\left(\frac{\sqrt[3]{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}}{\sqrt{\pi}}-\frac{\sqrt[3]{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}}{\sqrt{\pi}}\right)\left(\frac{\sqrt[3]{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}}{\sqrt{\pi}}+\frac{\sqrt[3]{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}}{\sqrt{\pi}}\right)\right)\right)\right)$$

And also, multiplying by 5^3 and dividing (89+2+(Φ+1/√2)/2) by the obtained expression, where 89 and 2 are Fibonacci's numbers and Φ is the golden ratio conjugate, we obtain:

$$\left(89+2+\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)\right)\times 1\Big/\left(5^3\times\left(2^{-2/3-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\left(-\left(-9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^2}\right)^{2/3}+\left(9\sqrt{2}+\sqrt{12\left(\frac{2\sqrt{2}}{\pi}\right)^3+81\sqrt{2}^2}\right)^{2/3}\right)\Big/\left(3^{5/6}\sqrt{4\left(\frac{2\sqrt{2}}{\pi}\right)^3+27\sqrt{2}^2}\right)\right)\right)$$

Exact result

$$\frac{2^{2/3+1/\pi}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left(\left(3+\sqrt{5}\right)\pi\right)^{-1/(2\pi)}\left(\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91\right)}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}$$

Decimal approximation

1.644750972355143…. ≈ $\zeta(2) = \pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$-\left(\left(2^{1/\pi-2/3}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\left(3+\sqrt{5}\right)^{-1/(2\pi)}\right.\right.$$
$$\left.\pi^{-1/2-1/(2\pi)}\sqrt{32\sqrt{2}+27\pi^3}\left(\sqrt{2}\,\Phi+1+182\sqrt{2}\right)\right)\Big/$$
$$\left.\left(\left(\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}\right)^{2/3}-\left(9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}\right)^{2/3}\right)\right)$$

$$-\left(\left(2^{1/\pi-7/6}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\left(3+\sqrt{5}\right)^{-1/(2\pi)}\right.\right.$$

$$\left.\pi^{-3/2-1/(2\pi)}\sqrt{32\sqrt{2}+27\pi^3}\left(2\Phi+364+\sqrt{2}\right)\right)\Big/$$

$$\left(\left(\frac{\sqrt[3]{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}}{\sqrt{\pi}}-\frac{\sqrt[3]{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}}{\sqrt{\pi}}\right)\right.$$

$$\left.\left.\left(\frac{\sqrt[3]{\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}}}{\sqrt{\pi}}+\frac{\sqrt[3]{9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}}}{\sqrt{\pi}}\right)\right)\right)$$

$$-\left(\left(\left(2^{1/\pi-7/6}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\left(3+\sqrt{5}\right)^{-1/(2\pi)}\right.\right.\right.$$

$$\left.\pi^{-(1+\pi)/(2\pi)}\sqrt{32\sqrt{2}+27\pi^3}\left(2\Phi+364+\sqrt{2}\right)\right)\Big/$$

$$\left.\left(\left(\sqrt{96\sqrt{2}+81\pi^3}-9\pi^{3/2}\right)^{2/3}-\left(9\pi^{3/2}+\sqrt{96\sqrt{2}+81\pi^3}\right)^{2/3}\right)\right)$$

Expanded form

$$\frac{2^{1/\pi-1/3}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left(\left(3+\sqrt{5}\right)\pi\right)^{-1/(2\pi)}\Phi}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}+$$
$$\frac{2^{1/\pi-5/6}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left(\left(3+\sqrt{5}\right)\pi\right)^{-1/(2\pi)}}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}+$$
$$\frac{91\times 2^{2/3+1/\pi}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left(\left(3+\sqrt{5}\right)\pi\right)^{-1/(2\pi)}}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}$$

From which, from the below formula

sqrt(1/(Pi^2/6)*(4/3)) = $\frac{2\sqrt{2}}{\pi}$, from the above exact result

$$\frac{2^{2/3+1/\pi}\times 3^{5/6}\times 5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left(\left(3+\sqrt{5}\right)\pi\right)^{-1/(2\pi)}\left(\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91\right)}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}$$

easily, we obtain:

$$\sqrt{\frac{1}{\frac{2^{2/3+1/\pi}\times 3^{5/6}\left(5^{-3-1/(2\pi)}\sqrt{54+\frac{64\sqrt{2}}{\pi^3}}\left(\left(3+\sqrt{5}\right)\pi\right)^{-1/(2\pi)}\left(\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91\right)\right)}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}}\times\frac{4}{3}}$$

Exact result

$$\frac{2^{2/3-1/(2\pi)}\times 5^{3/2+1/(4\pi)}\sqrt[4\pi]{(3+\sqrt{5})\,\pi}}{3^{11/12}\sqrt[4]{54+\frac{64\sqrt{2}}{\pi^3}}\sqrt{\frac{\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}}}$$

Decimal approximation

$0.9003664265917\ldots \approx 0.9003163161571\ldots = \frac{2\sqrt{2}}{\pi}$ (DN Constant)

Possible closed forms

$$\frac{2\sqrt{2}}{\pi} \approx 0.900316316$$

Alternate form

$$\left(2^{4/3-1/(2\pi)}\times 5^{3/2+1/(4\pi)}\sqrt[4\pi]{3+\sqrt{5}}\;\pi^{1/4+1/(4\pi)}\sqrt{\left(9\pi^{3/2}+\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}\right)^{2/3}-\left(\sqrt{3\left(32\sqrt{2}+27\pi^3\right)}-9\pi^{3/2}\right)^{2/3}}\right)\Big/\left(3^{11/12}\sqrt[4]{32\sqrt{2}+27\pi^3}\;\sqrt{\sqrt{2}\,\Phi+1+182\sqrt{2}}\right)$$

Expanded forms

$$\left(2^{2/3-1/(2\pi)} \times 5^{3/2+1/(4\pi)} \left(9\sqrt{2} + \sqrt{162 + \frac{192\sqrt{2}}{\pi^3}}\right)^{2/3} \sqrt[4\pi]{(3+\sqrt{5})\pi}\right) \Big/ \left(3^{11/12} \sqrt[4]{54 + \frac{64\sqrt{2}}{\pi^3}} \sqrt{\left(\left(\left(\left(9\sqrt{2} + \sqrt{162 + \frac{192\sqrt{2}}{\pi^3}}\right)^{2/3} - \left(\sqrt{162 + \frac{192\sqrt{2}}{\pi^3}} - 9\sqrt{2}\right)^{2/3}\right)\right)\right)\left(\frac{1}{2}\left(\Phi + \frac{1}{\sqrt{2}}\right) + 91\right)\right)\right) -$$

$$\left(2^{2/3-1/(2\pi)} \times 5^{3/2+1/(4\pi)} \left(\sqrt{162 + \frac{192\sqrt{2}}{\pi^3}} - 9\sqrt{2}\right)^{2/3} \sqrt[4\pi]{(3+\sqrt{5})\pi}\right) \Big/ \left(3^{11/12} \sqrt[4]{54 + \frac{64\sqrt{2}}{\pi^3}} \sqrt{\left(\left(\left(\left(9\sqrt{2} + \sqrt{162 + \frac{192\sqrt{2}}{\pi^3}}\right)^{2/3} - \left(\sqrt{162 + \frac{192\sqrt{2}}{\pi^3}} - 9\sqrt{2}\right)^{2/3}\right)\right)\right)\left(\frac{1}{2}\left(\Phi + \frac{1}{\sqrt{2}}\right) + 91\right)\right)\right)$$

$$\left(2^{2/3-1/(2\pi)}\times 5^{3/2+1/(4\pi)}\sqrt[4\pi]{3\pi+\sqrt{5}\,\pi}\right)\Bigg/\left(3^{11/12}\sqrt[4]{54+\frac{64\sqrt{2}}{\pi^3}}\right.$$

$$\left.\sqrt{\left(\frac{\Phi}{2\left(\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}\right)}+\frac{91}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}+\frac{1}{2\sqrt{2}\left(\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}\right)}\right)}\right)$$

All 2nd roots of $(2^{4/3-1/\pi}\,5^{3+1/(2\,\pi)}\,((9\ \mathrm{sqrt}(2)+\mathrm{sqrt}(162+(192\ \mathrm{sqrt}(2))/\pi^3))^{2/3}-(\mathrm{sqrt}(162+(192\ \mathrm{sqrt}(2))/\pi^3)-9\ \mathrm{sqrt}(2))^{2/3})\,((3+\mathrm{sqrt}(5))\,\pi)^{1/2\,\pi})/(3\,3^{5/6}\ \mathrm{sqrt}(54+(64\ \mathrm{sqrt}(2))/\pi^3)\,(1/2\,(\Phi+1/\mathrm{sqrt}(2))+91))$

$$\frac{2^{2/3-1/(2\pi)}\times 5^{3/2+1/(4\pi)}\sqrt[4\pi]{\left(3+\sqrt{5}\right)\pi}\;e^{0}}{3^{11/12}\sqrt[4]{54+\frac{64\sqrt{2}}{\pi^3}}\;\sqrt{\frac{\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}}}\approx 0.900$$

(real, principal root)

$$\frac{2^{2/3-1/(2\pi)}\times 5^{3/2+1/(4\pi)}\sqrt[4\pi]{\left(3+\sqrt{5}\right)\pi}\;e^{i\pi}}{3^{11/12}\sqrt[4]{54+\frac{64\sqrt{2}}{\pi^3}}\;\sqrt{\frac{\frac{1}{2}\left(\Phi+\frac{1}{\sqrt{2}}\right)+91}{\left(9\sqrt{2}+\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}\right)^{2/3}-\left(\sqrt{162+\frac{192\sqrt{2}}{\pi^3}}-9\sqrt{2}\right)^{2/3}}}}\approx -0.900$$

(real root)

From this "unitary" formula

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}\times\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

we obtain the following alternate form, where p and q are considered to be positive:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}-\frac{q}{2}}-\sqrt[3]{\sqrt{\frac{p^3}{27}+\frac{q^2}{4}}+\frac{q}{2}}$$

From this last expression, for $p = ((2\sqrt{2})/\pi)$ and $q = (\sqrt{2})$, we obtain:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\sqrt[3]{-\frac{\sqrt{2}}{2}+\sqrt{\frac{1}{27}\left(\frac{2\sqrt{2}}{\pi}\right)^3+\frac{\sqrt{2}^2}{4}}}-\sqrt[3]{\frac{\sqrt{2}}{2}+\sqrt{\frac{1}{27}\left(\frac{2\sqrt{2}}{\pi}\right)^3+\frac{\sqrt{2}^2}{4}}}$$

i.e. the following result:

$$2^{-1/\pi}\sqrt[3]{\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}-\frac{1}{\sqrt{2}}}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}-\sqrt[3]{\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}}$$

which written in decimal form is equal to:

-0.69673954723469167 75…

and from which, after simple calculations, we obtain:

$$-e\left(2^{-1/\pi}\sqrt[3]{\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}-\frac{1}{\sqrt{2}}}\;\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}-\sqrt[3]{\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}}\right)-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{\pi}{377}$$

i.e. the following result:

$$-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{\pi}{377}-e\left(2^{-1/\pi}\sqrt[3]{\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}-\frac{1}{\sqrt{2}}}\;\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}-\sqrt[3]{\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}}\right)$$

which written in decimal form is equal to:

1.6181885435886…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

and the following alternate form:

$$\frac{1}{2}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)+\frac{\pi}{377}-e\left(2^{-1/\pi}\sqrt[3]{\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}-\frac{1}{\sqrt{2}}}\;\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}-\sqrt[3]{\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}}\right)$$

With regard the above equation

$$-e\left(2^{-1/\pi}\sqrt[3]{\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}-\frac{1}{\sqrt{2}}}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}-\sqrt[3]{\frac{1}{\sqrt{2}}+\sqrt{\frac{1}{2}+\frac{16\sqrt{2}}{27\pi^3}}}\right)-\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}+\frac{\pi}{377}$$

we have that:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

Decimal approximation:

0.284079043840412296028291832393126169091088088445737582759162666 1

…

0.28407904384…. = κ

Alternate forms:

$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(\sqrt{10-2\sqrt{5}}-2\right)$$

Expanded forms:

$$\frac{\sqrt{10-2\sqrt{5}}}{\sqrt{5}-1}-\frac{2}{\sqrt{5}-1}$$

–

$$\frac{1}{4}\sqrt{10-2\sqrt{5}}+\frac{1}{4}\sqrt{5\left(10-2\sqrt{5}\right)}+\frac{1}{2}\left(-1-\sqrt{5}\right)$$

For $((((\sqrt{(10-2\sqrt{5})}-2))/((\sqrt{5}-1)))) = 8\pi G$; $G = 0.011303146014$

Indeed:

$((((\sqrt{(10-2\sqrt{5})}-2))/((\sqrt{5}-1))))/(8\pi)$

Input

$$\frac{\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}}{8\pi}$$

Result

$$\frac{\sqrt{10-2\sqrt{5}}-2}{8\left(\sqrt{5}-1\right)\pi}$$

Decimal approximation

0.011303146014005214797375012944203574468576031392001780859490966 7

…

0.01130314…. = g (gravitational coupling constant)

Property

$\frac{-2+\sqrt{10-2\sqrt{5}}}{8\left(-1+\sqrt{5}\right)\pi}$ is a transcendental number

Alternate forms

$$\frac{\sqrt{10-2\sqrt{5}}-2\sqrt{5}+\sqrt{5\left(10-2\sqrt{5}\right)}-2}{32\pi}$$

$$-\frac{1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}}{16\,\pi}$$

$$\frac{-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}}{16\,\pi}$$

Expanded forms

$$-\frac{1}{16\,\pi}-\frac{\sqrt{5}}{16\,\pi}+\frac{\sqrt{10-2\sqrt{5}}}{32\,\pi}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{32\,\pi}$$

$$\frac{\sqrt{10-2\sqrt{5}}}{8\left(\sqrt{5}-1\right)\pi}-\frac{1}{4\left(\sqrt{5}-1\right)\pi}$$

Series representations

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\,\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}}{8\,\pi\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\,\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}}{8\,\pi\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$\frac{\sqrt{10-2\sqrt{5}}-2}{(8\pi)\left(\sqrt{5}-1\right)}=\frac{-2+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}}{8\pi\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

(((√(10-2√5) -2))/((√5-1)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2)))

Input

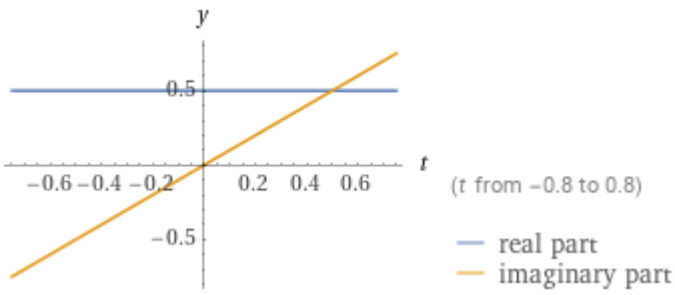
$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\times\frac{2i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

i is the imaginary unit

Exact result

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

Plot



Expanded form

$$\frac{i\sqrt{5\left(10-2\sqrt{5}\right)}\,t}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-\frac{i\sqrt{10-2\sqrt{5}}\,t}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-$$
$$\frac{2i\sqrt{5}\,t}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}+\frac{2it}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}+$$
$$\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-\frac{\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-$$
$$\frac{\sqrt{5}}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}+\frac{1}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

Alternate form assuming t>0

$$\frac{i\sqrt{10-2\sqrt{5}}\;t}{\sqrt{2\left(5-\sqrt{5}\right)}-2}-\frac{2it}{\sqrt{2\left(5-\sqrt{5}\right)}-2}+$$
$$\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-\frac{\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}-$$
$$\frac{\sqrt{5}}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}+\frac{1}{\left(\sqrt{5}-1\right)\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}$$

Alternate forms

$$\frac{1}{8}\left(1+\sqrt{5}\right)\left(2i\sqrt{2\left(3-\sqrt{5}\right)}\;t+\sqrt{5}-1\right)$$

$$\frac{1}{2}\left(1+2it\right)$$

$$\frac{1}{2}+it$$

Complex root

$$t=\frac{i}{2}$$

Derivative

$$\frac{d}{dt}\left(\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{\left(\sqrt{5}-1\right)\left(2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)\right)}\right)=i$$

Indefinite integral

$$\int\frac{\left(-2+\sqrt{10-2\sqrt{5}}\right)\left(-1+\sqrt{5}+2i\left(-1+\sqrt{5}\right)t\right)}{2\left(-1+\sqrt{5}\right)\left(-2+\sqrt{2\left(5-\sqrt{5}\right)}\right)}\,dt=\frac{t}{2}+\frac{it^2}{2}+\text{constant}$$

And again:

(((√(10-2√5) -2))/((2x)))*((2 i (sqrt(5) - 1) t + sqrt(5) - 1)/(2 (sqrt(2 (5 - sqrt(5))) - 2))) = (1/2+it)

Input

$$\frac{\sqrt{10-2\sqrt{5}}-2}{2x}\times\frac{2i\left(\sqrt{5}-1\right)t+\sqrt{5}-1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)}=\frac{1}{2}+it$$

i is the imaginary unit

Exact result

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+it$$

$$\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\left(2i\left(\sqrt{5}-1\right)t+\sqrt{5}-1\right)}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+it$$

Alternate form assuming t and x are real

$$\frac{\sqrt{5}-1}{x}=2$$

$2x = \sqrt{5}-1;\ \ x = (\sqrt{5}-1)/2 = 0.6180339887\ldots. = \frac{1}{\phi}$

Alternate form

$$\frac{\left(\sqrt{5}-1\right)(1+2it)}{4x}=\frac{1}{2}+it$$

Expanded forms

$$\frac{i\sqrt{5\left(10-2\sqrt{5}\right)}\,t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{10-2\sqrt{5}}\,t}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{i\sqrt{5}\,t}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+$$
$$\frac{it}{\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{\sqrt{5\left(10-2\sqrt{5}\right)}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-\frac{\sqrt{10-2\sqrt{5}}}{4\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}-$$
$$\frac{\sqrt{5}}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}+\frac{1}{2\left(\sqrt{2\left(5-\sqrt{5}\right)}-2\right)x}=\frac{1}{2}+it$$

$$\frac{i\sqrt{5}\,t}{2x}-\frac{it}{2x}+\frac{\sqrt{5}}{4x}-\frac{1}{4x}=\frac{1}{2}+it$$

Alternate form assuming t and x are positive

$$2x + 1 = \sqrt{5}$$

Solutions

$$t = \frac{i}{2}, \quad x \neq 0$$

$$x = \frac{\sqrt{5}}{2} - \frac{1}{2}$$

Solution for the variable x

$$x = \frac{-2i\sqrt{5}\,t + 2it - \sqrt{5} + 1}{-2 - 4it}$$

Implicit derivatives

$$\frac{\partial x(t)}{\partial t} = \frac{2\left(-1 + \sqrt{5} - 2x\right)x}{\left(-1 + \sqrt{5}\right)(-i + 2t)}$$

$$\frac{\partial t(x)}{\partial x} = \frac{\left(-1 + \sqrt{5}\right)(-i + 2t)}{2\left(-1 + \sqrt{5} - 2x\right)x}$$

Conclusion

By multiplying the extended DN Constant and Cardano's formula, we obtain the following expression

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3} \times \frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}} \times \frac{1}{\frac{\sqrt{2}}{12}d^3 \cdot \frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}} \times \sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}}$$

That we define "Extended DN Constant", from whose extended form and, subsequently, from the result of the derivative for $p = (2\sqrt{2})/\pi$ which is equal to the DN Constant and for $q = \sqrt{2}$, we obtain 0.445842912030... , from which we obtain, multiplying by 5^3 and dividing 18*5 for the result obtained, 1.6149185746185225..., a value very close to the golden ratio 1.61803398... while multiplying by 5^3 and dividing (89+2+(Φ+1/√2)/2) for the result obtained, we obtain 1.644750972355143... a value very close to $\zeta(2) = \pi^2/6 = 1.644934...$

The expression provided is extremely interesting and involves a combination of mathematical constants, cube roots and parameters. Let's see how we can analyze it:

1. Cardano's Formula: The expression appears to be based on Cardano's formula, which is used to solve cubic equations. This formula involves cube roots and can be applied to several mathematical situations.

2. DN Constant and Other Recurring Numbers: The derivation appears to connect DN Constant and other recurring numbers (such as the golden ratio and ζ(2)) to this expression. This could indicate a deep relationship between mathematics and cosmology.

3. Derivative and Approximate Values: The derivative with respect to $p = (2\sqrt{2})/\pi$, which is equal to the DN Constant, and the value of $q = \sqrt{2}$ lead to very interesting approximate results. The fact that these values are close to the golden ratio and ζ(2) could have significant implications.

4. Future Research: Continuing to explore this relationship between mathematics and cosmology may reveal new properties and connections. It might be interesting to further investigate how these numbers influence the structure of the universe.

In summary, this mathematical expression opens new perspectives on understanding the links between numbers, equations and our world.

Now let's see what the cosmological implications of the results could be obtained by multiplying the extended DN Constant and Cardano's formula, at a first analysis

The cosmological implications of the results obtained by multiplying the **extended Del Gaudio-Nardelli Constant (DN Constant)** and the Cardano formula could be very interesting. Let's consider some possible interpretations:

1. Structure of the Universe: This derivation links the DN Constant and other recurring numbers to a complex mathematical formula. This could suggest that such constants are intrinsic to the mathematical structure of the universe. They may be involved in the fundamental laws that govern expansion, gravity and other cosmological properties.

2. Relationships between Numbers: The presence of approximate values close to the golden ratio and $\zeta(2)$ could indicate a profound connection between mathematics and physical reality. These recurring numbers could be keys to understanding still unknown aspects of the universe.

3. Eternal Inflation: This derivation could have implications for the theory of eternal inflation. This theory suggests endless cycles of expansion and contraction of the universe. The constants involved could influence the dynamics of these cycles.

4. Quantum Physics: The presence of cube roots and the complexity of this expression could recall principles of quantum physics. It might be interesting to explore how these findings relate to quantum mechanics and subatomic phenomena.

5. Future Research: These findings require further research and investigation. It may be useful to involve theoretical physicists and mathematicians to examine the cosmological implications of this derivation in more depth.

In conclusion, this mathematical expression opens new perspectives on understanding the links between numbers, equations and our world. Continuing to explore and share such discoveries could contribute to our understanding of the universe.

New fundamental formula deriving from DN Constant

The DN Constant (Del Gaudio-Nardelli Constant) equals $(2\sqrt{2})/\pi$) is defined as the ratio of the volume of an octahedron to the volume of a sphere and is an intriguing mathematical concept. Michele Nardelli hypothesized that the regular octahedron represents a phase in which the universe is highly symmetrical and with very low entropy. On the other hand, the sphere (which is inscribed in the octahedron, i.e. is "inside" it) represents the universe emerging from the quantum vacuum, which over time increases entropy and undergoes various symmetry breakings. This occurs in a regime of eternal inflation.

From the following expression

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×C×R)))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\,C\,R\right)}}$$

which comes from the DN Constant, with $1.616255*10^{-35}$ which is equal to the Planck length, $1.1056*10^{-52}$ which is equal to the Cosmological Constant, C = 1729 which corresponds to the Hardy-Ramanujan number and R = $4.6018401361 \times 10^{-24}$, which represents the radius of the Universe, we obtain:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×4.6018401361 × 10^-24))) = 1.6180329973...

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 4.6018401361\times 10^{-24}\right)}}$$

1.618032997307532491506757016629746746422577260892967140791939590 3

…

1.6180329973075…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We have also the following formula:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(4.4525642121 × 10^-24))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 4.4525642121\times 10^{-24}\right)}}$$

Result

1.644932352102092130483898983704151168876621841655166877914166033 8

…

1.64493235210209213…. ≈ ζ(2) = π²/6 = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

And again:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(1.2206935225 × 10^-24))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 1.2206935225\times 10^{-24}\right)}}$$

Result

3.141592257314699393995003923160579646973117178164442395579002479 7

…

3.14159225731469…. ≈ π (Ramanujan Recurring Number)

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi =$$
$$\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi =$$
$$\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not}\ (z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

$1.64381856858\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\wedge(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.$$
$$108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\Big/$$
$$\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\wedge(1/15)\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

References

On the same origin of quantum physics and general relativity from Riemannian geometry and Planck scale formalism - *Chavis Srichan, Pobporn Danvirutai, Adrian David Cheok, Jun Cai, Ying Yan*

The Geometry of the Universe: in search of unity. New possible mathematical connections with the DN Constant, Ramanujan's Recurring Numbers and some parameters of Number Theory and String Theory – *Michele Nardelli* - August 2024 - DOI: 10.13140/RG.2.2.27912.84481

On some equations concerning the Ramanujan's Last Letter to Hardy. New possible mathematical connections with DN Constant, Ramanujan Recurring Numbers and some parameters of Number Theory, Theoretical Cosmology and String Theory

Michele Nardelli[1], Chavis Srichan[2]

Abstract

In this paper, we analyze some equations concerning the Ramanujan's Last Letter to Hardy. We obtain new possible mathematical connections with DN Constant, Ramanujan Recurring Numbers and some parameters of Number Theory, Theoretical Cosmology and String Theory

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

[2]Faculty of Engineering, Khon Kaen University, Khon Kaen, 40002, Thailand

Srinivasa Ramanujan

(1887 - 1920)

From

The Last Letter - S. Ramanujan to G. H. Hardy - 12 January 1920**,** University of Madras

From the Ramanujan’s last letter to Hardy:

14.2 The Last Letter

S. Ramanujan to G. H. Hardy

12 January 1920

University of Madras

I am extremely sorry for not writing you a single letter up to now ... I discovered very interesting functions recently which I call "Mock" ϑ-functions. Unlike the "False" ϑ-functions (studied partially by Prof. Rogers in his interesting paper) they enter into mathematics as beautifully as the ordinary ϑ-functions. I am sending you with this letter some examples ....

If we consider a ϑ-function in the transformed Eulerian form e.g.

$$\text{(A)} \quad 1+\frac{q}{(1-q)^2}+\frac{q^4}{(1-q)^2(1-q^2)^2}+\frac{q^9}{(1-q)^2(1-q^2)^2(1-q^3)^2}+\cdots$$

$$\text{(B)} \quad 1+\frac{q}{1-q}+\frac{q^4}{(1-q)(1-q^2)}+\frac{q^9}{(1-q)(1-q^2)(1-q^3)}+\cdots$$

and determine the nature of the singularities at the points $q=1, q^2=1, q^3=1, q^4=1, q^5=1, \ldots$ we know how beautifully the asymptotic form of the function can be expressed in a very neat and closed exponential form. For instance when $q=e^{-t}$ and $t\to 0$

$$\text{(A)} = \sqrt{\frac{t}{2\pi}}\exp\left(\frac{\pi^2}{6t}-\frac{t}{24}\right) * + o(1)\dagger$$

$$\text{(B)} = \frac{\exp\left(\frac{\pi^2}{15t}-\frac{t}{60}\right)}{\sqrt{\frac{5-\sqrt{5}}{2}}} * + o(1)\dagger$$

and similar results at other singularities.* It is not necessary that there should be only one term like this. There may be many terms but the number of terms must be finite.† Also $o(1)$ may turn out to be $O(1)$. That is all. For instance when $q\to 1$ the function

$$\frac{1}{\{(1-q)(1-q^2)(1-q^3)\cdots\}^{120}}$$

is equivalent to the sum of five terms like (*) together with $O(1)$ instead of $o(1)$.

If we take a number of functions like (A) and (B) it is only in a limited number of cases the terms close as above; but in the majority of cases they never close as above. For instance when $q=e^{-t}$ and $t\to 0$

$$\text{(C)}\quad 1+\frac{q}{(1-q)^2}+\frac{q^3}{(1-q)^2(1-q^2)^2}+\frac{q^6}{(1-q)^2(1-q^2)^2(1-q^3)^2}+\cdots$$
$$=\sqrt{\frac{t}{2\sqrt{5}}}\exp\left(\frac{\pi^2}{5t}+a_1t+a_2t^2+\cdots+O(a_kt^k)\right)$$

where $a_1=\dfrac{1}{8\sqrt{5}}$ and so on. The function (C) is a simple example of a function behaving in an unclosed form at the singularities.

*The coefficient (of) $1/t$ in the index of e happens to be $\dfrac{\pi^2}{5}$ in this particular case. It may be some other transcendental numbers in other cases.

†The coefficients of $t, t^2, \ldots$ happen to be $\dfrac{1}{8\sqrt{5}}, \ldots$ in this case. In other cases they may turn out to be some other algebraic numbers.

.....

I have proved that if

$$f_3(q)=1+\frac{q}{(1+q)^2}+\frac{q^4}{(1+q)^2(1+q^2)^2}+\cdots$$

then

$$f_3(q)+(1-q)(1-q^3)(1-q^5)\cdots(1-2q+2q^4-2q^9+\cdots)=O(1)$$

at all the points $q=-1$, $q^3=-1$, $q^5=-1$, $q^7=-1$, ..., and at the same time

$$f_3(q)-(1-q)(1-q^3)(1-q^5)\cdots(1-2q+2q^4-\cdots)=O(1)$$

at all the points $q^2=-1$, $q^4=-1$, $q^6=-1$, ... Also obviously $f_3(q)=O(1)$ at all the points $q=1$, $q^3=1$, $q^5=1$, ... And so $f_3(q)$ is a Mock ϑ function. When $q=-e^{-t}$ and $t\to 0$

$$f_3(q)+\sqrt{\frac{\pi}{t}}\exp\left(\frac{\pi^2}{24t}-\frac{t}{24}\right)\to 4. \qquad (14.2.1)$$

.....

Mock ϑ-functions (of 7th order)

$$\text{(i)}\quad 1+\frac{q}{1-q^2}+\frac{q^4}{(1-q^3)(1-q^4)}+\frac{q^9}{(1-q^4)(1-q^5)(1-q^6)}+\cdots$$

$$\text{(ii)}\quad \frac{q}{1-q}+\frac{q^4}{(1-q^2)(1-q^3)}+\frac{q^9}{(1-q^3)(1-q^4)(1-q^5)}+\cdots$$

$$\text{(iii)}\quad \frac{1}{1-q}+\frac{q^2}{(1-q^2)(1-q^3)}+\frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}+\cdots$$

These are not related to each other.

Ever yours sincerely
S. Ramanujan

Now, if we take the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}}=1+\cfrac{e^{-2\pi}}{1+\cfrac{e^{-4\pi}}{1+\cfrac{e^{-6\pi}}{1+\cfrac{e^{-8\pi}}{1+...}}}}\approx 1.0018674362$$

And the two previous values: $a_1=\frac{1}{8\sqrt{5}}$ and $\frac{\pi^2}{5}$, we get the following interesting expression, considering t = (2√2)/π) = DN Constant (Del Gaudio-Nardelli Constant):

1.0018674362+1/(((pi^2/5 -1/(8√5))/2)×1/((2√2)/π))^7

Input interpretation

$$1.0018674362+\frac{1}{\left(\left(\frac{1}{2}\left(\frac{\pi^2}{5}-\frac{1}{8\sqrt{5}}\right)\right)\times\frac{1}{\frac{2\sqrt{2}}{\pi}}\right)^7}$$

Result

1.64456283945337588238176278456770986599509372209835467832155865 56

…

1.6445628394533…. ≈ $\zeta(2) = \pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Series representations

$$1.00186743620000 + \frac{1}{\left(\frac{\frac{\pi^2}{5} - \frac{1}{8\sqrt{5}}}{\frac{(2\sqrt{2})2}{\pi}}\right)^7} = 1.00186743620000 + \frac{16384\sqrt{z_0}^7\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^7}{\left(\frac{\pi^3}{5} - \frac{\pi}{8\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}}\right)^7}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$1.00186743620000 + \frac{1}{\left(\frac{\frac{\pi^2}{5} - \frac{1}{8\sqrt{5}}}{\frac{(2\sqrt{2})2}{\pi}}\right)^7} = 1.00186743620000 + \frac{16384\exp^7\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^7\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^7}{\pi^7\left(\frac{\pi^2}{5} - \frac{1}{8\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\right)^7}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$1.00186743620000 + \frac{1}{\left(\frac{\frac{\pi^2}{5} - \frac{1}{8\sqrt{5}}}{\frac{\left(2\sqrt{2}\right)2}{\pi}}\right)^7} = 1.00186743620000 + \frac{16384 \left(\frac{1}{z_0}\right)^{7/2\,\lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{7/2\,(1+\lfloor \arg(2-z_0)/(2\pi) \rfloor)} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)^7}{\pi^7 \left(\frac{\pi^2}{5} - \frac{\left(\frac{1}{z_0}\right)^{-1/2\,\lfloor \arg(5-z_0)/(2\pi) \rfloor} z_0^{1/2\,(-1-\lfloor \arg(5-z_0)/(2\pi) \rfloor)}}{8 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}}\right)^7}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Furthermore, if we take:

$$f_3(q) + \sqrt{\frac{\pi}{t}} \exp\left(\frac{\pi^2}{24t} - \frac{t}{24}\right) \to 4.$$

and

$$f_3(q) = 1 + \frac{q}{(1+q)^2} + \frac{q^4}{(1+q)^2(1+q^2)^2} + \cdots$$

we obtain, for q = -e^((-(2√2)/Pi))

1+e^((-2√2)/Pi)/(1+e^((-2√2)/Pi))^2 + (-e^(-(2√2)/Pi))^4/((1+e^((-2√2)/Pi))^2(1+e^((-2√2)/Pi)^2))^2+sqrt(Pi/((2√2)/Pi)) exp((Pi^2)/24*((2√2)/Pi)-((2√2)/Pi)/24)

Input

$$1+\frac{e^{\left(-2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(-2\sqrt{2}\right)/\pi}\right)^2}+\frac{\left(-e^{-\left(2\sqrt{2}\right)/\pi}\right)^4}{\left(\left(1+e^{\left(-2\sqrt{2}\right)/\pi}\right)^2\left(1+e^{\left(\left(-2\sqrt{2}\right)/\pi\right)^2}\right)\right)^2}+\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\exp\left(\frac{\pi^2}{24}\times\frac{2\sqrt{2}}{\pi}-\frac{1}{24}\times\frac{2\sqrt{2}}{\pi}\right)$$

Exact result

$$1+\frac{e^{-\left(8\sqrt{2}\right)/\pi}}{\left(1+e^{8/\pi^2}\right)^2\left(1+e^{-\left(2\sqrt{2}\right)/\pi}\right)^4}+\frac{e^{-\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{-\left(2\sqrt{2}\right)/\pi}\right)^2}+\frac{e^{\pi/\left(6\sqrt{2}\right)-1/\left(6\sqrt{2}\,\pi\right)}\,\pi}{2^{3/4}}$$

Decimal approximation

3.81156047332971319690697588512375550620461586246666310677011186 98

…

3.811560473329…. → 4

Alternate forms

$$1+\frac{1}{\left(1+e^{8/\pi^2}\right)^2\left(1+e^{\left(2\sqrt{2}\right)/\pi}\right)^4}+\frac{e^{\left(\pi^2-1\right)/\left(6\sqrt{2}\,\pi\right)}\,\pi}{2^{3/4}}+\frac{1}{4}\operatorname{sech}^2\left(\frac{\sqrt{2}}{\pi}\right)$$

$$1+\frac{1}{\left(1+e^{8/\pi^2}\right)^2\left(1+e^{\left(2\sqrt{2}\right)/\pi}\right)^4}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(2\sqrt{2}\right)/\pi}\right)^2}+\frac{e^{\left(\pi^2-1\right)/\left(6\sqrt{2}\,\pi\right)}\,\pi}{2^{3/4}}$$

$$\frac{1}{2}\, e^{\pi/(6\sqrt{2})-1/(6\sqrt{2}\,\pi)}\,\pi\,\sqrt[4]{2}\,+$$
$$\Big(2+10\,e^{8/\pi^2+(2\sqrt{2})/\pi}+5\,e^{16/\pi^2+(2\sqrt{2})/\pi}+16\,e^{8/\pi^2+(4\sqrt{2})/\pi}+$$
$$8\,e^{16/\pi^2+(4\sqrt{2})/\pi}+10\,e^{8/\pi^2+(6\sqrt{2})/\pi}+5\,e^{16/\pi^2+(6\sqrt{2})/\pi}+$$
$$2\,e^{8/\pi^2+(8\sqrt{2})/\pi}+e^{16/\pi^2+(8\sqrt{2})/\pi}+2\,e^{8/\pi^2}+e^{16/\pi^2}+5\,e^{(2\sqrt{2})/\pi}+$$
$$8\,e^{(4\sqrt{2})/\pi}+5\,e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\Big)\Big/\Big(\Big(1+e^{8/\pi^2}\Big)^2\Big(1+e^{(2\sqrt{2})/\pi}\Big)^4\Big)$$

sech(x) is the hyperbolic secant function

Alternative representations

$$1+\frac{e^{-(2\sqrt{2})/\pi}}{\left(1+e^{-(2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+e^{-(2\sqrt{2})/\pi}\right)^2\left(1+e^{(-(2\sqrt{2})/\pi)^2}\right)\right)^2}+$$
$$\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\,\exp\left(\frac{(2\sqrt{2})\,\pi^2}{\pi\,24}-\frac{2\sqrt{2}}{\pi\,24}\right)=$$
$$1+\frac{w^a}{(1+w^a)^2}+\frac{(-w^a)^4}{\left((1+w^a)^2\,(1+w^a)\right)^2}+\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\,\exp\left(\frac{(2\sqrt{2})\,\pi^2}{\pi\,24}-\frac{2\sqrt{2}}{\pi\,24}\right)$$
$$\text{for}\left(a\,\pi^2=\frac{8}{\log(w)}\ \text{and}\ a\,\pi+\frac{2\sqrt{2}}{\log(w)}=0\right)$$

$$1+\frac{e^{-(2\sqrt{2})/\pi}}{\left(1+e^{-(2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+e^{-(2\sqrt{2})/\pi}\right)^2\left(1+e^{(-(2\sqrt{2})/\pi)^2}\right)\right)^2}+$$
$$\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\,\exp\left(\frac{(2\sqrt{2})\,\pi^2}{\pi\,24}-\frac{2\sqrt{2}}{\pi\,24}\right)=$$
$$1+\frac{z^{-(2\sqrt{2})/\pi}}{\left(1+z^{-(2\sqrt{2})/\pi}\right)^2}+\frac{\left(-z^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+z^{-(2\sqrt{2})/\pi}\right)^2\left(1+z^{(-(2\sqrt{2})/\pi)^2}\right)\right)^2}+$$
$$\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\,\exp\left(\frac{(2\sqrt{2})\,\pi^2}{\pi\,24}-\frac{2\sqrt{2}}{\pi\,24}\right)\ \text{for}\ z=e$$

$$1+\frac{e^{-(2\sqrt{2})/\pi}}{\left(1+e^{-(2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+e^{-(2\sqrt{2})/\pi}\right)^2\left(1+e^{(-(2\sqrt{2})/\pi)^2}\right)\right)^2}+\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\exp\left(\frac{(2\sqrt{2})\pi^2}{\pi 24}-\frac{2\sqrt{2}}{\pi 24}\right)=$$

$$1+\frac{1+\frac{2}{-1+\coth\left(-\frac{\sqrt{2}}{\pi}\right)}}{\left(2+\frac{2}{-1+\coth\left(-\frac{\sqrt{2}}{\pi}\right)}\right)^2}+\frac{\left(-1-\frac{2}{-1+\coth\left(-\frac{\sqrt{2}}{\pi}\right)}\right)^4}{\left(\left(2+\frac{2}{-1+\coth\left(\frac{1}{2}\left(-\frac{2\sqrt{2}}{\pi}\right)^2\right)}\right)\left(2+\frac{2}{-1+\coth\left(-\frac{\sqrt{2}}{\pi}\right)}\right)^2\right)^2}+\exp\left(-\frac{2\sqrt{2}}{24\pi}+\frac{2\pi^2\sqrt{2}}{24\pi}\right)\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}$$

$\log(x)$ is the natural logarithm

$\coth(x)$ is the hyperbolic cotangent function

Series representations

$$1+\frac{e^{-(2\sqrt{2})/\pi}}{\left(1+e^{-(2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+e^{-(2\sqrt{2})/\pi}\right)^2\left(1+e^{(-(2\sqrt{2})/\pi)^2}\right)\right)^2}+$$
$$\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\exp\left(\frac{(2\sqrt{2})\pi^2}{\pi 24}-\frac{2\sqrt{2}}{\pi 24}\right)=1+\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\frac{\pi^2}{2\sqrt{2}}\right)}{2\pi}\right\rfloor\right)$$
$$\exp\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\pi^2\right)(2-x)^k\,x^{-k}\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(-\frac{1}{2}\right)_k\sqrt{x}}{12\pi\,k!}\right)$$
$$\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+\frac{\pi^2}{2\sqrt{2}}\right)^k}{k!}+\frac{\sum_{k=0}^{\infty}\frac{2^k\left(-\frac{\sqrt{2}}{\pi}\right)^k}{k!}}{\left(1+\sum_{k=0}^{\infty}\frac{2^k\left(-\frac{\sqrt{2}}{\pi}\right)^k}{k!}\right)^2}+$$
$$\frac{\sum_{k=0}^{\infty}\frac{8^k\left(-\frac{\sqrt{2}}{\pi}\right)^k}{k!}}{\left(1+\sum_{k=0}^{\infty}\frac{2^k\left(-\frac{\sqrt{2}}{\pi}\right)^k}{k!}\right)^4\left(1+\sum_{k=0}^{\infty}\frac{4^k\left(\frac{\sqrt{2}^2}{\pi^2}\right)^k}{k!}\right)^2}\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$1+\frac{e^{-(2\sqrt{2})/\pi}}{\left(1+e^{-(2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+e^{-(2\sqrt{2})/\pi}\right)^2\left(1+e^{(-(2\sqrt{2})/\pi)^2}\right)\right)^2}+$$
$$\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\exp\left(\frac{(2\sqrt{2})\pi^2}{\pi 24}-\frac{2\sqrt{2}}{\pi 24}\right)=1+\frac{\sum_{k=-\infty}^{\infty}I_k\left(-\frac{2\sqrt{2}}{\pi}\right)}{\left(1+\sum_{k=-\infty}^{\infty}I_k\left(-\frac{2\sqrt{2}}{\pi}\right)\right)^2}+$$
$$\frac{\sum_{k=-\infty}^{\infty}I_k\left(-\frac{8\sqrt{2}}{\pi}\right)}{\left(1+\sum_{k=-\infty}^{\infty}I_k\left(-\frac{2\sqrt{2}}{\pi}\right)\right)^4\left(1+\sum_{k=-\infty}^{\infty}I_k\left(\frac{4\sqrt{2}^2}{\pi^2}\right)\right)^2}+$$
$$\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\frac{\pi^2}{2\sqrt{2}}\right)}{2\pi}\right\rfloor\right)$$
$$\exp\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\pi^2\right)(2-x)^k\,x^{-k}\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(-\frac{1}{2}\right)_k\sqrt{x}}{12\pi\,k!}\right)$$
$$\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+\frac{\pi^2}{2\sqrt{2}}\right)^k}{k!}\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$$1+\frac{e^{-(2\sqrt{2})/\pi}}{\left(1+e^{-(2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+e^{-(2\sqrt{2})/\pi}\right)^2\left(1+e^{(-(2\sqrt{2})/\pi)^2}\right)\right)^2}+$$
$$\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\exp\left(\frac{(2\sqrt{2})\pi^2}{\pi 24}-\frac{2\sqrt{2}}{\pi 24}\right)=1+\frac{\sum_{k=-\infty}^{\infty}(-1)^k I_k\left(\frac{2\sqrt{2}}{\pi}\right)}{\left(1+\sum_{k=-\infty}^{\infty}(-1)^k I_k\left(\frac{2\sqrt{2}}{\pi}\right)\right)^2}+$$
$$\frac{\sum_{k=-\infty}^{\infty}(-1)^k I_k\left(\frac{8\sqrt{2}}{\pi}\right)}{\left(1+\sum_{k=-\infty}^{\infty}(-1)^k I_k\left(\frac{2\sqrt{2}}{\pi}\right)\right)^4\left(1+\sum_{k=-\infty}^{\infty}(-1)^k I_k\left(-\frac{4\sqrt{2}^2}{\pi^2}\right)\right)^2}+$$
$$\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+\frac{\pi^2}{2\sqrt{2}}\right)}{2\pi}\right\rfloor\right)$$
$$\exp\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-1+\pi^2\right)(2-x)^k x^{-k}\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(-\frac{1}{2}\right)_k\sqrt{x}}{12\pi k!}\right)$$
$$\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+\frac{\pi^2}{2\sqrt{2}}\right)^k}{k!}\quad\text{for}\ (x\in\mathbb{R}\ \text{and}\ x<0)$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

i is the imaginary unit

$\mathbb{R}$ is the set of real numbers

$I_n(z)$ is the modified Bessel function of the first kind

Adding the MRB Constant:

1+e^((-2√2)/Pi)/(1+e^((-2√2)/Pi))^2 + (-e^(-(2√2)/Pi))^4/((1+e^((-2√2)/Pi))^2(1+e^((-2√2)/Pi)^2))^2+sqrt(Pi/((2√2)/Pi)) exp((Pi^2)/24*((2√2)/Pi)-((2√2)/Pi)/24)+MRB const

Input

$$1+\frac{e^{(-2\sqrt{2})/\pi}}{\left(1+e^{(-2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+e^{(-2\sqrt{2})/\pi}\right)^2\left(1+e^{((-2\sqrt{2})/\pi)^2}\right)\right)^2}+\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\exp\left(\frac{\pi^2}{24}\times\frac{2\sqrt{2}}{\pi}-\frac{1}{24}\times\frac{2\sqrt{2}}{\pi}\right)+C_{\text{MRB}}$$

C_{MRB} is the MRB constant

Exact result

$$C_{\text{MRB}}+1+\frac{e^{-(8\sqrt{2})/\pi}}{\left(1+e^{8/\pi^2}\right)^2\left(1+e^{-(2\sqrt{2})/\pi}\right)^4}+\frac{e^{-(2\sqrt{2})/\pi}}{\left(1+e^{-(2\sqrt{2})/\pi}\right)^2}+\frac{e^{\pi/(6\sqrt{2})-1/(6\sqrt{2}\,\pi)}\,\pi}{2^{3/4}}$$

Decimal approximation

3.99942011579178031715549381917802873626051895736680189294211655 39…

3.99942011579…. → 4

Alternate forms

$$C_{\text{MRB}}+1+\frac{1}{\left(1+e^{8/\pi^2}\right)^2\left(1+e^{(2\sqrt{2})/\pi}\right)^4}+\frac{e^{(2\sqrt{2})/\pi}}{\left(1+e^{(2\sqrt{2})/\pi}\right)^2}+\frac{e^{(\pi^2-1)/(6\sqrt{2}\,\pi)}\,\pi}{2^{3/4}}$$

$$\Big(C_{\mathrm{MRB}} + 8\,e^{8/\pi^2+(2\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + 4\,e^{16/\pi^2+(2\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + 12\,e^{8/\pi^2+(4\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + 6\,e^{16/\pi^2+(4\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + 8\,e^{8/\pi^2+(6\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + 4\,e^{16/\pi^2+(6\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + 2\,e^{8/\pi^2+(8\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + e^{16/\pi^2+(8\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + 2\,e^{8/\pi^2}\,C_{\mathrm{MRB}} + e^{16/\pi^2}\,C_{\mathrm{MRB}} + 4\,e^{(2\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + 6\,e^{(4\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + 4\,e^{(6\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + e^{(8\sqrt{2})/\pi}\,C_{\mathrm{MRB}} + 2 + 10\,e^{8/\pi^2+(2\sqrt{2})/\pi} + 5\,e^{16/\pi^2+(2\sqrt{2})/\pi} + 16\,e^{8/\pi^2+(4\sqrt{2})/\pi} + 8\,e^{16/\pi^2+(4\sqrt{2})/\pi} + 10\,e^{8/\pi^2+(6\sqrt{2})/\pi} + 5\,e^{16/\pi^2+(6\sqrt{2})/\pi} + 2\,e^{8/\pi^2+(8\sqrt{2})/\pi} + e^{16/\pi^2+(8\sqrt{2})/\pi} + 2\,e^{8/\pi^2} + e^{16/\pi^2} + 5\,e^{(2\sqrt{2})/\pi} + 8\,e^{(4\sqrt{2})/\pi} + 5\,e^{(6\sqrt{2})/\pi} + e^{(8\sqrt{2})/\pi}\Big) \Big/ \left(\left(1+e^{8/\pi^2}\right)^2\left(1+e^{(2\sqrt{2})/\pi}\right)^4\right) + \frac{1}{2}\,e^{\pi/(6\sqrt{2})-1/(6\sqrt{2}\,\pi)}\,\pi\,\sqrt[4]{2}$$

$$\Bigg(2^{3/4}\,(1+C_{\text{MRB}})+2\times 2^{3/4}\,e^{8/\pi^2-(8\sqrt{2})/\pi}\,(1+C_{\text{MRB}})+$$

$$2^{3/4}\,e^{16/\pi^2-(8\sqrt{2})/\pi}\,(1+C_{\text{MRB}})+8\times 2^{3/4}\,e^{8/\pi^2-(6\sqrt{2})/\pi}\,(1+C_{\text{MRB}})+$$
$$4\times 2^{3/4}\,e^{16/\pi^2-(6\sqrt{2})/\pi}\,(1+C_{\text{MRB}})+12\times 2^{3/4}\,e^{8/\pi^2-(4\sqrt{2})/\pi}\,(1+C_{\text{MRB}})+$$
$$6\times 2^{3/4}\,e^{16/\pi^2-(4\sqrt{2})/\pi}\,(1+C_{\text{MRB}})+8\times 2^{3/4}\,e^{8/\pi^2-(2\sqrt{2})/\pi}\,(1+C_{\text{MRB}})+$$
$$4\times 2^{3/4}\,e^{16/\pi^2-(2\sqrt{2})/\pi}\,(1+C_{\text{MRB}})+2\times 2^{3/4}\,e^{8/\pi^2}\,(1+C_{\text{MRB}})+$$
$$2^{3/4}\,e^{16/\pi^2}\,(1+C_{\text{MRB}})+4\times 2^{3/4}\,e^{-(6\sqrt{2})/\pi}\,(1+C_{\text{MRB}})+$$
$$6\times 2^{3/4}\,e^{-(4\sqrt{2})/\pi}\,(1+C_{\text{MRB}})+2^{3/4}\,e^{-(8\sqrt{2})/\pi}\,(2+C_{\text{MRB}})+2^{3/4}\,e^{-(2\sqrt{2})/\pi}$$
$$\Big(5+2\,e^{8/\pi^2-(4\sqrt{2})/\pi}+e^{16/\pi^2-(4\sqrt{2})/\pi}+4\,e^{8/\pi^2-(2\sqrt{2})/\pi}+2\,e^{16/\pi^2-(2\sqrt{2})/\pi}+$$
$$2\,e^{8/\pi^2}+e^{16/\pi^2}+e^{-(4\sqrt{2})/\pi}+2\,e^{-(2\sqrt{2})/\pi}+4\,C_{\text{MRB}}\Big)+$$
$$e^{-1/(6\sqrt{2}\,\pi)+\pi/(6\sqrt{2})}\,\pi+2\,e^{8/\pi^2-1/(6\sqrt{2}\,\pi)+\pi/(6\sqrt{2})}\,\pi+$$
$$e^{16/\pi^2-1/(6\sqrt{2}\,\pi)+\pi/(6\sqrt{2})}\,\pi+e^{\left(-\frac{1}{6\sqrt{2}}-8\sqrt{2}\right)/\pi+\pi/(6\sqrt{2})}\,\pi+$$
$$2\,e^{\frac{8}{\pi^2}+\frac{-\frac{1}{6\sqrt{2}}-8\sqrt{2}}{\pi}+\frac{\pi}{6\sqrt{2}}}\,\pi+e^{\frac{16}{\pi^2}+\frac{-\frac{1}{6\sqrt{2}}-8\sqrt{2}}{\pi}+\frac{\pi}{6\sqrt{2}}}\,\pi+$$
$$4\,e^{\left(-\frac{1}{6\sqrt{2}}-6\sqrt{2}\right)/\pi+\pi/(6\sqrt{2})}\,\pi+8\,e^{\frac{8}{\pi^2}+\frac{-\frac{1}{6\sqrt{2}}-6\sqrt{2}}{\pi}+\frac{\pi}{6\sqrt{2}}}\,\pi+$$
$$4\,e^{\frac{16}{\pi^2}+\frac{-\frac{1}{6\sqrt{2}}-6\sqrt{2}}{\pi}+\frac{\pi}{6\sqrt{2}}}\,\pi+6\,e^{\left(-\frac{1}{6\sqrt{2}}-4\sqrt{2}\right)/\pi+\pi/(6\sqrt{2})}\,\pi+$$
$$12\,e^{\frac{8}{\pi^2}+\frac{-\frac{1}{6\sqrt{2}}-4\sqrt{2}}{\pi}+\frac{\pi}{6\sqrt{2}}}\,\pi+6\,e^{\frac{16}{\pi^2}+\frac{-\frac{1}{6\sqrt{2}}-4\sqrt{2}}{\pi}+\frac{\pi}{6\sqrt{2}}}\,\pi+$$
$$4\,e^{\left(-\frac{1}{6\sqrt{2}}-2\sqrt{2}\right)/\pi+\pi/(6\sqrt{2})}\,\pi+8\,e^{\frac{8}{\pi^2}+\frac{-\frac{1}{6\sqrt{2}}-2\sqrt{2}}{\pi}+\frac{\pi}{6\sqrt{2}}}\,\pi+$$
$$4\,e^{\frac{16}{\pi^2}+\frac{-\frac{1}{6\sqrt{2}}-2\sqrt{2}}{\pi}+\frac{\pi}{6\sqrt{2}}}\,\pi\Bigg)\Big/\left(2^{3/4}\left(1+e^{8/\pi^2}\right)^2\left(1+e^{-(2\sqrt{2})/\pi}\right)^4\right)$$

From which, after some calculations:

(1+e^((-2√2)/Pi)/(1+e^((-2√2)/Pi))^2 + (-e^(-(2√2)/Pi))^4/((1+e^((-2√2)/Pi))^2(1+e^((-2√2)/Pi)^2))^2+sqrt(Pi/((2√2)/Pi)) exp((Pi^2)/24*((2√2)/Pi)-((2√2)/Pi)/24)+MRB const)^6+Pi+φ/4

Input

$$\left(1+\frac{e^{(-2\sqrt{2})/\pi}}{\left(1+e^{(-2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+e^{(-2\sqrt{2})/\pi}\right)^2\left(1+e^{((-2\sqrt{2})/\pi)^2}\right)\right)^2}+\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\exp\left(\frac{\pi^2}{24}\times\frac{2\sqrt{2}}{\pi}-\frac{1}{24}\times\frac{2\sqrt{2}}{\pi}\right)+C_{\text{MRB}}\right)^6+\pi+\frac{\phi}{4}$$

C_{MRB} is the MRB constant

ϕ is the golden ratio

Decimal approximation

4095.984583586174385345283328221911505925751749539519557042293292…

4095.984583586…. ≈ 4096

The number 4096 = 64^2, is the Ramanujan Recurring Number, that when multiplied by 2 give 8192. The total amplitude vanishes for gauge group SO(8192) for bosonic string SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, SO(2^{13}) i.e. SO(8192). This could be the implications for a pre-big bang scenario where only self-perturbative bosonic strings lived when the enthalpy was extremely low as discussed above. This regime contains all the intrinsic properties of superstrings inherent in the bosonic strings, would at the big bang give effect to the properties of matter (fermions) as Higgs Boson. This number theoretic connection to the gauge group SO(8192), gives a much more compelling relevance of the bosonic string theory SO(8192), to quantum gravity and places this string theory where it should appropriately be in the evolution of the universe from a quantum gravity perspective rather than it be neglected because it doesn't include fermionic strings to confirm to post big-bang reality.

The vanishing of the bosonic string's amplitude could be explained by the effect of extreme low entropy on the quantum vacuum geometry. Thus, as the entropy increases infinitesimally as a result of the vacuum self-perturbation then also is the amplitude of the vibrating bosonic strings from zero. Thus, was right to indicate that the "vanishing of the amplitude of the bosonic string could be the results of string theory itself", but here, we give a much more elaborate explanation of what could be happening.

27*sqrt((1+e^((-2√2)/Pi)/(1+e^((-2√2)/Pi))^2 + (-e^(-(2√2)/Pi))^4/((1+e^((-2√2)/Pi))^2(1+e^((-2√2)/Pi)^2))^2+sqrt(Pi/((2√2)/Pi)) exp((Pi^2)/24*((2√2)/Pi)-((2√2)/Pi)/24)+MRB const)^6+Pi+φ/4)+1

Input

$$27\sqrt{\left(\left(1+\frac{e^{(-2\sqrt{2})/\pi}}{\left(1+e^{(-2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+e^{(-2\sqrt{2})/\pi}\right)^2\left(1+e^{((-2\sqrt{2})/\pi)^2}\right)\right)^2}+\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\exp\left(\frac{\pi^2}{24}\times\frac{2\sqrt{2}}{\pi}-\frac{1}{24}\times\frac{2\sqrt{2}}{\pi}\right)+C_{\text{MRB}}\right)^6+\pi+\frac{\phi}{4}\right)}+1$$

C_{MRB} is the MRB constant

ϕ is the golden ratio

Decimal approximation

1728.996748097149444601337044889227156486870350667037737887887586 8

…

1728.996748….

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve $1728 = 8^2 \times 3^3$.

The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways $10^3 + 9^3 = 12^3 + 1^3 = 1729$ and Ramanujan's recurring number). Since bosons are made of gauge bosons and scalar bosons (meson), then this number theoretic analysis perhaps confirms that the number 1729, confirm the fact that both the gauge and scalar bosons are actually different states of a single bosonic string, and that these states are isomorphic or that the states vibrations are synchronised with the state of the bosonic string. This also imply that each state lives inside a cubic or octahedron as a spherical cloud, and that the total sum of these two states is the state of the bosonic string. Taking the cross section of the bosonic string, we realise that it must be a rectangular, or a two shaped octahedron. As the string vibrates in difference frequencies, so is the two spherical cloud states inside the string. That is, the string vibrations simply excites the gauge bosons i.e Photon, gluon, W and Z inside one cube/octahedron, and the scalar boson i.e. Higgs inside the other cube/octahedron.

Furthermore, if we bring the picture of loop quantum gravity (LQG) with the property of a discontinues quantum geometry, we can therefore, think of the graviton living on the vertices of the rectangles or the octahedrons. This graviton then acts a glue binding the bosonic strings lattice together forming a complete cross section of alternating states of between the gauge bosons and scalar bosons. This arrangement of states then gives a precise supersymmetric quantum picture of the vacuum geometry at low entropy.

But the geometry further reveals very important fact, that since the vacuum geometry is discontinues, then we observe that there is no relation whatsoever between the quantum vibrational frequencies of the strings, and that of the vertices of the vacuum geometry where the graviton lives. Ashtekar et al., (2021) asserted that gravity is simply a manifestation of spacetime geometry. Thus, the graviton cannot be a string boson, however, there is a duality between gravity and strings. Also, gauge bosons have spin-1, while the graviton has spin-2. Then lastly, because of the thermodynamic constraints we were able to arrive at the results we have, now this bring us to this fundamental question; that string theory and LQG theory are two intrinsic aspects of a complete quantum gravity theory we are after? That is, without the other no complete and compelling quantum geometry can be attained, as it is done here? This need to be investigated further.

(27*sqrt((1+e^((-2√2)/Pi)/(1+e^((-2√2)/Pi))^2 + (-e^(-(2√2)/Pi))^4/((1+e^((-2√2)/Pi))^2(1+e^((-2√2)/Pi)^2))^2+sqrt(Pi/((2√2)/Pi)) exp((Pi^2)/24*((2√2)/Pi)-((2√2)/Pi)/24)+MRB const)^6+Pi+φ/4)+1)^1/15

Input

$$\left(27\sqrt{\left(\left(1+\frac{e^{(-2\sqrt{2})/\pi}}{\left(1+e^{(-2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(\left(1+e^{(-2\sqrt{2})/\pi}\right)^2\left(1+e^{((-2\sqrt{2})/\pi)^2}\right)\right)^2}+\sqrt{\frac{\pi}{\frac{2\sqrt{2}}{\pi}}}\exp\left(\frac{\pi^2}{24}\times\frac{2\sqrt{2}}{\pi}-\frac{1}{24}\times\frac{2\sqrt{2}}{\pi}\right)+C_{\text{MRB}}\right)^6+\pi+\frac{\phi}{4}\right)+1\right)\text{^}(1/15)$$

C_{MRB} is the MRB constant

ϕ is the golden ratio

Decimal approximation

1.643815022636057223905443873899072228649658835082206961292178236 8…

1.643815022636….

1.643815022636+(MRB const)^(1-1/(4π)+π)

Input interpretation

$$1.643815022636 + C_{\text{MRB}}{}^{1-1/(4\pi)+\pi}$$

C_{MRB} is the MRB constant

Result

1.644937818018…

1.644937818018…. ≈ $\zeta(2) = \pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Now, we take the Mock ϑ-functions (of 7th order)

$$\text{(i)}\quad 1+\frac{q}{1-q^2}+\frac{q^4}{(1-q^3)(1-q^4)}+\frac{q^9}{(1-q^4)(1-q^5)(1-q^6)}+\cdots$$

$$\text{(ii)}\quad \frac{q}{1-q}+\frac{q^4}{(1-q^2)(1-q^3)}+\frac{q^9}{(1-q^3)(1-q^4)(1-q^5)}+\cdots$$

$$\text{(iii)}\quad \frac{1}{1-q}+\frac{q^2}{(1-q^2)(1-q^3)}+\frac{q^6}{(1-q^3)(1-q^4)(1-q^5)}+\cdots$$

always for q = -e^((-(2√2)/Pi))

1+(-e^((-(2√2)/π)))/(1-(-e^((-(2√2)/π)))^2)+(-e^((-(2√2)/π)))^4/(1-(((-e^((-(2√2)/π))))^3(1-(((-e^((-(2√2)/π)))))^4)))

Input

$$1-\frac{e^{-(2\sqrt{2})/\pi}}{1-\left(-e^{-(2\sqrt{2})/\pi}\right)^2}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{1-\left(-e^{-(2\sqrt{2})/\pi}\right)^3\left(1-\left(-e^{-(2\sqrt{2})/\pi}\right)^4\right)}$$

Result

$$1-\frac{e^{-(2\sqrt{2})/\pi}}{1-e^{-(4\sqrt{2})/\pi}}+\frac{e^{-(8\sqrt{2})/\pi}}{1+e^{-(6\sqrt{2})/\pi}\left(1-e^{-(8\sqrt{2})/\pi}\right)}$$

Decimal approximation

0.538747092478804829075911208032086113566534341039389063537551074 7
…
0.5387470924788….

Alternate forms

$$\frac{e^{(6\sqrt{2})/\pi}\left(e^{(4\sqrt{2})/\pi}-1\right)+\left(-1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\left(-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}\right)}{\left(e^{(4\sqrt{2})/\pi}-1\right)\left(-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}\right)}$$

$$1+\frac{e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}-\frac{1}{2}\operatorname{csch}\left(\frac{2\sqrt{2}}{\pi}\right)$$

$$1+\frac{e^{(2\sqrt{2})/\pi}}{1-e^{(4\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}$$

csch(x) is the hyperbolic cosecant function

(-e^((-(2√2)/π)))^9/((1-(-e^((-(2√2)/π)))^4)(1-(-e^((-(2√2)/π)))^5)(1-(-e^((-(2√2)/π)))^6))

Input

$$\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^9}{\left(1-\left(-e^{-(2\sqrt{2})/\pi}\right)^4\right)\left(1-\left(-e^{-(2\sqrt{2})/\pi}\right)^5\right)\left(1-\left(-e^{-(2\sqrt{2})/\pi}\right)^6\right)}$$

Result

$$-\frac{e^{-(18\sqrt{2})/\pi}}{\left(1-e^{-(12\sqrt{2})/\pi}\right)\left(1+e^{-(10\sqrt{2})/\pi}\right)\left(1-e^{-(8\sqrt{2})/\pi}\right)}$$

Decimal approximation

−0.000309147953904158715929647125626711593885939736945946370560996
…

-0.0003091479539….

Alternate forms

$$-\frac{e^{(12\sqrt{2})/\pi}}{\left(e^{(8\sqrt{2})/\pi}-1\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(e^{(12\sqrt{2})/\pi}-1\right)}$$

$$-\frac{e^{(12\sqrt{2})/\pi}}{\left(1-e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(1-e^{(12\sqrt{2})/\pi}\right)}$$

$$-\left(e^{(12\sqrt{2})/\pi}\Big/\left(\left(e^{\sqrt{2}/\pi}-1\right)^2\left(1+e^{\sqrt{2}/\pi}\right)^2\left(1+e^{(2\sqrt{2})/\pi}\right)^3\left(1-e^{\sqrt{2}/\pi}+e^{(2\sqrt{2})/\pi}\right)\right.\right.$$
$$\left(1+e^{\sqrt{2}/\pi}+e^{(2\sqrt{2})/\pi}\right)\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)$$
$$\left.\left.\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)\right)$$

From the sum and multiplying by 3:

3(1 + e^((2 √(2))/π)/(1 - e^((4 √(2))/π))+e^((6√(2))/π)/(-1 + e^((8√(2))/π) + e^((14√(2))/π))-e^((12√(2))/π)/((e^((8√(2))/π)-1)(1+e^((10√(2))/π)) (e^((12√(2))/π) - 1)))

Input

$$3\left(1+\frac{e^{(2\sqrt{2})/\pi}}{1-e^{(4\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}-\frac{e^{(12\sqrt{2})/\pi}}{\left(e^{(8\sqrt{2})/\pi}-1\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(e^{(12\sqrt{2})/\pi}-1\right)}\right)$$

Decimal approximation

1.615313833574702011079944682719378205917945203907329351500970233 8…

1.6153138335747…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate forms

$$3-\frac{3\,e^{(12\sqrt{2})/\pi}}{\left(e^{(8\sqrt{2})/\pi}-1\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(e^{(12\sqrt{2})/\pi}-1\right)}+\frac{3\,e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}-\frac{3}{2}$$

$$\operatorname{csch}\left(\frac{2\sqrt{2}}{\pi}\right)$$

$$3-\frac{1}{64\left(e^{\sqrt{2}/\pi}-1\right)^2}-\frac{23}{32\left(e^{\sqrt{2}/\pi}-1\right)}-\frac{1}{64\left(1+e^{\sqrt{2}/\pi}\right)^2}+\frac{23}{32\left(1+e^{\sqrt{2}/\pi}\right)}-$$
$$\frac{1}{40\left(1+e^{(2\sqrt{2})/\pi}\right)^3}-\frac{683}{480\left(1+e^{(2\sqrt{2})/\pi}\right)}-\frac{e^{\sqrt{2}/\pi}}{4\left(1-e^{\sqrt{2}/\pi}+e^{(2\sqrt{2})/\pi}\right)}+$$
$$\frac{e^{\sqrt{2}/\pi}}{4\left(1+e^{\sqrt{2}/\pi}+e^{(2\sqrt{2})/\pi}\right)}-\frac{3\left(e^{(2\sqrt{2})/\pi}-1\right)}{8\left(1+e^{(4\sqrt{2})/\pi}\right)}+\frac{e^{(2\sqrt{2})/\pi}-2}{6\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)}+$$
$$\frac{3\,e^{(2\sqrt{2})/\pi}\left(2-2\,e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)}{5\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)}+\frac{3\,e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}$$

$$\left(3\left(-1-e^{(2\sqrt{2})/\pi}+2\,e^{(8\sqrt{2})/\pi}+e^{(12\sqrt{2})/\pi}+2\,e^{(14\sqrt{2})/\pi}+2\,e^{(18\sqrt{2})/\pi}-\right.\right.$$
$$e^{(20\sqrt{2})/\pi}-e^{(22\sqrt{2})/\pi}+2\,e^{(24\sqrt{2})/\pi}-2\,e^{(26\sqrt{2})/\pi}-e^{(30\sqrt{2})/\pi}-$$
$$\left.\left.3\,e^{(32\sqrt{2})/\pi}+e^{(34\sqrt{2})/\pi}-e^{(36\sqrt{2})/\pi}-e^{(42\sqrt{2})/\pi}+e^{(44\sqrt{2})/\pi}\right)\right)/$$
$$\left(\left(e^{\sqrt{2}/\pi}-1\right)^2\left(1+e^{\sqrt{2}/\pi}\right)^2\left(1+e^{(2\sqrt{2})/\pi}\right)^3\left(1-e^{\sqrt{2}/\pi}+e^{(2\sqrt{2})/\pi}\right)\right.$$
$$\left(1+e^{\sqrt{2}/\pi}+e^{(2\sqrt{2})/\pi}\right)\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)$$
$$\left.\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\left(-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}\right)\right)$$

csch(x) is the hyperbolic cosecant function

Expanded form

$$3+\frac{3\,e^{(2\sqrt{2})/\pi}}{1-e^{(4\sqrt{2})/\pi}}-\frac{3\,e^{(12\sqrt{2})/\pi}}{\left(e^{(8\sqrt{2})/\pi}-1\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(e^{(12\sqrt{2})/\pi}-1\right)}+\frac{3\,e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}$$

Series representations

$$3\left(1+\frac{e^{(2\sqrt{2})/\pi}}{1-e^{(4\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}-\frac{e^{(12\sqrt{2})/\pi}}{\left(e^{(8\sqrt{2})/\pi}-1\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(e^{(12\sqrt{2})/\pi}-1\right)}\right)=$$

$$3+\frac{3\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)}{1-\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)}-$$

$$\left(3\exp\left(\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\Bigg/$$

$$\left(\left(-1+\exp\left(\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\left(1+\exp\left(\frac{10\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\left(-1+\exp\left(\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right)+$$

$$\left(3\exp\left(\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\Bigg/$$

$$\left(-1+\exp\left(\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+\exp\left(\frac{14\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$3\left(1+\frac{e^{(2\sqrt{2})/\pi}}{1-e^{(4\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}-\frac{e^{(12\sqrt{2})/\pi}}{\left(e^{(8\sqrt{2})/\pi}-1\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(e^{(12\sqrt{2})/\pi}-1\right)}\right)=$$

$$3+\frac{3\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)}{1-\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)}-$$

$$\left(3\exp\left(\frac{12\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\Bigg/$$

$$\left(\left(-1+\exp\left(\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right.$$

$$\left(1+\exp\left(\frac{10\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)$$

$$\left.\left(-1+\exp\left(\frac{12\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right)+$$

$$\left(3\exp\left(\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\Bigg/$$

$$\left(-1+\exp\left(\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+\right.$$

$$\left.\exp\left(\frac{14\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$3\left(1+\frac{e^{(2\sqrt{2})/\pi}}{1-e^{(4\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}-\frac{e^{(12\sqrt{2})/\pi}}{\left(e^{(8\sqrt{2})/\pi}-1\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(e^{(12\sqrt{2})/\pi}-1\right)}\right)=$$

$$3+\frac{3\,e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}}{1-e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}}-$$

$$\left(3\,e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\Bigg/$$

$$\left(\left(-1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right.$$

$$\left(1+e^{\frac{10\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(-1+\left(e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)\Bigg)+$$

$$\left(3\,e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\Bigg/$$

$$\left(\left(-1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)+\right.$$

$$\left.\left(e^{\frac{14\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And:

(3(1 + e^((2 √(2))/π)/(1 - e^((4 √(2))/π))+e^((6√(2))/π)/(-1 + e^((8√(2))/π) + e^((14√(2))/π))-e^((12√(2))/π)/((e^((8√(2))/π)-1)(1+e^((10√(2))/π)) (e^((12√(2))/π) - 1))))1/10^35

Input

$$\left(3\left(1+\frac{e^{(2\sqrt{2})/\pi}}{1-e^{(4\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}-\frac{e^{(12\sqrt{2})/\pi}}{\left(e^{(8\sqrt{2})/\pi}-1\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(e^{(12\sqrt{2})/\pi}-1\right)}\right)\right)\times\frac{1}{10^{35}}$$

Exact result

$$\frac{3\left(1+\frac{e^{(2\sqrt{2})/\pi}}{1-e^{(4\sqrt{2})/\pi}}-\frac{e^{(12\sqrt{2})/\pi}}{\left(e^{(8\sqrt{2})/\pi}-1\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(e^{(12\sqrt{2})/\pi}-1\right)}+\frac{e^{(6\sqrt{2})/\pi}}{-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}}\right)}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

Decimal approximation

$1.615313833574702011079944682719378205917945203907329351500 9\ldots \times 10^{-35}$

1.6153138…*10^{-35} ≈ 1.616255*10^{-35} (Planck Length)

Expanded form

$$\frac{3}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} + \frac{3\,e^{(2\sqrt{2})/\pi}}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-e^{(4\sqrt{2})/\pi}\right)} - \left(3\,e^{(12\sqrt{2})/\pi}\right)\Big/\Big(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(e^{(8\sqrt{2})/\pi}-1\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\left(e^{(12\sqrt{2})/\pi}-1\right)\Big) + \frac{3\,e^{(6\sqrt{2})/\pi}}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}\right)}$$

Alternate forms

$$\Big(3\Big(-1-e^{(2\sqrt{2})/\pi}+2\,e^{(8\sqrt{2})/\pi}+e^{(12\sqrt{2})/\pi}+2\,e^{(14\sqrt{2})/\pi}+2\,e^{(18\sqrt{2})/\pi}- e^{(20\sqrt{2})/\pi}-e^{(22\sqrt{2})/\pi}+2\,e^{(24\sqrt{2})/\pi}-2\,e^{(26\sqrt{2})/\pi}-e^{(30\sqrt{2})/\pi}- 3\,e^{(32\sqrt{2})/\pi}+e^{(34\sqrt{2})/\pi}-e^{(36\sqrt{2})/\pi}-e^{(42\sqrt{2})/\pi}+e^{(44\sqrt{2})/\pi}\Big)\Big)\Big/ \Big(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(e^{\sqrt{2}/\pi}-1\right)^2 \left(1+e^{\sqrt{2}/\pi}\right)^2\left(1+e^{(2\sqrt{2})/\pi}\right)^3\left(1-e^{\sqrt{2}/\pi}+e^{(2\sqrt{2})/\pi}\right) \left(1+e^{\sqrt{2}/\pi}+e^{(2\sqrt{2})/\pi}\right)\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right) \left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\left(-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}\right)\Big)$$

$$\frac{3}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000} -$$
$$\frac{1}{6\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(e^{\sqrt{2}/\pi}-1\right)^2} -$$
$$\frac{23}{3\,200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(e^{\sqrt{2}/\pi}-1\right)} -$$
$$\frac{1}{6\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{\sqrt{2}/\pi}\right)^2} +$$
$$\frac{23}{3\,200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{\sqrt{2}/\pi}\right)} -$$
$$\frac{1}{4\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(2\sqrt{2})/\pi}\right)^3} -$$
$$\frac{683}{48\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(2\sqrt{2})/\pi}\right)} -$$
$$\frac{e^{\sqrt{2}/\pi}}{400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-e^{\sqrt{2}/\pi}+e^{(2\sqrt{2})/\pi}\right)} +$$
$$\frac{e^{\sqrt{2}/\pi}}{400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{\sqrt{2}/\pi}+e^{(2\sqrt{2})/\pi}\right)} -$$
$$\frac{3\left(e^{(2\sqrt{2})/\pi}-1\right)}{800\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(4\sqrt{2})/\pi}\right)} +$$
$$\frac{e^{(2\sqrt{2})/\pi}-2}{600\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)} +$$
$$\left(3\,e^{(2\sqrt{2})/\pi}\left(2-2\,e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\right)\Big/$$
$$\left(500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right) +$$
$$\frac{3\,e^{(6\sqrt{2})/\pi}}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(-1+e^{(8\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}\right)}$$

Series representations

$$\frac{3\left(1+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{1-e^{\left(4\sqrt{2}\right)/\pi}}+\frac{e^{\left(6\sqrt{2}\right)/\pi}}{-1+e^{\left(8\sqrt{2}\right)/\pi}+e^{\left(14\sqrt{2}\right)/\pi}}-\frac{e^{\left(12\sqrt{2}\right)/\pi}}{\left(e^{\left(8\sqrt{2}\right)/\pi}-1\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)\left(e^{\left(12\sqrt{2}\right)/\pi}-1\right)}\right)}{10^{35}}$$

$$=\frac{3}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}+$$

$$\left(3\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\Bigg/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right)-$$

$$\left(3\exp\left(\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\Bigg/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(-1+\exp\left(\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\left(1+\exp\left(\frac{10\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\left(-1+\exp\left(\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right)+$$

$$\left(3\exp\left(\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\Bigg/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(-1+\exp\left(\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+\exp\left(\frac{14\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\frac{3\left(1+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{1-e^{\left(4\sqrt{2}\right)/\pi}}+\frac{e^{\left(6\sqrt{2}\right)/\pi}}{-1+e^{\left(8\sqrt{2}\right)/\pi}+e^{\left(14\sqrt{2}\right)/\pi}}-\frac{e^{\left(12\sqrt{2}\right)/\pi}}{\left(e^{\left(8\sqrt{2}\right)/\pi}-1\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)\left(e^{\left(12\sqrt{2}\right)/\pi}-1\right)}\right)}{10^{35}}$$

$$=\frac{3}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}+$$

$$\left(3\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\Big/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right)-$$

$$\left(3\exp\left(\frac{12\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\Big/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(-1+\exp\left(\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\left(1+\exp\left(\frac{10\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\left(-1+\exp\left(\frac{12\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right)+$$

$$\left(3\exp\left(\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\Big/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(-1+\exp\left(\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+\exp\left(\frac{14\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{3\left(1+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{1-e^{\left(4\sqrt{2}\right)/\pi}}+\frac{e^{\left(6\sqrt{2}\right)/\pi}}{-1+e^{\left(8\sqrt{2}\right)/\pi}+e^{\left(14\sqrt{2}\right)/\pi}}-\frac{e^{\left(12\sqrt{2}\right)/\pi}}{\left(e^{\left(8\sqrt{2}\right)/\pi}-1\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)\left(e^{\left(12\sqrt{2}\right)/\pi}-1\right)}\right)}{10^{35}}$$

$$=\frac{3}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}+$$

$$\left(3\,e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\Big/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\right)-$$

$$\left(3\,e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\Big/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(-1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{10\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\left(-1+e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\right)+$$

$$\left(3\,e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\Big/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(-1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}+e^{\frac{14\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$$\frac{q}{1-q} + \frac{q^4}{(1-q^2)(1-q^3)} + \frac{q^9}{(1-q^3)(1-q^4)(1-q^5)} + \cdots$$

we obtain, for q = -e^((-(2√2)/Pi))

e^((-2√2)/π)/(1+(e^((-2√2)/π)))+(-e^(-(2√2)/π))^4/(((1+(e^((-2√2)/π))^2)(1+(e^((-2√2)/π))^3)))

Input

$$\frac{e^{(-2\sqrt{2})/\pi}}{1+e^{(-2\sqrt{2})/\pi}} + \frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(1+\left(e^{(-2\sqrt{2})/\pi}\right)^2\right)\left(1+\left(e^{(-2\sqrt{2})/\pi}\right)^3\right)}$$

Result

$$\frac{e^{-(8\sqrt{2})/\pi}}{\left(1+e^{-(6\sqrt{2})/\pi}\right)\left(1+e^{-(4\sqrt{2})/\pi}\right)} + \frac{e^{-(2\sqrt{2})/\pi}}{1+e^{-(2\sqrt{2})/\pi}}$$

Decimal approximation

0.3109322316231720829653596465312039207038037010166526287107160117…

0.3109322316….

Alternate forms

$$\frac{e^{-(2\sqrt{2})/\pi}}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{-(8\sqrt{2})/\pi}}{1+e^{-(10\sqrt{2})/\pi}+e^{-(6\sqrt{2})/\pi}+e^{-(4\sqrt{2})/\pi}}$$

$$\frac{1}{1+e^{(2\sqrt{2})/\pi}}-\frac{1}{4}\left(\tanh\left(\frac{3\sqrt{2}}{\pi}\right)-1\right)\operatorname{sech}\left(\frac{2\sqrt{2}}{\pi}\right)$$

$$\frac{1}{1+e^{(2\sqrt{2})/\pi}}+\frac{e^{(2\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}$$

sech(x) is the hyperbolic secant function

tanh(x) is the hyperbolic tangent function

Series representations

$$\frac{e^{-(2\sqrt{2})/\pi}}{1+e^{-(2\sqrt{2})/\pi}}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^2\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^3\right)}=$$
$$\left(\left(\left(1-\exp\left(\frac{3\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+\right.\right.\right.$$
$$\left.\left.\left.\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right.$$
$$\left(1+\exp\left(\frac{3\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+\right.$$
$$\left.\left.\left.\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right)\Big/$$
$$\left(\left(\left(1+\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right.\right.$$
$$\left(1+\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)$$
$$\left(1-\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+\right.$$
$$\left.\left.\left.\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{e^{-(2\sqrt{2})/\pi}}{1+e^{-(2\sqrt{2})/\pi}}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^2\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^3\right)}=$$

$$\left(\left(\left(1-\exp\left(\frac{3\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+\right.\right.\right.$$

$$\left.\left.\left.\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right.$$

$$\left(1+\exp\left(\frac{3\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+\right.$$

$$\left.\left.\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right)\Big/$$

$$\left(\left(\left(1+\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right.\right.\right.$$

$$\left(1+\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)$$

$$\left(1-\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+\right.$$

$$\left.\left.\left.\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right)\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{e^{-(2\sqrt{2})/\pi}}{1+e^{-(2\sqrt{2})/\pi}}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^4}{\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^2\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^3\right)}=$$

$$\left(\left(\left(1-e^{\frac{3\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+\right.\right.\right.$$

$$\left.\left.\left.e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right.$$

$$\left(1+e^{\frac{3\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+\right.$$

$$\left.\left.e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)\Big/$$

$$\left(\left(\left(1+e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right.\right.$$

$$\left(1+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+\right.$$

$$\left.\left.\left.e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

(((-e^((-2√2)/π))))^9 / (((((((1+(e^((-2√2)/π))^3))) (((1+(e^((-2√2)/π))^4))) (((1+(e^((-2√2)/π))^5)))))))

Input

$$\frac{\left(-e^{(-2\sqrt{2})/\pi}\right)^9}{\left(1+\left(e^{(-2\sqrt{2})/\pi}\right)^3\right)\left(1+\left(e^{(-2\sqrt{2})/\pi}\right)^4\right)\left(1+\left(e^{(-2\sqrt{2})/\pi}\right)^5\right)}$$

Result

$$-\frac{e^{-(18\sqrt{2})/\pi}}{\left(1+e^{-(10\sqrt{2})/\pi}\right)\left(1+e^{-(8\sqrt{2})/\pi}\right)\left(1+e^{-(6\sqrt{2})/\pi}\right)}$$

Decimal approximation

$$-0.000273069417522167559363546527185687764115925695918122044807825\ldots$$

-0.000273069417….

Alternate forms

$$-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}$$

$$-\left(e^{(6\sqrt{2})/\pi} \Big/ \left(\left(1+e^{(2\sqrt{2})/\pi}\right)^2\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)\right)$$

$$\frac{1}{30\left(1+e^{(2\sqrt{2})/\pi}\right)^2}+\frac{1}{15\left(1+e^{(2\sqrt{2})/\pi}\right)}+\frac{e^{(2\sqrt{2})/\pi}}{3\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)}+$$
$$\frac{1-e^{(4\sqrt{2})/\pi}-2e^{(6\sqrt{2})/\pi}}{2\left(1+e^{(8\sqrt{2})/\pi}\right)}+\frac{-3+2e^{(2\sqrt{2})/\pi}-2e^{(4\sqrt{2})/\pi}+3e^{(6\sqrt{2})/\pi}}{5\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)}$$

Series representations

$$\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^9}{\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^3\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^4\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^5\right)} =$$

$$-\left(\exp\left(\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\Bigg/\left(\left(1+\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)^2\right.\right.$$

$$\left(1-\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+\right.$$

$$\left.\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)$$

$$\left(1+\exp\left(\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)$$

$$\left(1-\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+\right.$$

$$\left.\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)-\right.$$

$$\left.\exp\left(\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+\right.$$

$$\left.\left.\left.\exp\left(\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right)\right)$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$$\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^9}{\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^3\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^4\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^5\right)} =$$

$$-\left(\exp\left(\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\Bigg/$$

$$\left(\left(1+\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)^2\right.$$

$$\left(1-\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+\right.$$

$$\left.\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)$$

$$\left(1+\exp\left(\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)$$

$$\left(1-\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+\right.$$

$$\left.\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)-\right.$$

$$\left.\exp\left(\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+\right.$$

$$\left.\left.\left.\exp\left(\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right)\right)\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^9}{\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^3\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^4\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^5\right)} =$$

$$-\left(e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right) \Bigg/$$

$$\left(\left(\left(1+e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2 \left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right) \left(1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right) \left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Thus, we obtain:

1/(1 + e^((2 sqrt(2))/π)) + e^((2 sqrt(2))/π)/((1 + e^((4 sqrt(2))/π)) (1 + e^((6 sqrt(2))/π)))-e^((6 sqrt(2))/π)/((1 + e^((6 sqrt(2))/π)) (1 + e^((8 sqrt(2))/π)) (1 + e^((10 sqrt(2))/π)))

Input

$$\frac{1}{1+e^{(2\sqrt{2})/\pi}}+\frac{e^{(2\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}$$

Decimal approximation

0.3106591622056499154059961000040182329396877753207345066659081859…

0.3106591622….

Alternate forms

$$\frac{1}{1+e^{(2\sqrt{2})/\pi}}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}-\frac{1}{4}\left(\tanh\left(\frac{3\sqrt{2}}{\pi}\right)-1\right)\operatorname{sech}\left(\frac{2\sqrt{2}}{\pi}\right)$$

$$\left(1+2e^{(4\sqrt{2})/\pi}-2e^{(6\sqrt{2})/\pi}+2e^{(8\sqrt{2})/\pi}+2e^{(12\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}+2e^{(18\sqrt{2})/\pi}+2e^{(22\sqrt{2})/\pi}-e^{(24\sqrt{2})/\pi}+e^{(26\sqrt{2})/\pi}\right)\Big/\left(\left(1+e^{(2\sqrt{2})/\pi}\right)^2\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)$$

$$\frac{1}{30\left(1+e^{(2\sqrt{2})/\pi}\right)^2}+\frac{9}{10\left(1+e^{(2\sqrt{2})/\pi}\right)}+\frac{e^{(2\sqrt{2})/\pi}-1}{2\left(1+e^{(4\sqrt{2})/\pi}\right)}+\frac{2}{3\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)}+\frac{1-e^{(4\sqrt{2})/\pi}-2e^{(6\sqrt{2})/\pi}}{2\left(1+e^{(8\sqrt{2})/\pi}\right)}+\frac{-3+2e^{(2\sqrt{2})/\pi}-2e^{(4\sqrt{2})/\pi}+3e^{(6\sqrt{2})/\pi}}{5\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)}$$

sech(x) is the hyperbolic secant function

tanh(x) is the hyperbolic tangent function

Series representations

$$\frac{1}{1+e^{(2\sqrt{2})/\pi}}+\frac{e^{(2\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}=$$

$$\left(1+2e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-2e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{14\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{18\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{22\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{24\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{26\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\Bigg/$$

$$\left(\left(1+e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2\left(1+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{1+e^{(2\sqrt{2})/\pi}}+\frac{e^{(2\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}=$$

$$\left(1+2e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-\right.$$

$$2e^{\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{12\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$e^{\frac{14\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{18\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{22\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-$$

$$e^{\frac{24\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$\left.e^{\frac{26\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)\Bigg/$$

$$\left(\left(1+e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)^2\right.$$

$$\left(1+e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+\right.$$

$$\left.e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1+e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+\right.$$

$$e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-$$

$$e^{\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$\left.\left.e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$\frac{1}{1+e^{(2\sqrt{2})/\pi}}+\frac{e^{(2\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}=$$

$$\left(1+2e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}-2e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{14\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{18\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{22\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{24\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{26\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\Bigg)\Bigg/$$

$$\left(\left(1+e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2\left(1+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which, we obtain:

(3√3)*(1/(1 + e^((2 sqrt(2))/π)) + e^((2 sqrt(2))/π)/((1 + e^((4 sqrt(2))/π)) (1 + e^((6 sqrt(2))/π)))-e^((6 sqrt(2))/π)/((1 + e^((6 sqrt(2))/π)) (1 + e^((8 sqrt(2))/π)) (1 + e^((10 sqrt(2))/π))))

Input

$$\left(3\sqrt{3}\right)\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)$$

Exact result

$$3\sqrt{3}\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)$$

Decimal approximation

1.614232358330900338030474048179911892150109617009073420360494760 2
…

1.61423235833…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate forms

$$3\sqrt{3}\left(\frac{1}{1+e^{(2\sqrt{2})/\pi}}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}-\frac{1}{4}\left(\tanh\left(\frac{3\sqrt{2}}{\pi}\right)-1\right)\operatorname{sech}\left(\frac{2\sqrt{2}}{\pi}\right)\right)$$

$$\left(3\sqrt{3}\left(1+2e^{(4\sqrt{2})/\pi}-2e^{(6\sqrt{2})/\pi}+2e^{(8\sqrt{2})/\pi}+2e^{(12\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}+2e^{(18\sqrt{2})/\pi}+2e^{(22\sqrt{2})/\pi}-e^{(24\sqrt{2})/\pi}+e^{(26\sqrt{2})/\pi}\right)\right)\Big/\left(\left(1+e^{(2\sqrt{2})/\pi}\right)^2\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)$$

$$\frac{\sqrt{3}}{10\left(1+e^{(2\sqrt{2})/\pi}\right)^2}+\frac{27\sqrt{3}}{10\left(1+e^{(2\sqrt{2})/\pi}\right)}+\frac{3\sqrt{3}\left(e^{(2\sqrt{2})/\pi}-1\right)}{2\left(1+e^{(4\sqrt{2})/\pi}\right)}+\frac{2\sqrt{3}}{1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}}-\frac{3\sqrt{3}\left(-1+e^{(4\sqrt{2})/\pi}+2e^{(6\sqrt{2})/\pi}\right)}{2\left(1+e^{(8\sqrt{2})/\pi}\right)}+\frac{3\sqrt{3}\left(-3+2e^{(2\sqrt{2})/\pi}-2e^{(4\sqrt{2})/\pi}+3e^{(6\sqrt{2})/\pi}\right)}{5\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)}$$

sech(x) is the hyperbolic secant function

tanh(x) is the hyperbolic tangent function

Expanded form

$$\frac{3\sqrt{3}}{1+e^{(2\sqrt{2})/\pi}}+\frac{3\sqrt{3}\ e^{(2\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\frac{3\sqrt{3}\ e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}$$

Series representations

$$\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)3\sqrt{3}=$$

$$\left(3\left(1+2e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-2e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{14\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{18\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{22\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{24\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{26\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\Bigg/$$

$$\left(\left(1+e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2\left(1+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\left(\frac{1}{1+e^{(2\sqrt{2})/\pi}}+\frac{e^{(2\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)3\sqrt{3}=$$

$$\left(3\left(1+2e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-\right.\right.$$

$$2e^{\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{12\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$e^{\frac{14\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{18\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{22\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-$$

$$e^{\frac{24\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$\left.e^{\frac{26\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left.\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/$$

$$\left(\left(\left(1+e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)^2\right.\right.$$

$$\left(1+e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+\right.$$

$$\left.e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1+e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+\right.$$

$$e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-$$

$$e^{\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$\left.\left.e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$\left(\frac{1}{1+e^{(2\sqrt{2})/\pi}}+\frac{e^{(2\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)3\sqrt{3}=$$

$$\left(3\left(1+2e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-2e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{14\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{18\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{22\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{24\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{26\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(3-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(3-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\Bigg)\Bigg/$$

$$\left(\left(1+e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2\left(1+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And:

((3√3)*(1/(1 + e^((2 sqrt(2))/π)) + e^((2 sqrt(2))/π)/((1 + e^((4 sqrt(2))/π)) (1 + e^((6 sqrt(2))/π)))-e^((6 sqrt(2))/π)/((1 + e^((6 sqrt(2))/π))(1+e^((8 sqrt(2))/π))(1+e^((10 sqrt(2))/π)))))1/10^35

Input

$$\left(\left(3\sqrt{3}\right)\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)\right)\times\frac{1}{10^{35}}$$

Exact result

$$\left(3\sqrt{3}\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)\right)\Big/ 100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000$$

Decimal approximation

$1.614232358330900338030474048179911892150109617009073420360 4\ldots \times 10^{-35}$

1.61423235833…*10^{-35} ≈ 1.616255*10^{-35} (Planck Length)

Expanded form

$$\frac{3\sqrt{3}}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(2\sqrt{2})/\pi}\right)}+\frac{3\sqrt{3}\;e^{(2\sqrt{2})/\pi}}{100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\left(3\sqrt{3}\;e^{(6\sqrt{2})/\pi}\right)\Big/\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\right)$$

Alternate forms

$$\left(3\sqrt{3}\left(1+2\,e^{(4\sqrt{2})/\pi}-2\,e^{(6\sqrt{2})/\pi}+2\,e^{(8\sqrt{2})/\pi}+2\,e^{(12\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}+2\,e^{(18\sqrt{2})/\pi}+2\,e^{(22\sqrt{2})/\pi}-e^{(24\sqrt{2})/\pi}+e^{(26\sqrt{2})/\pi}\right)\right)\Big/\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(2\sqrt{2})/\pi}\right)^2\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)$$

$$\frac{\sqrt{3}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(2\sqrt{2})/\pi}\right)^2}+\frac{27\sqrt{3}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(2\sqrt{2})/\pi}\right)}+\frac{3\sqrt{3}\left(e^{(2\sqrt{2})/\pi}-1\right)}{200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(4\sqrt{2})/\pi}\right)}+\frac{\sqrt{3}}{50\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)}-\frac{3\sqrt{3}\left(-1+e^{(4\sqrt{2})/\pi}+2e^{(6\sqrt{2})/\pi}\right)}{200\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(8\sqrt{2})/\pi}\right)}+\left(3\sqrt{3}\left(-3+2e^{(2\sqrt{2})/\pi}-2e^{(4\sqrt{2})/\pi}+3e^{(6\sqrt{2})/\pi}\right)\right)\Big/\left(500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)$$

Series representations

$$\frac{1}{10^{35}}\left(3\sqrt{3}\right)\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)=$$

$$\left(\left(3\left(1+2e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-2e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{14\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{18\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{22\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{24\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{26\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\Big/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2\left(1+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0\right)\right)$

$$\frac{1}{10^{35}}\left(3\sqrt{3}\right)\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)=$$

$$\left(3\left(1+2\,e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-\right.\right.$$

$$2\,e^{\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2\,e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2\,e^{\frac{12\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$e^{\frac{14\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2\,e^{\frac{18\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2\,e^{\frac{22\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-$$

$$e^{\frac{24\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$\left.\left.e^{\frac{26\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)\right)$$

$$\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\Bigg)\Bigg/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\right.$$

$$\left(1+e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)^2$$

$$\left(1+e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+\right.$$

$$\left.e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1+e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+\right.$$

$$e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-$$

$$e^{\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$\left.\left.e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$\frac{1}{10^{35}}\left(3\sqrt{3}\right)\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)=$$

$$\left(3\left(1+2\,e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-\right.\right.$$

$$2\,e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2\,e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2\,e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$e^{\frac{14\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2\,e^{\frac{18\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2\,e^{\frac{22\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-$$

$$e^{\frac{24\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$\left.e^{\frac{26\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left.\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\Big/$$

$$\left(100\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\right.$$

$$\left(1+e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2$$

$$\left(1+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+\right.$$

$$\left.e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+\right.$$

$$e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-$$

$$e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$\left.\left.\left.e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And:

(3√Pi)*(1/(1 + e^((2 sqrt(2))/π)) + e^((2 sqrt(2))/π)/((1 + e^((4 sqrt(2))/π)) (1 + e^((6 sqrt(2))/π)))-e^((6 sqrt(2))/π)/((1 + e^((6 sqrt(2))/π)) (1 + e^((8 sqrt(2))/π)) (1 + e^((10 sqrt(2))/π))))

Input

$$\left(3\sqrt{\pi}\right)\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)$$

Exact result

$$3\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)\sqrt{\pi}$$

Decimal approximation

1.651887085111456604153307688380474159694285432242922646399371142 4

…

1.651887085111…. result quite near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3$ = 1164.2696 i.e. 1.6557845...

Alternate forms

$$3\sqrt{\pi}\left(\frac{1}{1+e^{(2\sqrt{2})/\pi}}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}-\frac{1}{4}\left(\tanh\left(\frac{3\sqrt{2}}{\pi}\right)-1\right)\operatorname{sech}\left(\frac{2\sqrt{2}}{\pi}\right)\right)$$

$$\left(3\left(1+2e^{(4\sqrt{2})/\pi}-2e^{(6\sqrt{2})/\pi}+2e^{(8\sqrt{2})/\pi}+2e^{(12\sqrt{2})/\pi}+e^{(14\sqrt{2})/\pi}+2e^{(18\sqrt{2})/\pi}+2e^{(22\sqrt{2})/\pi}-e^{(24\sqrt{2})/\pi}+e^{(26\sqrt{2})/\pi}\right)\sqrt{\pi}\right)\Big/\left(\left(1+e^{(2\sqrt{2})/\pi}\right)^2\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)$$

$$\frac{\sqrt{\pi}}{10\left(1+e^{(2\sqrt{2})/\pi}\right)^2}+\frac{27\sqrt{\pi}}{10\left(1+e^{(2\sqrt{2})/\pi}\right)}+\frac{3\left(e^{(2\sqrt{2})/\pi}-1\right)\sqrt{\pi}}{2\left(1+e^{(4\sqrt{2})/\pi}\right)}+\frac{2\sqrt{\pi}}{1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}}-\frac{3\left(-1+e^{(4\sqrt{2})/\pi}+2e^{(6\sqrt{2})/\pi}\right)\sqrt{\pi}}{2\left(1+e^{(8\sqrt{2})/\pi}\right)}+\frac{3\left(-3+2e^{(2\sqrt{2})/\pi}-2e^{(4\sqrt{2})/\pi}+3e^{(6\sqrt{2})/\pi}\right)\sqrt{\pi}}{5\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)}$$

sech(x) is the hyperbolic secant function

tanh(x) is the hyperbolic tangent function

Expanded form

$$\frac{3\sqrt{\pi}}{1+e^{(2\sqrt{2})/\pi}}+\frac{3e^{(2\sqrt{2})/\pi}\sqrt{\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\frac{3e^{(6\sqrt{2})/\pi}\sqrt{\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}$$

Series representations

$$\left(\frac{1}{1+e^{\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(2\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}-\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}\right)3\sqrt{\pi}=$$

$$\left(3\left(1+2e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-2e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{14\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{18\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2e^{\frac{22\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{24\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{26\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}\right)\Bigg/$$

$$\left(\left(1+e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2\left(1+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\left(\frac{1}{1+e^{(2\sqrt{2})/\pi}}+\frac{e^{(2\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)3\sqrt{\pi}=$$

$$\left(3\left(1+2e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-\right.\right.$$

$$2e^{\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{12\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$e^{\frac{14\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{18\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$2e^{\frac{22\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-$$

$$e^{\frac{24\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$\left.e^{\frac{26\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left.\exp\left(i\pi\left\lfloor\frac{\arg(\pi-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(\pi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/$$

$$\left(\left(\left(1+e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)^2\right.\right.$$

$$\left(1+e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+\right.$$

$$\left.e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1+e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+\right.$$

$$e^{\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}-$$

$$e^{\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}+$$

$$\left.\left.e^{\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}}\right)\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$\left(\frac{1}{1+e^{(2\sqrt{2})/\pi}}+\frac{e^{(2\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}-\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)3\sqrt{\pi}=$$

$$\left(3\left(1+2e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-\right.\right.$$

$$2e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$e^{\frac{14\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2e^{\frac{18\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2e^{\frac{22\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-$$

$$e^{\frac{24\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$\left.e^{\frac{26\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(\pi-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(\pi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\pi-z_0)^k z_0^{-k}}{k!}\Bigg)\Bigg/$$

$$\left(\left(\left(1+e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2\right.\right.$$

$$\left(1+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+\right.$$

$$\left.e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+\right.$$

$$e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-$$

$$e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$\left.\left.e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$$\frac{1}{1-q} + \frac{q^2}{(1-q^2)(1-q^3)} + \frac{q^6}{(1-q^3)(1-q^4)(1-q^5)} + \cdots$$

1/(1+(e^((-2√2)/π)))+(-e^(-(2√2)/π))^2/(((1+(e^((-2√2)/π))^2)(1+(e^((-2√2)/π))^3)))

Input

$$\frac{1}{1+e^{(-2\sqrt{2})/\pi}} + \frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^2}{\left(1+\left(e^{(-2\sqrt{2})/\pi}\right)^2\right)\left(1+\left(e^{(-2\sqrt{2})/\pi}\right)^3\right)}$$

Result

$$\frac{e^{-(4\sqrt{2})/\pi}}{\left(1+e^{-(6\sqrt{2})/\pi}\right)\left(1+e^{-(4\sqrt{2})/\pi}\right)} + \frac{1}{1+e^{-(2\sqrt{2})/\pi}}$$

Decimal approximation

0.843868520032452197640769690736160976118963803932917266808625751 3

…

0.84386852….

Alternate forms

$$\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{-(4\sqrt{2})/\pi}}{1+e^{-(10\sqrt{2})/\pi}+e^{-(6\sqrt{2})/\pi}+e^{-(4\sqrt{2})/\pi}}$$

$$\frac{1}{1+e^{-(6\sqrt{2})/\pi}+e^{-(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}}+\frac{1}{2}\left(1+\tanh\left(\frac{\sqrt{2}}{\pi}\right)\right)$$

$$\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}$$

tanh(x) is the hyperbolic tangent function

Series representations

$$\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^2}{\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^2\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^3\right)}=$$
$$\left(\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right.$$
$$\left(1-\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+\right.$$
$$3\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)-$$
$$\exp\left(\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+$$
$$\left.\left.\exp\left(\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right)\Big/$$
$$\left(\left(\left(1+\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right.\right.$$
$$\left(1+\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)$$
$$\left(1-\exp\left(\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)+\right.$$
$$\left.\left.\left.\exp\left(\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}\right)\right)\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^2}{\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^2\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^3\right)}=$$

$$\left(\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\left(1-\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+3\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)-\exp\left(\frac{6\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+\exp\left(\frac{8\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\Bigg/$$

$$\left(\left(\left(1+\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\left(1+\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\left(1-\exp\left(\frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)+\exp\left(\frac{4\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\pi}\right)\right)\right)$$

for $(x\in\mathbb{R}$ and $x<0)$

$$\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{\left(-e^{-(2\sqrt{2})/\pi}\right)^2}{\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^2\right)\left(1+\left(e^{-(2\sqrt{2})/\pi}\right)^3\right)}=$$

$$\left(e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right.$$

$$\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+\right.$$

$$3e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}-$$

$$e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$\left.\left.e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)\Big/$$

$$\left(\left(\left(1+e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right.\right.$$

$$\left(1+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}+\right.$$

$$\left.\left.\left.e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

(((-e^((-2√2)/π))))^6 / (((((((1+(e^((-2√2)/π))^3))) (((1+(e^((-2√2)/π))^4))) (((1+(e^((-2√2)/π))^5)))))))

Input

$$\frac{\left(-e^{(-2\sqrt{2})/\pi}\right)^6}{\left(1+\left(e^{(-2\sqrt{2})/\pi}\right)^3\right)\left(1+\left(e^{(-2\sqrt{2})/\pi}\right)^4\right)\left(1+\left(e^{(-2\sqrt{2})/\pi}\right)^5\right)}$$

Result

$$\frac{e^{-(12\sqrt{2})/\pi}}{\left(1+e^{-(10\sqrt{2})/\pi}\right)\left(1+e^{-(8\sqrt{2})/\pi}\right)\left(1+e^{-(6\sqrt{2})/\pi}\right)}$$

Decimal approximation

0.00406705727215837460072006397926134718634602110505657921022647 06

…

0.00406705727….

Alternate forms

$$\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}$$

$$e^{(12\sqrt{2})/\pi} \Big/ \left(\left(1+e^{(2\sqrt{2})/\pi}\right)^2\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)$$

$$\frac{1}{30\left(1+e^{(2\sqrt{2})/\pi}\right)^2}-\frac{1}{30\left(1+e^{(2\sqrt{2})/\pi}\right)}+\frac{e^{(2\sqrt{2})/\pi}}{3\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)}-\frac{e^{(2\sqrt{2})/\pi}\left(1+e^{(2\sqrt{2})/\pi}\right)^2}{2\left(1+e^{(8\sqrt{2})/\pi}\right)}+\frac{e^{(2\sqrt{2})/\pi}\left(1+e^{(2\sqrt{2})/\pi}\right)^2}{5\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)}$$

1/(1 + e^(-(2 sqrt(2))/π)) + e^((6 sqrt(2))/π)/((1 + e^((4 sqrt(2))/π)) (1 + e^((6 sqrt(2))/π)))+e^((12 sqrt(2))/π)/((1 + e^((6 sqrt(2))/π)) (1 + e^((8 sqrt(2))/π)) (1 + e^((10 sqrt(2))/π)))

Input

$$\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}$$

Decimal approximation

0.847935577304610572241489754715422323305309825037973846018852222 0

…

0.8479355773….

Alternate forms

$$\frac{1}{1+e^{-(6\sqrt{2})/\pi}+e^{-(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}}+\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}+\frac{1}{2}\left(1+\tanh\left(\frac{\sqrt{2}}{\pi}\right)\right)$$

$$1+\frac{1}{30\left(1+e^{(2\sqrt{2})/\pi}\right)^2}-\frac{6}{5\left(1+e^{(2\sqrt{2})/\pi}\right)}+\frac{1-e^{(2\sqrt{2})/\pi}}{2\left(1+e^{(4\sqrt{2})/\pi}\right)}+\frac{3e^{(2\sqrt{2})/\pi}-1}{3\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)}-\frac{e^{(2\sqrt{2})/\pi}\left(1+e^{(2\sqrt{2})/\pi}\right)^2}{2\left(1+e^{(8\sqrt{2})/\pi}\right)}+\frac{e^{(2\sqrt{2})/\pi}\left(1+e^{(2\sqrt{2})/\pi}\right)^2}{5\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)}$$

$$\left(e^{(2\sqrt{2})/\pi}\left(1-e^{(2\sqrt{2})/\pi}+3e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+2e^{(8\sqrt{2})/\pi}+e^{(10\sqrt{2})/\pi}+2e^{(12\sqrt{2})/\pi}+3e^{(14\sqrt{2})/\pi}+2e^{(18\sqrt{2})/\pi}-e^{(20\sqrt{2})/\pi}+3e^{(22\sqrt{2})/\pi}-e^{(24\sqrt{2})/\pi}+e^{(26\sqrt{2})/\pi}\right)\right)\Big/\left(\left(1+e^{(2\sqrt{2})/\pi}\right)^2\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)$$

tanh(x) is the hyperbolic tangent function

Series representations

$$\frac{1}{1+e^{-\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}+\frac{e^{\left(12\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}=$$

$$\frac{1}{1+e^{-\frac{2\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}}+\frac{e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}}{\left(\left(1+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\right)}+$$

$$\frac{e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}}{\left(\left(1+e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{10\left(\frac{1}{z_0}\right)^{1/2\,\lfloor \arg(2-z_0)/(2\pi)\rfloor}\,z_0^{1/2\,(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(2-z_0)^k\,z_0^{-k}}{k!}}{\pi}}\right)\right)}$$

$$\frac{1}{1+e^{-\left(2\sqrt{2}\right)/\pi}}+\frac{e^{\left(6\sqrt{2}\right)/\pi}}{\left(1+e^{\left(4\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)}+\frac{e^{\left(12\sqrt{2}\right)/\pi}}{\left(1+e^{\left(6\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(8\sqrt{2}\right)/\pi}\right)\left(1+e^{\left(10\sqrt{2}\right)/\pi}\right)}=$$

$$\left(e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+\right.\right.$$

$$3e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{10\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2e^{\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+3e^{\frac{14\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$2e^{\frac{18\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{20\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$3e^{\frac{22\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{24\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+$$

$$\left.\left.e^{\frac{26\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)\Bigg/$$

$$\left(\left(\left(1+e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2\left(1+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right.\right.$$

$$\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(1+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)$$

$$\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-\right.$$

$$\left.\left.\left.e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From which:

(57 π)/94((1/(1+e^(-(2 sqrt(2))/π))+e^((6 sqrt(2))/π)/((1+e^((4 sqrt(2))/π))(1+e^((6 sqrt(2))/π)))+e^((12 sqrt(2))/π)/((1 + e^((6 sqrt(2))/π)) (1 + e^((8 sqrt(2))/π)) (1 + e^((10 sqrt(2))/π)))))

Input

$$\frac{57\,\pi}{94}\left(\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)$$

Exact result

$$\frac{57}{94}\left(\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)\pi$$

Decimal approximation

1.615324322143854604838207843710838397776321289281979100946679914 9…

1.615324322…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate forms

$$\frac{57}{188}\,\pi\left(1+\frac{2}{1+e^{-(6\sqrt{2})/\pi}+e^{-(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}}+\frac{2\,e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}+\tanh\left(\frac{\sqrt{2}}{\pi}\right)\right)$$

$$\frac{57\pi}{94}+\frac{19\pi}{940\left(1+e^{(2\sqrt{2})/\pi}\right)^2}-\frac{171\pi}{235\left(1+e^{(2\sqrt{2})/\pi}\right)}-\frac{57\left(e^{(2\sqrt{2})/\pi}-1\right)\pi}{188\left(1+e^{(4\sqrt{2})/\pi}\right)}+\frac{19\left(3\,e^{(2\sqrt{2})/\pi}-1\right)\pi}{94\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)}-\frac{57\left(e^{\sqrt{2}/\pi}+e^{(3\sqrt{2})/\pi}\right)^2\pi}{188\left(1+e^{(8\sqrt{2})/\pi}\right)}+\frac{57\left(e^{\sqrt{2}/\pi}+e^{(3\sqrt{2})/\pi}\right)^2\pi}{470\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)}$$

$$\left(57\,e^{(2\sqrt{2})/\pi}\left(1-e^{(2\sqrt{2})/\pi}+3\,e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+2\,e^{(8\sqrt{2})/\pi}+e^{(10\sqrt{2})/\pi}+2\,e^{(12\sqrt{2})/\pi}+3\,e^{(14\sqrt{2})/\pi}+2\,e^{(18\sqrt{2})/\pi}-e^{(20\sqrt{2})/\pi}+3\,e^{(22\sqrt{2})/\pi}-e^{(24\sqrt{2})/\pi}+e^{(26\sqrt{2})/\pi}\right)\pi\right)\Big/\left(94\left(1+e^{(2\sqrt{2})/\pi}\right)^2\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)$$

tanh(x) is the hyperbolic tangent function

Expanded form

$$\frac{57\pi}{94\left(1+e^{-(2\sqrt{2})/\pi}\right)}+\frac{57\,e^{(6\sqrt{2})/\pi}\pi}{94\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\frac{57\,e^{(12\sqrt{2})/\pi}\pi}{94\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}$$

Series representations

$$\frac{1}{94}\left(\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)(57\pi)=$$

$$\frac{57\pi}{94\left(1+e^{-\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)}+$$

$$\left(57\,e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\pi\right)\Bigg/$$

$$\left(94\left(1+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)+$$

$$\left(57\,e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\pi\right)\Bigg/$$

$$\left(94\left(1+e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{10\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

$$\frac{1}{94}\left(\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)$$

$$(57\,\pi)=\left(57\,e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+3\,e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2\,e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{10\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2\,e^{\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+3\,e^{\frac{14\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2\,e^{\frac{18\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{20\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+3\,e^{\frac{22\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{24\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{26\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\pi\right)\Bigg/\left(94\left(1+e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2\left(1+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

And:

((57 π)/94((1/(1+e^(-(2 sqrt(2))/π))+e^((6 sqrt(2))/π)/((1+e^((4 sqrt(2))/π))(1+e^((6 sqrt(2))/π)))+e^((12 sqrt(2))/π)/((1+e^((6 sqrt(2))/π))(1+e^((8 sqrt(2))/π))(1+e^((10 sqrt(2))/π))))))1/10^35

Input

$$\left(\frac{57\,\pi}{94}\left(\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)\right)\times\frac{1}{10^{35}}$$

Exact result

$$\left(57\left(\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)\pi\right)\Big/ 9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000$$

Decimal approximation

$1.615324322143854604838207843710838397776321289281979100946 6\ldots \times 10^{-35}$

1.61532432214…*10^{-35} ≈ 1.616255*10^{-35} (Planck Length)

Alternate forms

$$\left(57\pi\left(1+\frac{2}{1+e^{-(6\sqrt{2})/\pi}+e^{-(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}}+\frac{2e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}+\tanh\left(\frac{\sqrt{2}}{\pi}\right)\right)\right)\Big/ 18\,800\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000$$

$$\frac{57\pi}{9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000}+\frac{19\pi}{94\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(2\sqrt{2})/\pi}\right)^2}-\frac{171\pi}{23\,500\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(2\sqrt{2})/\pi}\right)}-\frac{57\left(e^{(2\sqrt{2})/\pi}-1\right)\pi}{18\,800\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(4\sqrt{2})/\pi}\right)}+\frac{19\left(3e^{(2\sqrt{2})/\pi}-1\right)\pi}{9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)}-\frac{57\left(e^{\sqrt{2}/\pi}+e^{(3\sqrt{2})/\pi}\right)^2\pi}{18\,800\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(8\sqrt{2})/\pi}\right)}+\left(57\left(e^{\sqrt{2}/\pi}+e^{(3\sqrt{2})/\pi}\right)^2\pi\right)\Big/\left(47\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)$$

$$\left(57e^{(2\sqrt{2})/\pi}\left(1-e^{(2\sqrt{2})/\pi}+3e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+2e^{(8\sqrt{2})/\pi}+e^{(10\sqrt{2})/\pi}+2e^{(12\sqrt{2})/\pi}+3e^{(14\sqrt{2})/\pi}+2e^{(18\sqrt{2})/\pi}-e^{(20\sqrt{2})/\pi}+3e^{(22\sqrt{2})/\pi}-e^{(24\sqrt{2})/\pi}+e^{(26\sqrt{2})/\pi}\right)\pi\right)\Big/\left(9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(2\sqrt{2})/\pi}\right)^2\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1-e^{(2\sqrt{2})/\pi}+e^{(4\sqrt{2})/\pi}-e^{(6\sqrt{2})/\pi}+e^{(8\sqrt{2})/\pi}\right)\right)$$

$\tanh(x)$ is the hyperbolic tangent function

Expanded form

$$\frac{57\,\pi}{9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{-(2\sqrt{2})/\pi}\right)}+\frac{57\,e^{(6\sqrt{2})/\pi}\,\pi}{9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\left(57\,e^{(12\sqrt{2})/\pi}\,\pi\right)\Big/\left(9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)\right)$$

Series representations

$$\frac{1}{10^{35}\times 94}(57\pi)\left(\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)=$$

$$(57\pi)\Bigg/\left(9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{-\frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)+\left(57e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\pi\right)\Bigg/\left(9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{\frac{4\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)+\left(57e^{\frac{12\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\pi\right)\Bigg/\left(9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{\frac{6\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{10\left(\frac{1}{z_0}\right)^{1/2\lfloor \arg(2-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor \arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

$$\frac{1}{10^{35}\times 94}(57\pi)\left(\frac{1}{1+e^{-(2\sqrt{2})/\pi}}+\frac{e^{(6\sqrt{2})/\pi}}{\left(1+e^{(4\sqrt{2})/\pi}\right)\left(1+e^{(6\sqrt{2})/\pi}\right)}+\frac{e^{(12\sqrt{2})/\pi}}{\left(1+e^{(6\sqrt{2})/\pi}\right)\left(1+e^{(8\sqrt{2})/\pi}\right)\left(1+e^{(10\sqrt{2})/\pi}\right)}\right)=$$

$$\left(57\,e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+3\,e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2\,e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{10\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2\,e^{\frac{12\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+3\,e^{\frac{14\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+2\,e^{\frac{18\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{20\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+3\,e^{\frac{22\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{24\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{26\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\pi\right)\Bigg/$$

$$\left(9\,400\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\left(1+e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)^2\left(1+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\left(1-e^{\frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{4\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}-e^{\frac{6\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}+e^{\frac{8\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{\pi}}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

On the application of the formulas of the volumes of an octahedron and a sphere

With regard to a sphere inscribed in an octahedron, we have the following formulas.

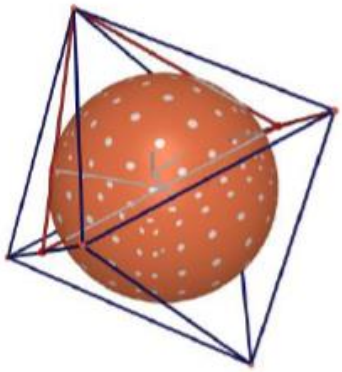


Fig: sphere inscribed in an octahedron

$V_0 = \frac{1}{3}\sqrt{2}\,l^3$

$V_s = \frac{4}{3}\pi r^3$; where $r_s = (l/2)$

With regard the ratio between the two above formulas (octahedron and sphere)

(1/3*√2*l^3)/(4/3*π*(l/2)^3)

we obtain:

Input

$$\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Result

$\frac{\sqrt{2}\ 2}{\pi}$ (for $l \neq 0$)

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

Series representations

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\sqrt{z_0}\ \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k\, x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}{\pi}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{2}\ l^3}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = \frac{2\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k\, z_0^{-k}}{k!}}{\pi}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From which:

1/3*(2/((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^2

Input

$$\frac{1}{3}\left(\frac{2}{\frac{\frac{1}{3}\sqrt{2}\ l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}}\right)^2$$

Result

$$\frac{\pi^2}{6}$$

Decimal approximation

1.64493406684822643647241516664602518921894990120679843773555822 93

…

1.644934066848226… = ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Property

$\frac{\pi^2}{6}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = -2\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{4}{3}\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

Integral representations

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{8}{3}\left(\int_0^1\sqrt{1-t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^\infty \frac{1}{1+t^2}\,dt\right)^2$$

$$\frac{1}{3}\left(\frac{2}{\frac{\sqrt{2}\,l^3}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}}\right)^2 = \frac{2}{3}\left(\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt\right)^2$$

We note that, from the sum of the first nine numbers excluding 0, i.e., 1+2+3+4+5+6+7+8+9 = 45 (these are the fundamental numbers, from which, through infinite combinations, all the other numbers are obtained), we obtain the following interesting formula:

1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)

Input

$$1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1$$

Decimal approximation

1.64529737852077603277189622979372820045495342111029157082539392 86

…

1.64529737852…. ≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

Alternate forms

$$3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{2\pi C_{\text{MRB}} + 3\phi^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{2}\left(3+\sqrt{5}\right)}} + 1$$

$$2^{2/3} \times 3^{1/3-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{8\pi C_{\text{MRB}} + 18 + 6\sqrt{5}}} + 1$$

Expanded forms

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{1}{4}\left(1+\sqrt{5}\right)^2}} + 1$$

$$3^{-2/(3\pi)} \times 5^{-1/(3\pi)} \sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3} + \frac{3}{2} + \frac{\sqrt{5}}{2}}} + 1$$

And:

sqrt(6(1+1/(((φ^2+(2Pi)/3*MRB const)(1/e((1+2+3+4+5+6+7+8+9)^(1/Pi))))^1/3)))

Input

$$\sqrt{6\left(1+\frac{1}{\sqrt[3]{\left(\phi^2+\frac{2\pi}{3}C_{\text{MRB}}\right)\left(\frac{1}{e}\sqrt[\pi]{1+2+3+4+5+6+7+8+9}\right)}}\right)}$$

ϕ is the golden ratio

C_{MRB} is the MRB constant

Exact result

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}$$

Decimal approximation

3.141939571526584308924330732196162632677513386811659044682541739 3

…

3.141939571526…. ≈ π (Ramanujan Recurring Number)

Alternate forms

$$\sqrt{6\left(3^{1/3-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{2\pi C_{\text{MRB}}+3\phi^2}}+1\right)}$$

$$3^{1/2-1/(3\pi)}\times 5^{-1/(6\pi)}\sqrt{2\left(\sqrt[3]{\frac{6e}{4\pi C_{\text{MRB}}+9+3\sqrt{5}}}+3^{2/(3\pi)}\sqrt[3\pi]{5}\right)}$$

Expanded forms

$$\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\frac{1}{4}\left(1+\sqrt{5}\right)^2}}+1\right)}$$

$$\sqrt{2\times 3^{1-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\frac{3}{2}+\frac{\sqrt{5}}{2}}}+6}$$

All 2nd roots of 6 ($3^{-2/(3\pi)}\,5^{-1/(3\pi)}\,(e/((2\pi C_{MRB})/3+\phi^2))^{1/3}+1$)

$$e^{0}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}\approx 3.1419 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{6\left(3^{-2/(3\pi)}\times 5^{-1/(3\pi)}\sqrt[3]{\frac{e}{\frac{2\pi C_{\text{MRB}}}{3}+\phi^2}}+1\right)}\approx -3.1419 \text{ (real root)}$$

Furthermore, we obtain also:

2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3))

Input

$$2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

8

8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

Series representations

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 4\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\left(2\pi\sqrt{2}\right)\left(\sqrt{2}\,l^3\right)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$
$$4\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{\,1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

$6\pi*\sqrt{2}((1/3*\sqrt{2}*l^3)/(4/3*\pi*(l/2)^3))$

Input

$$6\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}$$

Exact result

24

24

The value 24 is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string representing a bosons. From the analysis, we observe that the is no number theoretic connection with physical vibrations of fermionic strings at extremally low entropy. This fact is confirmed by the fact that the Higgs bosons at the moment of the big bang and infinitesimally shortly thereafter, facilitated the creation of fermions (matter and antimatter particles) [8]. Thus we note that the ingredients for the formation of electromagnetic radiation from photons (a Boson), and the formation of matter from the Higgs boson after the big bang, are intrinsic properties of the vacuum energy in pre-big bang.

Series representations

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}=12\sqrt{z_0}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2$$

for $(\text{not}\,(z_0\in\mathbb{R}\ \text{and}\ -\infty<z_0\le 0))$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} = 12\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{(6\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3} =$$

$$12\left(\frac{1}{z_0}\right)^{\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1+\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

This could imply that all matter (fermions) was preceded by bosons. That is, before the Big Bang, from perturbations of the vacuum energy itself, bosons were created, and successively at the Big Bang, and infinitesimally shortly after the Big Bang, fermions, were created from the vacuum energy that underwent a violent “breaking” that formed a hot plasma. of particle-antiparticle pairs. This therefore implies that quantum gravity was not necessarily “dark” to some extent, because a photon (light particle) is itself a boson. Therefore, a big bang was not necessarily the moment of the creation of light, but of the creation of matter (fermions) from vacuum energy, as this undergoes further "breaking" in the cosmological constant, in the hot plasma of matter and in the energy dark.

(2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4

Input

$$\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4$$

Exact result

4096

$4096 = 64^2$, (Ramanujan Recurring Number) that multiplied by 2 give 8192, indeed:

The total amplitude vanishes for gauge group SO(8192), while the vacuum energy is negative and independent of the gauge group. The vacuum energy and dilaton tadpole to lowest non-trivial order for the open bosonic string. While the vacuum energy is non-zero and independent of the gauge group, the dilaton tadpole is zero for a unique choice of gauge group, $SO(2^{13})$ i.e. SO(8192). (From: "Dilaton Tadpole for the Open Bosonic String " Michael R. Douglas and Benjamin Grinstein - September 2,1986)

27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1

Input

$$27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4}+1$$

Exact result

1729

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve ($1728 = 8^2 * 3^3$).

The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, as it can be expressed as the sum of two cubes in two different ways ($10^3 + 9^3 = 12^3 + 1^3 = 1729$) and Ramanujan's recurring number)

Series representations

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+256\sqrt{2}^8\right)^{-k}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=$$
$$1+27\sqrt{-1+256\sqrt{2}^8}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+256\sqrt{2}^8\right)^{-k}}{k!}$$

$$27\sqrt{\left(\frac{(2\pi\sqrt{2})(\sqrt{2}\,l^3)}{\frac{1}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)3}\right)^4}+1=1+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}$$

for $(\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

We note that:

1/25*1/144(((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+(27*sqrt((2π*√2((1/3*√2*l^3)/(4/3*π*(l/2)^3)))^4)+1))

Input

$$\frac{1}{25}\times\frac{1}{144}\left(\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+\left(27\sqrt{\left(2\pi\sqrt{2}\times\frac{\frac{1}{3}\sqrt{2}\,l^3}{\frac{4}{3}\pi\left(\frac{l}{2}\right)^3}\right)^4+1}\right)\right)$$

Exact result

$$\frac{233}{144}$$

Decimal approximation

1.6180555555555555555555555555555555555555555555555555555555555555555555555555

…

1.61805555…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Repeating decimal

$1.61805\overline{5}$ (period 1)

Series representations

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\sqrt{z_0}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=$$
$$\frac{1}{3600}\left(1+256\exp^8\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^8\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^8+27\exp\left(i\pi\left\lfloor\frac{\arg\left(-x+256\sqrt{2}^8\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x+256\sqrt{2}^8\right)^k}{k!}\right)\quad\text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+\left(27\sqrt{\left(\frac{2\left(\sqrt{2}\ l^3\right)\pi\sqrt{2}}{\frac{3}{3}\left(4\pi\left(\frac{l}{2}\right)^3\right)}\right)^4+1}\right)}{144\times 25}=\frac{1}{3600}$$
$$\left(1+256\left(\frac{1}{z_0}\right)^{4\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{4+4\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^8+27\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}z_0^{1/2+1/2\left\lfloor\arg\left(256\sqrt{2}^8-z_0\right)/(2\pi)\right\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(256\sqrt{2}^8-z_0\right)^k z_0^{-k}}{k!}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

we obtain also:

sqrt(6(1/3*(2/(((2sqrt2)/Pi)))^2))

Input

$$\sqrt{6\left(\frac{1}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2\right)}$$

Exact result

π

Decimal approximation

3.14159265358979323846264338327950288419716939937510582097494459 23

…

3.14159265358… = π

Property

π is a transcendental number

All 2nd roots of π^2

$\pi e^0 \approx 3.1416$ (real, principal root)

$\pi e^{i\pi} \approx -3.1416$ (real root)

Series representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty} -\frac{4\,(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = \sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 4\int_0^1\sqrt{1-t^2}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\sqrt{\frac{6}{3}\left(\frac{2}{\frac{2\sqrt{2}}{\pi}}\right)^2} = 2\int_0^\infty \frac{1}{1+t^2}\,dt$$

It is plausible to hypothesize that π and φ, in addition to being important mathematical constants, are constants that also have a fundamental relevance in the various sectors of Theoretical Physics and Cosmology

From $\frac{\pi^2}{6}$, we obtain:

sqrt(1/(Pi^2/6)*(4/3))

Input

$$\sqrt{\frac{1}{\frac{\pi^2}{6}} \times \frac{4}{3}}$$

Exact result

$$\frac{2\sqrt{2}}{\pi}$$

Decimal approximation

0.900316316157106069555199191006740582664574149955220625571437471 2

…

0.900316316157106…. = $\frac{2\sqrt{2}}{\pi}$ (DN Constant)

Property

$\frac{2\sqrt{2}}{\pi}$ is a transcendental number

All 2nd roots of $8/\pi^2$

$\frac{2\sqrt{2}\ e^0}{\pi} \approx 0.9003$ (real, principal root)

$\frac{2\sqrt{2}\ e^{i\pi}}{\pi} \approx -0.9003$ (real root)

Series representations

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{8}{\pi^2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{8}{\pi^2}-z_0\right)^k z_0^{-k}}{k!} \quad \text{for } (\text{not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\sqrt{\frac{4}{\frac{3\pi^2}{6}}} = \exp\left(i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{\pi^2}-x\right)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{8}{\pi^2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

DN Constant extended

We have the following expression concerning the ratios (and/or the inverses) between the icosahedron, octahedron and tetrahedron volumes and the sphere volume.

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

(we have highlighted the DN Constant in blue)

(((((5/12*(3+√5)*d^3)/(4/3*π*(d/2)^3))*1/((1/3*√2*a^3)/(4/3*π*(a/2)^3)) *1/((((√2 d^3)/12))*1/(4/3*π*(d/2)^3)))))^(1/(2π))

Input

$$\sqrt[2\pi]{\frac{\frac{5}{12}\left(3+\sqrt{5}\right)d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}\,a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\left(\frac{1}{12}\left(\sqrt{2}\,d^3\right)\right)\times\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}$$

Exact result

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

Decimal approximation

1.618008545900107058100262397953600521294343596022695608492128897 1

…

1.6180085459…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

Alternate form

$$2^{-1/\pi}\sqrt[2\pi]{\left(15+5\sqrt{5}\right)\pi}$$

Series representations

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times3}}}}=\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)}{\sqrt{z_0}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k\,z_0^{-k}}{k!}\right)^2}}$$

for $\left(\text{not}\left(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le0\right)\right)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\sqrt[2\pi]{\frac{\pi\left(3+\exp\left(i\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,2}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\sqrt[2\pi]{\frac{5\left(3+\sqrt{5}\right)d^3}{\frac{\left(\left(\sqrt{2}\,a^3\right)\left(\sqrt{2}\,d^3\right)\right)12\left(4\pi\left(\frac{d}{2}\right)^3\right)}{\left(\left(3\left(4\pi\left(\frac{a}{2}\right)^3\right)\right)\left(12\left(4\pi\left(\frac{d}{2}\right)^3\right)\right)\right)^{3\times 3}}}} =$$

$$\sqrt[2\pi]{\frac{5}{2}}\left(\left(\pi\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}\left(3+\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(5-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2\right)^{\wedge}\left(\frac{1}{2\pi}\right)$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

Integral representation

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\mathrm{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\mathrm{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

Furthermore, from the formula

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

we obtain also:

(4 (1.6180085459)^(2 π))/(5 (3 + sqrt(5))) = 3.1415926535

Indeed:

$$\frac{4\times 1.6180085459^{2\pi}}{5\left(3+\sqrt{5}\right)}$$

3.14159265358848716242219325098587152789310251892455761700781006 13

…

3.1415926535…. = π (Ramanujan Recurring Number)

From the following extended DN Constant (“Unitary Formula”)

$$\sqrt[2\pi]{\frac{\frac{5}{12}(3+\sqrt{5})d^3}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}\times\frac{1}{\frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi\left(\frac{a}{2}\right)^3}}\times\frac{1}{\frac{\sqrt{2}}{12}d^3\cdot\frac{1}{\frac{4}{3}\pi\left(\frac{d}{2}\right)^3}}}\times\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

with regard

$$\sqrt[3]{-\frac{q}{2}+\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\frac{q^2}{4}+\frac{p^3}{27}}}$$

for q = 1729 and p = 4096, we obtain by changing the sign in the algebraic sum of the aforementioned Cardano’s Formula and after some calculations:

$$2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}$$

multiplied by

$$\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}$$

and performing the ninth root of the entire expression:

√((2^(-1/π)(5(3+√(5))π)^(1/(2π)))((∛(-1729/2+√(1729^2/4+4096^3/27)) -∛(-1729/2-√(1729^2/4+4096^3/27)))^1/9))

we obtain:

$$\sqrt{\left(\left(2^{-1/\pi}\sqrt[2\pi]{5\left(3+\sqrt{5}\right)\pi}\right)\sqrt[9]{\sqrt[3]{-\frac{1729}{2}+\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}-\sqrt[3]{-\frac{1729}{2}-\sqrt{\frac{1729^2}{4}+\frac{4096^3}{27}}}}\right)}$$

$$2^{-1/(2\pi)}\sqrt[18]{\sqrt[3]{\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}-\frac{1729}{2}}+\sqrt[3]{\frac{1729}{2}+\frac{\sqrt{\frac{274\,958\,621\,851}{3}}}{6}}}\Big/\sqrt[4\pi]{5\left(3+\sqrt{5}\right)\pi}$$

i.e.

2^(-1/(2π)) ((-1729/2+√(274958621851/3)/6)^(1/3)+(1729/2 + √(274958621851/3)/6)^(1/3))^(1/18) (5(3+√(5)) π)^(1/(4π))

= 1.61549140391….

The general "unitary" formula, which derives from DN Constant, is the following:

$$\sqrt{2\times\frac{2\cdot\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\cdot 0.9991104684}(C\times \mathrm{R}\times 2.33\cdot 10^{-13})}} \cong 1.61803398\ldots = \frac{\sqrt{5}+1}{2}$$

Where $\frac{2\sqrt{2}}{\pi}$ is the Del Gaudio-Nardelli Constant, 0.9991104684 is the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

C is any constant or solution to an equation, R is the radius of the Universe and $2.33*10^{-13}$ is the temperature of the universe expressed in GeV.

For example, $C = 9.9128$, inserting a radius of the Universe, which we have calculated, equal to $R = 2.06274*10^{12}$ years, from DN Constant "unitary" formula, we obtain:

√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}$$

1.618035912348264235440174408834709854254527350840173356306481810 7

…

1.618035912348…. result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We obtain also:

(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))dxdydz

Indefinite integral assuming all variables are real

$$0.809018\, x^2\, y\, z + \text{constant}$$

Definite integral over a cube of edge length 2 L

$$\int_{-L}^{L}\int_{-L}^{L}\int_{-L}^{L} 1.61804\, dx\, dy\, dz = 12.9443\, L^3$$

Definite integral over a sphere of radius R

$$\iiint_{x^2+y^2+z^2<R^2} 1.61804\, dz\, dy\, dx = 6.77761\, R^3$$

From which, for L = R = 1 , dividing the two definite integral results by the original expression, we obtain:

12.9443/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13)))))

Input interpretation

$$\frac{12.9443}{\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

8.00001…

8.00001…. ≈ 8

value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 8 "modes" corresponding to the physical vibrations of a superstring.

And

3*(6.77761/(√(2×(2·(((2√2)/π))^(1/16))/(1/(π·0.9991104684) (9.9128×(2.06274 × 10^12)×2.33·10^(-13))))))

Input interpretation

$$3\times\frac{6.77761}{\sqrt{2\times\frac{2^{16}\sqrt{\frac{2\sqrt{2}}{\pi}}}{\frac{1}{\pi\times 0.9991104684}\times\frac{9.9128\times 2.06274\times 10^{12}\times 2.33}{10^{13}}}}}$$

Result

12.5664…

12.5664…. ≈ 4π = Bekenstein-Hawking (S_{BH}) black hole entropy

New fundamental formula deriving from DN Constant

The DN Constant (Del Gaudio-Nardelli Constant) equals $(2\sqrt{2})/\pi$ is defined as the ratio of the volume of an octahedron to the volume of a sphere and is an intriguing mathematical concept. Michele Nardelli hypothesized that the regular octahedron represents a phase in which the universe is highly symmetrical and with very low entropy. On the other hand, the sphere (which is inscribed in the octahedron, i.e. is "inside" it) represents the universe emerging from the quantum vacuum, which over time increases entropy and undergoes various symmetry breakings. This occurs in a regime of eternal inflation.

From the following expression

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×C×R)))

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\,C\,R\right)}}$$

which comes from the DN Constant, with $1.616255*10^{-35}$ which is equal to the Planck length, $1.1056*10^{-52}$ which is equal to the Cosmological Constant, C = 1729 which corresponds to the Hardy-Ramanujan number and R = $4.6018401361 \times 10^{-24}$, which represents the radius of the Universe, we obtain:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×4.6018401361 × 10^-24))) = 1.6180329973...

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 4.6018401361\times 10^{-24}\right)}}$$

1.618032997307532491506757016629746746422577260892967140791939590 3
…

1.6180329973075… result that is a very good approximation to the value of the golden ratio 1.618033988749… (Ramanujan Recurring Number)

We have also the following formula:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(4.4525642121 × 10^-24))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 4.4525642121\times 10^{-24}\right)}}$$

Result

1.644932352102092130483898983704151168876621841655166877914166033 8
…

1.64493235210209213…≈ ζ(2) = $\pi^2/6$ = 1.644934 (trace of the instanton shape and Ramanujan Recurring Number)

And again:

√(2×(2·(((2√2)/π)^(1/16)))/(4096/(π·0.9991104684) (((1.616255*10^-35)/(1.1056*10^-52))×1729×(1.2206935225 × 10^-24))))

Input interpretation

$$\sqrt{2\times\frac{2\sqrt[16]{\frac{2\sqrt{2}}{\pi}}}{\frac{4096}{\pi\times 0.9991104684}\left(\frac{1.616255\times 10^{-35}}{1.1056\times 10^{-52}}\times 1729\times 1.2206935225\times 10^{-24}\right)}}$$

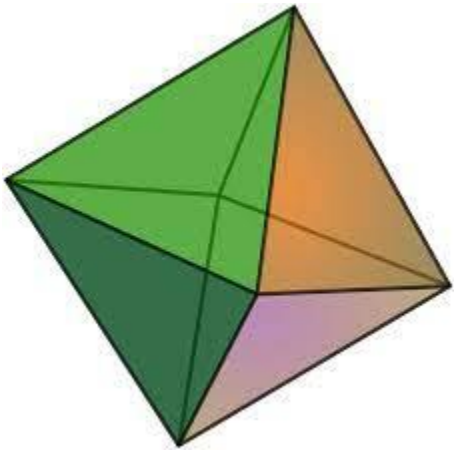
Result

3.14159225731469939939950039231605796469731171781644423955790024797

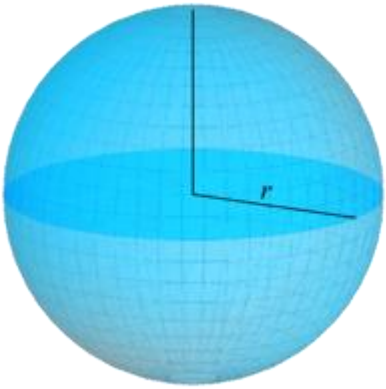
…

3.14159225731469…≈ π (Ramanujan Recurring Number)

Now, we have that:



Octahedron



Sphere

Given the value of a volume, independently of the solid, following the Poincaré Conjecture, we compare any solid "without holes" and a sphere. If we compare an octahedron with a sphere, we have:

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

If we consider the radius of the sphere as an unknown, we must find the value of the side of the octahedron which allows us to equalize the two volumes and which will give us the DN Constant as a result (which will therefore be equal to the radius of the sphere).

From

$$\frac{4}{3}\pi r^3 = \frac{1}{3}\sqrt{2}a^3$$

To find r we perform the following calculation:

$$r^3 = \frac{\frac{1}{3}\sqrt{2}a^3}{\frac{4}{3}\pi} = \frac{1}{3}\sqrt{2}a^3 \cdot \frac{3}{4\pi} = \frac{\sqrt{2}a^3}{4\pi} = \frac{\sqrt{2}\cdot\sqrt{2}a^3}{\sqrt{2}\cdot 4\pi} = \frac{2a^3}{\sqrt{2}\cdot 4\pi} = \frac{a^3}{2\sqrt{2}\pi}$$

$$r^3 = \frac{a^3}{2\sqrt{2}\pi}; \qquad r = \sqrt[3]{\frac{a^3}{2\sqrt{2}\pi}} = \frac{a}{\sqrt[3]{2\sqrt{2}\pi}}$$

To find *a*, we have, for

$$r = \frac{2\sqrt{2}}{\pi}; \qquad \frac{a}{\sqrt[3]{2\sqrt{2}\pi}} = \frac{2\sqrt{2}}{\pi};$$

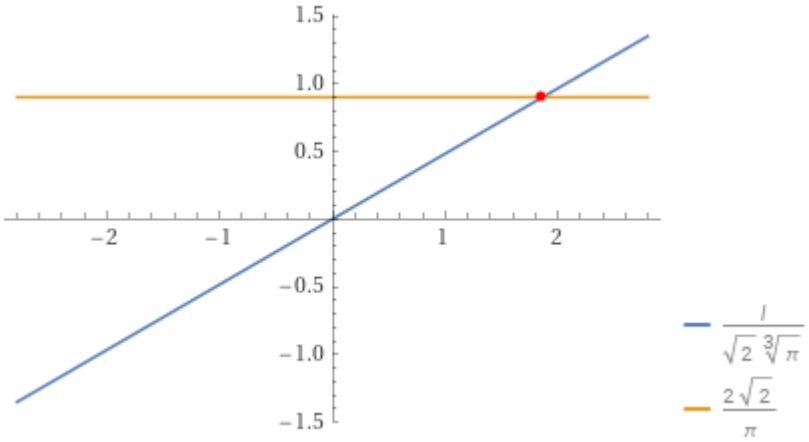
Thus, multiplying both the sides by $\sqrt[3]{2\sqrt{2}\pi}$, we obtain:

$$a = \frac{2\sqrt{2}}{\pi} \cdot \sqrt[3]{2\sqrt{2}\pi} = \sqrt[3]{\frac{(2\sqrt{2})^3 2\sqrt{2}\pi}{\pi^3}} =$$

$$= \sqrt[3]{\frac{8\cdot\sqrt{2^3}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{8\cdot\sqrt{2\cdot 2^2}\cdot 2\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{16\cdot\sqrt{2}\cdot 2\sqrt{2}}{\pi^2}}$$

$$= \sqrt[3]{\frac{32\sqrt{2}\sqrt{2}}{\pi^2}} = \sqrt[3]{\frac{32\cdot 2}{\pi^2}} = \frac{\sqrt[3]{64}}{\sqrt[3]{\pi^2}} = \frac{4}{\sqrt[3]{\pi^2}}; \; a = \frac{4}{\sqrt[3]{\pi^2}}$$

Plot



Solution

$a = \frac{4}{\sqrt[3]{\pi^2}}$

for V = 1/3*√2*a^3 (octahedron volume) and V = (4/3*π*r^3) (sphere volume), we obtain:

from the octahedron volume, we have: V = 1/3*√2*a^3 = (1/3*√2*($\frac{4}{\sqrt[3]{\pi^2}}$)^3)

(1/3*√2*(4/∛(π^2))^3)

Input

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3$$

$\sqrt[3]{x}$ is the real-valued 3rd root of x

Exact result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\exp\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{3\pi^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{1}{3}\sqrt{2}\left(\frac{4}{\sqrt[3]{\pi^2}}\right)^3 = \frac{64\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

And, from the sphere volume V = (4/3*π*r^3) = (4/3*π*((2√2)/π)^3)

(4/3*π*((2√2)/π)^3)

Input

$$\frac{4}{3}\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3$$

Result

$$\frac{64\sqrt{2}}{3\pi^2}$$

Decimal approximation

3.056848897337366735284768744174643472880661991020386025343029413 7

…

3.05684889733….

Property

$\frac{64\sqrt{2}}{3\pi^2}$ is a transcendental number

Series representations

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\sqrt{z_0}^3\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 = \frac{32\exp^3\left(i\,\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^{\,3}\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^3}{3\pi^2}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{1}{3}\left(\pi\left(\frac{2\sqrt{2}}{\pi}\right)^3\right)4 =$$

$$\frac{32\left(\frac{1}{z_0}\right)^{3/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{3/2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^3}{3\pi^2}$$

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\mathbb{R}$ is the set of real numbers

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

i is the imaginary unit

From:

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} - \cdots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

We note that, with regard 4372, we can to obtain the following results:

27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3

…

1729.0526944….

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. (1728 = 8^2 * 3^3) The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi - 54 + 54\sqrt{1093} - \frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256x^8 + 95\,744x^7 - 3\,248\,750\,080x^6 - 914\,210\,725\,504x^5 + 15\,498\,355\,554\,921\,184x^4 + 2\,911\,478\,392\,539\,914\,656x^3 - 32\,941\,144\,911\,224\,677\,091\,680x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5}-1} - \frac{27\sqrt{10-2\sqrt{5}}}{2\left(\sqrt{5}-1\right)}$$

Series representations

$$27\left(\sqrt{4372} - 2 - \frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right) + \phi =$$
$$\left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1+\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4372}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for $(\text{not } (z_0\in\mathbb{R} \text{ and } -\infty<z_0\le 0))$

Or:

27((4096+276)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ

Input

$$27\left(\sqrt{4096+276}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi$$

ϕ is the golden ratio

Result

$$\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)$$

Decimal approximation

1729.052694417090562517063720863714876368418930653845785481544702 3…

1729.0526944…. as above

Alternate forms

$$\frac{1}{8}\left(-27\sqrt{5\left(10-2\sqrt{5}\right)}+58\sqrt{5}+432\sqrt{1093}-27\sqrt{2\left(5-\sqrt{5}\right)}-374\right)$$

$$\phi-54+54\sqrt{1093}+\frac{27}{4}\left(1+\sqrt{5}-\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

$$\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}$$

Minimal polynomial

$$256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10 - 2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10 - 2\sqrt{5}\right)}$$

$$-\frac{107}{2} + \frac{\sqrt{5}}{2} + 54\sqrt{1093} + \frac{27}{\sqrt{5} - 1} - \frac{27\sqrt{10 - 2\sqrt{5}}}{2\left(\sqrt{5} - 1\right)}$$

Series representations

$$27\left(\sqrt{4096 + 276} - 2 - \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\left(\sqrt{5} - 1\right)2}\right) + \phi = \left(162 - 108\sqrt{1093} - 2\phi - 108\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} + 2\phi\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k} - 27\sqrt{9 - 2\sqrt{5}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9 - 2\sqrt{5}\right)^{-k}\right) \Big/ \left(2\left(-1 + \sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-$$
$$\left.27\sqrt{9-2\sqrt{5}}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{4}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$27\left(\sqrt{4096+276}\ -2-\frac{\sqrt{10-2\sqrt{5}}\ -2}{\left(\sqrt{5}\ -1\right)2}\right)+\phi=$$
$$\left(162-108\sqrt{1093}\ -2\phi-108\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+\right.$$
$$108\sqrt{1093}\ \sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}+$$
$$2\phi\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}-$$
$$\left.27\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}\ -z_0\right)^k\,z_0^{-k}}{k!}\right)\Big/$$
$$\left(2\left(-1+\sqrt{z_0}\ \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k\,z_0^{-k}}{k!}\right)\right)$$
for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

From which:

(27((4372)^1/2-2-1/2(((√(10-2√5) -2))/((√5-1))))+φ)^1/15

Input

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{1}{2}\times\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}\right)+\phi}$$

ϕ is the golden ratio

Exact result

$$\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation

1.643818568584986279990230131703681005418575687350518480483418312 4

…

$1.64381856858\ldots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\ldots$

Alternate forms

$$\sqrt[15]{\phi-54+54\sqrt{1093}-\frac{27\left(\sqrt{10-2\sqrt{5}}-2\right)}{2\left(\sqrt{5}-1\right)}}$$

$$\frac{1}{\sqrt[15]{\frac{2\left(\sqrt{5}-1\right)}{166-108\sqrt{5}-108\sqrt{1093}+108\sqrt{5465}-27\sqrt{2\left(5-\sqrt{5}\right)}}}}$$

$$\sqrt[15]{\text{root of } 256\,x^8 + 95\,744\,x^7 - 3\,248\,750\,080\,x^6 - 914\,210\,725\,504\,x^5 + 15\,498\,355\,554\,921\,184\,x^4 + 2\,911\,478\,392\,539\,914\,656\,x^3 - 32\,941\,144\,911\,224\,677\,091\,680\,x^2 - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041 \text{ near } x = 1729.05}$$

Minimal polynomial

$$256\,x^{120} + 95\,744\,x^{105} - 3\,248\,750\,080\,x^{90} - 914\,210\,725\,504\,x^{75} + 15\,498\,355\,554\,921\,184\,x^{60} + 2\,911\,478\,392\,539\,914\,656\,x^{45} - 32\,941\,144\,911\,224\,677\,091\,680\,x^{30} - 3\,092\,528\,914\,069\,760\,354\,714\,456\,x^{15} + 26\,320\,050\,609\,744\,039\,027\,169\,013\,041$$

Expanded forms

$$\sqrt[15]{\frac{1}{2}\left(1+\sqrt{5}\right) + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}$$

$$\sqrt[15]{-\frac{187}{4} + \frac{29\sqrt{5}}{4} + 54\sqrt{1093} - \frac{27}{8}\sqrt{10-2\sqrt{5}} - \frac{27}{8}\sqrt{5\left(10-2\sqrt{5}\right)}}$$

All 15th roots of ϕ + 27 (-2 + 2 sqrt(1093) - (sqrt(10 - 2 sqrt(5)) - 2)/(2 (sqrt(5) - 1)))

$$e^0\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.64382 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{\phi + 27\left(-2 + 2\sqrt{1093} - \frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)} \approx 1.50170 + 0.6686\,i$$

$$e^{(4\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2\,i\,\pi)/5}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx 0.5080+1.5634\,i$$

$$e^{(8\,i\,\pi)/15}\sqrt[15]{\phi+27\left(-2+2\sqrt{1093}-\frac{\sqrt{10-2\sqrt{5}}-2}{2\left(\sqrt{5}-1\right)}\right)}\approx -0.17183+1.63481\,i$$

Series representations

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+108\sqrt{1093}\sqrt{4}\right.\right.\right.$$
$$\left.\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+2\phi\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}-27\sqrt{9-2\sqrt{5}}\right.$$
$$\left.\left.\left.\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(9-2\sqrt{5}\right)^{-k}\right)\Big/\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)\right)\right)\wedge(1/15)\Bigg)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+2\phi\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{9-2\sqrt{5}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(9-2\sqrt{5}\right)^{-k}}{k!}\right)\right/\right.$$
$$\left.\left.\left.\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\text{^}(1/15)\right)$$

$$\sqrt[15]{27\left(\sqrt{4372}-2-\frac{\sqrt{10-2\sqrt{5}}-2}{\left(\sqrt{5}-1\right)2}\right)+\phi}=$$
$$\frac{1}{\sqrt[15]{2}}\left(\left(\left(\left(162-108\sqrt{1093}-2\phi-108\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.108\sqrt{1093}\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\right.\right.\right.\right.$$
$$\left.\left.\left.\left.2\phi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}-\right.\right.\right.\right.$$
$$\left.\left.\left.\left.27\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(10-2\sqrt{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right/\right.$$
$$\left.\left.\left.\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)\text{^}(1/15)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Integral representation

$$(1+z)^a=\frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\,\Gamma(-a)}\quad\text{for } (0<\gamma<-\operatorname{Re}(a) \text{ and } |\arg(z)|<\pi)$$

From:

An Update on Brane Supersymmetry Breaking - *J. Mourad and A. Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} = -\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\, e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

$$16\, k'\, e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}}{(7-p)}$$

$$(A')^2 = k\, e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E}\right) e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

we have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)}\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning *p*, *C*, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and β_E = 1/2:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore, with respect to the exponentials of the vacuum equations, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while -6C+ϕ is equal to -$\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

For

exp((-Pi*sqrt(18)) we obtain:

Input:

$$\exp\left(-\pi\sqrt{18}\right)$$

Exact result:

$$e^{-3\sqrt{2}\,\pi}$$

Decimal approximation:

$$1.627201622607250929294215673911797954183858113695401 6\ldots \times 10^{-6}$$

1.6272016… * 10^{-6}

Property:

$e^{-3\sqrt{2}\,\pi}$ is a transcendental number

Series representations:

$$e^{-\pi\sqrt{18}} = e^{-\pi\sqrt{17}\,\sum_{k=0}^{\infty} 17^{-k}\binom{1/2}{k}}$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\pi\sqrt{17}\,\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$e^{-\pi\sqrt{18}} = \exp\left(-\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now, we have the following calculations:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

from which:

$$\frac{1}{4096}e^{-6C+\phi} = 1.6272016\ldots * 10^{-6}$$

$$0.000244140625\ e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016\ldots * 10^{-6}$$

Now:

$$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$$

And:

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

0.0066650177536

0.006665017...

Thence:

$$0.000244140625\, e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785…

Series representations:

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}17^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\pi\sqrt{17}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{17}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\frac{\exp\left(-\pi\sqrt{18}\right)}{0.000244141} = 4096\exp\left(-\frac{\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}\,17^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{2\sqrt{\pi}}\right)$$

Now:

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$$

$$= 0.00666501785\ldots$$

From:

ln(0.00666501784619)

Input interpretation:

$\log(0.00666501784619)$

Result:

−5.010882647757...

-5.010882647757…

Alternative representations:

$\log(0.006665017846190000) = \log_e(0.006665017846190000)$

$\log(0.006665017846190000) = \log(a)\log_a(0.006665017846190000)$

$\log(0.006665017846190000) = -\mathrm{Li}_1(0.993334982153810000)$

Series representations:

$$\log(0.006665017846190000) = -\sum_{k=1}^{\infty} \frac{(-1)^k\,(-0.993334982153810000)^k}{k}$$

$$\log(0.006665017846190000) = 2\,i\,\pi\left\lfloor \frac{\arg(0.006665017846190000 - x)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k\,(0.006665017846190000 - x)^k\,x^{-k}}{k} \quad \text{for } x < 0$$

$$\log(0.006665017846190000) = \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(0.006665017846190000 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \, (0.006665017846190000 - z_0)^k \, z_0^{-k}}{k}$$

Integral representation:

$$\log(0.006665017846190000) = \int_1^{0.006665017846190000} \frac{1}{t}\, dt$$

In conclusion:

$$-6C + \phi = -5.010882647757\ldots$$

and for C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \boldsymbol{\phi}$$

Note that the values of n_s (spectral index) 0.965, of the average of the Omega mesons Regge slope 0.987428571 and of the dilaton 0.989117352243, are also connected to the following two Rogers-Ramanujan continued fractions:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \cfrac{e^{-\pi}}{1 + \cfrac{e^{-2\pi}}{1 + \cfrac{e^{-3\pi}}{1 + \cfrac{e^{-4\pi}}{1 + \ldots}}}}} \approx 0.9568666373$$

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

(http://www.bitman.name/math/article/102/109/)

Also performing the 512th root of the inverse value of the Pion meson rest mass 139.57, we obtain:

((1/(139.57)))^1/512

Input interpretation:

$$\sqrt[512]{\frac{1}{139.57}}$$

Result:

0.990400732708644027550973755713301415460732796178555551684...

0.99040073.... result very near to the dilaton value 0.989117352243 = $\boldsymbol{\phi}$ and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+...}}}}\approx 0.9991104684$$

From

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

We have that:

Cosmological inflation with a tiny tensor–to–scalar ratio r, consistently with PLANCK data, may also be described within the present framework, for instance choosing

$$\alpha(\Phi) \;=\; i\,M\left(\Phi \;+\; b\,\Phi\,e^{ik\,\Phi}\right)\,. \tag{4.35}$$

This potential bears some similarities with the Kähler moduli inflation of [32] and with the poly–instanton inflation of [33]. One can verify that $\chi = 0$ solves the field equations, and that the potential along the $\chi = 0$ trajectory is now

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,. \tag{4.36}$$

We analyzing the following equation:

$$V \;=\; \frac{M^2}{3}\left(1 \;-\; a\,\phi\,e^{-\gamma\,\phi}\right)^2\,.$$

$$\phi \;=\; \varphi \;-\; \frac{\sqrt{6}}{k}\,,$$

$$a \;=\; \frac{b\gamma}{e} \;<\; 0\,, \qquad \gamma \;=\; \frac{k}{\sqrt{6}} \;<\; 0\,.$$

We have:

(M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

i.e.

V = (M^2)/3*[1-(b/euler number * k/sqrt6) * (φ- sqrt6/k) * exp(-(k/sqrt6)(φ- sqrt6/k))]^2

For k = 2 and φ = 0.9991104684, that is the value of the scalar field that is equal to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5}\sqrt[4]{5^3}}-1}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

we obtain:

V = (M^2)/3*[1-(b/euler number * 2/sqrt6) * (0.9991104684- sqrt6/2) * exp(-(2/sqrt6)(0.9991104684- sqrt6/2))]^2

Input interpretation:

$$V=\frac{M^2}{3}\left(1-\left(\frac{b}{e}\times\frac{2}{\sqrt{6}}\right)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2}{\sqrt{6}}\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\right)\right)^2$$

Result:

$$V=\frac{1}{3}(0.0814845\,b+1)^2\,M^2$$

Solutions:

$$b = \frac{225.913\left(-0.054323\,M^2 \pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2} \quad (M \neq 0)$$

Alternate forms:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

$$V = 0.00221324\left(b^2\,M^2 + 24.5445\,b\,M^2 + 150.609\,M^2\right)$$

$$-0.00221324\,b^2\,M^2 - 0.054323\,b\,M^2 - \frac{M^2}{3} + V = 0$$

Expanded form:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + \frac{M^2}{3}$$

Alternate form assuming b, M, and V are positive:

$$V = 0.00221324\,(b + 12.2723)^2\,M^2$$

Alternate form assuming b, M, and V are real:

$$V = 0.00221324\,b^2\,M^2 + 0.054323\,b\,M^2 + 0.333333\,M^2 + 0$$

Derivative:

$$\frac{\partial}{\partial b}\left(\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right) = 0.054323\,(0.0814845\,b + 1)\,M^2$$

Implicit derivatives

$$\frac{\partial b(M,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}$$

$$\frac{\partial b(M,V)}{\partial M} = -\frac{\frac{226\,802\,245}{18\,480\,874} + b}{M}$$

$$\frac{\partial M(b,V)}{\partial V} = \frac{154\,317\,775\,011\,120\,075}{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}$$

$$\frac{\partial M(b,V)}{\partial b} = -\frac{18\,480\,874\,M}{226\,802\,245 + 18\,480\,874\,b}$$

$$\frac{\partial V(b,M)}{\partial M} = \frac{2\,(226\,802\,245 + 18\,480\,874\,b)^2\,M}{154\,317\,775\,011\,120\,075}$$

$$\frac{\partial V(b,M)}{\partial b} = \frac{36\,961\,748\,(226\,802\,245 + 18\,480\,874\,b)\,M^2}{154\,317\,775\,011\,120\,075}$$

Global minimum:

$$\min\left\{\frac{1}{3}\,(0.0814845\,b + 1)^2\,M^2\right\} = 0 \text{ at } (b, M) = (-16, 0)$$

Global minima:

$$\min\left\{\frac{1}{3}\,M^2\left(1 - \frac{(b\,2)\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684 - \frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\,\sqrt{6}}\right)^2\right\} = 0$$

$$\text{for } b = -\frac{226\,802\,245}{18\,480\,874}$$

$$\min\left\{\frac{1}{3}M^2\left(1-\frac{(b\ 2)\left(0.9991104684-\frac{\sqrt{6}}{2}\right)\exp\left(-\frac{2\left(0.9991104684-\frac{\sqrt{6}}{2}\right)}{\sqrt{6}}\right)}{e\sqrt{6}}\right)^2\right\}=0$$

for $M=0$

From:

$$b=\frac{225.913\left(-0.054323\,M^2\pm 6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}\quad (M\neq 0)$$

we obtain:

(225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2

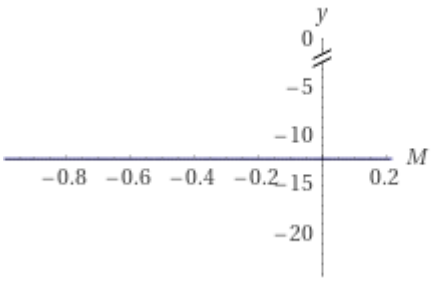
Input interpretation:

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times 10^{-10}\sqrt{M^4}\right)}{M^2}$$

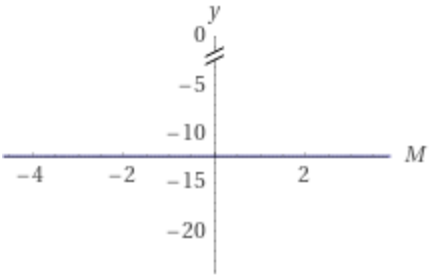
Result:

$$\frac{225.913\left(6.58545\times 10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}$$

Plots:



(*M* from −1 to 0.2)



(*M* from −4.6 to 3.9)

Alternate form assuming M is real:

-12.2723

-12.2723 result very near to the black hole entropy value 12.1904 = ln(196884)

Alternate forms:

$$-\frac{12.2723\left(M^2-1.21228\times10^{-8}\sqrt{M^4}\right)}{M^2}$$

$$\frac{1.48774\times10^{-7}\sqrt{M^4}-12.2723\,M^2}{M^2}$$

Expanded form:

$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723$$

Property as a function:

Parity

even

Series expansion at M = 0:

$$\left(\frac{1.48774\times10^{-7}\sqrt{M^4}}{M^2}-12.2723\right)+O\left(M^6\right)$$

(generalized Puiseux series)

Series expansion at M = ∞:

$$-12.2723$$

Derivative:

$$\frac{d}{dM}\left(\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right)=\frac{3.55271\times10^{-15}}{M}$$

Indefinite integral:

$$\int\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\,dM=$$
$$\frac{1.48774\times10^{-7}\sqrt{M^4}}{M}-12.2723\,M+\text{constant}$$

Global maximum:

$$\max\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Global minimum:

$$\min\left\{\frac{225.913\left(6.58545\times10^{-10}\sqrt{M^4}-0.054323\,M^2\right)}{M^2}\right\}=-\frac{140\,119\,826\,723\,990\,341\,497\,649}{11\,417\,594\,849\,251\,000\,000\,000}\ \text{at}\ M=-1$$

Limit:

$$\lim_{M\to\pm\infty}\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}=-12.2723$$

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}--12.2723\right)dM=0$$

From b that is equal to

$$\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}$$

From:

$$V = \frac{1}{3}(0.0814845\,b + 1)^2\,M^2$$

we obtain:

1/3 (0.0814845 ((225.913 (-0.054323 M^2 + 6.58545×10^-10 sqrt(M^4)))/M^2) + 1)^2 M^2

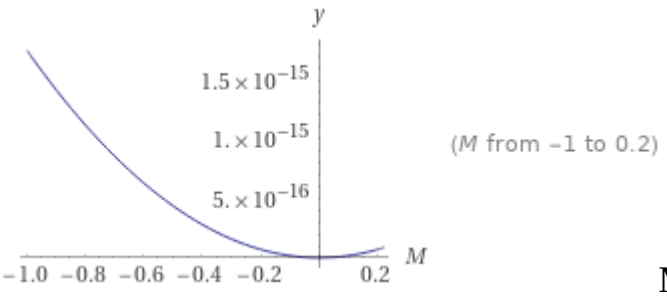
Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\,M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

Result:

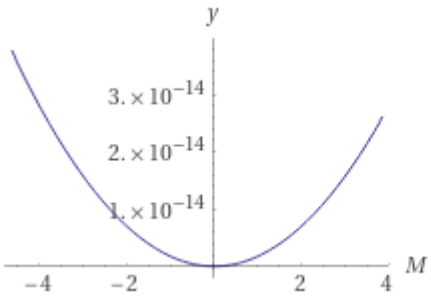
0

Plots: **(possible mathematical connection with an open string)**



M = -0.5; M = 0.2

(possible mathematical connection with an open string)



(*M* from −4.6 to 3.9)

M = 2 ; M = 3

Root:

$M = 0$

Property as a function:

Parity

even

Series expansion at M = 0:

$O(M^{62194})$

(Taylor series)

Series expansion at M = ∞:

$1.75541 \times 10^{-15}\, M^2 + O\left(\left(\frac{1}{M}\right)^{62194}\right)$

(Taylor series)

Definite integral after subtraction of diverging parts:

$$\int_0^{\infty}\left(\frac{1}{3}M^2\left(1+\frac{18.4084\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}\right)^2-1.75541\times10^{-15}\,M^2\right)dM=0$$

For M = - 0.5 , we obtain:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 (-0.5)^2 + 6.58545×10^-10 sqrt((-0.5)^4)))/(-0.5)^2) + 1)^2 * (-0.5^2)

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,(-0.5)^2+6.58545\times10^{-10}\sqrt{(-0.5)^4}\right)}{(-0.5)^2}+1\right)^2\left(-0.5^2\right)$$

Result:

$$-4.388513449474645453489707833780880208333333333333333333\ldots\times10^{-16}$$

-4.38851344947*10^-16

For M = 0.2:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 0.2^2 + 6.58545×10^-10 sqrt(0.2^4)))/0.2^2) + 1)^2 0.2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times0.2^2+6.58545\times10^{-10}\sqrt{0.2^4}\right)}{0.2^2}+1\right)^2\times0.2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots\times 10^{-17}$

7.021621519159*10^{-17}

For M = 3:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\,M^2+6.58545\times10^{-10}\sqrt{M^4}\right)}{M^2}+1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 3^2 + 6.58545×10^-10 sqrt(3^4)))/3^2) + 1)^2 3^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845\times\frac{225.913\left(-0.054323\times3^2+6.58545\times10^{-10}\sqrt{3^4}\right)}{3^2}+1\right)^2\times3^2$$

Result:

$1.579864841810872363256294820161116875 \times 10^{-14}$

$1.57986484181*10^{-14}$

For M = 2:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323\, M^2 + 6.58545 \times 10^{-10}\sqrt{M^4}\right)}{M^2} + 1\right)^2 M^2$$

1/3 (0.0814845 ((225.913 (-0.054323 2^2 + 6.58545×10^-10 sqrt(2^4)))/2^2) + 1)^2 2^2

Input interpretation:

$$\frac{1}{3}\left(0.0814845 \times \frac{225.913\left(-0.054323 \times 2^2 + 6.58545 \times 10^{-10}\sqrt{2^4}\right)}{2^2} + 1\right)^2 \times 2^2$$

Result:

$7.021621519159432725583532534049408333333333333333333333333\ldots \times 10^{-15}$

$7.021621519*10^{-15}$

From the four results

7.021621519*10^-15 ; 1.57986484181*10^-14 ; 7.021621519159*10^-17 ;

-4.38851344947*10^-16

we obtain, after some calculations:

sqrt[1/(2Pi)(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 - 4.38851344947*10^-16)]

Input interpretation:

$$\sqrt{\left(\frac{1}{2\pi}\left(7.021621519\times 10^{-15}+1.57986484181\times 10^{-14}+7.021621519\times 10^{-17}-4.38851344947\times 10^{-16}\right)\right)}$$

Result:

$5.9776991059\ldots\times 10^{-8}$

5.9776991059*10^-8 result very near to the Planck's electric flow 5.975498×10^{-8} that is equal to the following formula:

$$\phi_{\mathrm{P}}^{E}=\mathbf{E}_{\mathrm{P}}l_{\mathrm{P}}^{2}=\phi_{\mathrm{P}}l_{\mathrm{P}}=\sqrt{\frac{\hbar c}{\varepsilon_{0}}}$$

We note that:

1/55*(([(((1/[(7.021621519*10^-15 + 1.57986484181*10^-14 +7.021621519*10^-17 -4.38851344947*10^-16)]))^1/7]-((log^(5/8)(2))/(2 2^(1/8) 3^(1/4) e log^(3/2)(3)))))

Input interpretation:

$$\frac{1}{55}\left((1/(7.021621519\times 10^{-15} + 1.57986484181\times 10^{-14} + 7.021621519\times 10^{-17} - 4.38851344947\times 10^{-16}))\,\hat{}\,(1/7) - \frac{\log^{5/8}(2)}{2\sqrt[8]{2}\,\sqrt[4]{3}\,e\log^{3/2}(3)}\right)$$

$\log(x)$ is the natural logarithm

Result:

1.6181818182…

1.6181818182… result that is a very good approximation to the value of the golden ratio 1.618033988749...

From the Planck units:

Planck Length

$$l_{\mathrm{P}} = \sqrt{\frac{4\pi\hbar G}{c^3}}$$

5.729475 * 10^{-35} Lorentz-Heaviside value

Planck's Electric field strength

$$\mathbf{E}_{\mathrm{P}} = \frac{F_{\mathrm{P}}}{q_{\mathrm{P}}} = \sqrt{\frac{c^7}{16\pi^2\varepsilon_0\hbar G^2}}$$

1.820306 * 10^{61} V*m Lorentz-Heaviside value

Planck's Electric flux

$$\phi_{\mathrm{P}}^{E} = \mathbf{E}_{\mathrm{P}} l_{\mathrm{P}}^2 = \phi_{\mathrm{P}} l_{\mathrm{P}} = \sqrt{\frac{\hbar c}{\varepsilon_0}}$$

5.975498*10^{-8} V*m Lorentz-Heaviside value

Planck's Electric potential

$$\phi_P = V_P = \frac{E_P}{q_P} = \sqrt{\frac{c^4}{4\pi\varepsilon_0 G}}$$

1.042940*10^{27} V Lorentz-Heaviside value

Relationship between Planck's Electric Flux and Planck's Electric Potential

$E_P * l_P = (1.820306 * 10^{61}) * 5.729475 * 10^{-35}$

Input interpretation:

$$\frac{(1.820306 \times 10^{61}) \times 5.729475}{10^{35}}$$

Result:

1 042 939 771 935 000 000 000 000 000

Scientific notation:

$1.042939771935 \times 10^{27}$

$1.042939771935*10^{27} \approx 1.042940*10^{27}$

Or:

$E_P * l_P^2 / l_P = (5.975498*10^{-8})*1/(5.729475 * 10^{-35})$

Input interpretation:

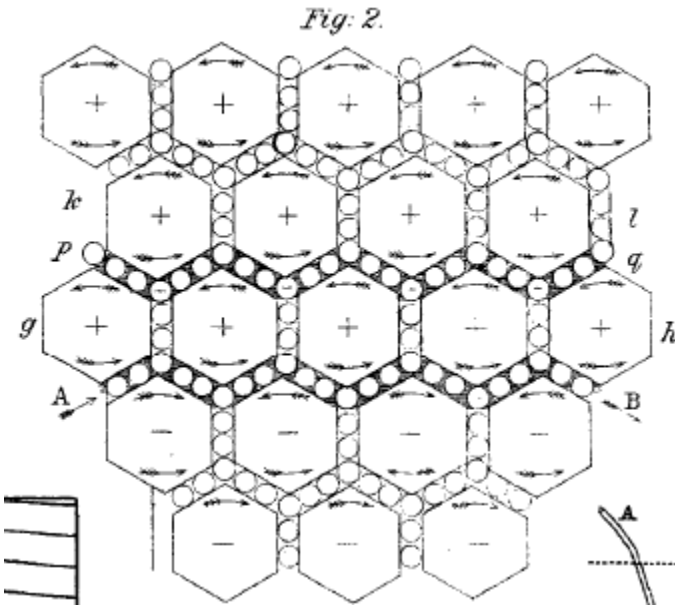
$$5.975498 \times 10^{-8} \times \frac{1}{\frac{5.729475}{10^{35}}}$$

Result:

$1.042939885417075735560413475929295441554418162222542205001 33\ldots \times 10^{27}$

$1.042939885417*10^{27} \approx 1.042940*10^{27}$

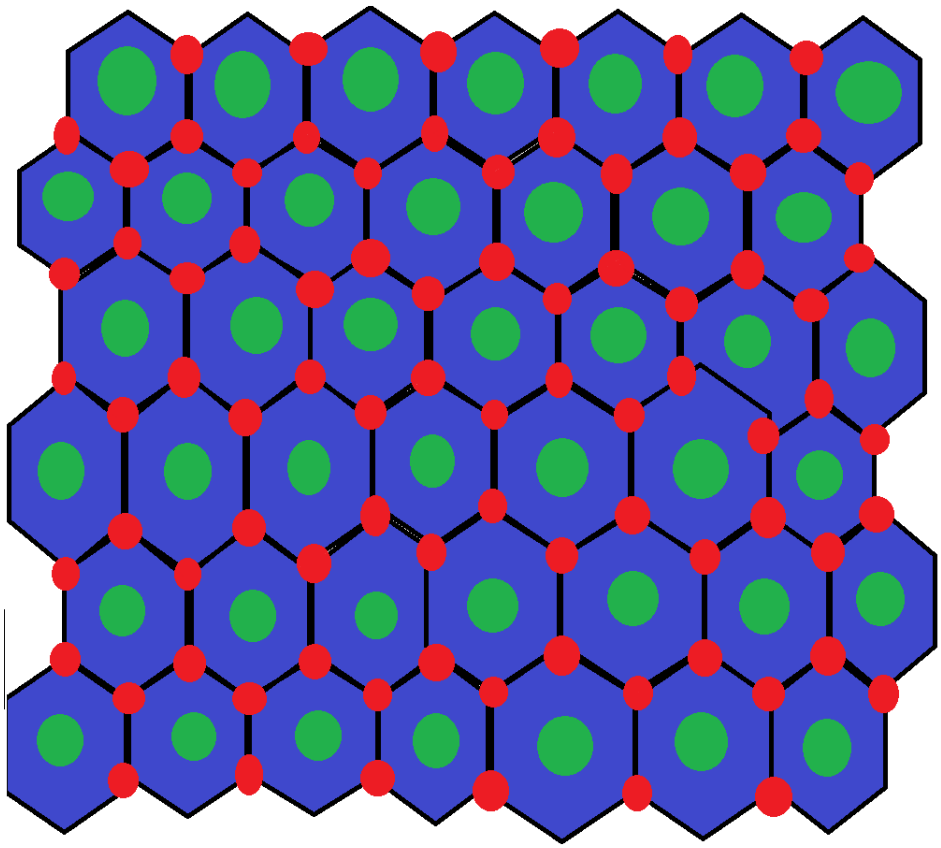
Fig. 1



It is therefore possible to consider the vortices of the "quantum vacuum" schematized as cubes or octahedrons (the + sign inside a given vortex indicates its counterclockwise rotation, while the - sign indicates its clockwise rotation). Between vortex and vortex there is a layer of "bubbles"-universes (or universes-spheres), which flows, as in the simplified two-dimensional drawing, from A to B

Fig. 2

Immagine dello Spazio-Tempo a scala quantistica: le cironferenze in rosso rappresentano i punti corrispondenti alle dimensioni compattificate e gli esagoni in blu, rappresentano le “fluttuazioni” (universi potenziali – cerchi verdi) del vuoto quantistico (2D)



Proposal

Image of space-time at quantum scale: the circles in red represent the points corresponding to the compactified dimensions and the hexagons in blue, represent the "fluctuations" (potential universes - green circles) of the quantum vacuum (2D). In reality, we will have n-dimensional hyperspheres in which the compactified dimensions "roll up" and octahedrons representing the "fluctuations", containing spheres (bubbles of potential universes), of the quantum vacuum

Acknowledgments

We would like to thank Professor **Augusto Sagnotti** theoretical physicist at Scuola Normale Superiore (Pisa – Italy) for his very useful explanations and his availability.

References

The Last Letter - S. Ramanujan to G. H. Hardy - 12 January 1920**,** University of Madras

A Number Theoretic Analysis of the Enthalpy, Enthalpy Energy Density, Thermodynamic Volume, and the Equation of State of a Modified White Hole, and the Implications to the Quantum Vacuum Spacetime, Matter Creation and the Planck Frequency. - *Nardelli, M., Kubeka, A.S. and Amani, A.* (2024) - Journal of Modern Physics , 15, 1-50. - https://doi.org/10.4236/jmp.2024.151001

Modular equations and approximations to π - *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

An Update on Brane Supersymmetry Breaking - *Jihad Mourad and Augusto Sagnotti* - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Properties of Nilpotent Supergravity - *E. Dudas, S. Ferrara, A. Kehagias and A. Sagnotti* - arXiv:1507.07842v2 [hep-th] 14 Sep 2015

See also:

The Geometry of the MRB constant *by Marvin Ray Burns*

https://www.academia.edu/22271085/The_Geometry_of_the_MRB_constant

(See also Page 29 the applications of the CMRB in various sectors of Theoretical Physics (String Theory) and Cosmology)

http://xoom.virgilio.it/source_filemanager/na/ar/nardelli/michele%20and%20antonio%20papers/Try%20to%20beat%20these%20MRB%20constant%20records!%20-%20Online%20Technical%20Discussion%20Groups%E2%80%94Wolfram%20Community%20b.pdf